GRAVITATIONAL WAVES
and
GRAVITY BEYOND GENERAL RELATIVITY

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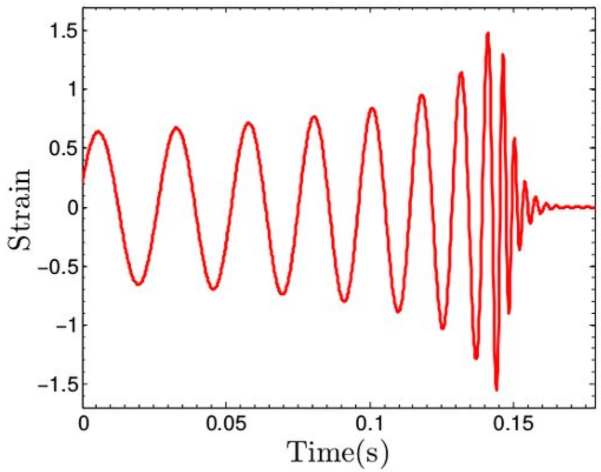
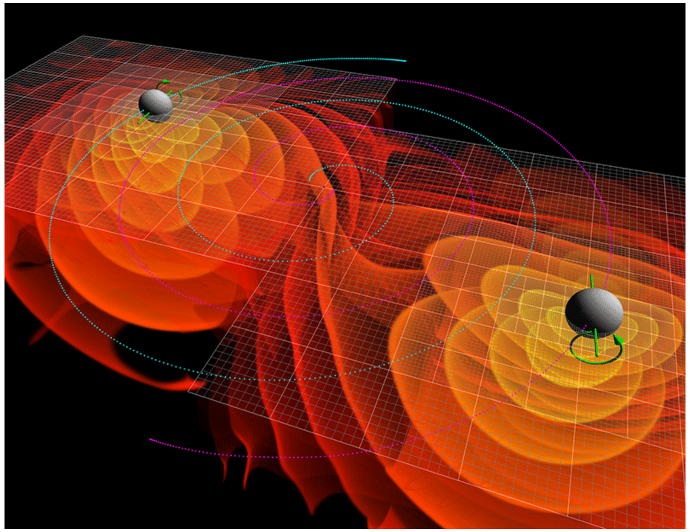
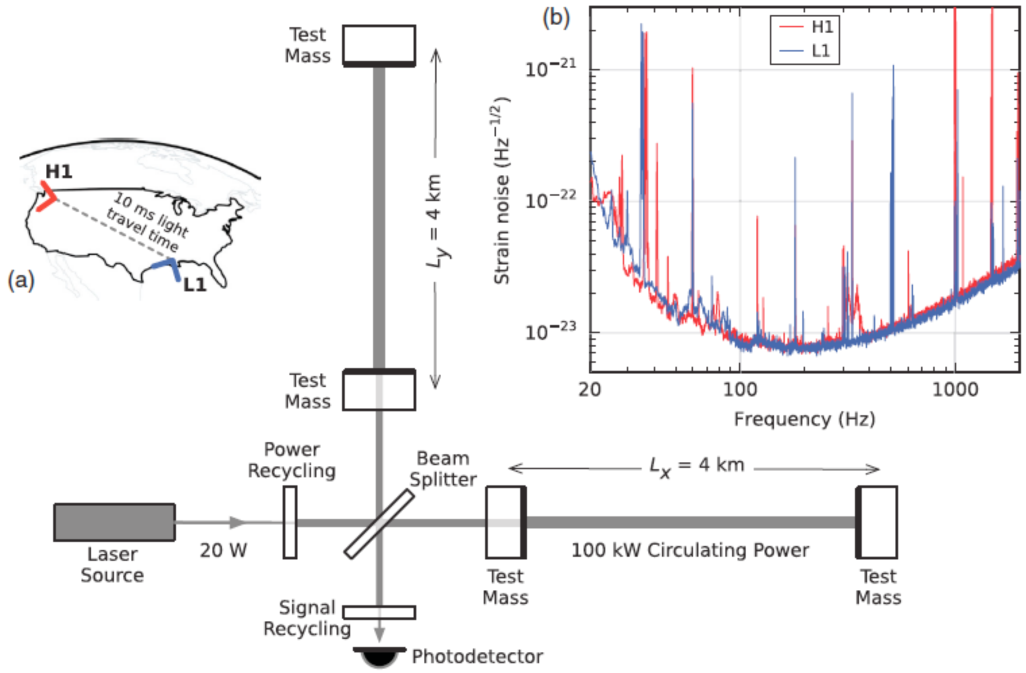




