Exact $T\bar{T}$ deformation of 2d Yang–Mills theory

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TFI 2022
Venice, June 14

Based on **Phys.Rev.Lett. 128 (2022) 22, 221601** , *[2204.xxxxx]*
in collaboration with L. Griguolo, R. Panerai and D. Seminara

Introduction and motivation: $T\bar{T}$ deformation

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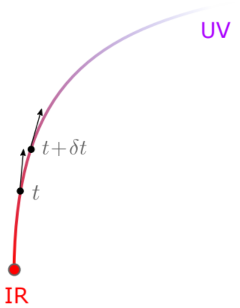

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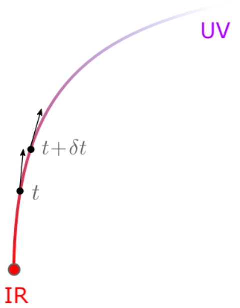

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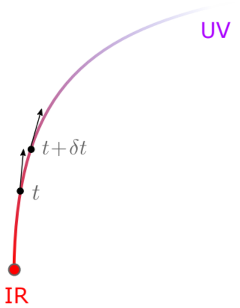

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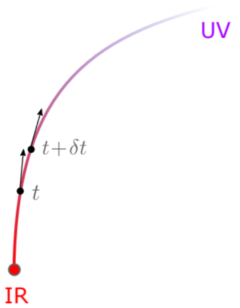

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Two possible signs for the deformation parameter:

- $t > 0$ ("good" sign): the density of states in a deformed CFT interpolates between Cardy $\log \rho \sim \sqrt{E}$ and Hagedorn $\log \rho \sim E$ (nonlocal QFT).

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Despite recent activity on the subject, certain aspects of TT-deformed theories are still enigmatic.

Our goal: investigate a simple example of $T\bar{T}$-deformed QFT_2, YM_2, and see if we can make sense of it in both ranges of the deformation parameter.

The undeformed theory: Yang–Mills in two dimensions

The action of Yang-Mills theory with gauge group $U(N)$ on a two dimensional manifold Σ is

$$S_{\mathrm{YM}_2} = \frac{1}{4g_{\mathrm{YM}^2}} \int_\Sigma d^2x \, \sqrt{h}\,\mathrm{Tr}\left(F^{ab}F_{ab}\right) \tag{2}$$

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The partition function localizes on a sum over irreps, [Migdal '75; Witten '92; Rusakov '90]

$$Z(\alpha) = \sum_R (\dim R)^{2-2g} \, e^{-\frac{\alpha}{2N} C_2(R)} \tag{4}$$

where $c_2(R)$ and $\dim R$ are the quadratic Casimir and the dimension of R.

$T\bar{T}$-deformed Yang–Mills: a first step

In the first order formalism, we expect the following form of the deformed action S_τ

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Similarly, one can compute the $T\bar{T}$-deformed Lagrangian

$$\mathcal{L}_\tau = \frac{3}{8\tau}\left[{}_3F_2\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{4};\frac{1}{3},\frac{2}{3};\frac{256}{27}2\tau\mathcal{L}_0\right)-1\right] \;. \tag{9}$$

which holds for the Abelian $U(1)$ case. [Conti, Iannella, Negro, Tateo '18]

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For $\tau < 0$

- we expect instanton-like corrections in the deformation parameter to cure the divergence of $Z(\alpha,\tau)$
- not clear how such nonperturbative contributions could arise and be unambiguously fixed

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Via Poisson resummation, we can find its dual representation

$$Z(\alpha;0) = \sum_{\mathfrak{m}=-\infty}^{\infty} \sqrt{\frac{2\pi}{\alpha}}\, e^{-2\pi^2\mathfrak{m}^2/\alpha} \tag{12}$$

where

$$S_{\text{class}}(\mathfrak{m}) = \frac{2\pi^2\mathfrak{m}^2}{\alpha} \qquad \frac{1}{2\pi}\int_\Sigma d^2x\sqrt{h}\, F_{12} = \mathfrak{m} \in \mathbb{Z} \tag{13}$$

is the classical instanton action for configurations of quantized magnetic flux $\mathfrak{m}$.

