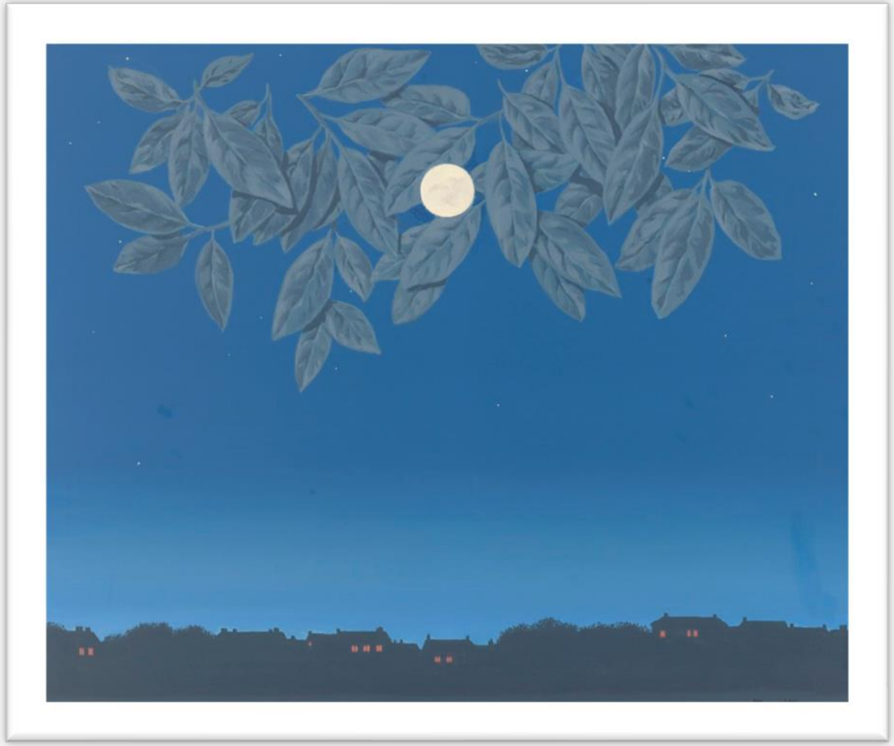
Towards
Celestial Holography



René Magritte, *La page blanche* (1967)

Laura Donnay (TU Wien & SISSA)

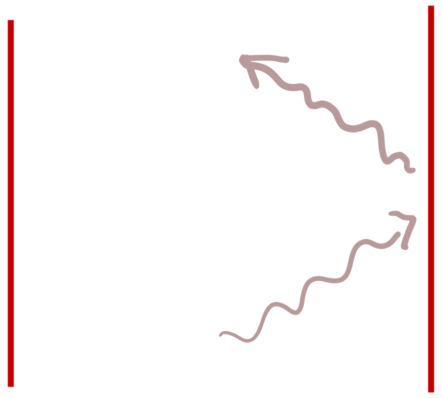
7^{th} **INFN meeting** – Theories of the Fundamental Interactions 2022
15.06.2022

Intro and motivations

- **Jan de Boer**'s talk

  Quantum gravity in a **box**

AdS



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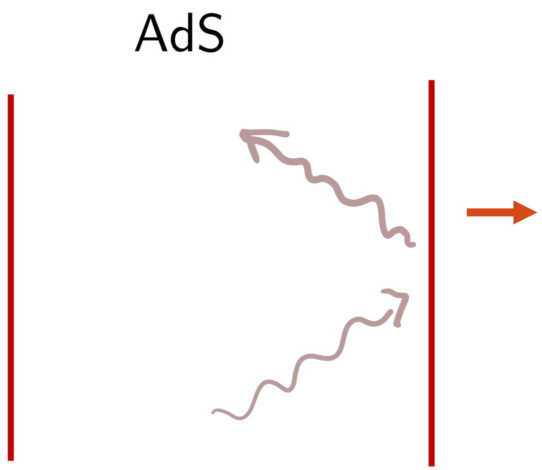

The holographic principle

Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

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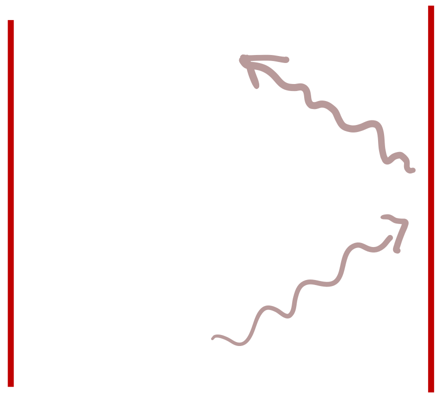

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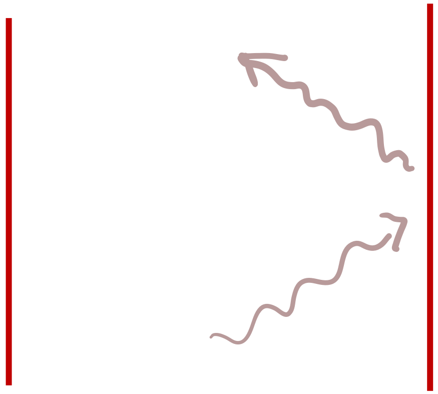

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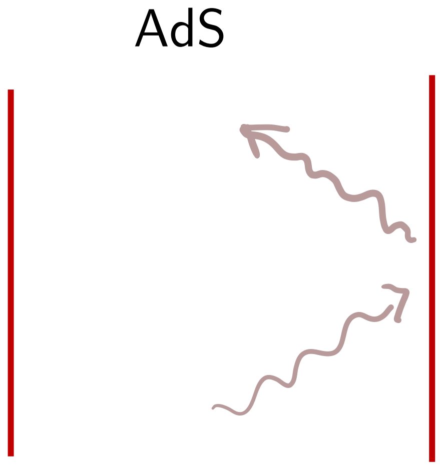

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Go beyond the canonical cases!

Anti-de Sitter	vs	**Flat**
$\Lambda < 0$		$\Lambda = 0$
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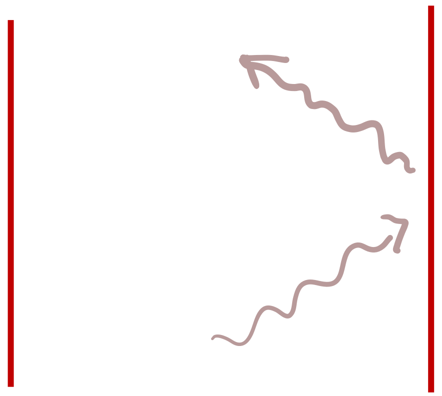

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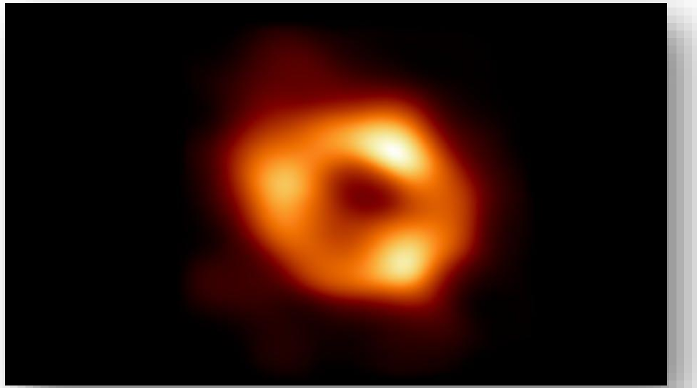
(Image credit: EHT Collaboration, CC BY-SA)



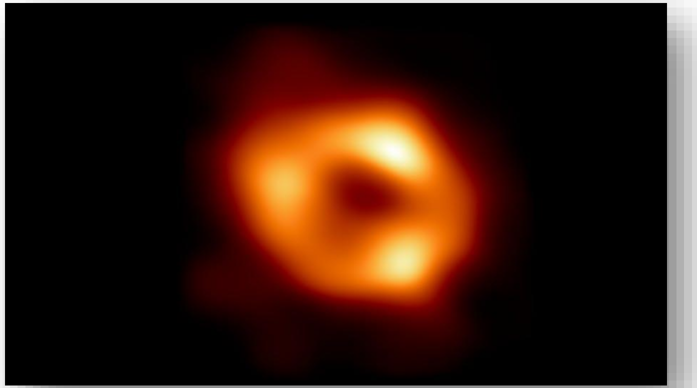
Intro and motivations

(Image credit: EHT Collaboration, CC BY-SA)

Black holes

$$\boxed{S_{BH} = \frac{Ac^3}{4G\hbar}}$$

[Bekenstein][Hawking]



"Primordial holographic relationship"

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(Image credit: EHT Collaboration, CC BY-SA)

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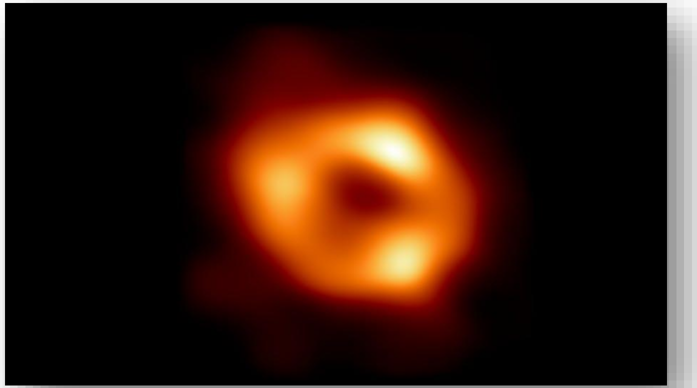


"Primordial holographic relationship"

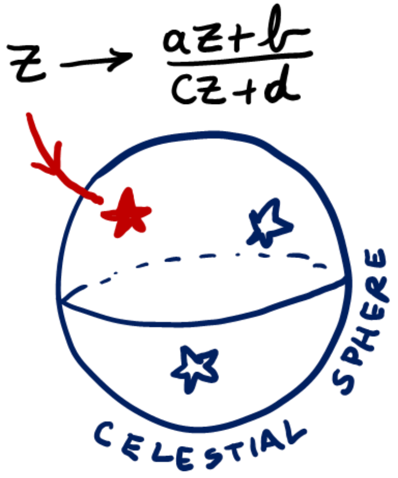
Realistic black holes (e.g. Kerr solution) do not possess an AdS geometry.

$\longrightarrow$ need to develop a **holographic correspondence** for **asymptotically *flat* spacetimes**

The aim of this review talk is to present :

Recent advances in the holographic formulation of **quantum gravity** in 4-dimensional asymptotically **flat** spacetimes called

"**celestial holography**"



This proposal is motivated by recent surprising discoveries about the **infrared structure** of **gravity** (and **gauge theories**)...

Outline of the talk

1. **Infrared structure of gravity**

2. **Celestial holography**

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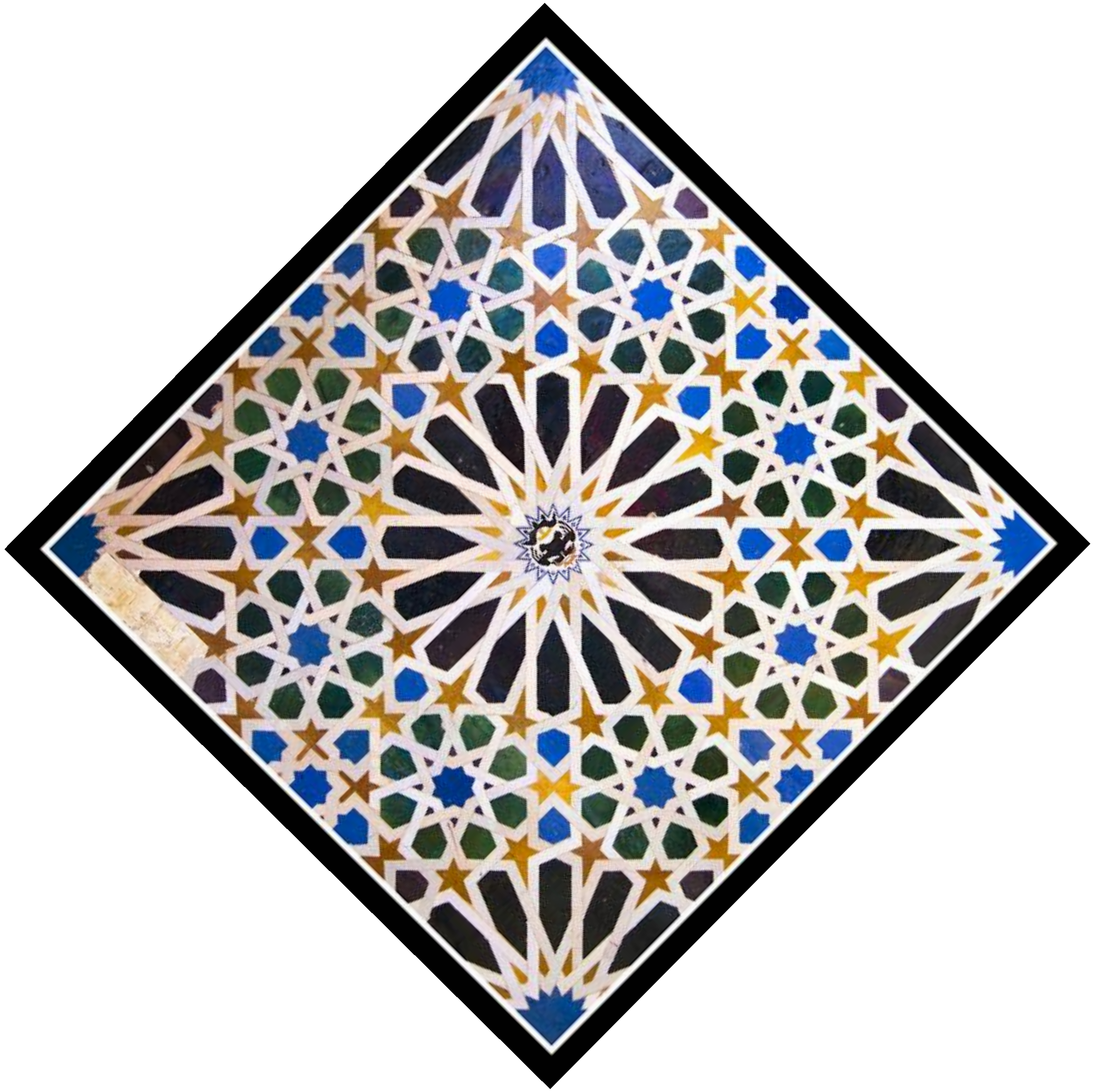
1. **Infrared structure of gravity**

2. **Celestial holography**

Road map: symmetries.

