Charting the space of superconformal theories from string theory

Simone Giacomelli

University of Milano-Bicocca

TFI2022, June 14th 2022

Based on: 2001.00533, 2007.00647, 2010.03943, 2010.05889, 2205.08578 plus work in progress...

Field theory from geometry

In recent years we have significantly improved our understanding of non-perturbative phenomena in field theory.

Geometry is a key ingredient in these developments:

- We can enlarge the landscape of known theories;
- We gain insight into the dynamics of field theories from the structure of their space of vacua (moduli space).

Questions we wish to answer:

- Can we identify the set of data specifying a field theory?
- Can we study the dynamics of the theory using these data?

We can tackle these questions in the case of theories with eight supercharges.

Field theory from geometry

In recent years we have significantly improved our understanding of non-perturbative phenomena in field theory.

Geometry is a key ingredient in these developments:

- We can enlarge the landscape of known theories;
- We gain insight into the dynamics of field theories from the structure of their space of vacua (moduli space).

Questions we wish to answer:

- Can we identify the set of data specifying a field theory?
- Can we study the dynamics of the theory using these data?

We can tackle these questions in the case of theories with eight supercharges.

Field theory from geometry

In recent years we have significantly improved our understanding of non-perturbative phenomena in field theory.

Geometry is a key ingredient in these developments:

- We can enlarge the landscape of known theories;
- We gain insight into the dynamics of field theories from the structure of their space of vacua (moduli space).

Questions we wish to answer:

- Can we identify the set of data specifying a field theory?
- Can we study the dynamics of the theory using these data?

We can tackle these questions in the case of theories with eight supercharges.

$\mathcal{N} = 2$ theories and their moduli space of vacua

4d $\mathcal{N} = 2$ SCFTs have a $SU(2)_R \times U(1)_R$ symmetry and display an intricate moduli space of vacua:

- **Coulomb Branch (CB):** where vector multiplet scalars have nonzero vev ($SU(2)_R$ unbroken);
- **Higgs Branch (HB):** where hypermultiplet scalars have nonzero vev ($U(1)_R$ unbroken).

While the HB is not quantum corrected, the CB is: its (quantum) geometry is determined by the SW solution. Seiberg, Witten '94.
This allows us to study **analytically** confinement and chiral symmetry breaking in supersymmetric gauge theories!

$\mathcal{N} = 2$ theories and their moduli space of vacua

4d $\mathcal{N} = 2$ SCFTs have a $SU(2)_R \times U(1)_R$ symmetry and display an intricate moduli space of vacua:

- **Coulomb Branch (CB):** where vector multiplet scalars have nonzero vev ($SU(2)_R$ unbroken);
- **Higgs Branch (HB):** where hypermultiplet scalars have nonzero vev ($U(1)_R$ unbroken).

While the HB is not quantum corrected, the CB is: its (quantum) geometry is determined by the SW solution. Seiberg, Witten '94.
This allows us to study **analytically** confinement and chiral symmetry breaking in supersymmetric gauge theories!

Data defining a $\mathcal{N} = 2$ SCFT

Based on the currently known landscape of theories, it seems that $\mathcal{N} = 2$ SCFTs are characterized by:

- The spectrum of local (CB) operators;
- The global symmetry **group**;
- The ‘t Hooft anomalies.

$\mathcal{N} = 2$ theories are usually characterized by their **rank** ($=$ CB dimension) and we can attempt classifying them at low rank (done for rank 1, in part for rank 2).

We can improve our understanding of SCFTs of arbitrary rank with the help of stringy constructions.
In this talk I will focus on brane setups in Type IIB.

Data defining a $\mathcal{N} = 2$ SCFT

Based on the currently known landscape of theories, it seems that $\mathcal{N} = 2$ SCFTs are characterized by:

- The spectrum of local (CB) operators;
- The global symmetry **group**;
- The 't Hooft anomalies.

$\mathcal{N} = 2$ theories are usually characterized by their **rank** (= CB dimension) and we can attempt classifying them at low rank (done for rank 1, in part for rank 2).

