Black hole perturbations from Liouville correlators

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Outline

- Introduction
- A brief introduction to Liouville CFT
- 3 punctured sphere: the hypergeometric equation
- 4 punctured sphere: the Heun equation
- AdS_5 black holes perturbations and holography

This talk is based on

- G. Bonelli, C.I., D. Panea Lichtig and A. Tanzini. "Irregular Liouville correlators and connection formulae for Heun functions". In: (Jan. 2022). arXiv: 2201.04491 [hep-th]
- M. Dodelson, A. Grassi, C.I., Lichtig, D. Panea Lichtig and A. Zhiboedov. In: (2022). arXiv: 2206.xxxxx [hep-th]

Introduction

Solving for black holes perturbations means solving the PDE

$$(\Box - m^2)\Phi = 0$$

If $\Box$ is separable this reduces to a 2nd order ODE.

Imposing boundary conditions:

$$\begin{aligned}\Phi = \Phi_{in}^{h} &= (r - R_+)^{\#}(1 + \mathcal{O}(r - R_+)) = \\ &= \mathcal{A}\Phi_1^{\infty} + \mathcal{B}\Phi_2^{\infty}\,.\end{aligned}$$

$\mathcal{A}$ and $\mathcal{B}$ are the so called connection coefficients: they analytically continue the local solution near the horizon to the local solution at infinity.

Introduction

A lot of physics is contained in $\mathcal{A}$ and $\mathcal{B}$:

- quasinormal modes ($\mathcal{B}(\omega_n) = 0$),
- response functions ($\sim \mathcal{A}/\mathcal{B}$),
- Love numbers,
- ...

To compute the connection coefficients of an ODE is a hard problem, that has surprising relations with 2d CFT (and 4d gauge theories).

This connection will allow us to find $\mathcal{A}$ and $\mathcal{B}$ in terms of CFT data.

To see how the story goes, let us take a step back.

A brief introduction to Liouville CFT

- It is an interacting CFT with coupling b,
- central charge $c = 1 + 6Q^2$ with $Q = b + b^{-1}$,
- diagonal and continuous spectrum of primaries of weight $\Delta_\alpha = \frac{Q^2}{4} - \alpha^2$ with $\alpha \in i\mathbb{R}$,
- degenerate Verma modules for $2\alpha_{m,n} = -mb - nb^{-1}$,
- the 3 point functions are known explicitly (DOZZ formulas) in terms of special functions Υ ($\sim$ double gamma functions).

A brief introduction to Liouville CFT

Product of operators can be series expanded (OPE):

$$V_{\alpha_t}(t)V_{\alpha_0}(0) = \int_{i\mathbb{R}} d\alpha\, C^{\alpha}_{\alpha_t,\alpha_0} t^{\Delta-\Delta_1-\Delta_0} \left(V_\alpha(0) + c_1 t L_{-1} V_\alpha(0) + \mathcal{O}(t^2)\right)$$

Inserting the OPE in a correlator we find the conformal block (CB) expansion

$$\langle V_{\alpha_\infty}(\infty)V_{\alpha_1}(1)V_{\alpha_t}(t)V_{\alpha_0}(0)\rangle =$$
$$= \int_{i\mathbb{R}} d\alpha\, C_{\alpha_\infty,\alpha_1,\alpha} C^{\alpha}_{\alpha_t,\alpha_0} \left| t^{\Delta-\Delta_1-\Delta_0}(1+\mathcal{O}(t))\right|^2 .$$

Different expansions (e.g. $t \sim 0$ and $t \sim 1$) of the correlators agree (**crossing symmetry**).

A brief introduction to Liouville CFT

Null vectors decouple from correlators.

$$\langle \prod_{i=1}^{n} V_{\alpha_i}(z_i) \left(b^{-2} L_{-1}^2(z) + L_{-2}(z) \right) V_{\alpha_{2,1}}(z) \rangle = 0 \,.$$

This turns into a PDE for $\langle \prod_{i=1}^{n} V_{\alpha_i}(z_i) V_{\alpha_{2,1}}(z) \rangle$ (BPZ equation).
In the semiclassical limit

$$b \to 0, \;\; b\alpha_i = a_i \text{ fixed}$$

it reduces to a 2nd order ODE with $(z - z_i)^{-2}$ singularities.
OPE with $V_{\alpha_{2,1}}$ is easy:

$$V_{\alpha_{2,1}}(z) V_{\alpha_i}(z_i) = \sum_{\pm} C_{\alpha_{2,1},\alpha_i}^{\alpha_i \pm \frac{b}{2}} (z - z_i)^{\frac{bQ}{2} \mp \alpha_i} V_{\alpha_{i\mp}}(0) + \mathcal{O}\left((z - z_i)^{\frac{bQ}{2} \mp \alpha_i + 1} \right)$$

and provides local expansions of the solution of the ODE.