Symmetries control universal phenomena, simplify the analysis and render otherwise intractable calculations feasible.

What are the symmetries of asymptotically flat spacetimes?

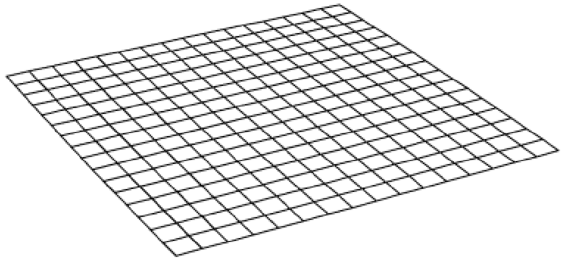


Alhambra الحَمْراء tile (13^{th} century)

A surprise in flat spacetimes

The BMS symmetries [Bondi-Metzner-van der Burg; Sachs, ‘62]

Minkowski metric (flat spacetime) in 4D



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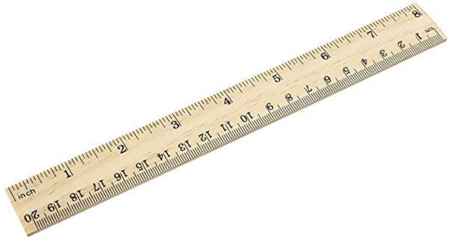
(measure of distance in flat spacetime)



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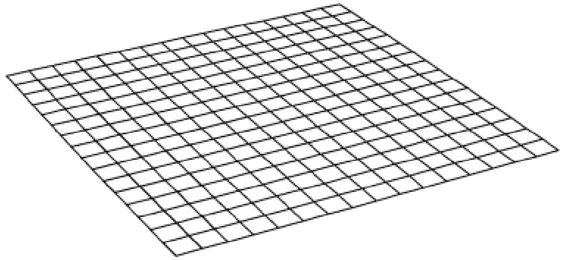

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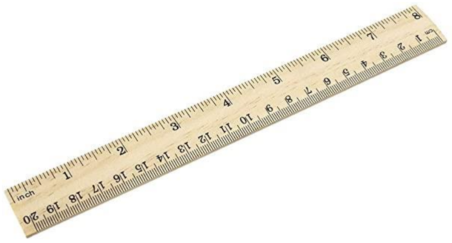

Change to *Bondi coordinates* $(u, r, z, \bar{z})$

$$\boxed{\begin{aligned} t &= u + r, \quad x_1 = \frac{r(z+\bar{z})}{1+z\bar{z}} \\ x_2 &= \frac{-ir(z-\bar{z})}{1+z\bar{z}}, \quad x_3 = \frac{r(1-z\bar{z})}{1+z\bar{z}} \end{aligned}}$$

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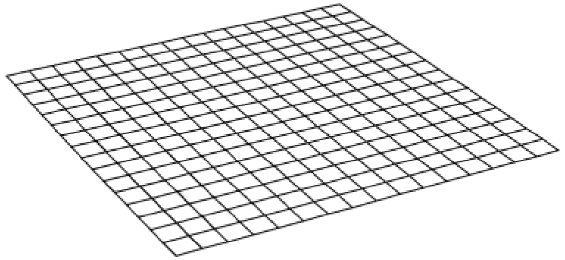

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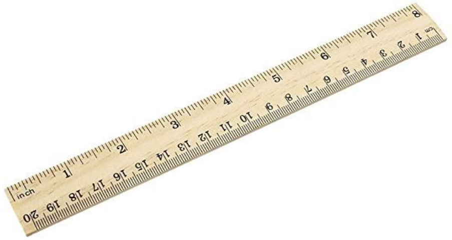
$$r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$



$$z = e^{i\phi}\cot\frac{\theta}{2}$$

$$\bar{z} = e^{-i\phi}\cot\frac{\theta}{2}$$

$$\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

sphere angles

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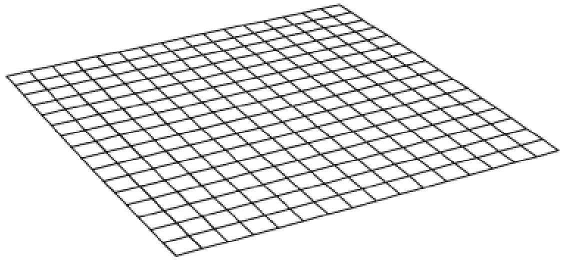

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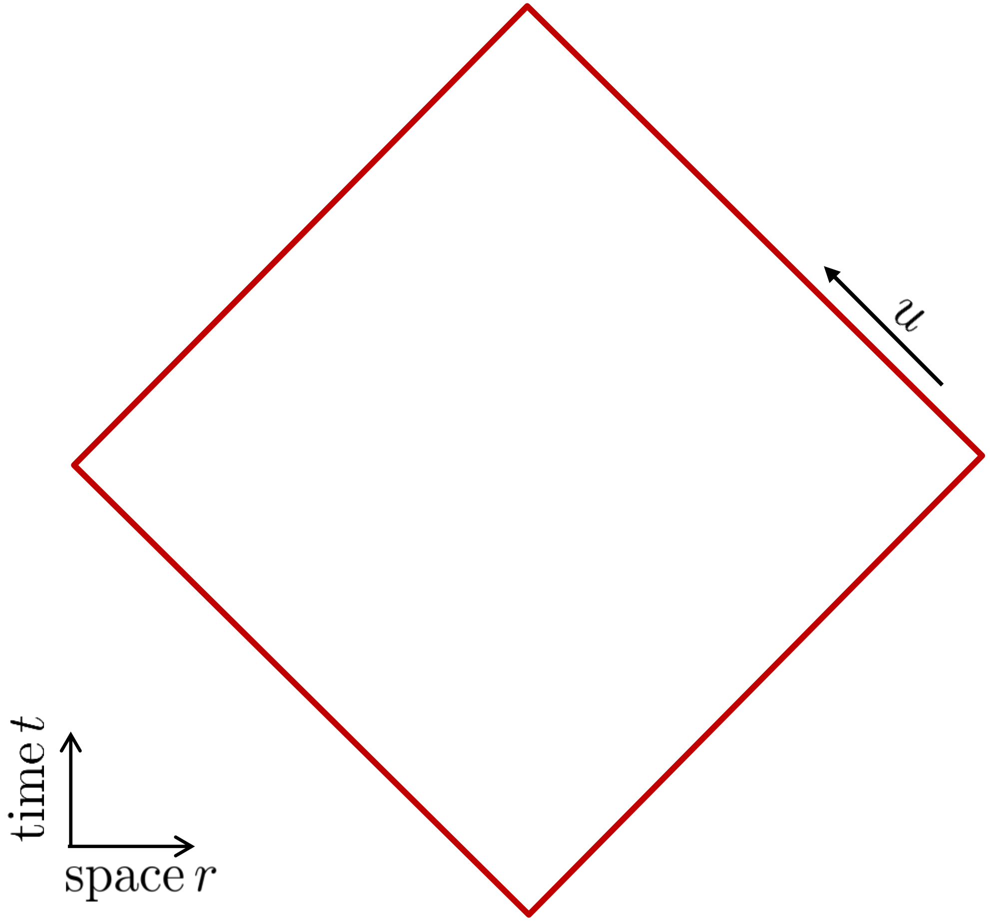
$u = t - r$: 'retarded' null time



Penrose diagram of Minkowski

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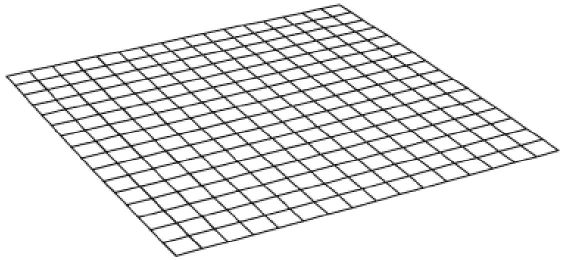

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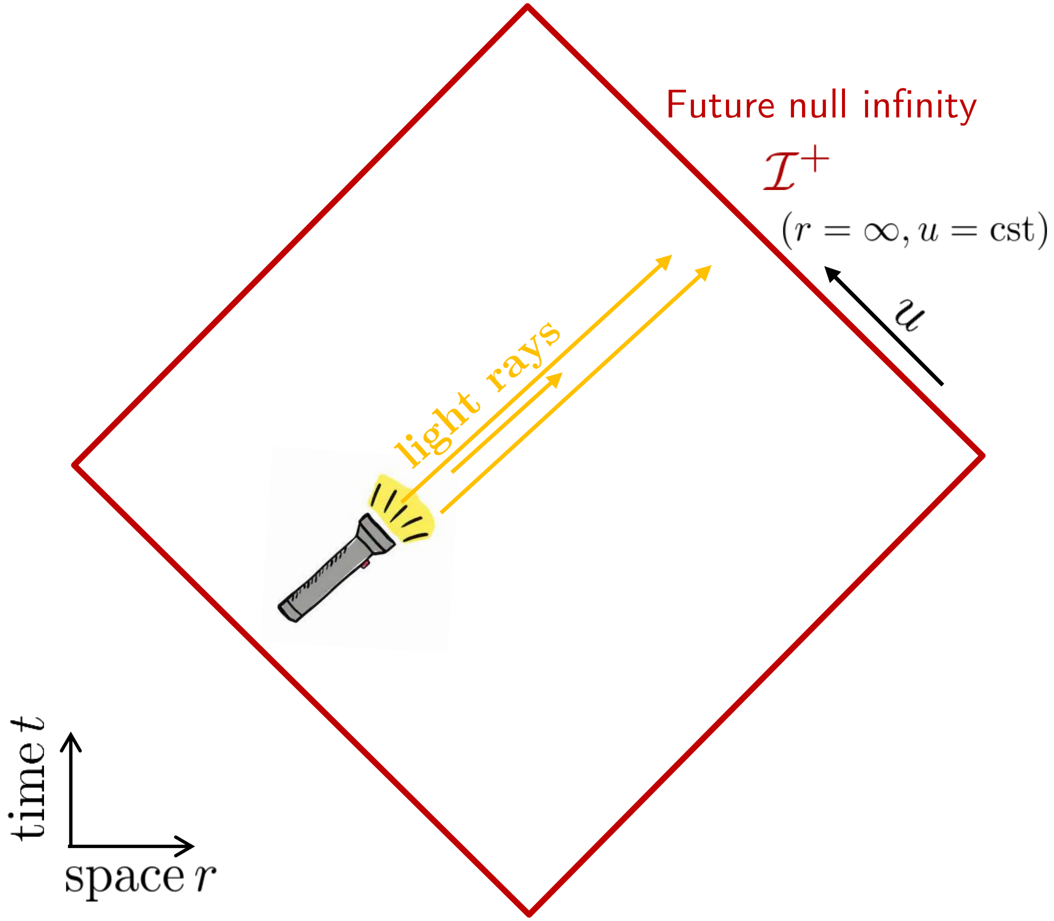

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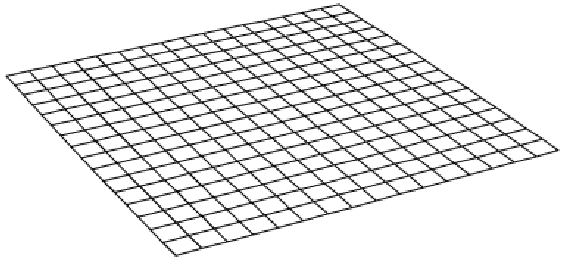

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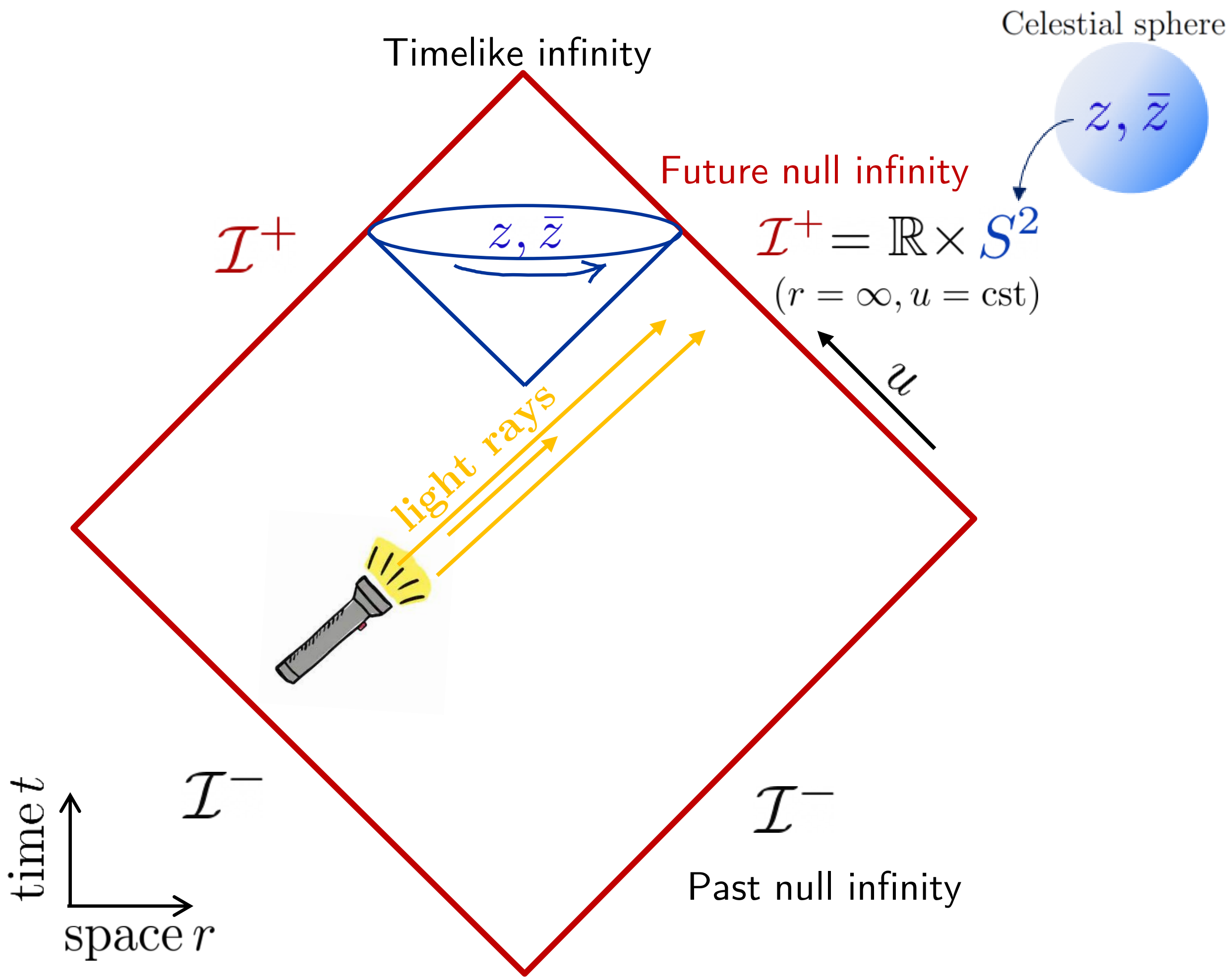