We can improve our understanding of SCFTs of arbitrary rank with the help of stringy constructions.
In this talk I will focus on brane setups in Type IIB.

$\mathcal{N} = 2$ instanton theories from $D3$ branes

We consider 7-branes with constant axio-dilaton. Mukhi, Dasgupta '96.

G	$\emptyset$	$SU(2)$	$SU(3)$	$SO(8)$	E_6	E_7	E_8
Δ_7	$6/5$	$4/3$	$3/2$	2	3	4	6

The angular variable around the 7-brane has periodicity $2\pi/\Delta_7$.

We probe the 7-brane with a stack of N $D3$ branes:

	0	1	2	3	4	5	6	7	8	9
7-brane	x	x	x	x	x	x	x	x		
D3 brane	x	x	x	x						

The gauge symmetry G on the 7-brane becomes the global symmetry in 4d.

$\mathcal{N} = 2$ instanton theories from $D3$ branes

We consider 7-branes with constant axio-dilaton. Mukhi, Dasgupta '96.

G	$\emptyset$	$SU(2)$	$SU(3)$	$SO(8)$	E_6	E_7	E_8
Δ_7	$6/5$	$4/3$	$3/2$	2	3	4	6

The angular variable around the 7-brane has periodicity $2\pi/\Delta_7$.

We probe the 7-brane with a stack of N $D3$ branes:

	0	1	2	3	4	5	6	7	8	9
7-brane	x	x	x	x	x	x	x	x		
D3 brane	x	x	x	x						

The gauge symmetry G on the 7-brane becomes the global symmetry in 4d.

7-branes on singular spaces

We can refine the construction by letting the G-type 7-brane wrap an orbifold singularity. New ingredients: SG, Moleti, Savelli '22.

- We need to specify the holonomy for the G gauge fields at the origin and at infinity;
- The global symmetry of the 4d theory is the subgroup of G commuting with the holonomy.

S-fold: Wrap the 7-brane on $\mathbb{C}^2/\mathbb{Z}_\ell$ together with a 'non geometric' quotient: Embed $\mathbb{Z}_{\ell\Delta_7}$ in $SL(2,\mathbb{Z})$ Apruzzi, SG, Schafer-Nameki '20.

↓

S-Duality group of Type IIB

7-branes on singular spaces

We can refine the construction by letting the G-type 7-brane wrap an orbifold singularity. New ingredients: SG, Moleti, Savelli '22.

- We need to specify the holonomy for the G gauge fields at the origin and at infinity;
- The global symmetry of the 4d theory is the subgroup of G commuting with the holonomy.

S-fold: Wrap the 7-brane on $\mathbb{C}^2/\mathbb{Z}_\ell$ together with a 'non geometric' quotient: Embed $\mathbb{Z}_{\ell\Delta_7}$ in $SL(2,\mathbb{Z})$ Apruzzi, SG, Schafer-Nameki '20.

$\downarrow$

S-Duality group of Type IIB

New theories from Type IIB 7-branes

Probing S-folds with N D_3 branes we find rank N $\mathcal{N} = 2$ theories.

Notable special cases:

- For $\ell = 1$ we recover the $\mathcal{N} = 2$ instanton theories.
- If we remove the 7-brane $(\Delta_7 = 1)$ the 4d theory on the probes has $\mathcal{N} = 3$ susy. Garcia-Etxebarria, Regalado '15; Aharony, Tachikawa '16.

A geometric realization of rank 1 theories

For $N = 1$ we recover the classification of rank 1 theories and for $N > 1$ we find higher-rank generalizations of these.

New theories from Type IIB 7-branes

Probing S-folds with N D_3 branes we find rank N $\mathcal{N} = 2$ theories.

Notable special cases:

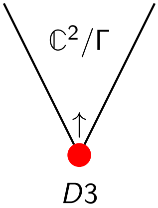
- For $\ell = 1$ we recover the $\mathcal{N} = 2$ instanton theories.
- If we remove the 7-brane ($\Delta_7 = 1$) the 4d theory on the probes has $\mathcal{N} = 3$ susy. Garcia-Etxebarria, Regalado '15; Aharony, Tachikawa '16.