A brief introduction to Liouville CFT

Crossing symmetry for degenerate correlators requires

$$\langle\prod_{i=1}^{n} V_{\alpha_i}(z_i) V_{\alpha_{2,1}}(z)\rangle \sim$$

$$\sim \text{(DOZZ factors)} \times \left|\sum_{\pm}(z - z_1)^{\frac{bQ}{2} \pm b\alpha_1} + \dots\right|^2$$

$$\sim \text{(DOZZ factors)} \times \left|\sum_{\pm}(z - z_2)^{\frac{bQ}{2} \pm b\alpha_2} + \dots\right|^2$$

Liouville CFT analytically continues the series from z_1 to z_2.

Can we exploit crossing symmetry constraints to solve connection problems?

A brief introduction to Liouville CFT

The takeaway message is that the semiclassical Liouville correlator

$$\langle \prod_{i=1}^{n} V_{\alpha_i}(z_i) V_{\alpha_{2,1}}(z) \rangle_{sc}$$

solves an ODE with n regular singularities at z_i.

OPE provides a local expansion around the z_i with leading exponent α_i.

DOZZ coefficients make sure that these expansions glue (connection formulas?).

Remark:

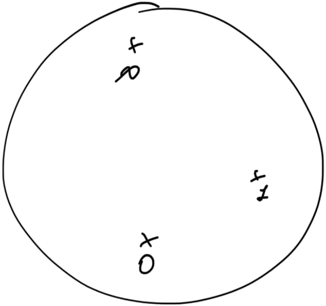
These correlators are (**AGT**) dual to 4d $\mathcal{N} = 2$ partition functions. This provides an explicit combinatorial expression.

3 punctured sphere: the hypergeometric equation

The BPZ equation of

$$\langle V_{\alpha_\infty}(\infty) V_{\alpha_1}(1) V_{\alpha_{2,1}}(z) V_{\alpha_0}(0) \rangle$$

is the **hypergeometric equation**.



Hypergeometrics defined as

$$ {}_2F_1(a, b; c, z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \, . $$

Local solutions near 0, 1 are

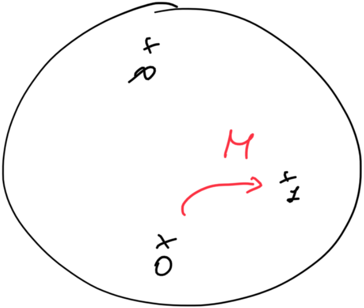
$$ z^{\frac{bQ}{2} \pm \alpha_0} (1-z)^{\frac{bQ}{2} + \alpha_1} {}_2F_1 \left(\frac{1}{2} + b\,(\alpha_1 \pm \alpha_0 + \alpha_\infty) \, , \frac{1}{2} + b\,(\alpha_1 \pm \alpha_0 - \alpha_\infty) \, ; 1 \pm 2b\alpha_0, z \right) \, , $$

$$ z^{\frac{bQ}{2} + \alpha_0} (1-z)^{\frac{bQ}{2} \pm \alpha_1} {}_2F_1 \left(\frac{1}{2} + b\,(\alpha_0 \pm \alpha_1 + \alpha_\infty) \, , \frac{1}{2} + b\,(\alpha_0 \pm \alpha_1 - \alpha_\infty) \, ; 1 \pm 2b\alpha_1, 1 - z \right) $$

The 3 punctured sphere

The series converges up the next insertion (singularity), but it can be analytically continued:

$$
\begin{aligned}
&{}_2F_1(a,b;c,z) = \\
&= M_{++}\,{}_2F_1(\ldots,1-z) + \\
&+ M_{+-}\,(1-z)^{c-a-b}{}_2F_1(\ldots,1-z)\,,
\end{aligned}
$$



where

$$
M_{++} = \frac{\Gamma(c)\,\Gamma(c-a-b)}{\Gamma(c-a)\,\Gamma(c-b)},\quad M_{+-} = \frac{\Gamma(c)\,\Gamma(a+b-c)}{\Gamma(a)\,\Gamma(b)}
$$

The 3 punctured sphere

M_{ab} are the **connection coefficients** ($\sim \mathcal{A}$, $\mathcal{B}$) for hypergeometric eqn.