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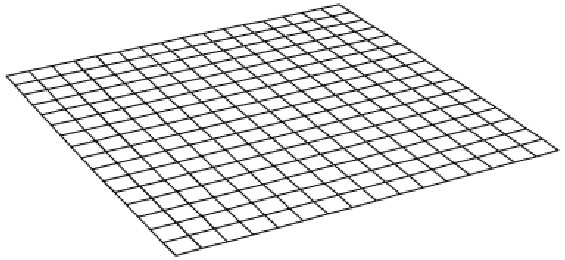

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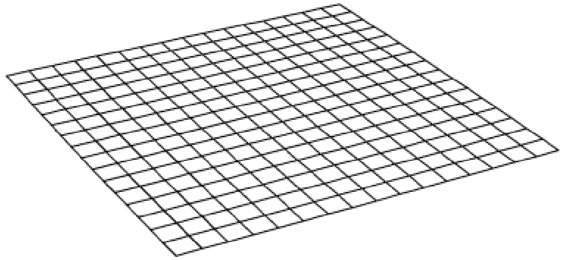

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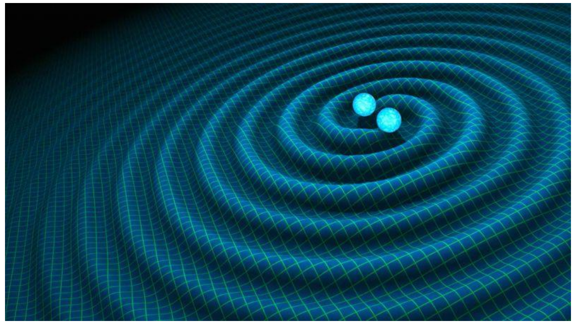

Curved spacetime that looks flat seen from a far distance. The deviation from Minkowski is dictated by boundary conditions for the metric.

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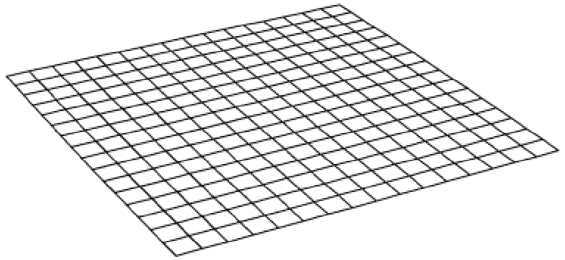

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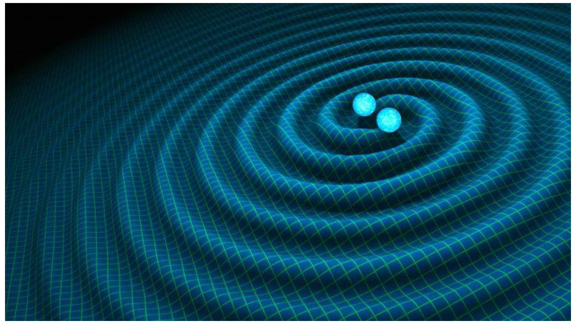

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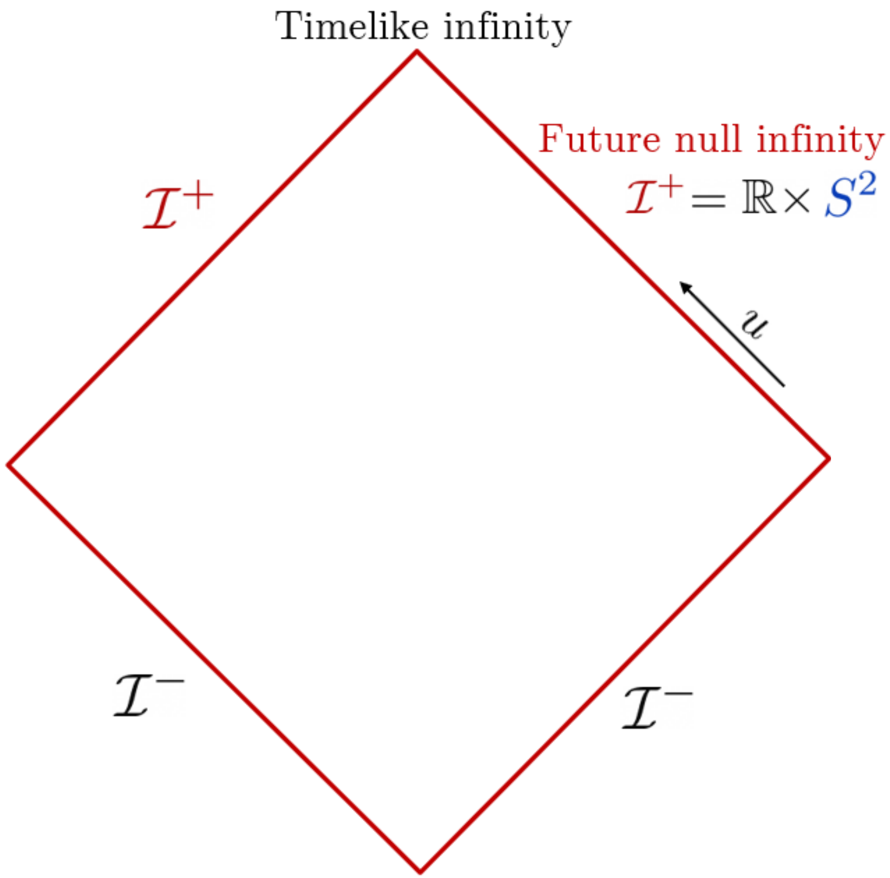

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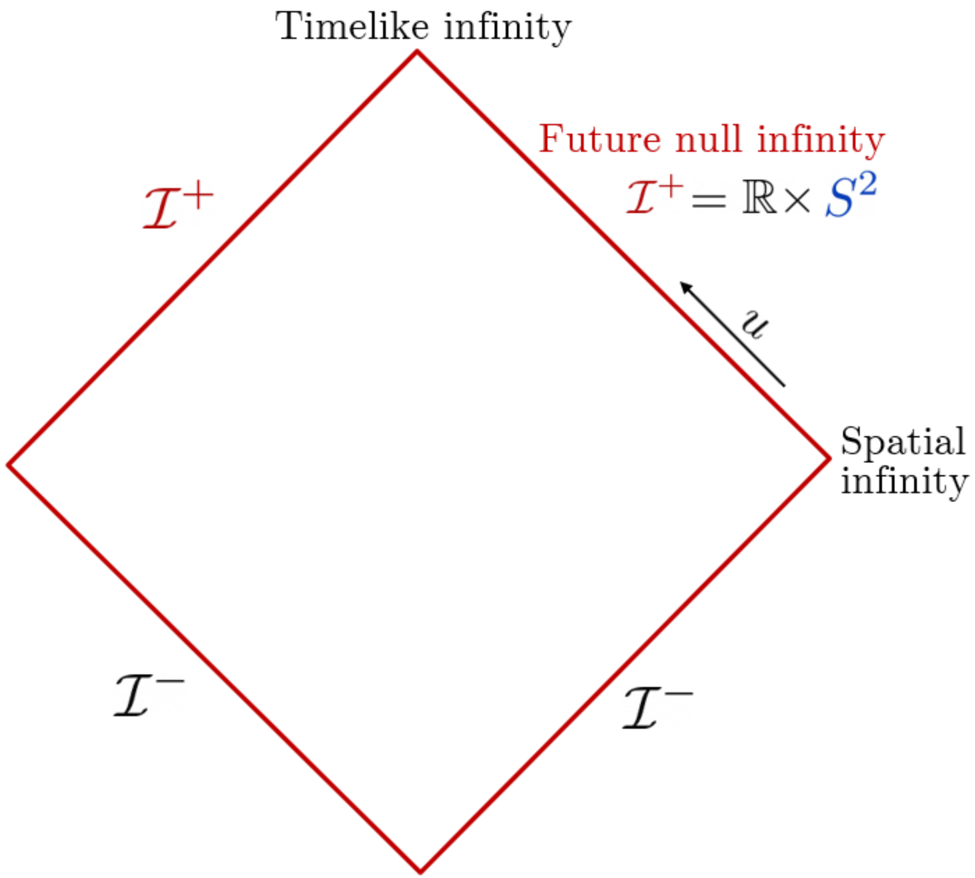

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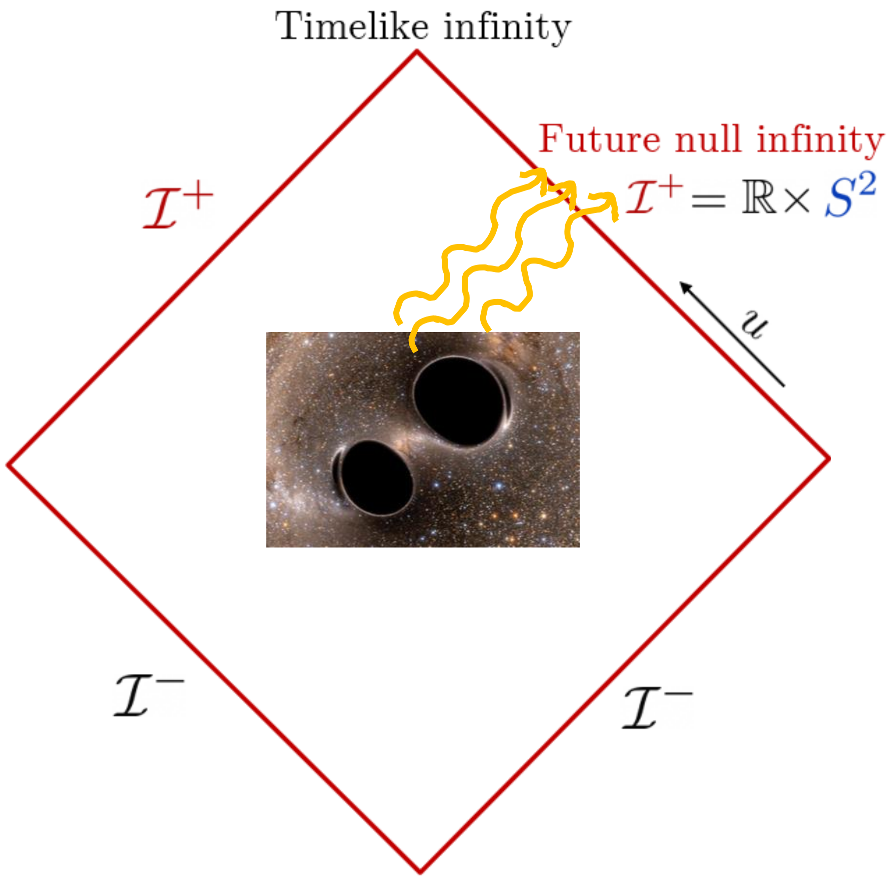

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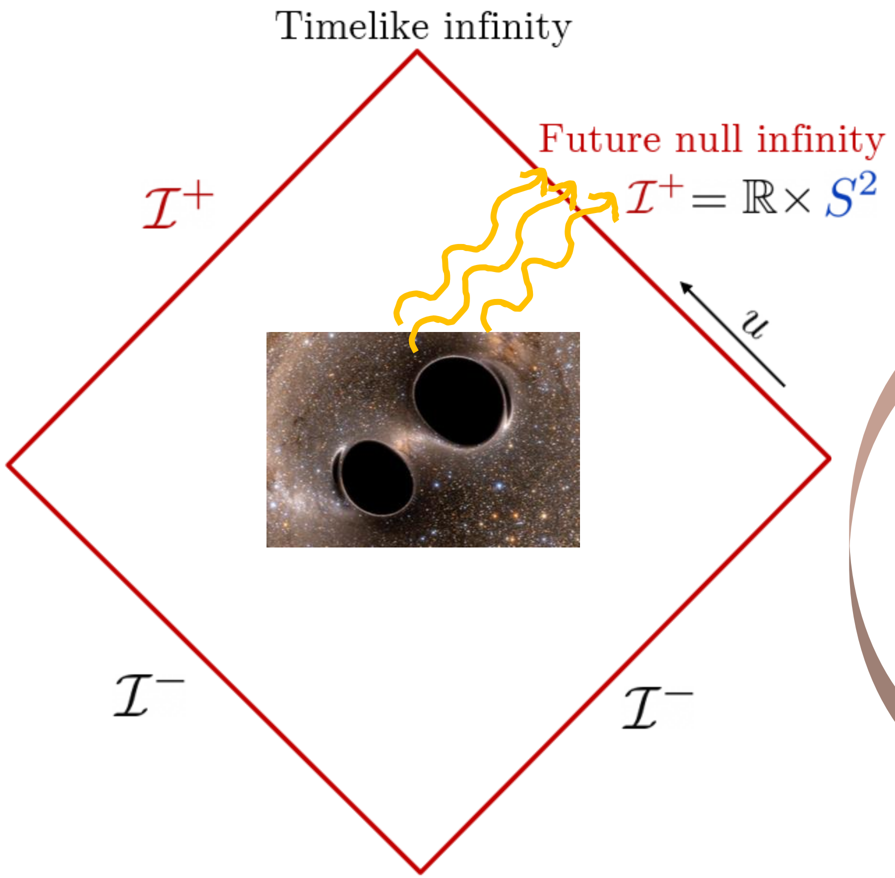

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Mathematical description of a **radiating spacetime**

What are the symmetries of asymptotically flat spacetimes?

