3d N=2 dualities for SQCD with D-type superpotential

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Based on 2204.06961 w/ Simone Rota
(see also 1809.03796,1509.02199 &1409.8623)

Motivations

SUSY dualities: why are they still important/useful?

- Links with mathematics
- Non perturbative dynamics
- Conformal window and AdS/CFT
- 3d bosonization
- Connections among dualities in different dimensions
- SUSY and global symmetry enhancements
- Relating quantum aspects in different dimensions (e.g. role of the monopoles)

Plan

- 3d reduction of 4d dualities
- Basic dualities and basic web
- The web for the A-type superpotential
- D-type
  - 4d aspects
  - The web
  - Three sphere partition function
- Conclusions

4d/3d reduction

This work fits in the program of reduction of 4d Seiberg-like dualities to 3d (ARSW, Aharony-Razamat-Seiberg-Willett `13).

Original case: SU(N) SQCD with F flavors and F>N+1 (Seiberg `94)

Then extension to SO/USp SQCD (Intriligator-Seiberg `95 and Intriligator-Pouliot `95) and to cases w/ one two-index tensor (Adj, Symm. or Anti-Symm) and A-type superpotential (Kutasov-Schwimmer-Seiberg `95, Intriligator-Leigh-Strassler `95).

The ARSW procedure has been (or can be) applied to all these cases. Often the reduction can be studied with the aim of localization.

$$
\begin{array}{cc}
\widehat{O} & W_{\widehat{O}} = 0 \\
\widehat{A} & W_{\widehat{A}} = \mathrm{Tr} Y^2 \\
\widehat{D} & W_{\widehat{D}} = \mathrm{Tr} X Y^2 \\
\widehat{E} & W_{\widehat{E}} = \mathrm{Tr} Y^3 \\
A_k & W_{A_k} = \mathrm{Tr}(X^{k+1} + Y^2) \\
D_{k+2} & W_{D_{k+2}} = \mathrm{Tr}(X^{k+1} + XY^2) \\
E_6 & W_{E_6} = \mathrm{Tr}(Y^3 + X^4) \\
E_7 & W_{E_7} = \mathrm{Tr}(Y^3 + YX^3) \\
E_8 & W_{E_8} = \mathrm{Tr}(Y^3 + X^5).
\end{array}
$$

Arnold's ADE classification of singularities, which precisely coincides with the possible relevant deformation superpotentials, listed as A_k, D_{k+2} and $E_{6,7,8}$

D-type superpotential? Curious, uncommon and more delicate dualities obtained by Brodie `96 and Brodie-Strassler `96.

4d and 3d SUSY with four supercharges: a brief overview

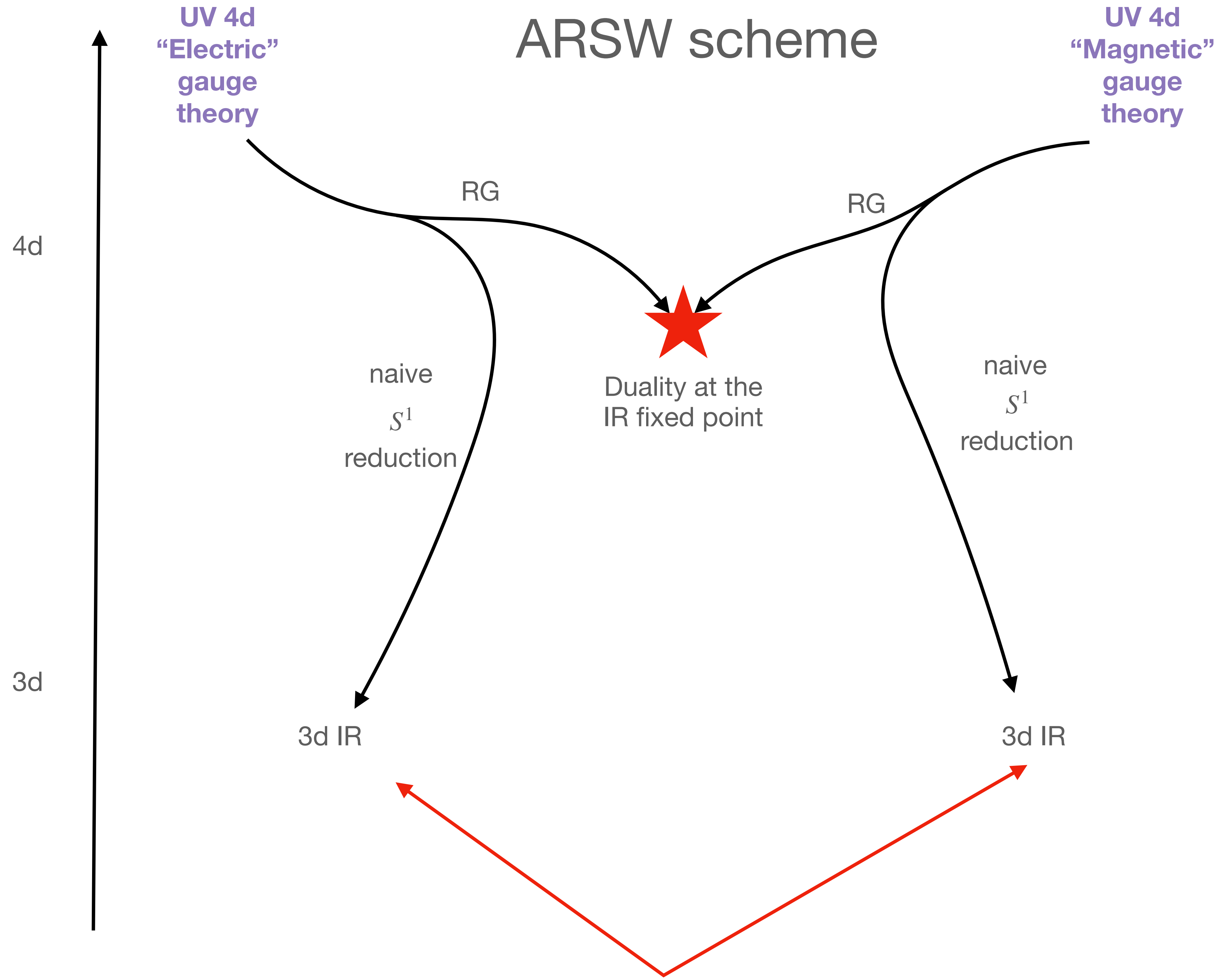
4d

- Global Anomalies
- Absence of axial symmetry
- U(1) gauge groups are IR free
- Exact R by a-maximization
- SYM action
- $V = (A, \lambda, D)$
- $\Lambda_{holo} = Exp(1/g_4^2)$
- Instanton: $W_{eff} = N\Lambda_{holo}^3$