Theories of the Fundamental Interactions - TFI 2022
7th Meeting of the INFN Networks GAST, GSS and ST&FI
Istituto Veneto di Scienze - Palazzo Franchetti,
13-15 June 2022, Venice, Italy

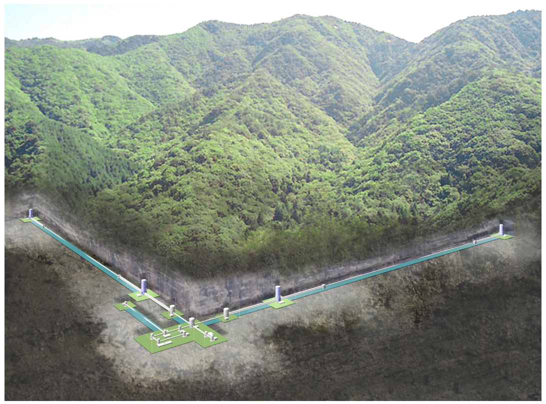
STARTING FROM 14 SEPT 2015: GRAVITATIONAL WAVE (GW) DETECTIONS BY TWO LIGO (+ VIRGO+KAGRA+...) GW DETECTORS



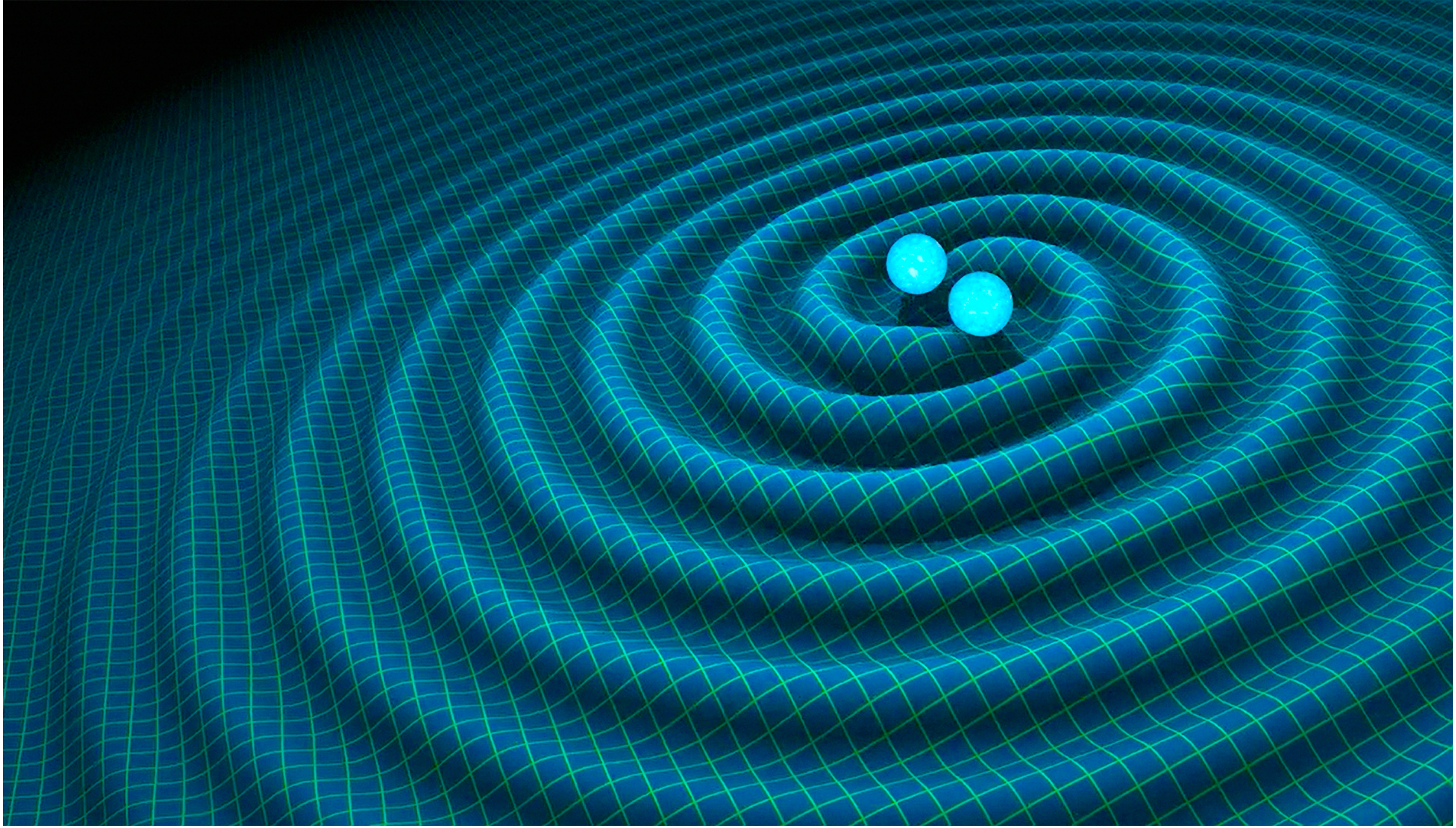
LIGO Hanford

LIGO Livingston

KAGRA



Virgo (IT)