What are the symmetries of asymptotically flat spacetimes?

what was expected

Poincaré

4 spacetime translations
6 Lorentz transformations

what was found

Bondi-Metzner-Sachs (BMS) (‘62)

Infinite-dimensional extension!

Supertranslations

[Bondi, van der Burg, Metzner '62] [Sachs '62]
[Barnich, Troessaert '10]

Asymptotically flat spacetimes in Bondi gauge:

$$r \to \infty \qquad\qquad (u, r, x^A)\,, \quad x^A = (z, \bar{z})$$

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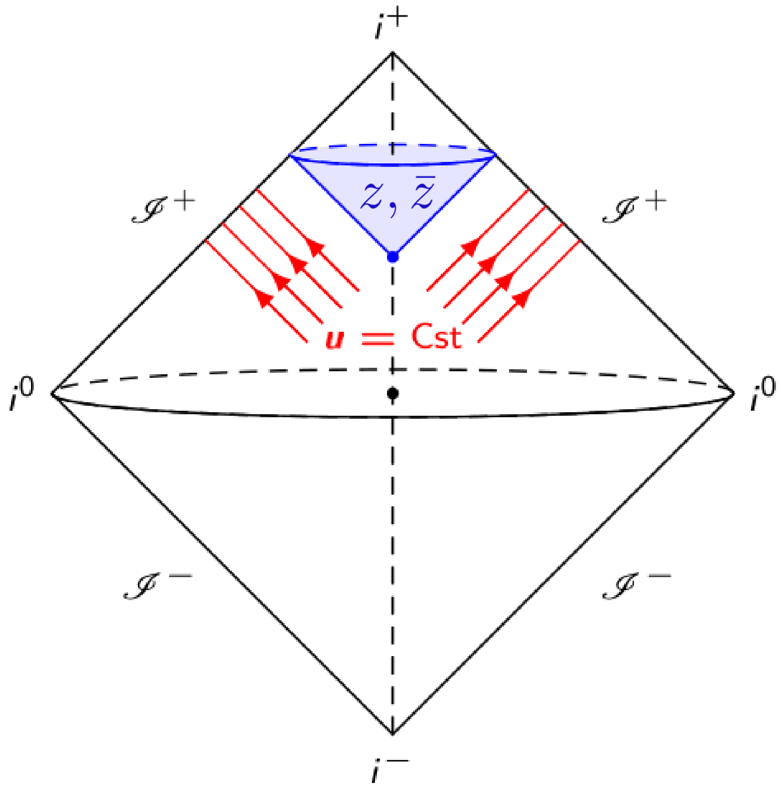

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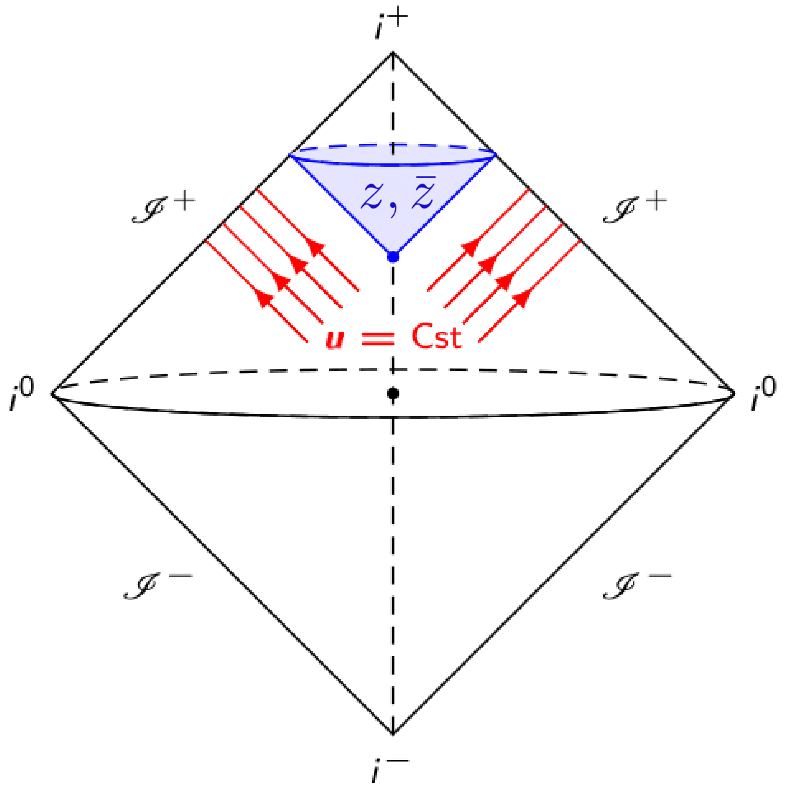

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$$\xi = \boxed{\mathcal{T}(z, \bar{z})}\partial_u + \cdots$$

arbitrary function
on the celestial sphere

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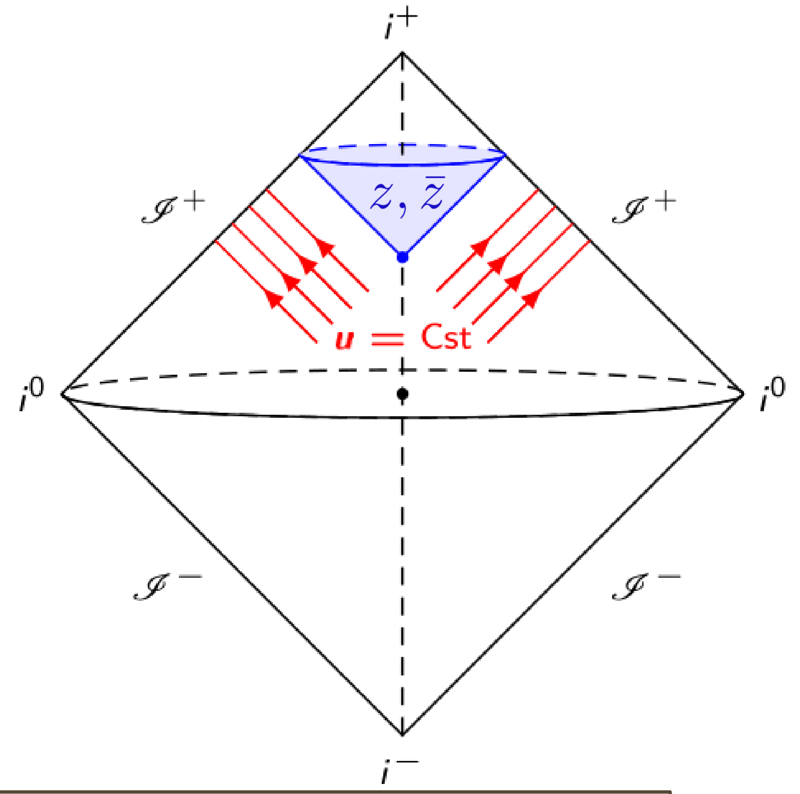

4 Poincaré translations

↓ *Symmetry enhancement*

∞ BMS *super*translations

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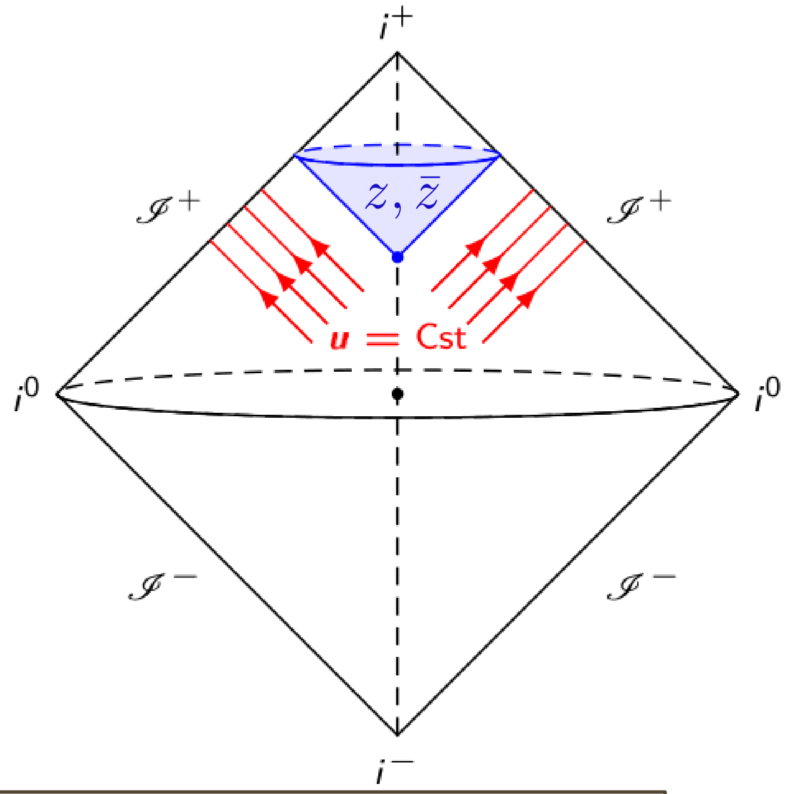

BMS supertranslation symmetries:

$$\xi = \mathcal{T}(z, \bar{z})\partial_u + \cdots$$

act non-trivially on the gravitational solution space

$$\delta_\xi M = \cdots \qquad \delta_\xi C_{zz} = (\cdots)C_{zz} - 2D_z^2\mathcal{T}$$

4 Poincaré translations

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[Sachs '62]

But...

Conceivably α is a blessing in disguise and the representations of the group (3·12) are more interesting than the representations of the Lorentz group.

BMS and the scattering problem

Seminal observation: BMS symmetries constrain the gravitational scattering problem! [Strominger '14]

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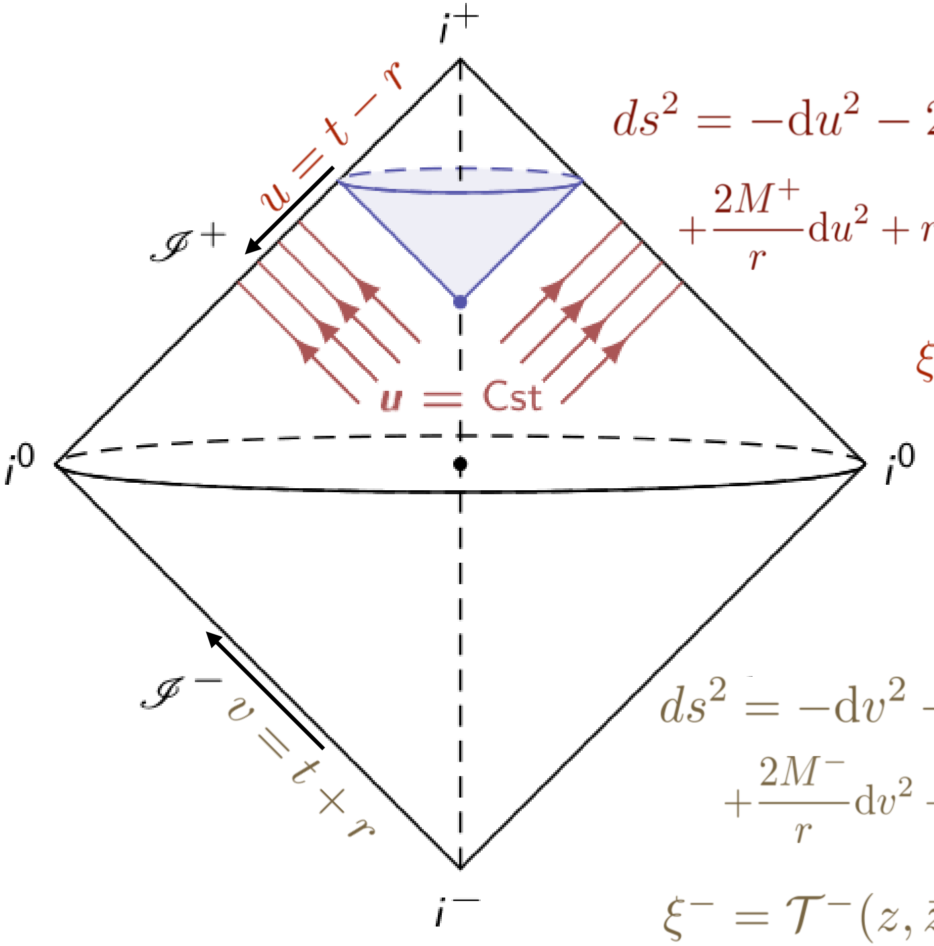