3d

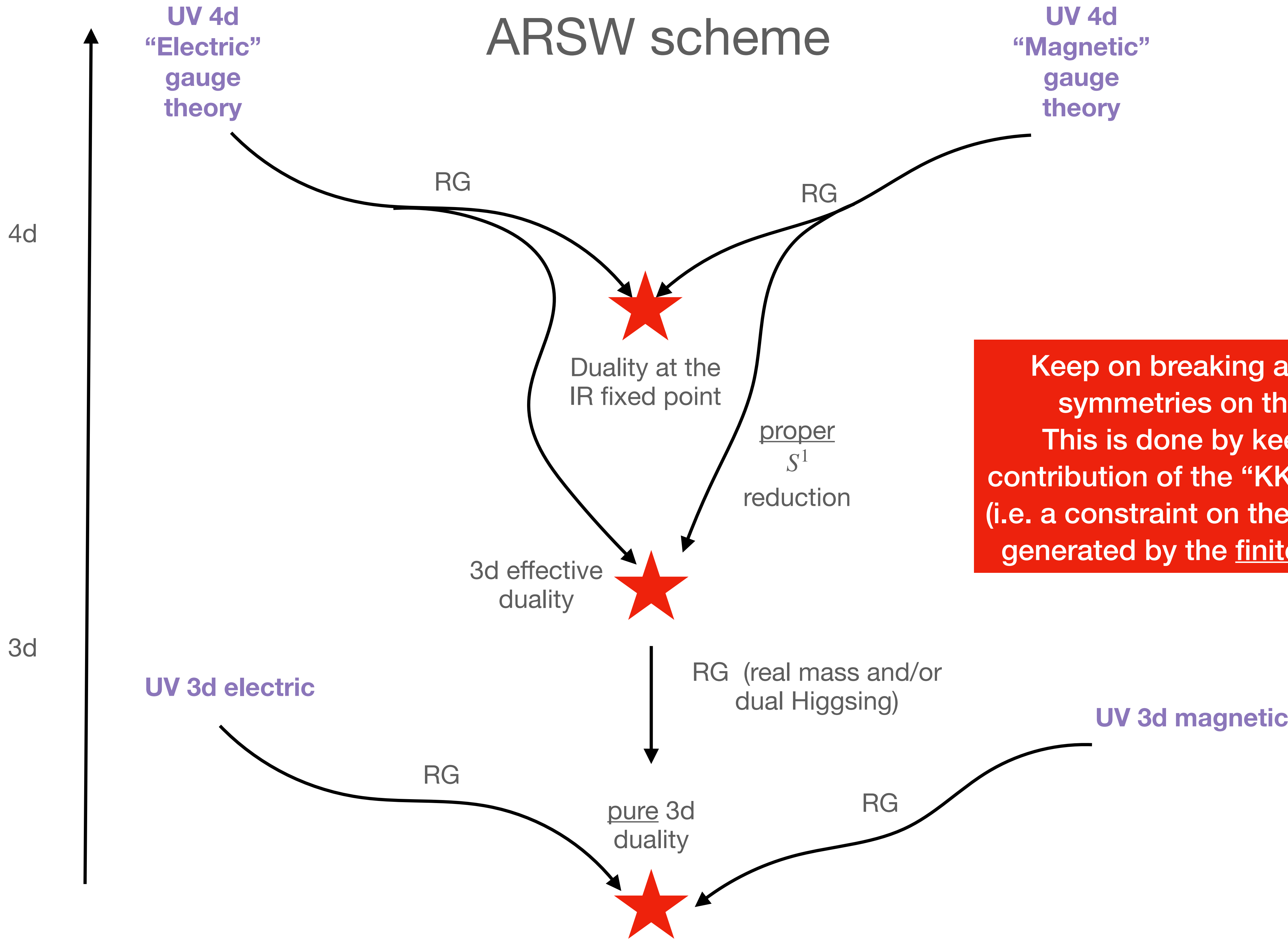
- Parity Anomalies
- Possible axial symmetry (broken by monopoles)
- Interacting U(1) gauge groups in the IR
- Exact R by F-maximization
- SYM and CS action
- $V = (A, \lambda, D, \sigma) \quad w/ \quad \sigma \propto A_4 \quad \& \quad \varphi = d * F$
- $CB : Y \propto Exp(\sigma/g_3^2 + i\varphi)$
- BPS Monopoles: $W_{eff} \propto \sum 1/Y_i$