They interplay with the 3 point functions to ensure **crossing symmetry**:

$$\langle V_{\alpha_\infty}(\infty) V_{\alpha_1}(1) V_{\alpha_{2,1}}(z) V_{\alpha_0}(0)\rangle =$$

$$= \sum_{\pm} C_{\alpha_\infty,\alpha_1,\alpha_{0\pm}} C^{\alpha_{0\pm}}_{\alpha_{2,1},\alpha_0} \left| z^{\frac{bQ}{2}\pm\alpha_0}(1-z)^{\frac{bQ}{2}+\alpha_1} {}_2F_1(\dots,z)\right|^2 =$$

$$= \sum_{\pm} C_{\alpha_\infty,\alpha_1,\alpha_{0\pm}} C^{\alpha_{0\pm}}_{\alpha_{2,1},\alpha_0} \left| \sum_{\pm'} M_{\pm\pm'}(1-z)^{\frac{bQ}{2}\pm'\alpha_1} z^{\frac{bQ}{2}+\alpha_0} {}_2F_1(\dots,1-z)\right|^2 =$$

$$= \sum_{\pm} C_{\alpha_\infty,\alpha_0,\alpha_{1\pm}} C^{\alpha_{1\pm}}_{\alpha_{2,1},\alpha_1} \left| (1-z)^{\frac{bQ}{2}\pm\alpha_1} z^{\frac{bQ}{2}+\alpha_0} {}_2F_1(\dots,1-z)\right|^2 .$$

The 3 punctured sphere

The requirement of crossing symmetry puts strong constraints on the $M_{\pm\pm'}$.

In fact, from the C's one can compute the $M_{\pm\pm'}$ (and viceversa).

connection formulas for the CBs $\sim$ crossing symmetry

The C's ensure crossing symmetry of correlators with arbitrarily many insertions, so we can play the same game with more complicated correlators to obtain connection coefficients for more complicated ODEs.

4 punctured sphere: the Heun equation

We now add an insertion

$$\Psi(z,t) = \langle V_{\alpha_\infty}(\infty) V_{\alpha_1}(1) V_{\alpha_{2,1}}(z) V_{\alpha_t}(t) V_{\alpha_0}(0)\rangle$$

It satisfies the BPZ equation

$$\Big(\partial_z^2 + b^2 \frac{\Delta_1}{(z-1)^2} - b^2 \frac{\Delta_1 + t\partial_t + \Delta_t + z\partial_z + \Delta_{2,1} + \Delta_0 - \Delta_\infty}{z(z-1)} + \\ + b^2 \frac{\Delta_t}{(z-t)^2} + b^2 \frac{t}{z(z-t)}\partial_t - b^2 \frac{1}{z}\partial_z + b^2 \frac{\Delta_0}{z^2}\Big)\Psi = 0\,.$$

This is a PDE, but (since $b^2\Delta_{2,1} = \mathcal{O}(b^2)$, while $b^2\Delta_i = \mathcal{O}(1)$)

$$b^2 t\partial_t \Psi(z,t) = t\partial_t b^2 \langle V_{\alpha_\infty}(\infty) V_{\alpha_1}(1) V_{\alpha_t}(t) V_{\alpha_0}(0)\rangle + \mathcal{O}(b^2)\,.$$

The 4 punctured sphere

To evaluate $t\partial_t$ we expand in CB for $t \sim 0$ (WLOG) and select an intermediate momentum α

$$\begin{aligned} u :=& t\partial_t b^2 \langle V_{\alpha_\infty}(\infty) V_{\alpha_1}(1) \Pi_\alpha V_{\alpha_t}(t) V_{\alpha_0}(0) \rangle + \mathcal{O}(b^2) = \\ =& b^2 \left(\Delta_\alpha - \Delta_t - \Delta_0 \right) + \mathcal{O}(t) + \mathcal{O}(b^2) \,. \end{aligned}$$

This gives $u(a = b\alpha)$ (Matone relation).