A geometric realization of rank 1 theories

For $N = 1$ we recover the classification of rank 1 theories and for $N > 1$ we find higher-rank generalizations of these.

7-branes on singular spaces and instanton theories

Moving the probe $D3$ branes away from the Γ fixed point implements a motion along the Higgs branch.

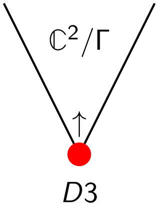


Once the probes have been moved, they see only the 7-brane and do not perceive the Γ quotient anymore.

All the models we are studying can be higgsed to the corresponding G instanton theory!

7-branes on singular spaces and instanton theories

Moving the probe $D3$ branes away from the Γ fixed point implements a motion along the Higgs branch.



Once the probes have been moved, they see only the 7-brane and do not perceive the Γ quotient anymore.

All the models we are studying can be higgsed to the corresponding G instanton theory!

An example with exceptional 7-branes

Consider a 7-brane of type D_4 on $\mathbb{C}^2/\mathbb{Z}_2$.
When we probe it with a single $D3$ brane we find:

$$\boxed{2} - SU(2) - SU(2) - \boxed{2}$$

What is the 4d SCFT when the 7-brane is of type E_8?

- Should have rank 2;
- Flows to rank 1 E_8 instanton theory upon higgsing;
- Flows to the $SU(2)^2$ gauge theory upon mass deformation.

There is a theory with these properties and symmetry Martone '21

$$SO(16) \times SU(2)$$

E_8 subgroup ← $SO(16)$; $SU(2)$ → Isometry

An example with exceptional 7-branes

Consider a 7-brane of type D_4 on $\mathbb{C}^2/\mathbb{Z}_2$.
When we probe it with a single $D3$ brane we find:

$$\boxed{2} - SU(2) - SU(2) - \boxed{2}$$

What is the 4d SCFT when the 7-brane is of type E_8?

- Should have rank 2;
- Flows to rank 1 E_8 instanton theory upon higgsing;
- Flows to the $SU(2)^2$ gauge theory upon mass deformation.

There is a theory with these properties and symmetry Martone '21

$$SO(16) \times SU(2)$$

$SO(16)$ → E_8 subgroup

$SU(2)$ → Isometry

Type IIB theories and holography

't Hooft anomalies of SCFTs on the worldvolume of $D3$ branes are determined via holography. Aharony, Tachikawa '07; Apruzzi, SG, Schafer-Nameki '20

The holographic dual is given by $AdS_5 \times M_5$ ($M_5 = \widetilde{S}^5/\Gamma$) where $\widetilde{S}^5$ is $\{|x|^2 + |y|^2 + |z|^2 = 1\} \subset \mathbb{C}^3$ with $\theta_z \in [0, 2\pi/\Delta_7]$.

In particular a and c central charges read:

$$2a - c = AN^2 + BN; \quad c - a = CN,$$

- A is proportional to $1/Vol(M_5)$;
- B and C are proportional to $Vol(M_3)/Vol(M_5)$, where M_3 is the manifold wrapped by the 7-brane.

Type IIB theories and holography

't Hooft anomalies of SCFTs on the worldvolume of $D3$ branes are determined via holography. Aharony, Tachikawa '07; Apruzzi, SG, Schafer-Nameki '20

The holographic dual is given by $AdS_5 \times M_5$ $(M_5 = \widetilde{S}^5/\Gamma)$ where $\widetilde{S}^5$ is $\{|x|^2 + |y|^2 + |z|^2 = 1\} \subset \mathbb{C}^3$ with $\theta_z \in [0, 2\pi/\Delta_7]$.

In particular a and c central charges read:

$$2a - c = AN^2 + BN; \quad c - a = CN,$$

- A is proportional to $1/Vol(M_5)$;
- B and C are proportional to $Vol(M_3)/Vol(M_5)$, where M_3 is the manifold wrapped by the 7-brane.

An alternative construction from M-theory

Two realizations of rank N E_8 instanton theory: Minahan, Nemeschansky '96.