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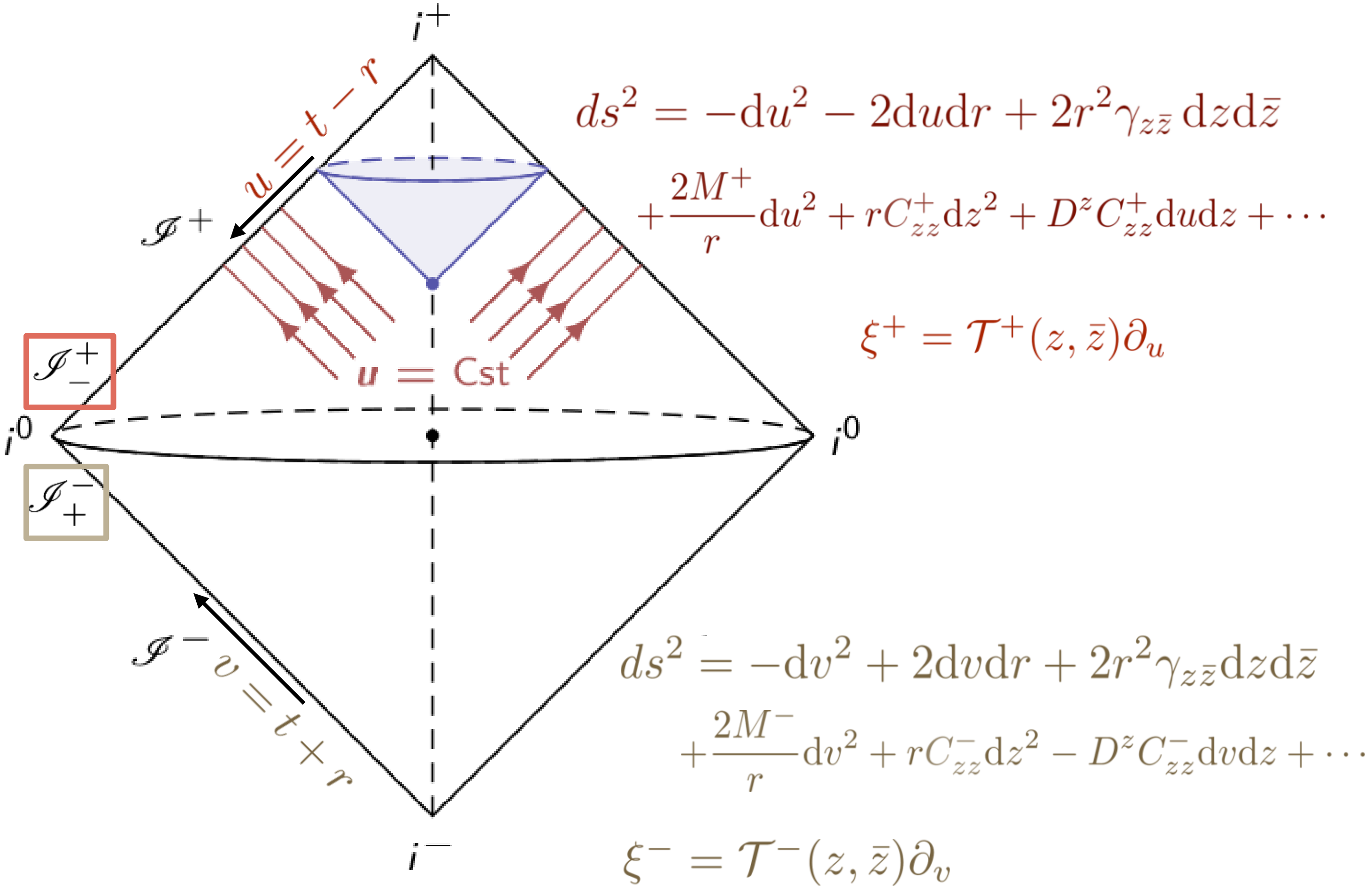

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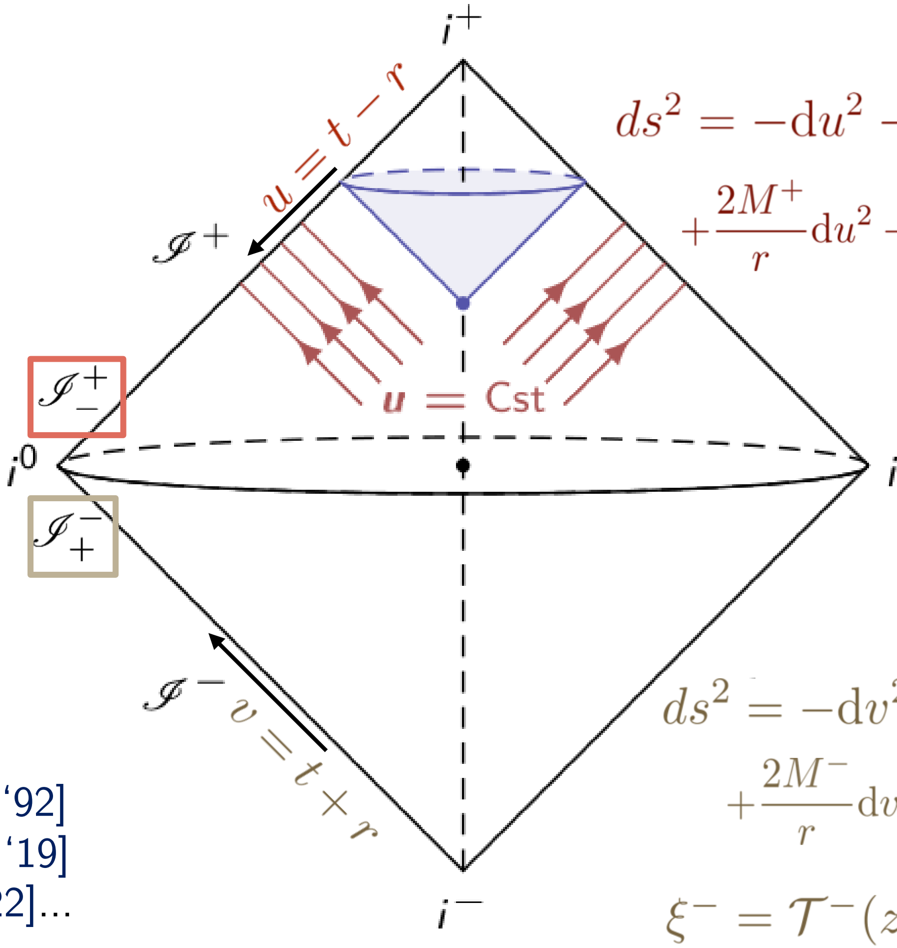
$$M^-(v,z,\bar z)|_{\mathscr{I}^-_+} = M^+(u,z,\bar z)|_{\mathscr{I}^+_-}$$

$$\mathcal{T}^-(z,\bar z)|_{\mathscr{I}^-_+} = \mathcal{T}^+(z,\bar z)|_{\mathscr{I}^+_-}$$

[Strominger '14]; see also [Herberthson, Ludvigsen '92] [Troessaert '18][Henneaux, Troessaert '18][Prabhu '19] [Kroon, Mohamed '21][Capone, Nguyen, Parisini '22]...



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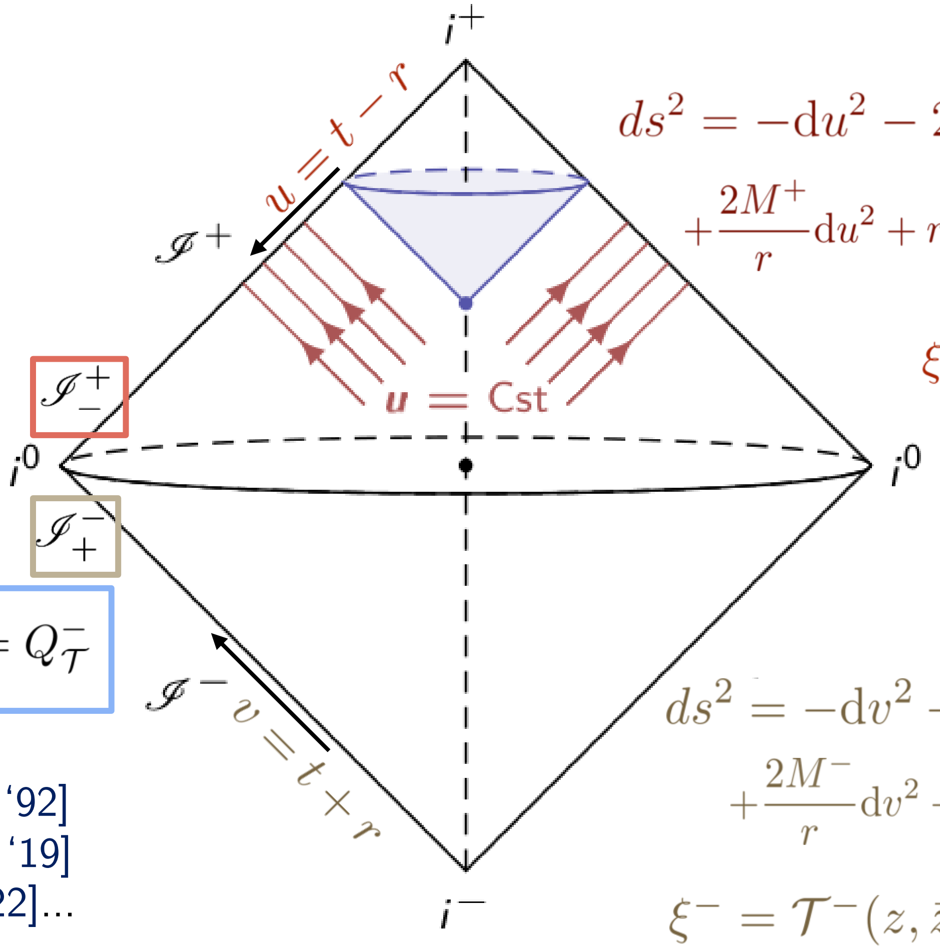

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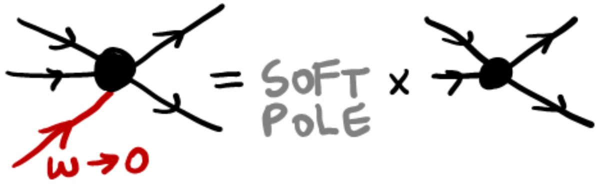
BMS and the scattering problem

Prime example:

The **leading soft graviton theorem** [Weinberg '65]

$\mathcal{A}_n = \langle out | S | in \rangle$

+soft particle (energy $\omega \to 0$)



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n hard particles (p_k) + external graviton (q)

$$\lim_{\omega \to 0} \mathcal{A}_{n+1}(q) = S^{(0)} \mathcal{A}_n + \mathcal{O}(q^0)$$

$$S^{(0)} = \sum_{k=1}^{n} \frac{p_k^\mu p_k^\nu \varepsilon_{\mu\nu}(q)}{p_k \cdot q}$$

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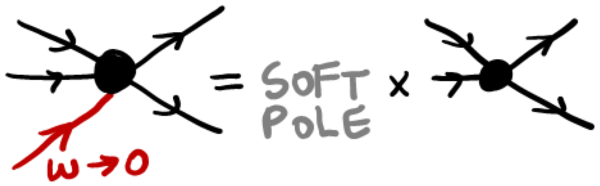

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is nothing but the **Ward identity** associated to **supertranslation** symmetry [He, Lysov, Mitra, Strominger '15]

$$\langle out | Q_{\mathcal{T}}^{+} \mathcal{S} - \mathcal{S} Q_{\mathcal{T}}^{-} | in \rangle = 0$$

supertranslation charge

$$Q_{\mathcal{T}} = \frac{1}{4\pi G} \int d^2 z \sqrt{\gamma}\, \mathcal{T} M$$

3 languages for the same IR physics [Strominger '18]

Asymptotic symmetries

General Relativity

supertranslations

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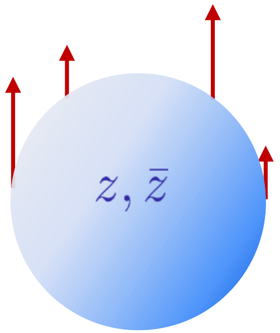

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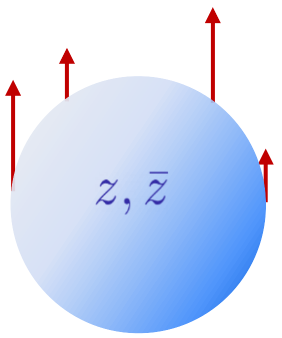

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Soft theorems

Quantum Field Theory

leading soft graviton theorem

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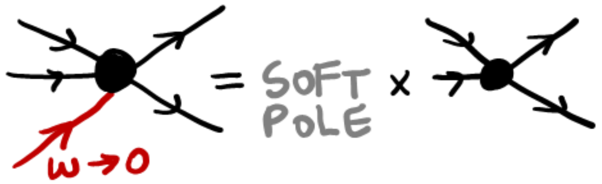

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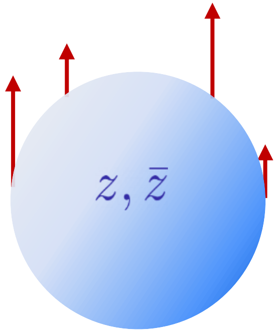

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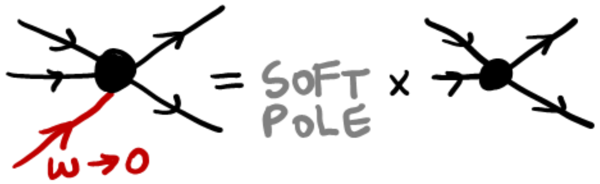
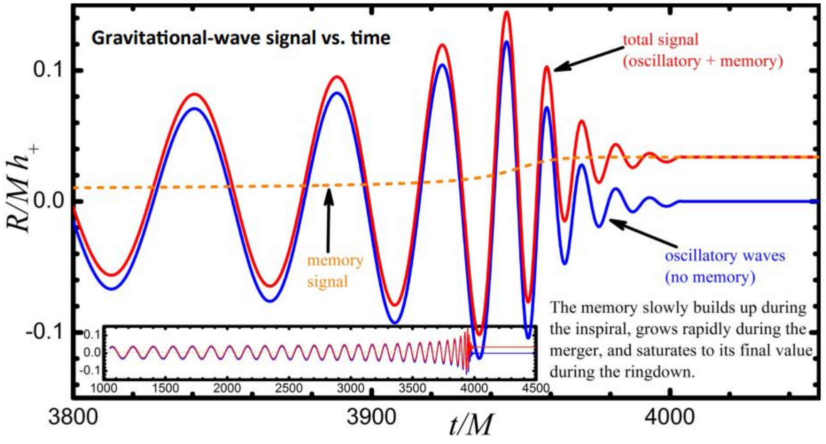


Memory effects

GW observation

displacement memory

[Zel'dovich, Polnarev, Braginskii, Thorne, Christodoulou] ... 70s – 90s



[Favata, '10]

Conclusions

Physics in the deep infrared is **much richer**, more subtle and **much less understood** than we previously thought.

The boundary of **flat space** exhibits an **infinite** amount of **symmetries** which constrain the scattering problem.