As $b \to 0$ the BPZ equation reduces to an ODE with 4 regular singularities at $0, t, 1, \infty$ (**Heun equation**).

u appears as a parameter that in the Heun literature is known as the *accessory parameter*.

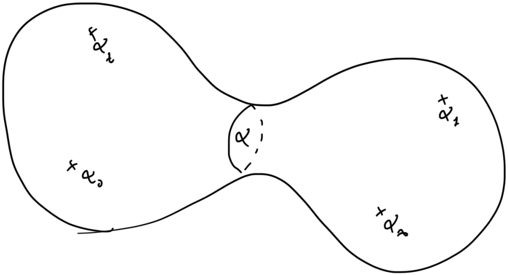
The 4 punctured sphere

The solution of the ODE is given by the $b \to 0$ limit of the CBs of

$$\Psi_\Delta = \langle V_{\alpha_\infty}(\infty) V_{\alpha_1}(1) V_{\alpha_{2,1}}(z) \Pi_\Delta V_{\alpha_t}(t) V_{\alpha_0}(0) \rangle$$

Ψ_Δ can be series expanded doing OPEs

$$V_{\alpha_{2,1}}(z) V_{\alpha_i}(z_i) \propto z^{\frac{bQ}{2} \pm \alpha_i}, \quad V_{\alpha_{2,1}}(z) \Pi_\Delta \propto z^{\frac{bQ}{2} \pm \alpha}$$



The 4 punctured sphere

Ψ_Δ is crossing symmetric:

$$\Psi_\Delta = \sum_{\pm} C_{\alpha_\infty,\alpha_1,\alpha} C^{\alpha}_{\alpha_t,\alpha_{0\pm}} C^{\alpha_{0\pm}}_{\alpha_{2,1},\alpha_0} \left|\mathfrak{F}^{(0)}_{\pm}(t,z)\right|^2 =$$

$$= \sum_{\pm} C_{\alpha_\infty,\alpha_1,\alpha} C^{\alpha}_{\alpha_0,\alpha_{t\pm}} C^{\alpha_{t\pm}}_{\alpha_{2,1},\alpha_t} \left|\mathfrak{F}^{(t)}_{\pm}(t,t-z)\right|^2 .$$

$\mathfrak{F}_\pm$ are the CB, and as $b \to 0$

$$\mathcal{F} \sim \lim_{b\to 0} \mathfrak{F}$$

satisfies the Heun equation.

The 4 punctured sphere

Plugging in the crossing relation the Ansatz

$$\mathfrak{F}^{(0)}_{\pm}(t,z) = \sum_{\pm'} M_{\pm\pm'} \mathfrak{F}^{(t)}_{\pm'}(t, t-z)$$

we find

$$\sum_{\pm} C^{\alpha}_{\alpha_t, \alpha_{0\pm}} C^{\alpha_{0\pm}}_{\alpha_{2,1}, \alpha_0} \left| \sum_{\pm'} M_{\pm\pm'} \mathfrak{F}^{(t)}_{\pm'}(t, t-z) \right|^2 =$$

$$= \sum_{\pm} C^{\alpha}_{\alpha_0, \alpha_{t\pm}} C^{\alpha_{t\pm}}_{\alpha_{2,1}, \alpha_t} \left| \mathfrak{F}^{(t)}_{\pm}(t, t-z) \right|^2 .$$

These constraints allow us to solve for M. As $b \to 0$ ...