- From Type IIB with N D3 branes probing a 7-brane of type E_8.
- Rank-N E-string on a torus: N M5 branes probing the M9 wall in M-theory.

In both cases a $\mathbb{R}^4$ transverse to the probes $(SU(2)_R \times SU(2)_F)$.

This duality persists in the general case, we need to orbifold the $\mathbb{R}^4$ in both descriptions. In the $\mathcal{S}$-**fold** case we need to turn on holonomies on the torus! SG, Martone, Tachikawa, Zafrir '20

An alternative construction from M-theory

Two realizations of rank N E_8 instanton theory: Minahan, Nemeschansky '96.

- From Type IIB with N D3 branes probing a 7-brane of type E_8.
- Rank-N E-string on a torus: N M5 branes probing the M9 wall in M-theory.

In both cases a $\mathbb{R}^4$ transverse to the probes $(SU(2)_R \times SU(2)_F)$.

This duality persists in the general case, we need to orbifold the $\mathbb{R}^4$ in both descriptions. In the $\mathcal{S}$-**fold** case we need to turn on holonomies on the torus! SG, Martone, Tachikawa, Zafrir '20

All 4d superconformal theories from six dimensions?

We find that all 4d models from Type IIB

- Either can be obtained via compactification of a 6d SCFT;
- Or can be described as a mass deformation thereof.

The above statement applies to all rank 1 theories and (almost all) known rank 2 theories. Is it true in general?

Since we have a classification of 6d SCFTs Heckman, Morrison, Rudelius, Vafa '15 this observation suggests a simple organizing principle for the space of 4d superconformal theories!

The Higgs branch of 6d SCFTs is known, and from this we can extract a detailed description of the moduli space of the 4d theory!

All 4d superconformal theories from six dimensions?

We find that all 4d models from Type IIB

- Either can be obtained via compactification of a 6d SCFT;
- Or can be described as a mass deformation thereof.

The above statement applies to all rank 1 theories and (almost all) known rank 2 theories. Is it true in general?

Since we have a classification of 6d SCFTs Heckman, Morrison, Rudelius, Vafa '15 this observation suggests a simple organizing principle for the space of 4d superconformal theories!

The Higgs branch of 6d SCFTs is known, and from this we can extract a detailed description of the moduli space of the 4d theory!

4d physics from 3d

The HB of 6d theories is equivalent to the CB of a 3d **lagrangian** theory called **magnetic quiver** Cabrera, Hanany, Sperling '19.

From the 6d result we can derive magnetic quivers for our 4d theories! Bourget, SG, Grimminger, Hanany, Sperling, Zhong '20- SG, Moleti, Savelli '22.

Since the Coulomb Branch of 3d lagrangian theories is well understood, Cremonesi, Hanany, Zaffaroni '13- Bullimore, Gaiotto, Dimofte '15.
from magnetic quivers we obtain several results, such as:

- Theories labelled by different choices of holonomy are related by RG flow (higgsing). SG, Moleti, Savelli '22.
- Mass deformations are mapped to relevant deformations in the 3d theory which we can study using e.o.m.'s! SG, Van Beest '21

4d physics from 3d

The HB of 6d theories is equivalent to the CB of a 3d **lagrangian** theory called **magnetic quiver** Cabrera, Hanany, Sperling '19.

From the 6d result we can derive magnetic quivers for our 4d theories! Bourget, SG, Grimminger, Hanany, Sperling, Zhong '20- SG, Moleti, Savelli '22.

Since the Coulomb Branch of 3d lagrangian theories is well understood, Cremonesi, Hanany, Zaffaroni '13- Bullimore, Gaiotto, Dimofte '15.
from magnetic quivers we obtain several results, such as:

- Theories labelled by different choices of holonomy are related by RG flow (higgsing). SG, Moleti, Savelli '22.
- Mass deformations are mapped to relevant deformations in the 3d theory which we can study using e.o.m.'s! SG, Van Beest '21

S-fold theories and new instanton moduli spaces

Since our 4d theories are engineered by $D3$ branes probing a 7-brane of type G, we expect the Higgs branch to coincide with the moduli space of G instantons.