Outline of the talk

1. Infrared structure of gravity

2. **Celestial holography**

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Flat space holography

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Early attempts:
[Susskind ‘99][Polchinski ‘99][Giddings ’99]
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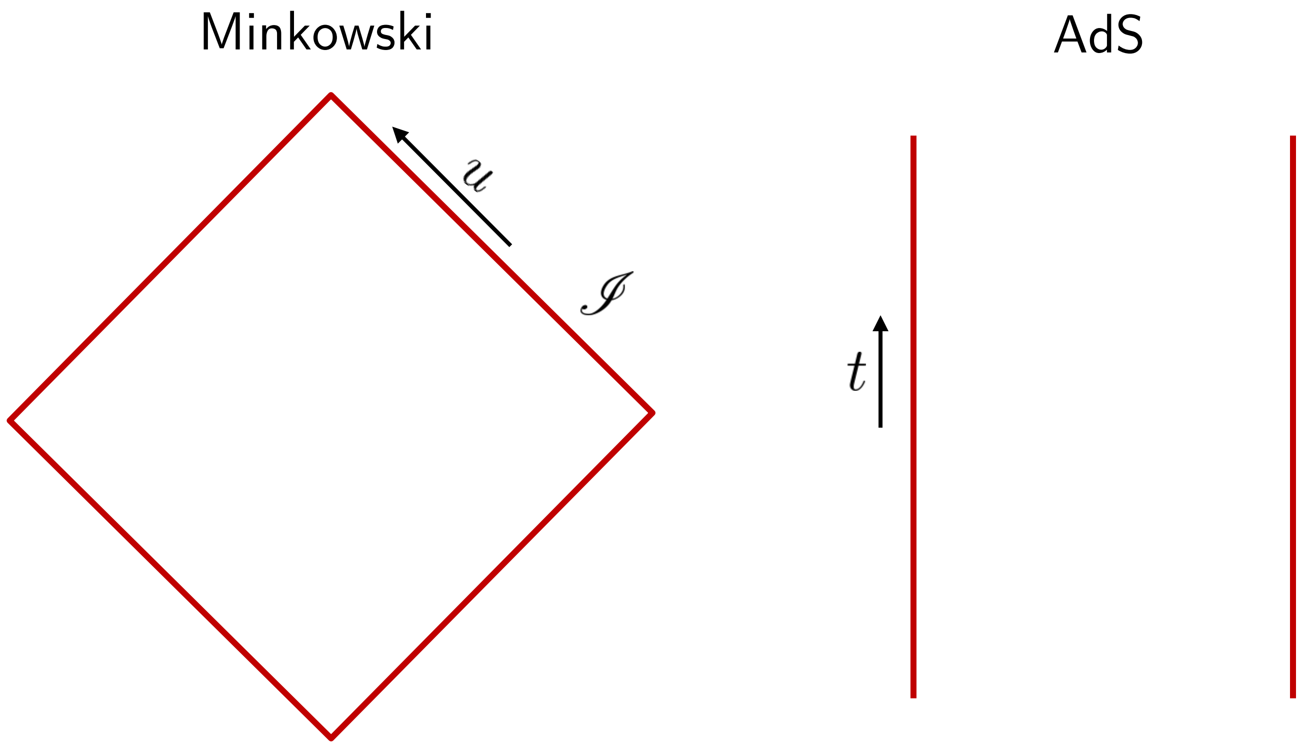
Main obstructions/difficulties:

1. The boundary is a **null** hypersurface



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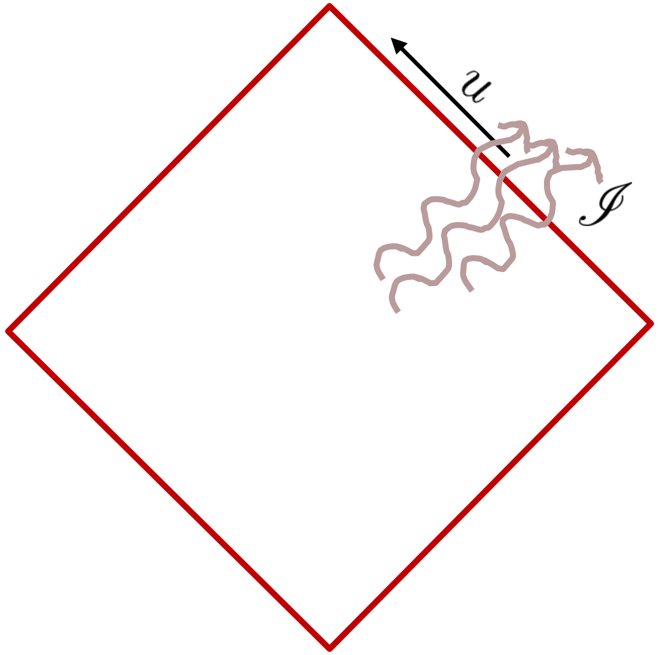
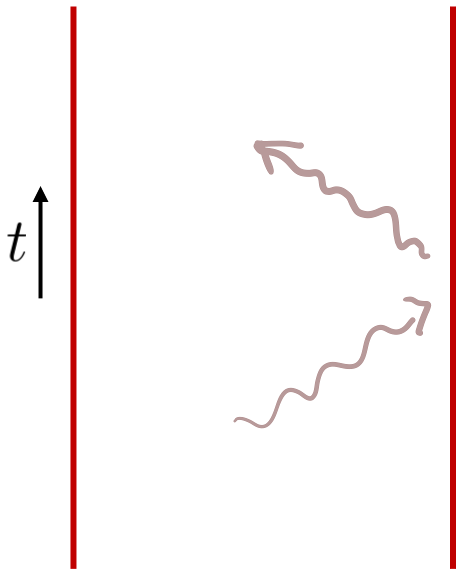
2. There are **fluxes** leaking out the boundary

Minkowski



AdS



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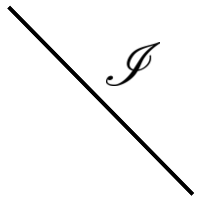
Holographic description of quantum gravity in 4d asymptotically flat spacetimes

two natural boundaries/proposals

null infinity

lighlike 3d hypersurface



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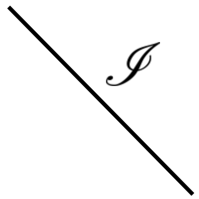

4d bulk/3d holography

Dual: 3d ‘BMS field theory’

[Arcioni, Dappiaggi ’03 ’04] [Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06] [Bagchi, Basu, Kakkar, Melhra ’16] [Bagchi, Melhra, Nandi ’20] [LD, Fiorucci, Herfray, Ruzziconi ’22][Bagchi, Banerjee, Basu, Dutta ‘22][...]

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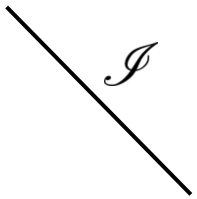

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celestial sphere

Euclidean 2-sphere

4d bulk/2d holography: 'celestial holography'

Dual: 2d 'celestial CFT'

[Strominger '17] [Pasterski, Shao, Strominger '17] [Pasterski, Shao '17] [...]

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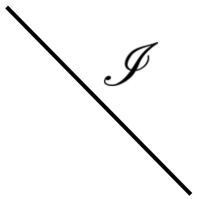

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Features: easier link to AdS/CFT ☺
treatment of fluxes ☹

celestial sphere

Euclidean 2-sphere

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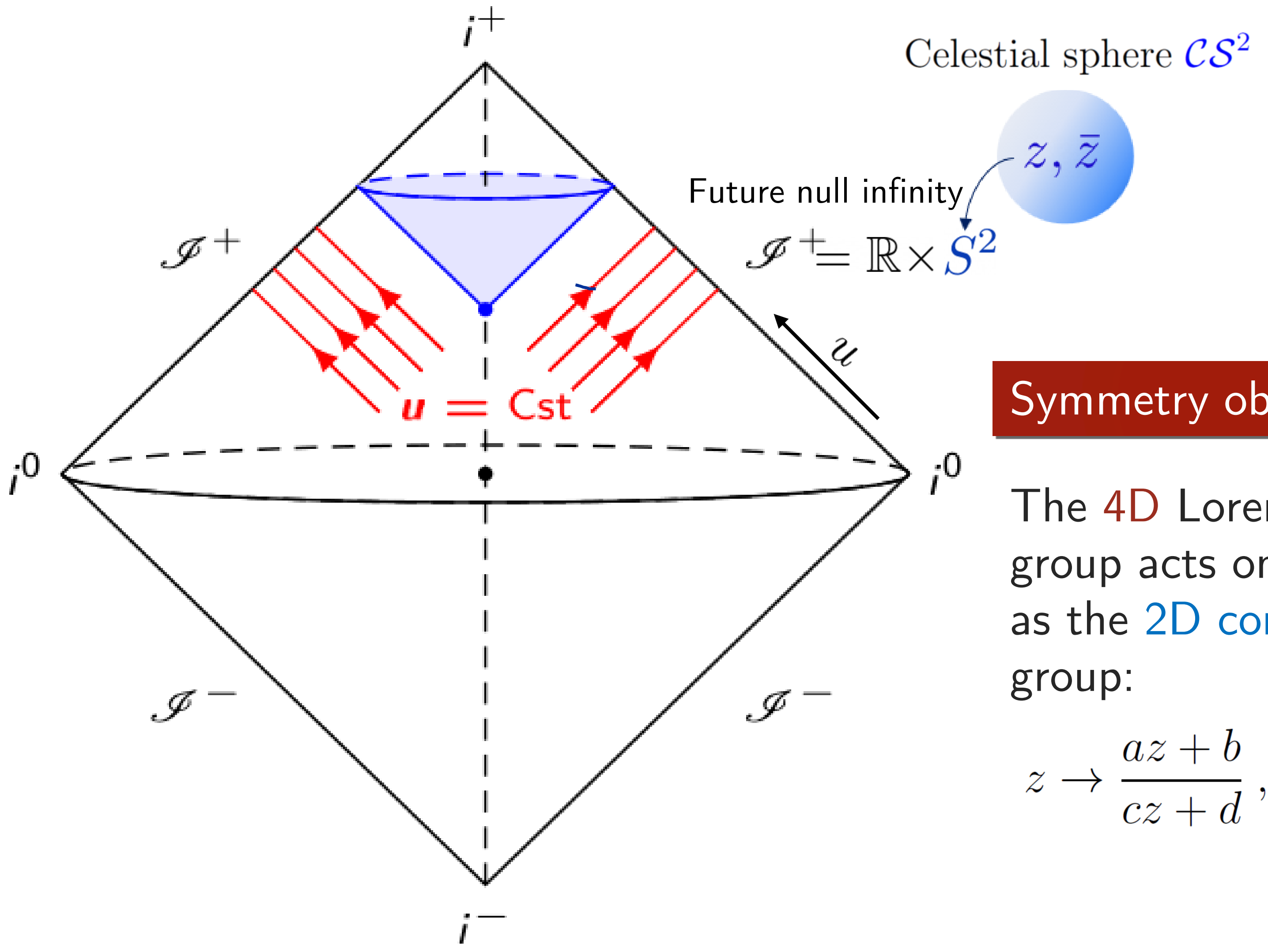
Dual: 2d ‘celestial CFT’

[Strominger ’17] [Pasterski, Shao, Strominger ’17] [Pasterski, Shao ’17] [...]

Features: powerful CFT techniques at hand ☺
role of translations obscured ☹

Towards *flat space* holography... 'celestial holography'



Symmetry observation

The 4D Lorentz group acts on $\mathcal{CS}^2$ as the 2D conformal group:

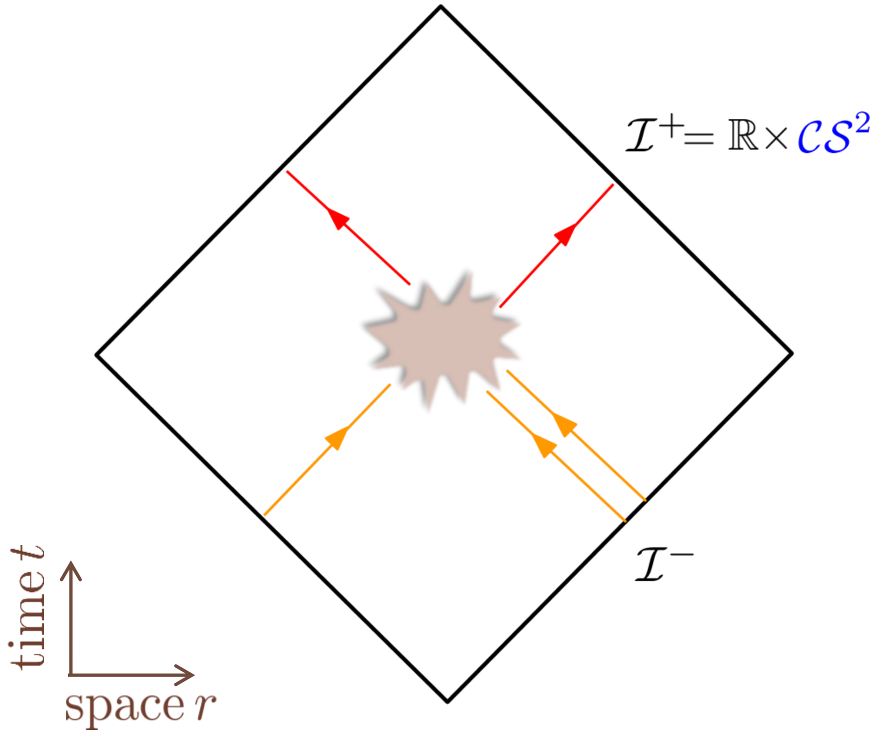
$$z \to \frac{az+b}{cz+d}\,, \qquad ad - bc = 1$$