Our strategy

Consider $Z(\alpha, \tau)$ as a formal expression obeying the **Flow equation**:

$$\mathrm{Flow}_{\alpha,\tau} Z(\alpha, \tau) = \left[\frac{\partial}{\partial \tau} + 2\alpha \frac{\partial^2}{\partial \alpha^2}\right] Z(\alpha, \tau) = 0 \tag{14}$$

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$$\mathrm{Flow}_{\alpha,\tau} z_\mathfrak{m}(\alpha,\tau) = 0 \qquad z_\mathfrak{m}(\alpha, 0) = \sqrt{\frac{2\pi}{\alpha}}\, e^{-2\pi^2\mathfrak{m}^2/\alpha} \tag{16}$$

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3. we make sure that the sum over deformed instantons is convergent and perform it

$$Z(\alpha,\tau) = \sum_\mathfrak{m} z_\mathfrak{m}(\alpha,\tau) \tag{17}$$

General solution and boundary conditions

The flow equation can be solved by separation of variables. The generic solution for the $\mathfrak{m}$ flux sector is a linear combination

$$z_{\mathfrak{m}}(\alpha, \tau) = \sum_{s \in \Gamma} \frac{\tau^{-s}}{s!} \left[p_s U\left(\frac{1}{2} + s, 0, \frac{\alpha}{2\tau}\right) + q_s \frac{\alpha}{2\tau} {}_1F_1\left(\frac{3}{2}s + 1, 2, \frac{\alpha}{2\tau}\right)\right] \tag{18}$$

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where the coefficients p_s, q_s and the set Γ are determined by imposing the following conditions:

- the solution must be real and finite

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where the coefficients p_s, q_s and the set Γ are determined by imposing the following conditions:

- the solution must be real and finite
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General solution and boundary conditions

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- $z_{\mathfrak{m}}(\alpha,\tau)$ must reproduce the semiclassical limit when $\alpha \to 0$ and $\sigma = \frac{4\pi^2\tau}{\alpha^2}$

$$-\log\left(z_{\mathfrak{m}}(\alpha,\tau)\right) \simeq \frac{3\pi^2}{2\alpha\sigma}\left[{}_3F_2\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{4};\frac{1}{3},\frac{2}{3};\frac{256}{27}\mathfrak{m}^2\sigma\right)-1\right] = S_{\text{cl}}(\mathfrak{m},\sigma) \tag{19}$$

  i.e. the deformed action evaluated on the classical solution associated to a monopole charge $\mathfrak{m}$.

$\tau > 0$

The correct choice is

$$z_{\mathfrak{m}}(\alpha,\tau) = \sqrt{\frac{\pi}{\tau}} \sum_{s=0}^{\infty} \frac{1}{s!} \left(-\frac{\pi^2 \mathfrak{m}^2}{\tau}\right)^s U\left(s+\frac{1}{2}, 0, \frac{\alpha}{2\tau}\right) \tag{20}$$

We perform the sum inside the integral representation of the Tricomi U function and we get:

$$z_{\mathfrak{m}}(\alpha,\tau) = \int_{-\infty}^{+\infty} dy\ e^{2\pi i \mathfrak{m} y} \phi(y) \qquad \phi(y) = e^{-\frac{\alpha y^2}{2\left(1-\tau y^2\right)}} \Theta\left(1-y^2\tau\right) \tag{21}$$

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Now, the sum of $e^{2\pi i \mathfrak{m} x}$ over $\mathfrak{m}$ simply yields the Dirac comb of period 1.
$\Longrightarrow$This localizes the integral over the contributions of integers n less or equal than the threshold imposed by the Θ function.

Result for $\tau > 0$

The deformed partition function for $\tau > 0$ is

$$Z(\alpha, \tau)_{U(1)} = \sum_{\ell=-\lfloor \frac{1}{\sqrt{\tau}} \rfloor}^{\lfloor \frac{1}{\sqrt{\tau}} \rfloor} e^{-\frac{\alpha}{2} \frac{\ell^2}{1-\tau\ell^2}} \tag{22}$$

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- The final result is similar to what one would write by using the naive prescription but the deformed spectrum has a cutoff.
- The deformation acts on the spectrum by "inflating" it and only a finite number of energy levels survive. All energy levels above such threshold drop out of the spectrum.
- When $\tau > 1$, almost nothing is left: the entire spectrum consists of the sole ground state and the partition function becomes trivial: $Z = 1$.

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Gluing and phase transitions

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We can easily generalize our result to an arbitrary topology with b boundaries:

$$Z_b(\alpha, \tau, \theta_1, \ldots, \theta_b) = \sum_{\ell=-\lfloor \frac{1}{\sqrt{\tau}} \rfloor}^{\lfloor \frac{1}{\sqrt{\tau}} \rfloor} \mathrm{e}^{-\frac{\alpha \ell^2}{2(1-\ell^2\tau)} + \mathrm{i}(\theta_1 \ell + \ldots + \theta_b \ell)} \,. \tag{23}$$

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where the θ's parametrize the boundary holonomies.