ARSW scheme



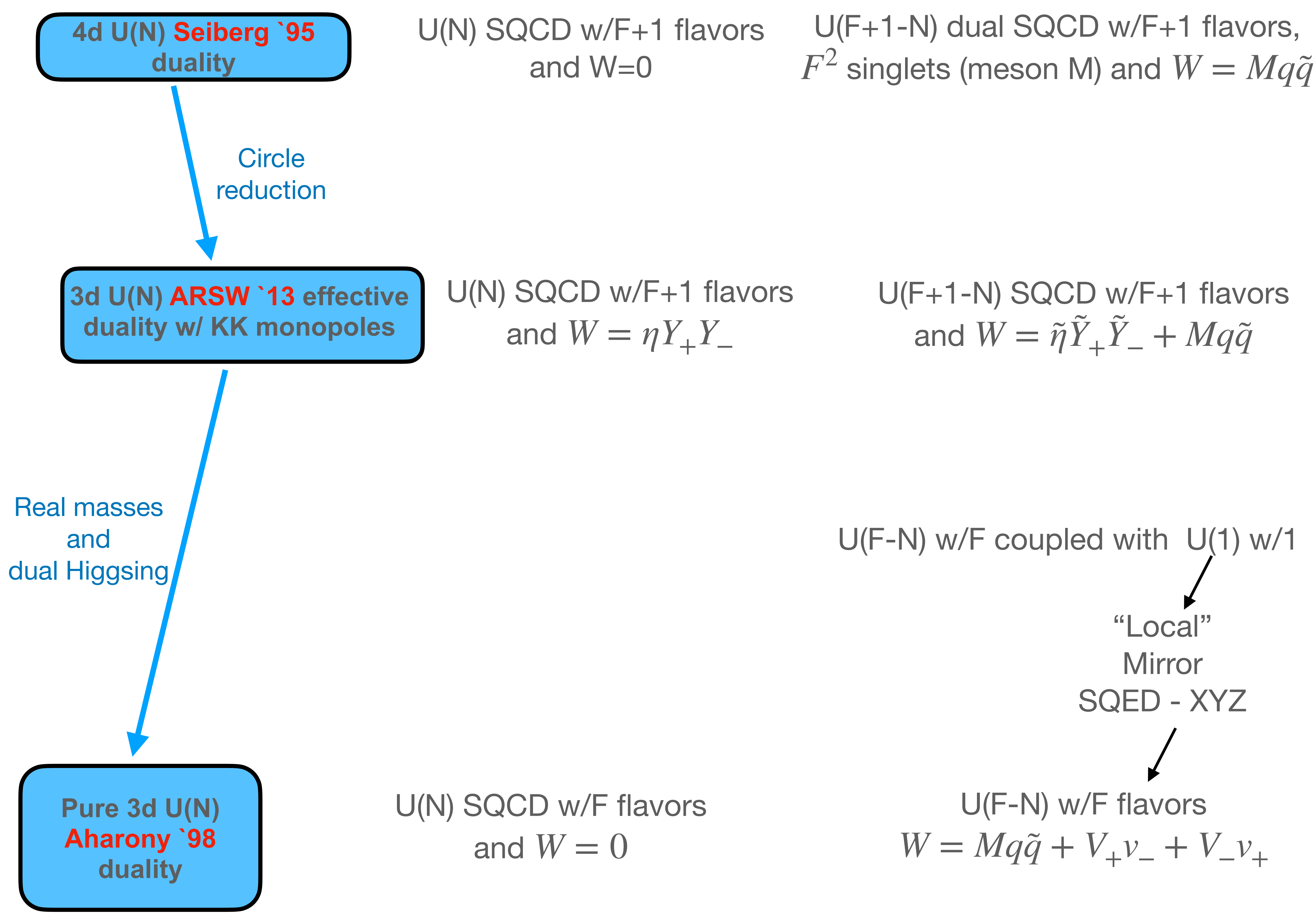
4d Duality not preserved (in general) in 3d by a “naive” dimensional reduction
Reason: extra symmetries anomalous in 4d (e.g. axial in SQCD) can spoil the duality
(e.g. different IR mixing with the R-symmetry in the two phases - check w/ F-max)

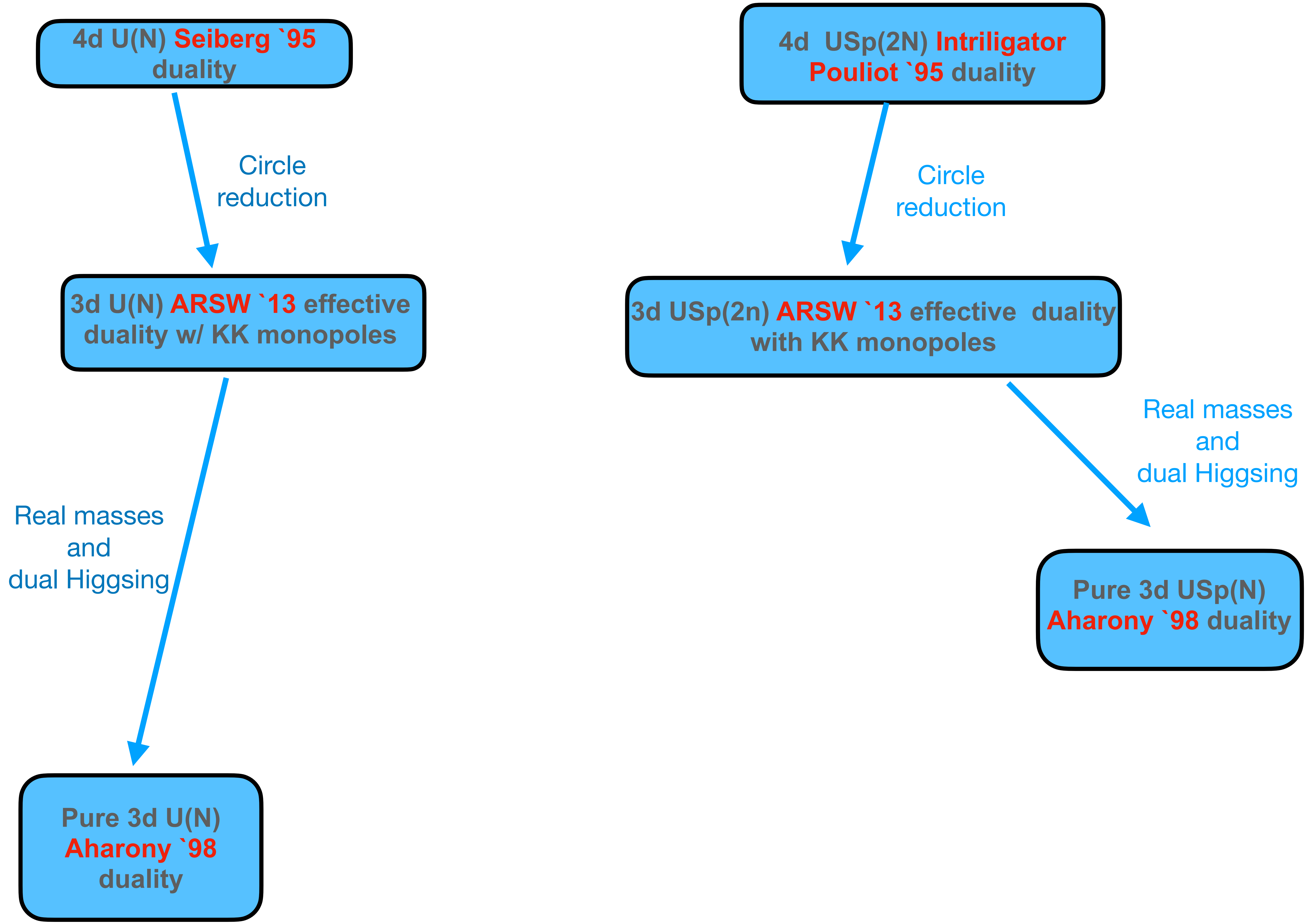
ARSW scheme



Keep on breaking anomalous symmetries on the circle.
This is done by keeping the contribution of the "KK" monopoles (i.e. a constraint on the compact CB generated by the finite size circle)

A bit naive: 3d RG does not necessarily eliminates all the finite size effects.
There can be more sophisticated structures involving the CB.