The 4 punctured sphere

we find the **connection coefficients**

$$\mathcal{F}_{\pm}^{(0)}(t,z) = \sum_{\pm'} M_{\pm\pm'} \mathcal{F}_{\pm'}^{(t)}(t, t-z)$$

$$M_{\pm\pm'} = \frac{\Gamma\left(\mp' 2a_t\right)\Gamma\left(1 \pm 2a_0\right)}{\Gamma\left(\frac{1}{2} \pm a_0 \mp' a_t + a\right)\Gamma\left(\frac{1}{2} \pm a_0 \mp' a_t - a\right)}$$

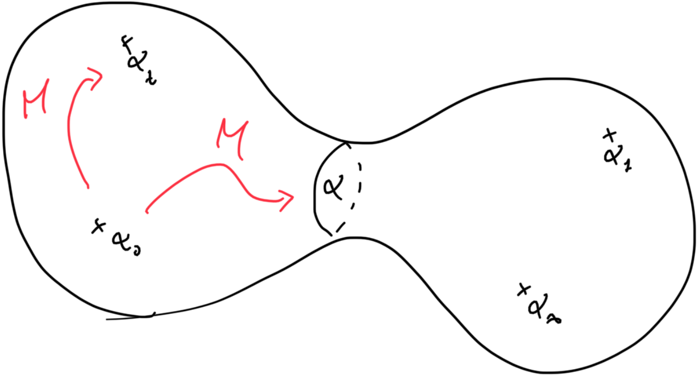
They are the hypergeometric ones, except for one crucial difference: the intermediate dimension $a(= b\alpha)$ appears.

In the ODE, a does not appear, but $u(a)$ does.

$$u = \sum_n c_n(a_i, a) t^n \Longrightarrow a = \sum_n k_n(a_i, u) t^n \,.$$

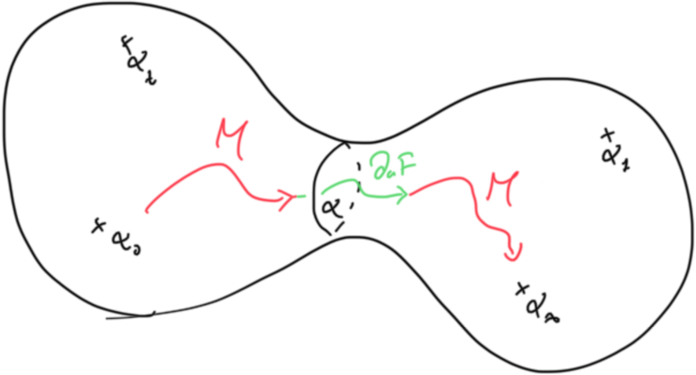
The 4 punctured sphere

One finds the same from $0 \sim \alpha_0$ to the thin tube region $\sim \alpha$



The 4 punctured sphere

When crossing the tube, one picks up two M's, and an extra factor.



The 4 punctured sphere

Explicitly the connection coefficients from 0 and ∞ reads

$$\sum_{\sigma=\pm}\frac{\Gamma(-2\sigma a)\Gamma(1+2\theta a_0)t^{-\sigma a}e^{-\frac{\sigma}{2}\partial_a F}}{\Gamma\left(\frac{1}{2}+\theta a_0-\sigma a+a_t\right)\Gamma\left(\frac{1}{2}+\theta a_0-\sigma a-a_t\right)}\times$$
$$\times\frac{\Gamma(1-2\sigma a)\Gamma(-2\theta' a_\infty)}{\Gamma\left(\frac{1}{2}-\sigma a-\theta' a_\infty+a_1\right)\Gamma\left(\frac{1}{2}-\sigma a-\theta' a_\infty-a_1\right)}\,.$$

where

$$F=\lim_{b\to 0} b^2\log\langle V_{\alpha_\infty}(\infty)V_{\alpha_1}(1)\Pi_\Delta V_{\alpha_t}(t)V_{\alpha_0}(0)\rangle\,.$$

The 4 punctured sphere

We found connection formulas for $\mathcal{F}$.

However $\mathcal{F}$ is only proportional to what are called Heun functions ($\sim$ Frobenius series) in literature.