The Higgs branch of S-fold theories can be interpreted as instanton moduli spaces on $\mathbb{C}^2/\mathbb{Z}_\ell$. Since the holonomy at infinity generically involves outer-automorphisms, this provides new examples of ALE instantons. Bourget, SG, Grimminger, Hanany, Sperling, Zhong '20.

This suggests the possibility of generalizing the construction of Kronheimer and Nakajima! Kronheimer, Nakajima '90.

S-fold theories and new instanton moduli spaces

Since our 4d theories are engineered by $D3$ branes probing a 7-brane of type G, we expect the Higgs branch to coincide with the moduli space of G instantons.

The Higgs branch of S-fold theories can be interpreted as instanton moduli spaces on $\mathbb{C}^2/\mathbb{Z}_\ell$. Since the holonomy at infinity generically involves outer-automorphisms, this provides new examples of ALE instantons. Bourget, SG, Grimminger, Hanany, Sperling, Zhong '20.

This suggests the possibility of generalizing the construction of Kronheimer and Nakajima! Kronheimer, Nakajima '90.

Summary

We have constructed a large new class of 4d superconformal theories by probing with $D3$ branes a 7-brane on a singular space. We provided an explicit construction of all rank 1 theories and found generalizations of these of arbitrary rank.

From the geometric description we can:

- Determine the CB spectrum of the 4d SCFTs;
- Identify the global symmetry and compute the corresponding 't Hooft anomalies;
- Study RG flows induced by higgsing and mass deformations;
- Determine the HB of the 4d theories, finding evidence for the existence of new instanton moduli spaces.

Summary

We have constructed a large new class of 4d superconformal theories by probing with $D3$ branes a 7-brane on a singular space. We provided an explicit construction of all rank 1 theories and found generalizations of these of arbitrary rank.

From the geometric description we can:

- Determine the CB spectrum of the 4d SCFTs;
- Identify the global symmetry and compute the corresponding 't Hooft anomalies;
- Study RG flows induced by higgsing and mass deformations;
- Determine the HB of the 4d theories, finding evidence for the existence of new instanton moduli spaces.

Concluding remarks

Several possible future directions:

- It would be important to generalize our construction and understand how general the connection between 6d and 4d theories is;
- We should understand whether it is possible or not to extract information about correlation functions from the CFT data we can access from the geometric construction;
- It would be important to explore the space of $\mathcal{N} = 1$ theories from Type IIB. Work in progress...

Thank You!

Concluding remarks

Several possible future directions:

- It would be important to generalize our construction and understand how general the connection between 6d and 4d theories is;
- We should understand whether it is possible or not to extract information about correlation functions from the CFT data we can access from the geometric construction;
- It would be important to explore the space of $\mathcal{N} = 1$ theories from Type IIB. Work in progress...

Thank You!

Almost commuting holonomies on T^2

Consider the 6d SCFT UV completing $SU(2)$ **with 10 flavors**.

The theory on T^2 gives a rank 2 SCFT with $G_F = SO(20)$.

Flavors transform as $(\mathbf{10}, \mathbf{2})$ under $USp(10) \times SU(2) \subset SO(20)$.
We embed in the $SU(2)$ factor the almost commuting holonomies

$$P = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}; \quad Q = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

To have all fields uncharged under $\mathbb{Z}_2$, we embed the holonomy in the $SU(2)$ gauge group as well.

We end up in 4d with a rank 1 theory with $USp(10)$ symmetry.

Almost commuting holonomies on T^2

Consider the 6d SCFT UV completing $SU(2)$ **with 10 flavors**.

The theory on T^2 gives a rank 2 SCFT with $G_F = SO(20)$.

Flavors transform as $(\mathbf{10}, \mathbf{2})$ under $USp(10) \times SU(2) \subset SO(20)$. We embed in the $SU(2)$ factor the almost commuting holonomies

$$P = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}; \quad Q = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

To have all fields uncharged under $\mathbb{Z}_2$, we embed the holonomy in the $SU(2)$ gauge group as well.

We end up in 4d with a rank 1 theory with $USp(10)$ symmetry.