4d U(N) Seiberg `95 duality
U(N) SQCD w/F+1 flavors and W=0
U(F+1-N) dual SQCD w/F+1 flavors, F^2 singlets (meson M) and $W = Mq\tilde{q}$
Circle reduction
3d U(N) ARSW `13 effective duality w/ KK monopoles
U(N) SQCD w/F+1 flavors and $W = \eta Y_+ Y_-$
U(F+1-N) SQCD w/F+1 flavors and $W = \tilde{\eta}\tilde{Y}_+\tilde{Y}_- + Mq\tilde{q}$
Real masses and dual Higgsing
U(F-N) w/F coupled with U(1) w/1
“Local” Mirror SQED - XYZ
Pure 3d U(N) Aharony `98 duality
U(N) SQCD w/F flavors and $W = 0$
U(F-N) w/F flavors
$W = Mq\tilde{q} + V_+v_- + V_-v_+$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4d USp(2N) Intriligator Pouliot `95 duality
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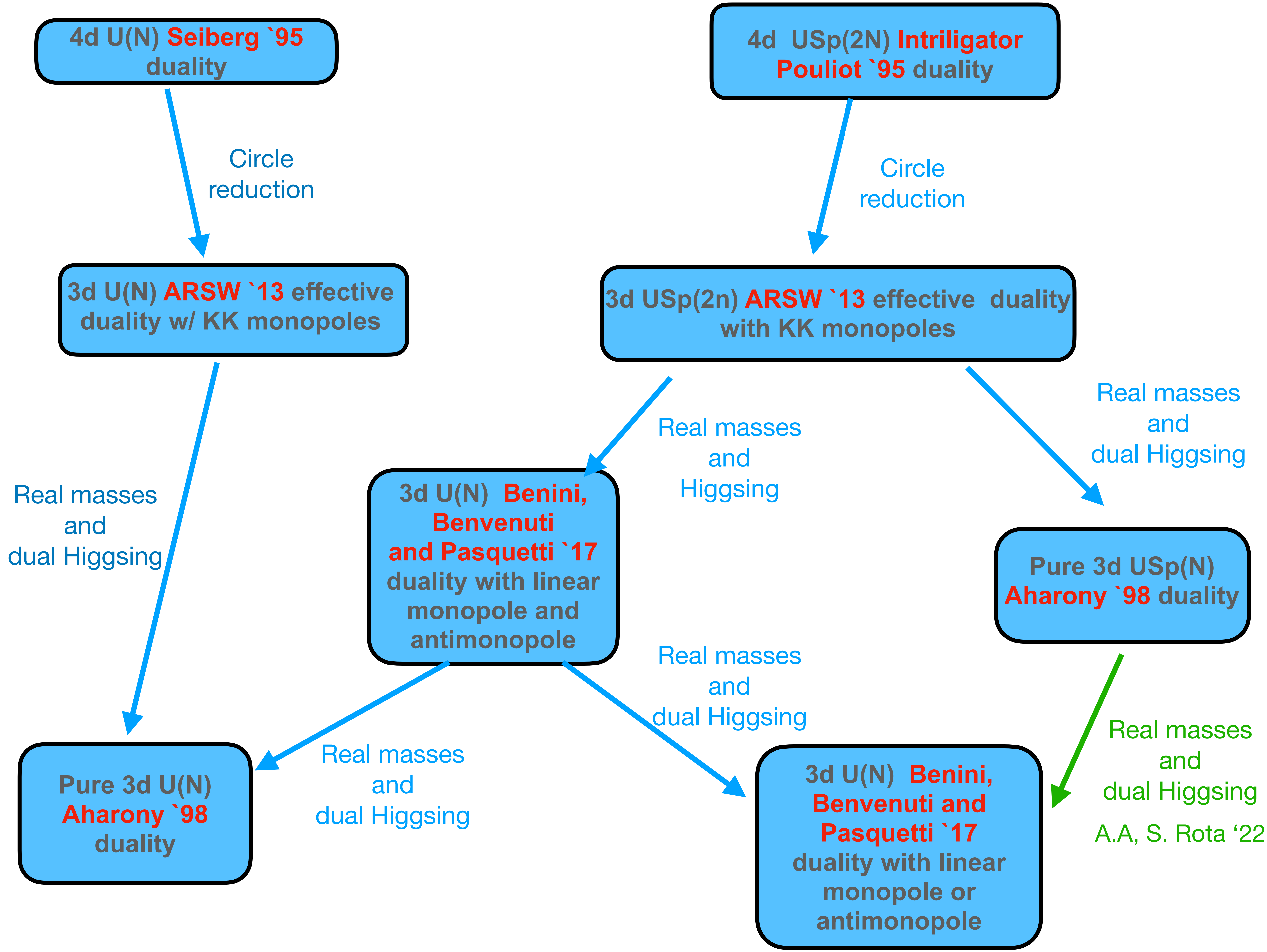
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3d U(N) Benini, Benvenuti and Pasquetti `17 duality with linear monopole and antimonopole
Pure 3d USp(N) Aharony `98 duality
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A.A, S. Rota '22
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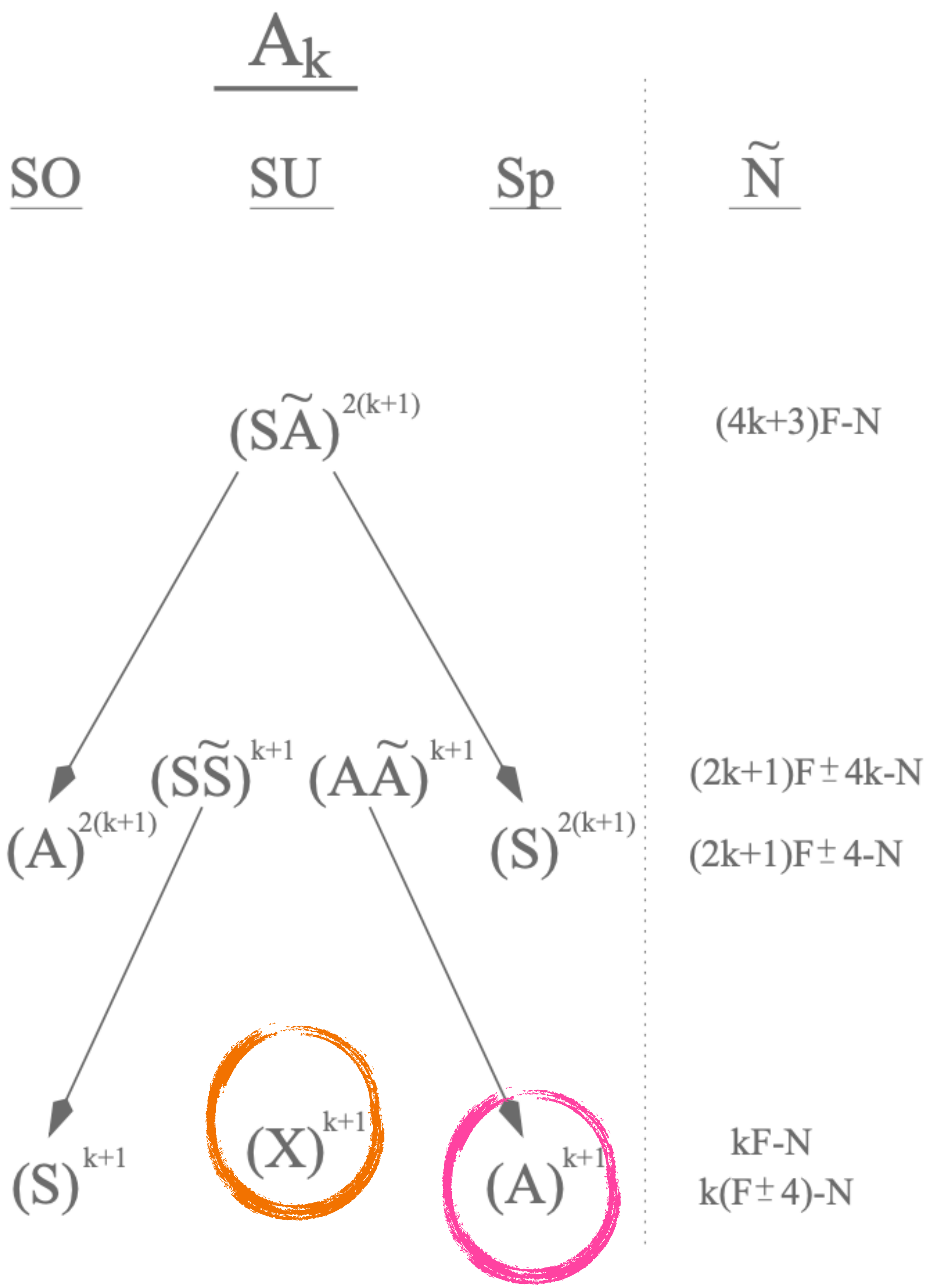
Let's borrow this picture from Brodie-Strassler `96, the orange and pink circles generalize Seiberg `94 and Intriligator-Pouliot `95 for the A_k type superpotential