Namely

$$\mathcal{F}_{\pm}^{(0)}(t,z) \sim e^{\frac{\mp}{2}\partial_{a_0}F} z^{\frac{1}{2}\pm a_0}(1+\mathcal{O}(t,z)) = e^{\frac{\mp}{2}\partial_{a_0}F} H_{\pm}^{(0)}(z,t)\,.$$

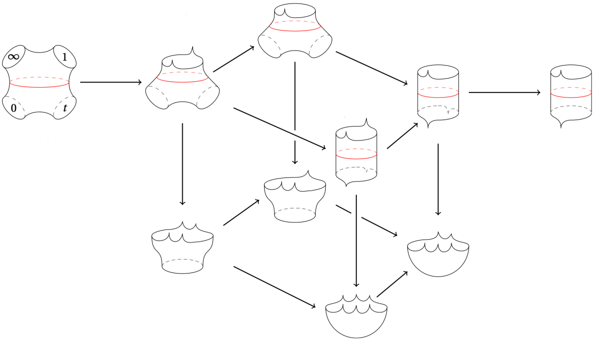
$H_{\pm}^{(0)}(z,t)$ is what you get plugging a series Ansatz in the Heun (semiclassical BPZ) equation.

$$H_{\pm}^{(0)}(z,t) = \sum_{\pm'} e^{\frac{\mp'}{2}\partial_{a_0}F} M_{\pm\pm'} H_{\pm'}^{(t)}(t-z,t)\,.$$

These are the **connection formulas for the Heun functions.**

The 4 punctured sphere

We can collide singularities and solve for the connection problem of confluent Heun functions



The 4 punctured sphere

Confluent cases involve irregular CFT states that behave like coherent states for the Virasoro algebra, that is

$$L_{-1}|\mu,\Lambda\rangle = \mu\Lambda|\mu,\Lambda\rangle, \quad L_{-2}|\mu,\Lambda\rangle = \Lambda^2|\mu,\Lambda\rangle$$

and

$$L_{-1}|\Lambda^2\rangle = \Lambda^2|\Lambda^2\rangle$$

They are obtained by colliding primary insertions.

Via the collision limit one can compute DOZZ factors of irregular correlators and find the connection coefficients.

The 4 punctured sphere

All this can be recasted in 4d gauge theory language, since

$$\langle V_{\alpha_\infty}(\infty) V_{\alpha_1}(1) \Pi_\Delta V_{\alpha_t}(t) V_{\alpha_0}(0) \rangle = \mathcal{Z}^{SU(2)}_{\mathcal{N}=2}(m_1, m_2, m_3, m_4, a, t) \,.$$

The relation $u(a)$ here appears as

$$\langle \Phi \rangle = a\sigma_3, \ \ u = \langle \mathrm{Tr}\Phi^2 \rangle = a^2 + \dots$$

And

$$F \sim \log \mathcal{Z} \ \text{ is the prepotential of the theory } \ .$$

Colliding primaries in the CFT $\sim$ decoupling masses in the gauge theory.

Remark:

The duality with gauge theory is useful since these $\mathcal{N} = 2$ partition functions are known exactly (**Nekrasov partition functions**).

The 4 punctured sphere

Let's summarize what we found:

- Semiclassical CB solve the Heun equation,
- Crossing symmetry + knowledge of 3 point functions allows us to **compute connection coefficients** for the Heun functions,
- Most of the complication lies in inverting $u(a)$ to find $a(u)$.

AdS_5 black holes perturbations

The spherically symmetric BH metric in AdS_5 is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega\,,$$

with

$$f(r) = \left(1 - \frac{R_+^2}{r^2}\right)(r^2 + R_+^2 + 1)\,.$$

Scalar perturbations on this background satisfy.

$$(\Box - m^2)\phi = 0\,.$$

AdS_5 black holes perturbations

The problem is separable.

$$\phi = \int d\omega \sum_{\ell,\vec{m}} e^{-i\omega t} Y_{\ell\vec{m}}\psi(r)\,,$$

gives

$$\left(\frac{1}{r^3}\partial_r\left(r^3 f(r)\partial_r\right) + \frac{\omega^2}{f(r)} - \frac{\ell(\ell+2)}{r^2} - m^2\right)\psi(r) = 0\,.$$

Imposing that the perturbation is ingoing in the horizon

$$\begin{aligned}\psi &= (r-R_+)^{-\frac{i\omega}{2}\frac{R_+}{2R_+^2+1}} + \cdots = \\ &= \mathcal{A}(\omega,\ell)(r^{-2-\sqrt{4+m^2}} + \dots) + \mathcal{B}(\omega,\ell)(r^{-2+\sqrt{4+m^2}} + \dots)\end{aligned}$$