$SL(2,\mathbb{C})$

Celestial Holography

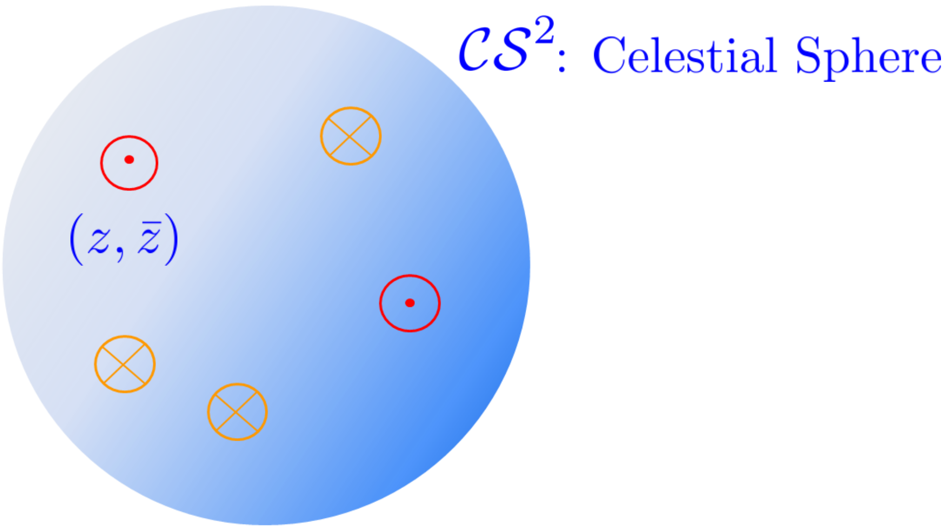
Towards a holographic description for quantum gravity in flat spacetimes

The 4D spacetime S-matrix is encoded in a 2D 'Celestial Conformal Field Theory'



$\langle out|S|in\rangle$

$=$



$\langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\ldots\mathcal{O}_{\Delta_n,J_n}(z_n,\bar{z}_n)\rangle$

Basis for celestial holography

standard formulation	**'celestial basis' formulation**
energy-momentum basis $\rightarrow$ plane waves	conformal basis $\rightarrow$ *Mellin transform* of plane waves
$e^{\pm i p \cdot X}$ $\quad$ $p^\mu = \omega q^\mu(z,\bar{z})$ ω: energy	$\Psi_\Delta(X;z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i p\cdot X}$
solutions labelled by p^μ 4d helicity ℓ	$\Delta = h + \bar{h}$: conformal dimension $(z,\bar{z})$: a point on $\mathcal{CS}^2$ 2d spin $J = h - \bar{h}$

$$z = \frac{p^1 + ip^2}{p^3 + p^0}\,,$$

de Boer, Solodukhin
Cheung, de la Fuente, Sundrum
Pasterski, Shao, Strominger

Basis for celestial holography

standard formulation	**'celestial basis' formulation**
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translation symmetry manifest *conformal properties obscured*	*conformal properties manifest* *translation symmetry obscured*

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Basis for celestial holography

- **Plane waves** (null momentum $p^\mu = \omega q^\mu(z,\bar{z})$) get mapped to

$$\Psi_{\Delta}(X; z, \bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i p \cdot X}$$

'conformal primary wavefunctions' of weights $(h, \bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$:

$$\Psi_{h,\bar{h}}(z,\bar{z}) \underset{SL(2,\mathbb{C})}{\rightarrow} (cz+d)^{2h}(\bar{c}\bar{z}+\bar{d})^{2\bar{h}}\Psi_{h,\bar{h}}(z,\bar{z})$$

$\Delta = h + \bar{h}$: conformal dimension

$(z, \bar{z})$: a point on $\mathcal{CS}^2$

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- **Celestial operators**

$$\boxed{\mathcal{O}_\Delta(z,\bar{z}) = (\Phi(X), \Psi_\Delta(X;z,\bar{z}))} \propto a_\Delta(z,\bar{z}) \equiv \int_0^\infty d\omega\omega^{\Delta-1} a(\omega,z,\bar{z})$$

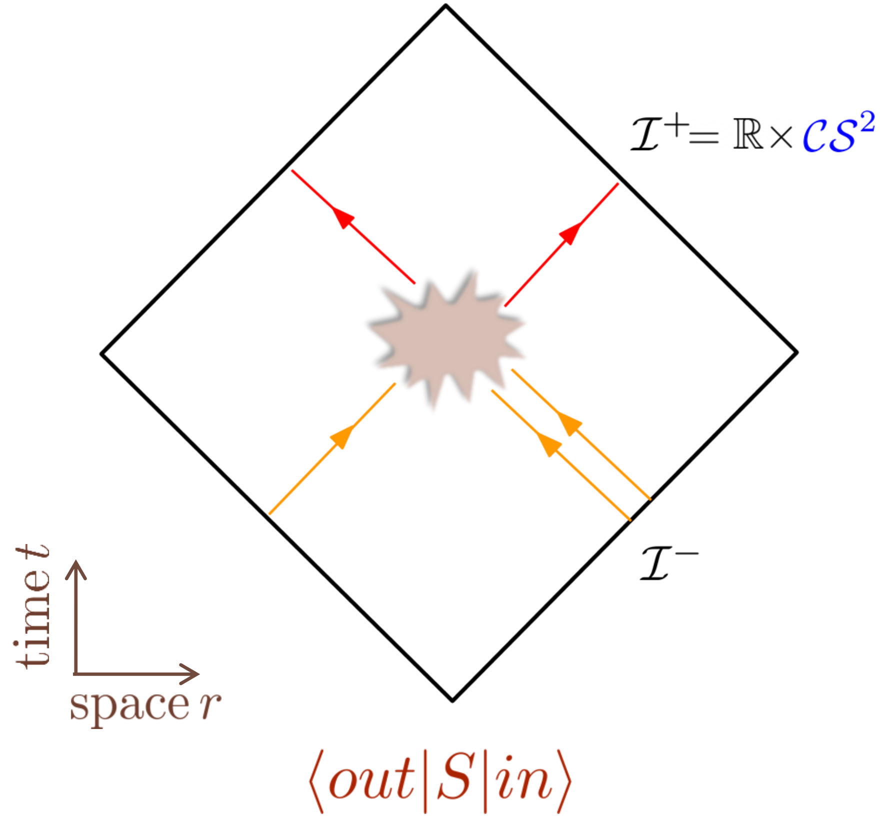
bulk operator   conformal primary wavefunction   usual ladder operator

[LD, Pasterski, Puhm '20]

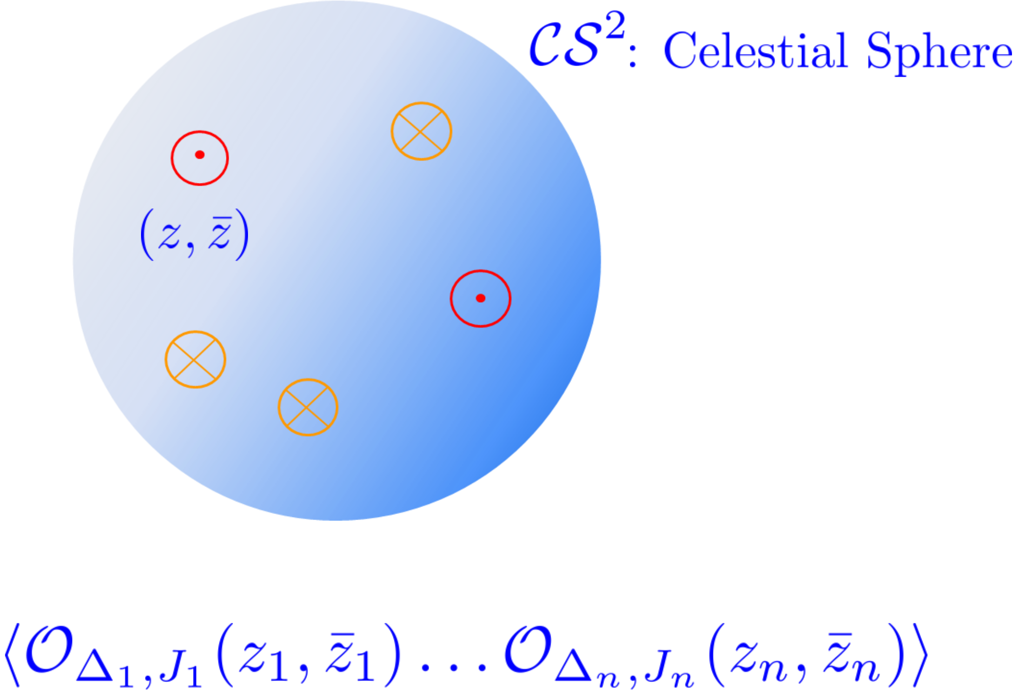
Celestial Holography

Towards a holographic description for quantum gravity in flat spacetimes

The 4D spacetime S-matrix is encoded in a 2D 'Celestial Conformal Field Theory'



$=$



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Celestial version of Weinberg's soft photon theorem!

cf. Ward identity in momentum basis of [He, Mitra, Porfyriadis, Strominger]

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Gravity $(J = 2)$**:**

- Supertranslation current

$$P(z, \bar{z}) = \partial_{\bar{z}} \mathcal{O}_{\Delta=1, J=2}$$

$$(\tfrac{3}{2}, -\tfrac{1}{2} + 1) = (\tfrac{3}{2}, \tfrac{1}{2})$$

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Celestial version of Weinberg's (leading) soft graviton theorem!

[Strominger][He, Lysov, Mitra, Strominger]
[LD, Puhm, Strominger][Puhm][Stieberger, Taylor]

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$$T(z) = \int d^2y \frac{1}{(z-y)^4}\mathcal{O}_{\Delta=0,J=-2}(y,\bar{y})$$: this is the *shadow transform* of a $\Delta = 0$ primary

[Kapec, Mitra, Raclariu, Strominger][Cheung, de la Fuente, Sundrum] [LD, Puhm, Strominger][Stieberger, Taylor]
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N.B.: Shadow transform

$$\widetilde{\mathcal{O}}_{2-\Delta,-J}(z, \bar{z}) = \int d^2y \frac{1}{(z-y)^{2-\Delta-J}} \frac{1}{(\bar{z}-\bar{y})^{2-\Delta+J}} \mathcal{O}_{\Delta,J}(y, \bar{y})$$

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Summary: celestial currents

$(J=1)$

Leading conformally soft photon operator

$\Delta \to 1 \qquad J(z) : (1,0)$

$(J=2)$

Leading conformally soft graviton operator

$\Delta \to 1 \qquad P(z,\bar{z}) : (\frac{3}{2},\frac{1}{2})$

Sub-leading conformally soft graviton operator

$\Delta \to 0 \qquad T(z) : (2,0)$

$$J(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h,\bar{h}}(w,\bar{w})$$

$$P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$$

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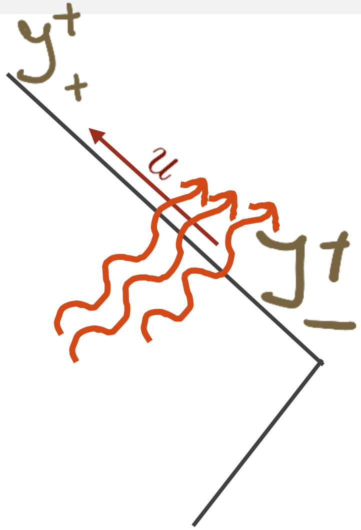
OPE: celestial encoding of soft theorems

Celestial currents

- Are naturally related to the objects of the gravitational solution space in terms of 'BMS fluxes' [LD, Ruzziconi '21]

$$\int_{\mathscr{I}^+} du \partial_u (\cdot) = (\cdot)\Big|_{\mathscr{I}^+_-}^{\mathscr{I}^+_+}$$

$$\begin{aligned} ds^2 &= -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\ &+ \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^z C_{zz}\mathrm{d}u\mathrm{d}z \\ &+ \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots \end{aligned}$$



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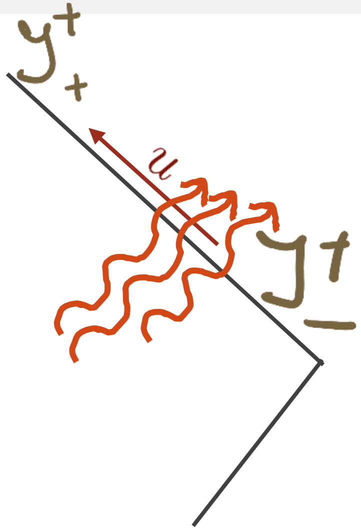

- Infinite tower of currents! $\Delta \to 2, 1, 0, -1, \cdots$

  reorganized in terms of a $w_{1+\infty}$ algebra (all positive helicity gravitons)

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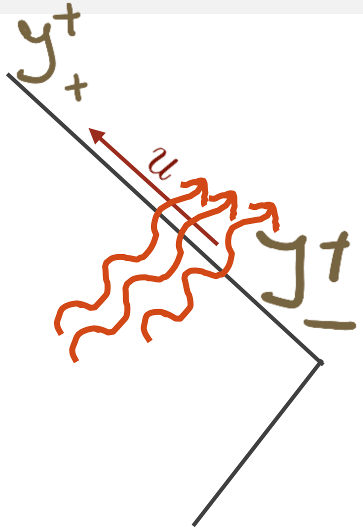

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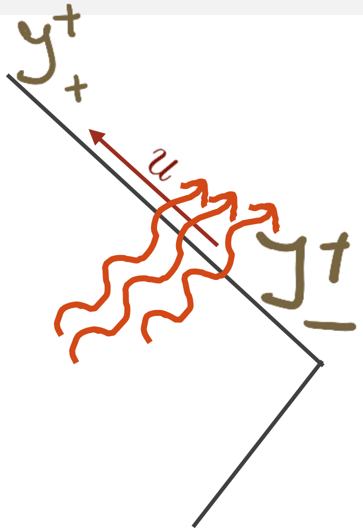

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→ Powerful organizing principle for the soft sector of the S-matrix

Celestial amplitudes

Mellin transformations of massless scattering amplitudes = celestial correlators

$$\prod_{k=1}^{n}\int_0^\infty d\omega_k\ \omega_k^{\Delta_k-1}\,\mathcal{A}(\omega_1,z_1,\bar{z}_1,\dots\omega_n,z_n,\bar{z}_n)=\langle\mathcal{O}_1(\Delta_1,z_1,\bar{z}_1,\dots,\Delta_n,z_n,\bar{z}_n)\rangle_{\mathrm{CCFT}_2}$$

[Adamo, Arkani-Hamed, Atanasov, Banerjee, Cardona, Casali, Fan, Fotopoulos, Ghosh, Gonzalez, Huang, Mason, Melton, Lam, Law, Pasterski, Pate, Paul, Puhm, Raclariu, Rojas, Sharma, Sharma, Shao, Schreiber, Strominger, Stieberger, Taylor, Volovich, Yuan, Zhu, Zlotnikov,...]