Kutasov-Schwimmer-Seiberg (KSS) `95

U(N) Adjoint SQCD,
w/ F flavors and
$W = TrX^{k+1}$

U(kF-N) Adjoint SQCD,
w/ F flavors and
$W = Trx^{k+1} + \sum_{j=0}^{k-1} M_j q x^{k-1-j} \tilde{q}$

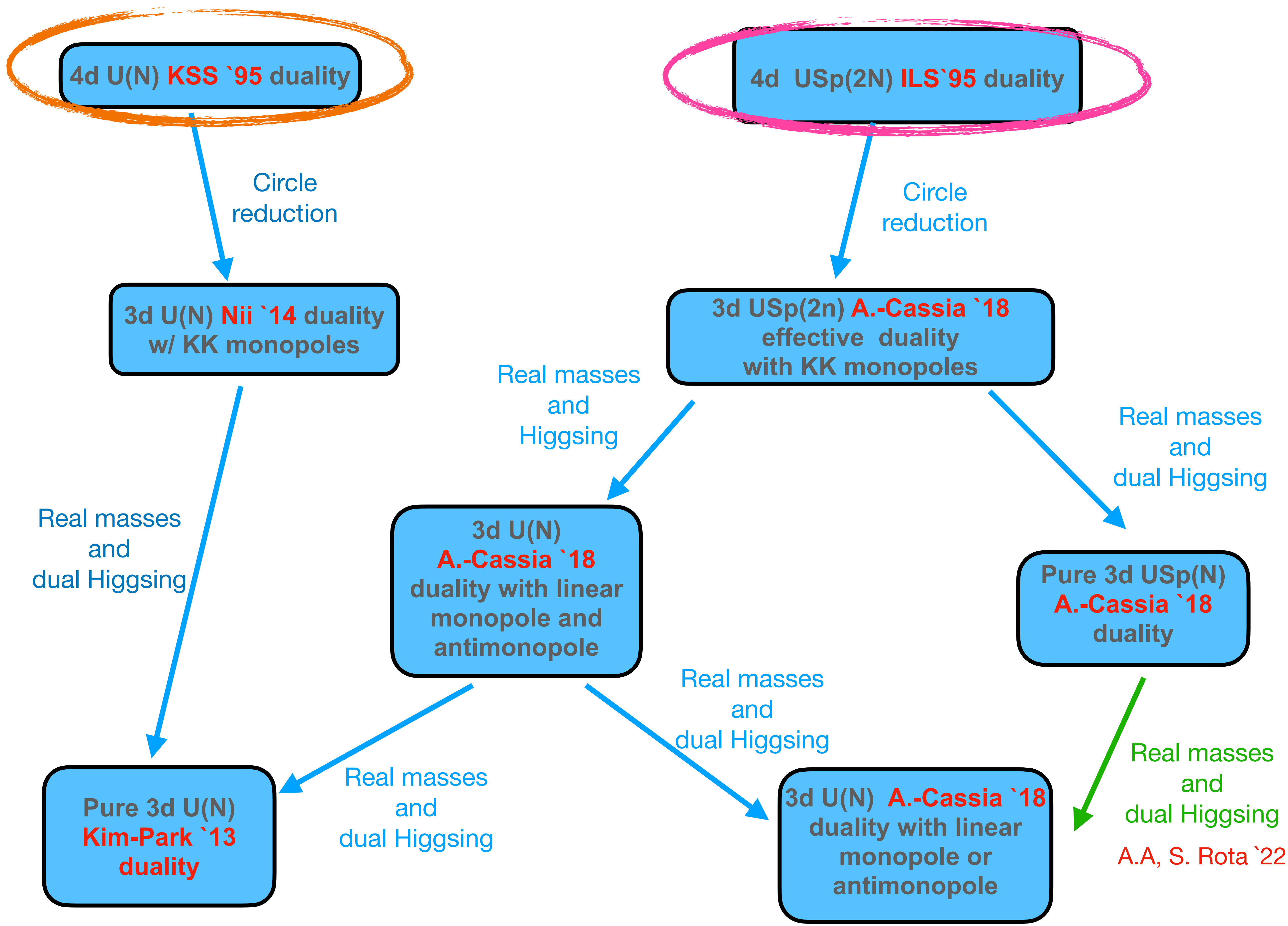
Bifundamental dressed mesons
$M_j = qX^j\tilde{q}$