AdS_5 black holes perturbations

After the change of variable

$$z = \frac{r^2}{r^2 + R_+^2 + 1}, \;\; \psi(r) = \left(r^3 f(r) z'(r)\right) \chi(z)$$

the radial equation reduces to the Heun equation with

$$t = \frac{R_+^2}{2R_+^2 + 1}, \;\; a_0 = 0, \;\; a_1 = \sqrt{1 + \frac{m^2}{4}}\,,$$

$$a_t = \frac{i\omega}{2} \frac{R_+}{2R_+^2 + 1}, \;\; a_\infty = \frac{\omega}{2} \frac{\sqrt{R_+^2 + 1}}{2R_+^2 + 1}\,,$$

$$u = \frac{\ell(\ell+2) + 2(R_+ - R_0) + R_+ \mu^2}{4R_0} + \frac{R_+}{R_0} \frac{\omega^2}{4(R_0 - R_+)} \Rightarrow a = -\frac{\ell+1}{2} + \ldots\,,$$

where $R_0 = -1 - R_+$ and $R_+ \sim M$ (the BH mass) for $M \ll 1$.

AdS_5 black holes perturbations

In AdS one can define the QNMs as the frequencies s.t. $\psi(\infty) = 0$.

$$\psi(r \to \infty) \sim \mathcal{B} r^{\sqrt{4+m^2}-2}$$

QNMs boundary conditions require $\mathcal{B} = 0$, that is

$$\Rightarrow \sum_{\sigma=\pm} \frac{\Gamma(1-2\sigma a)\Gamma(-2\sigma a)\Gamma\left(1-2\frac{i\omega}{2}\frac{\sqrt{R_+}}{1+2R_+}\right)\Gamma\left(2\sqrt{1+\frac{m^2}{4}}\right) t^{\sigma a} e^{-\frac{\sigma}{2}\partial_a F}}{\Gamma\left(\frac{1}{2}-\frac{i\omega}{2}\frac{\sqrt{R_+}}{1+2R_+}-\sigma a\right)^2 \Gamma\left(\frac{1}{2}-\sigma a+\sqrt{1+\frac{m^2}{4}}+\frac{\omega}{2}\frac{\sqrt{1+R_+}}{1+2R_+}\right)\Gamma\left(\frac{1}{2}-\sigma a+\sqrt{1+\frac{m^2}{4}}-\frac{\omega}{2}\frac{\sqrt{1+R_+}}{1+2R_+}\right)} = 0$$

To solve this is easier than it seems. One finds

$$\omega_{n\ell} = \omega_{n\ell}^{(0)} + \sum_{i\geq 1} \gamma_i M^i + i \sum_{k\geq 1} f_k M^{\ell+\frac{1}{2}+k}$$

with

AdS_5 black holes perturbations...

$$
\begin{aligned}
\omega_{n\ell}^{(0)} &= \Delta + \ell + 2n\,, \\
\gamma_1 &= -\frac{\Delta^2 + \Delta(6n-1) + 6n(n-1)}{2(\ell+1)}\,, \\
f_1 &= -\frac{2^{-4\ell\pi^2}}{(\ell+1)^2}\omega_{n\ell}^{(0)}\frac{\Gamma\left(\Delta+n+\ell\right)}{\Gamma\left(\Delta+n-1\right)}\frac{(n+\ell+1)!}{n!\Gamma\left(\frac{\ell+1}{2}\right)^4}\,, \\
&\dots
\end{aligned}
$$

where $m^2 = \Delta(\Delta - 4)$.

The same quantities have been computed in the holographic CFT via heavy-light light cone bootstrap to the first orders in $1/\ell$ [Li, Zhang; Karlsson, Kulaxizi, Parnachev, Tadic; ...].

Our results agree with the literature and easily extend to all orders.

...and holography

Moreover, $\mathcal{A}/\mathcal{B} \sim$ thermal 2-pt function of two light operators of weight Δ in the boundary CFT.

Our formula provides an explicit result, prone to analytic and numerical exploration.

More about this in our forthcoming work...

Conclusions and future directions

- Liouville CFT provides us an effective method to compute connection formulas of a class of ODEs,
- These formulas have a large number of applications in gravitational physics and their holographic counterpart,
- Many other black holes (and microstates) to explore,
- The same method can be applied to find connection formulas of a larger class of ODEs.

Thanks for the attention!