A correlator of two Polyakov loops is computed by gluing two cylinders together

$$\langle W_{n_1} W_{n_2} \rangle = \int_0^{2\pi} \frac{\mathrm{d}\theta_1}{2\pi} \int_0^{2\pi} \frac{\mathrm{d}\theta_2}{2\pi}\, \mathrm{e}^{\mathrm{i}(\theta_1 n_1 + \theta_2 n_2)}\, \frac{Z_2(\alpha_1, \tau, \theta_1, \theta_2)\, \overline{Z_2(\alpha_2, \tau, \theta_1, \theta_2)}}{Z(\alpha_1 + \alpha_2, \tau)} . \tag{24}$$

where n_1, $n_2 \in \mathbb{Z}$ label the $U(1)$ representations of the Wilson loops at the interface.

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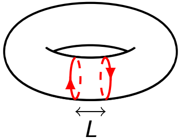

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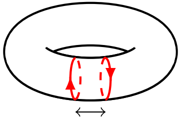

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In the decompactification limit, where $\alpha = \alpha_1 + \alpha_2 \to \infty$ while $\alpha_2 = e^2 L\beta$ is fixed.

$$\langle W_n \, W_{-n} \rangle \sim \mathrm{e}^{-e^2 L\beta \frac{n^2}{2(1-\tau n^2)}} \, \Theta(1 - \tau n^2) \, . \tag{25}$$

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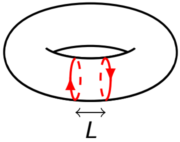

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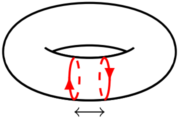

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- $\boldsymbol{\tau} > \boldsymbol{n}^{-2}$: the charged particles cannot exist as individual excitations, seemingly decoupling from the theory.

$\tau < 0$

The correct linear combination for $\tau < 0$ is instead

$$z_{\mathfrak{m}}\left(\alpha, \tau\right) = \frac{\pi\alpha}{2(-\tau)^{3/2}} \sum_{s=0}^{\infty} \frac{1}{s!} \left(-\frac{4\pi^2 \mathfrak{m}^2}{\tau}\right)^{s/2} {}_1F_1\left(\frac{s}{2} + \frac{3}{2}; 2; \frac{\alpha}{2\tau}\right) \tag{26}$$

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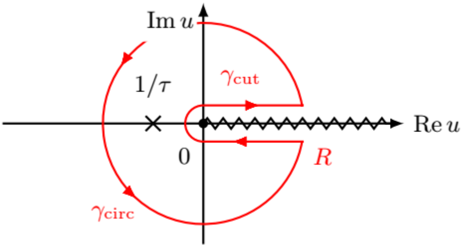
The integration over γ_{circle} vanishes in the limit of large R.

Instead, the contribution of γ_{cut} is evaluated by taking the discontinuity of the integrand across the cut of the square root.



Result for $\tau < 0$

Upon integrating by parts and setting $u = y^2$, we find

$$z_{\mathfrak{m}}(\alpha, \tau) = 2 \int_0^{+\infty} dy \left(e^{-\frac{\alpha}{2} \frac{y^2}{1-\tau y^2}} - e^{\frac{\alpha}{2\tau}} \right) \cos(2\pi \mathfrak{m} y) \tag{28}$$

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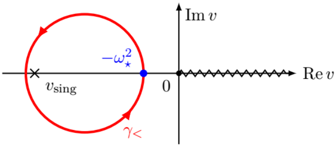
- The new term ensures the convergence of the above, since it precisely matches the asymptotic value of the upper part of the spectrum.



We can reorganize the contour integral representation for $z_{\mathfrak{m}}$ in this way

$$z_{\mathfrak{m}}\left(\alpha,\tau\right) \simeq \frac{i\pi\mathfrak{m}}{\alpha} \oint \frac{dv}{v} \sqrt{-v}\; e^{-\frac{4\pi^2\mathfrak{m}^2}{\alpha}\chi(v)} \tag{30}$$

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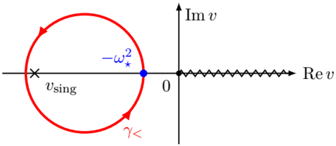

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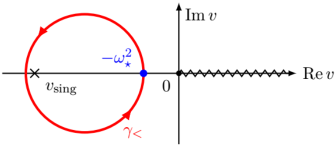

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For $\tau > 0$:

- we have dinamically proven that the spectrum of the theory undergoes a drastic reduction with only a finite number of states surviving.
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- generalize this construction to theories with matter.

Thanks for the attention!