Intriligator, Leigh-Strassler (ILS) `95

USp(2N) SQCD
w/. A (antisymmetric),
2F flavors and
$W = TrA^{k+1}$

USp(2(k(F-4)-N)) SQCD
w/. a (antisymmetric),
2F flavors and
$W = Tra^{k+1} + \sum_{j=0}^{k-1} M_j q a^{k-1-j} q$

Symmetric dressed mesons
$M_j = qA^jq$



4d U(N) KSS `95 duality
4d USp(2N) ILS`95 duality
Circle reduction
Circle reduction
3d U(N) Nii `14 duality w/ KK monopoles
3d USp(2n) A.-Cassia `18 effective duality with KK monopoles
Real masses and Higgsing
Real masses and dual Higgsing
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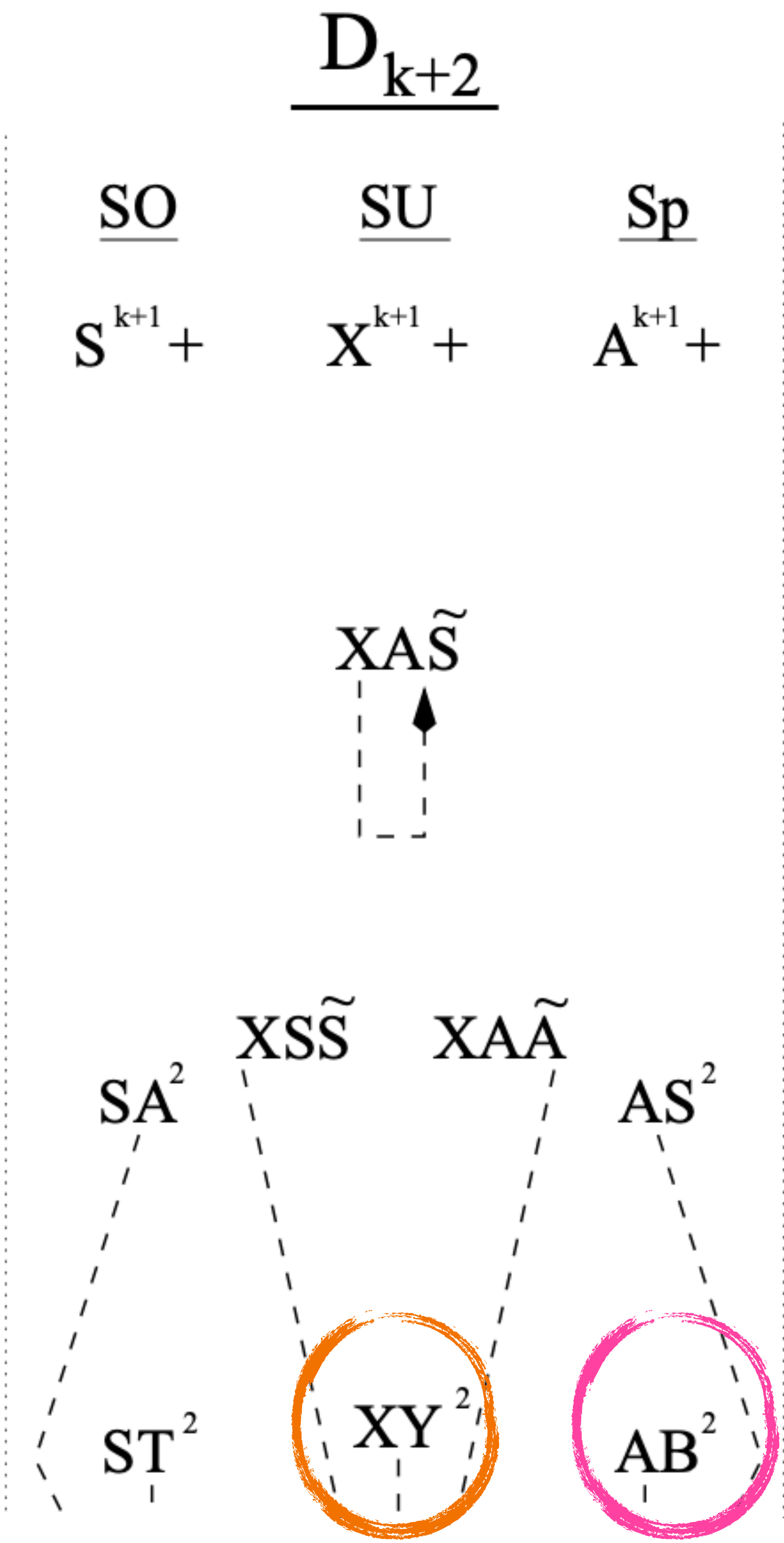
A.A, S. Rota `22



4d $SU(n)$ SQCD with F flavors Q and $\widetilde{Q}$ with two adjoints X and Y interacting through the superpotential

$$W = \mathrm{Tr}\, XY^2 + \mathrm{Tr}\, X^{k+1}$$

with k odd.

4d $SU(\tilde{n} = 3kF - n)$ SQCD with F dual flavors q and $\widetilde{q}$ with two adjoints x and y interacting through the superpotential

$$W = \mathrm{Tr}\, xy^2 + \mathrm{Tr}\, x^{k+1} + \sum_{j=0}^{k-1} \sum_{\ell=0}^{2} \mathrm{Tr}\, \mathcal{M}_{j,\ell}\, q x^{k-1-j} y^{\ell} \widetilde{q}$$

where the singlets $\mathcal{M}_{j,\ell}$ correspond under the duality map to the dressed mesons $QX^jY^\ell\tilde{Q}$ for $j = 0, \ldots, k-1$ and $\ell = 0, 1, 2$ of the electric phase.

$Usp(2n)$ SQCD with $2f$ fundamentals Q and two rank-two antisymmetric tensors A and B interacting through a superpotential

$$W = TrA^{k+1} + TrAB^2$$

$USp(2\tilde{n} = 2(3kf - n - 4k - 2))$ SQCD with $2f$ fundamentals q, two rank-two antisymmetric tensors a and b and the dressed mesons $M_{rs}^{(j,\ell)} = Q_r A^j B^\ell Q_s$ symmetric (antisymmetric) for $j\ell$ odd (even) with $j = 0, \ldots, k-1$ and $\ell = 0, 1, 2$. The superpotential of the dual phase is

$$W = Tra^{k+1} + Trab^2 + \sum_{j=0}^{k-1} \sum_{\ell=0}^{2} M_{j\ell}\, q a^{k-j-1} b^{2-\ell} q$$

where the singlets $\mathcal{M}_{j,\ell}$ correspond under the duality map to the dressed mesons $QX^jY^\ell Q$ for $j = 0, \ldots, k-1$ and $\ell = 0, 1, 2$ of the electric phase. Furthermore k here is required to be odd, otherwise the global anomalies do not match.