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- **collinear limits** $p_1^\mu \parallel p_2^\mu$ of 4d amplitudes $\leftrightarrow$ 2d **celestial OPEs** $z_1 - z_2 \to 0$
- **kinematic singularities** of low point celestial amplitudes (use of shadow or light transforms)
- **conformal block decomposition**
- **UV/IR mixing** (anti-Wilsonian paradigm), ...

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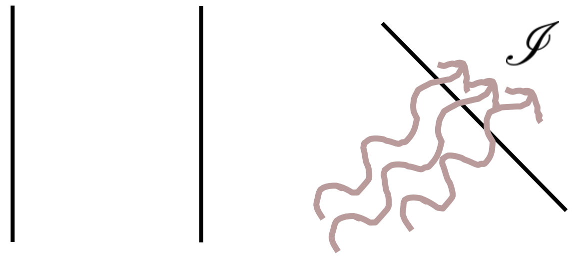
Conclusions

Celestial holography: beyond AdS holography

Holography in Anti-de Sitter: no escape of **radiation** (AdS acts like a box)!
Celestial holography allows for outgoing radiation.



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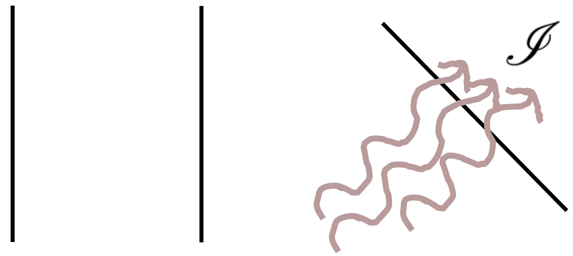

Infinitely many **symmetry constraints** beyond conformal invariance.

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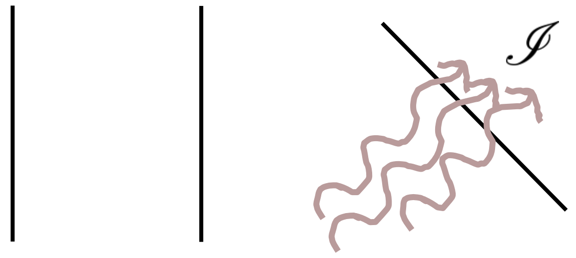

Holography in Anti-de Sitter: no escape of **radiation** (AdS acts like a box)!
Celestial holography allows for outgoing radiation.

Infinitely many **symmetry constraints** beyond conformal invariance.

e.g. Constraints coming from ***supertranslation symmetry*** have no analog in usual holography.

$$P(z)\mathcal{O}_\Delta(w,\bar{w}) \sim \frac{1}{z-w}\mathcal{O}_{\Delta+1}(w,\bar{w})$$

'Celestial conformal field theories' resemble/differ from usual CFTs
in ways that are yet to be fully understood!

Many things remain to be understood!

... we have to keep building up the celestial map.

Some of the outstanding challenges ahead:

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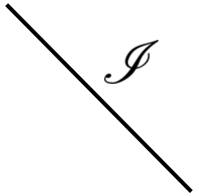
celestial holography & **black holes** ('soft hair', conservation laws...)

Another (mad) road to flat space holography...

two natural boundaries/proposals

null infinity

lighlike 3d hypersurface



4d bulk/3d holography

celestial sphere

Euclidean 2-sphere

4d bulk/2d holography: 'celestial holography'

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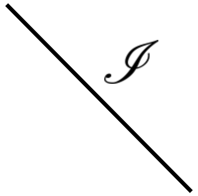

4d bulk/3d holography

$\downarrow$

3d sourced conformal Carrollian field theory

Features: more AdS/CFT like ☺ [LD, Fiorucci, Herfray, Ruzziconi '22]

treatment of fluxes ☹ ✓ **see Romain Ruzziconi's poster**

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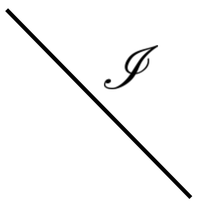

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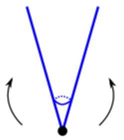
A Carrollian spacetime is *acausal* (time intervals are arbitrary) [Levy-Leblond '65]



$c \to 0$

A mad tea party, Lewis Carroll *(1865)*

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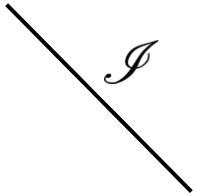

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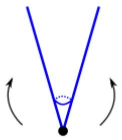

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Thank you for listening!

On the various extensions of BMS

$$\xi = \left(\mathcal{T}(z,\bar z) + \tfrac{u}{2}(\partial\mathcal{Y} + \bar\partial\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar\partial + \cdots$$

- Global BMS: $\boxed{\mathfrak{bms}_4^{\text{glob}} = \mathfrak{so}(3,1) \inplus \mathfrak{supertranslations}}$ [Bondi, van der Burg, Metzner '62] [Sachs '62]

  supertranslations needed to include radiation

- Extended BMS: $\boxed{\mathfrak{bms}_4^{\text{ext}} = (\text{Witt} \oplus \text{Witt}) \inplus \mathfrak{supertranslations}^*}$ [Barnich, Troessaert '10]

  allows for non-globally well-defined transformations on the celestial spher

- Generalized BMS: $\boxed{\mathfrak{bms}_4^{\text{gen}} = \mathfrak{diff}(S^2) \inplus \mathfrak{supertranslations}}$
  [Campiglia, Laddha '14]

  allows for fluctuations of the transverse boundary metric

- Weyl BMS: $\boxed{\mathfrak{bms}_4^{\text{Weyl}} = [\mathfrak{diff}(S^2) \inplus \text{Weyl}] \inplus \mathfrak{supertranslations}}$

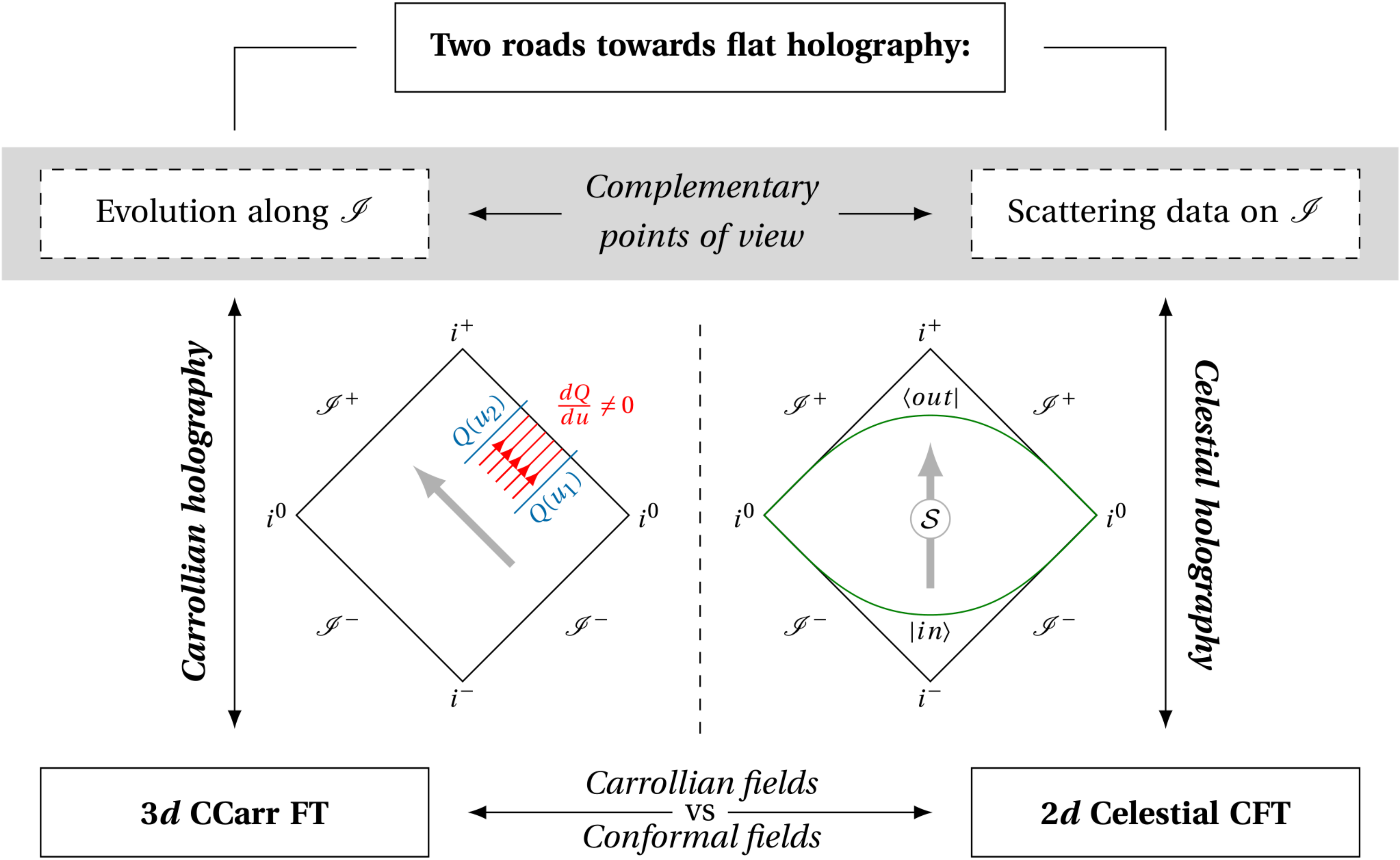
  [Barnich, Troessaert '10][Freidel, Oliveri, Pranzetti, Speziale '21]

  includes (on top of the rest) Weyl rescalings

3 languages for the same IR physics [Strominger '18]

Asymptotic symmetries	Soft theorems	Memory effects
General Relativity	Quantum Field Theory	GW observation
supertranslations [Bondi-Metzner-Sachs '62]	leading soft graviton theorem [Weinberg '65]	displacement memory [Zel'dovich, Polnarev, Braginskii, Thorne, Christodoulou] … 70s – 90s
superrotations [Barnich, Troessaert '10]	new *subleading* soft graviton theorem [Cachazo-Strominger '14]	spin memory [Pasterski-Strominger-Zhiboedov '15]

… and many more!



Two roads towards flat holography:
Evolution along $\mathscr{I}$
Complementary points of view
Scattering data on $\mathscr{I}$
Carrollian holography
i^+
$\mathscr{I}^+$
$\frac{dQ}{du} \neq 0$
$Q(u_2)$
$Q(u_1)$
i^0
i^0
$\mathscr{I}^-$
$\mathscr{I}^-$
i^-
i^+
$\mathscr{I}^+$
$\langle out|$
$\mathscr{I}^+$
i^0
$\mathcal{S}$
i^0
$\mathscr{I}^-$
$|in\rangle$
$\mathscr{I}^-$
i^-
Celestial holography
3d CCarr FT
Carrollian fields vs Conformal fields
2d Celestial CFT

Basis for celestial holography

$$\int_0^\infty d\omega\, \omega^{\Delta-1} \qquad p^\mu = \omega q^\mu(z,\bar{z})$$

- The statement that **plane waves** form a delta-function normalizable basis

$$(e^{ip\cdot X}, e^{ip'\cdot X}) = 2(2\pi)^3 p^0 \delta^{(3)}(\vec{p}-\vec{p}\,')$$

translates into requiring the conformal dimension to lie on the principal series [Pasterski, Shao '17]

$$\Delta = 1 + i\lambda\,,\ \ \lambda \in \mathbb{R} \qquad (\Psi_{1+i\lambda}(X;z,\bar{z}), \Psi_{1+i\lambda'}(X;z',\bar{z}')) = (2\pi)^4 \delta(\lambda+\lambda')\delta^{(2)}(z-z')$$

- **2d celestial operators**

$$\boxed{\mathcal{O}_\Delta(z,\bar{z}) = (\Phi(X), \Psi_\Delta(X;z,\bar{z}))} \sim a_\Delta(z,\bar{z}) \equiv \int_0^\infty d\omega \omega^{\Delta-1} a(\omega,z,\bar{z})$$

bulk operator — conformal primary wavefunction — usual ladder operator

[LD, Pasterski, Puhm '20]

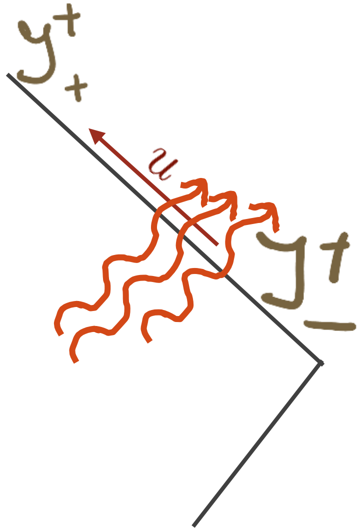
Celestial currents

- Can be organized with 'celestial diamonds' [Pasterski, Puhm, Trevisani '21]

- Are naturally related to the objects of the gravitational solution space in terms of 'BMS fluxes' [LD, Ruzziconi '21]

$$\int_{\mathcal{I}^+} du \partial_u \left(\cdot\right) = \left(\cdot\right)\Big|^{\mathcal{I}^+_+}_{\mathcal{I}^+_-}$$

$$\begin{aligned} ds^2 = &-\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\, \mathrm{d}z\mathrm{d}\bar{z} \\ &+ \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^z C_{zz}\mathrm{d}u\mathrm{d}z \\ &+ \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots \end{aligned}$$



- Infinite tower of currents! $\Delta \to 2, 1, 0, -1, \cdots$

  reorganized in terms of a $w_{1+\infty}$ algebra (all positive helicity gravitons)

  [Guevara, Himwich, Pate, Strominger '21] [Strominger '21][Himwich, Pate, Singh '21]

  natural appearance from twistor space! [Penrose '76][Newman '76][Adamo, Mason, Sharma '21]...

$\longrightarrow$ Powerful organizing principles for the soft sector of the S-matrix