	G_{ele}	G_{mag}
SQCD	**SU(N)**	**SU(F-N)**
A_k	**SU(N)**	**SU(kF-N)**
D_{k+2}	**SU(N)**	**SU(3kF-N)**

	G_{ele}	G_{mag}
SQCD	**USp(2N)**	**USp(F-N-4)**
A_k	**USp(N)**	**SU(k(F-4)—N)**
D_{k+2}	**USp(N)**	**SU(3k F-4k-2—N)**

Chiral ring truncation: classical for odd k, quantum (conjectured) in the case of even k

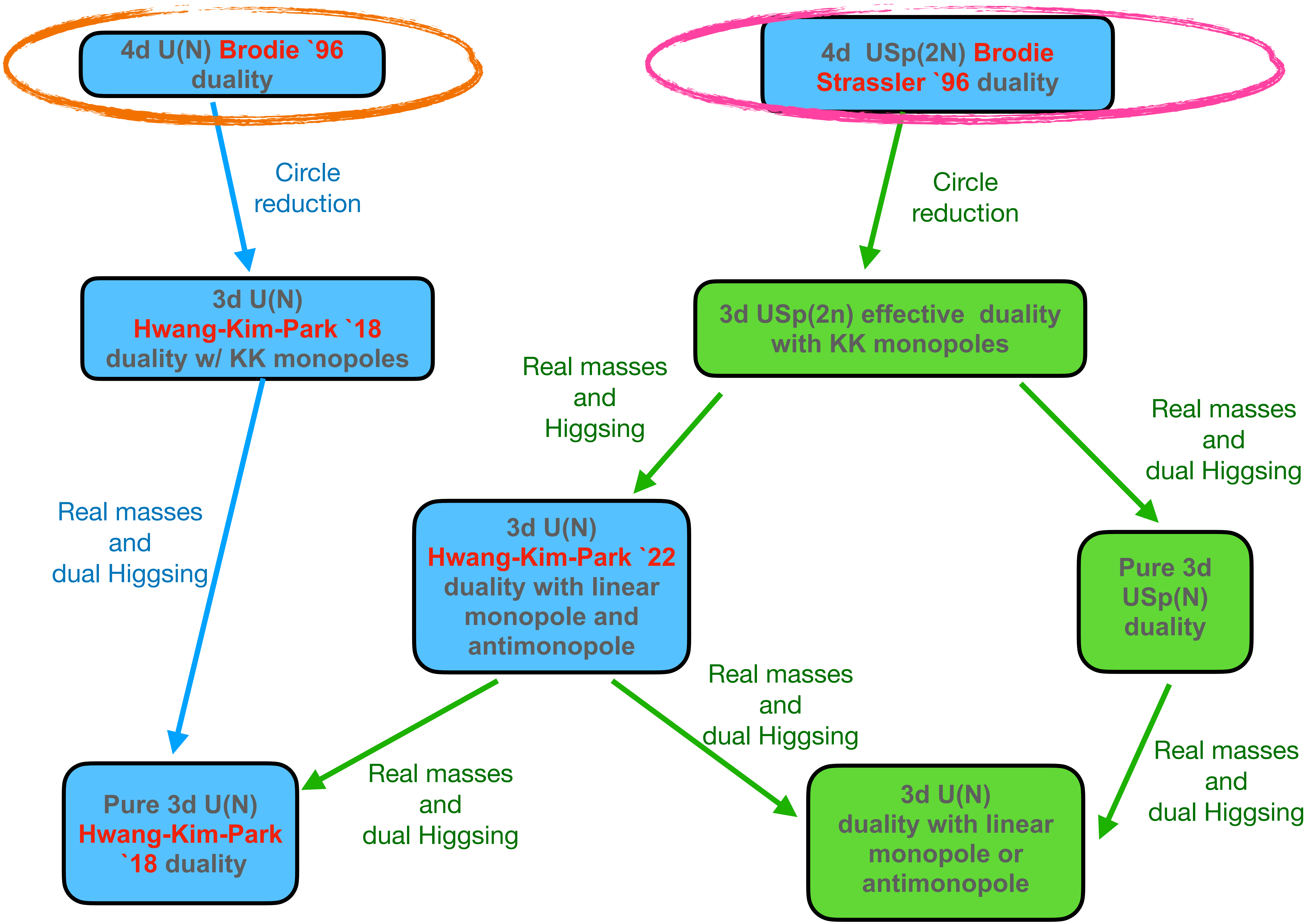
Duality only for even k (mismatch of 'Hooft anomalies for odd k)

$$F_Y = XY + YX = 0$$
$$F_X = Y^2 + X^k = 0$$

Use F_Y

$$YF_X + F_XY = 0 \rightarrow YX^k + X^kY =$$
$$= ((-1)^k + 1)X^kY = -2Y^3$$

NOTE: absence of a type IIA description of the D-type dualities in the Hanany-Witten setup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Hwang-Kim-Park `18:

S^1 reduction of U(n) Brodie duality by breaking the gauge symmetry in SQCD sectors
Aharony like dual W involving also flux-two monopoles $|\pm 1, \pm 1, 0, \ldots, 0\rangle$

Hwang-Kim-Park `22:

New U(n) dualities with linear monopole and anti-monopole W in both phases
4d origin not discussed

A.A.-S.Rota `22:

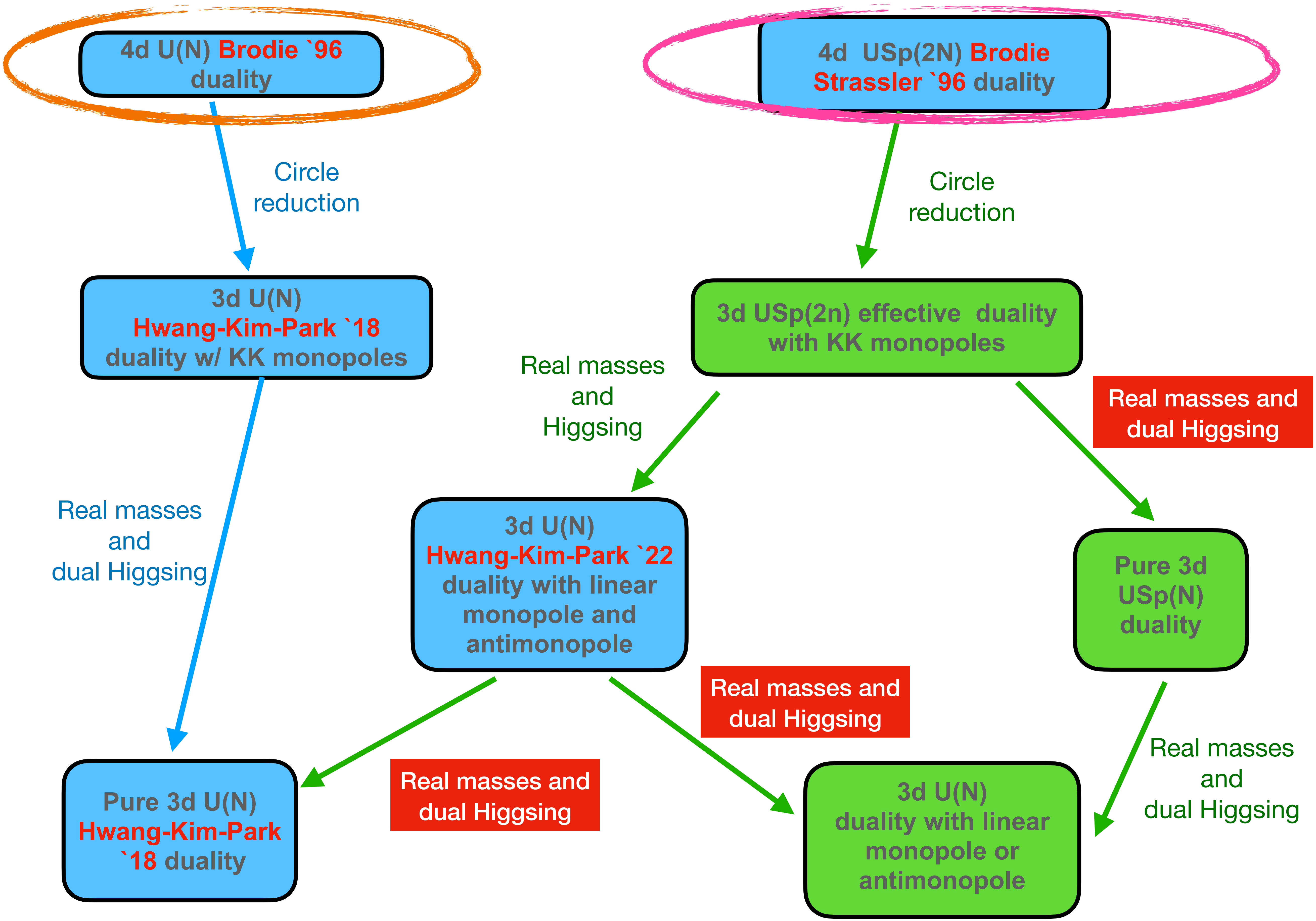
Unified picture, embedding of the various models in the general web

Novelty: non-trivial "topological" sectors, more complicated that in the other cases

Caveat: one **conjecture** necessary (w/o proof, but consistent w/ the whole web)

CONJECTURE: U(k-1) with $W = TrXY^2 + TrX^{k+1} + V_\pm$ is confining

(See Klein `98 for 4d confining limits of Brodie-Strassler `96 dualities)



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Results

	$\text{G}_{\text{electric}}$	$\text{G}_{\text{magnetic}}$	
W_{KK} on S^1 **A.A.,S. Rota `22**	$Usp(2n)$	$USp(2(3kf-n-4k-2))$	
	$U(n)$	$U(3kf-n-4k-2)$	$W_{mon^+}+W_{mon^-}$ **HKP `22**
$W_{mon^\pm}$ **A.A.,S. Rota `22**	$U(n)$	$U(3kf-n-2k-1)$	
	$U(n)$	$U(3kf-n)$	W_{KK} on S^1 **HKP `18**
Aharony-like W **A.A.,S. Rota `22**	$Usp(2n)$	$USp(2(3kf-n-2k-1))$	
	$U(n)$	$U(3kf-n)$	Aharony-like W **HKP `18**

A further check: the three sphere partition function

$$Z_{G;k}(\lambda;\vec{\mu}) = \frac{1}{|W|}\int \prod_{i=1}^{G} \frac{d\sigma_i}{\sqrt{-\omega_1\omega_2}} e^{\frac{k\pi i \sigma_i^2}{\omega_1\omega_2}+\frac{2\pi i\lambda\sigma_i}{\omega_1\omega_2}} \frac{\prod_I \Gamma_h\left(\omega\Delta_I+\rho_I(\sigma)+\widetilde{\rho}_I(\mu)\right)}{\prod_{\alpha\in G_+}\Gamma_h\left(\pm\alpha(\sigma)\right)};$$

$$\Gamma_h(x;\omega_1,\omega_2) \equiv \Gamma_h(x) \equiv e^{\frac{\pi i}{2\omega_1\omega_2}\left((x-\omega)^2-\frac{\omega_1^2+\omega_2^2}{12}\right)} \prod_{j=0}^{\infty} \frac{1-e^{\frac{2\pi i}{\omega_1}(\omega_2-x)}e^{\frac{2\pi i\omega_2 j}{\omega_1}}}{1-e^{-\frac{2\pi i}{\omega_2}x}e^{-\frac{2\pi i\omega_1 j}{\omega_2}}}$$

- Reduction of 4d SCI (or $Z_{S^3\times S^1}$) to Z_{S^3}
- 4d integral identities (not yet proven!!) become 3d identities
- Interpretation: effective 3d dualities on S^1
- Real mass flows & dual Higgsing
  - Cancellation of divergent contributions in the new (IR) identities
  - “Topological” vacua related to new gauge sectors
  - Eliminated by conjecturing an integral identity for U(k-1) w/ 1 flavor & $W = TrXY^2 + TrX^{k+1} + V_\pm$

In A.A.-S.Rota `22: we checked the consistency of the whole web (i.e. flows and dualities) using this strategy

Conclusions

- Prove the confining duality (using field theory methods and/or localization). Mirror symmetry?
- Dualities with linear superpotential for the flux-two monopole(s)
- Generalizations
  - Chern-Simons
  - Chiral matter
  - SU(n)
  - Other rank-two tensors
  - Orthogonal case
- F-maximization
  - Relevancy of the monopole deformations
  - Conformal window
- Brane picture
- Case of even k