$$m_1 = 36^{+5}_{-4} M_\odot$$

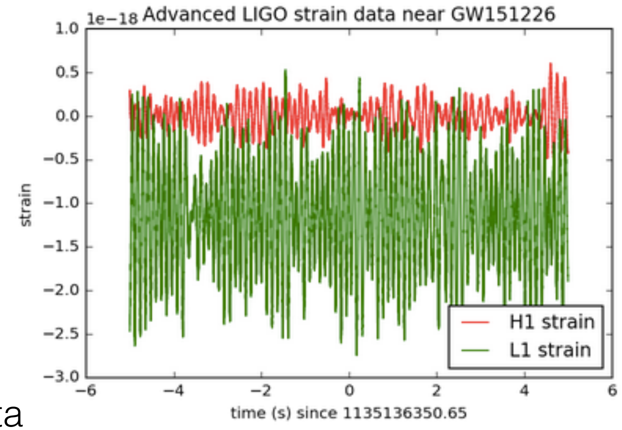
$$m_2 = 29^{+4}_{-4} M_\odot$$

$$\chi_{\rm eff} = -0.06^{+0.17}_{-0.18}$$

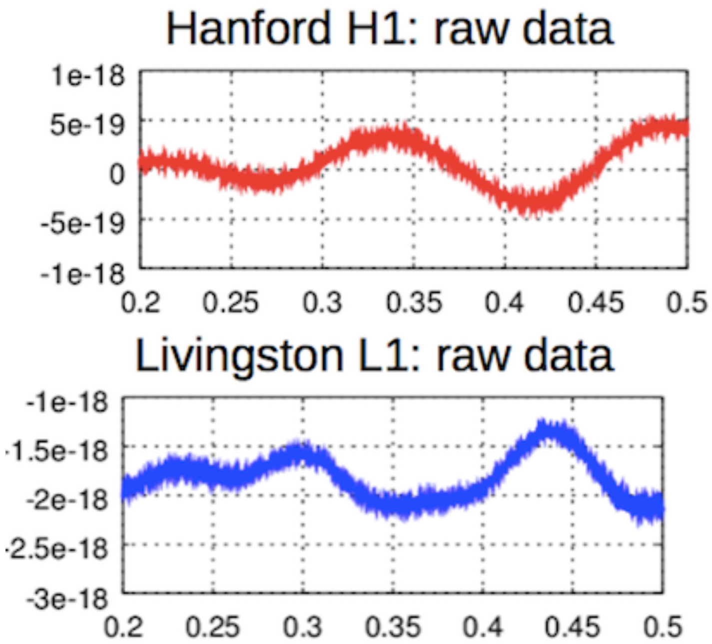
$$D_{\rm L} = 410^{+160}_{-180}{\rm Mpc}$$

GW150914, [LVT151012,]GW151226, GW170104,…: incredibly small signals lost in the broad-band noise

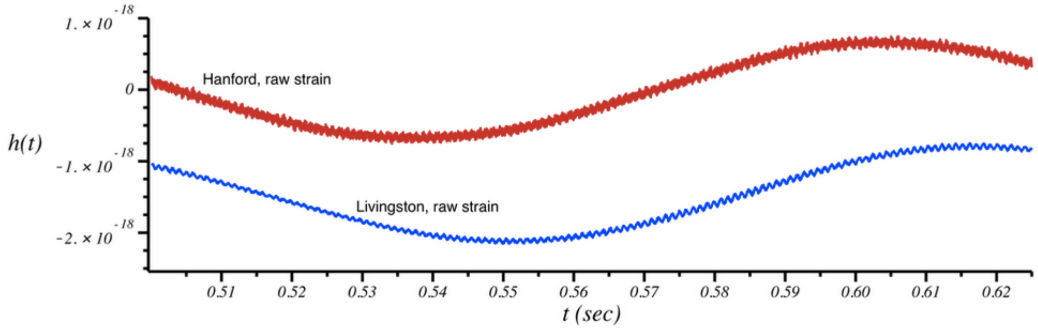
GW151226 from LIGO open data



GW150914, from LIGO open data



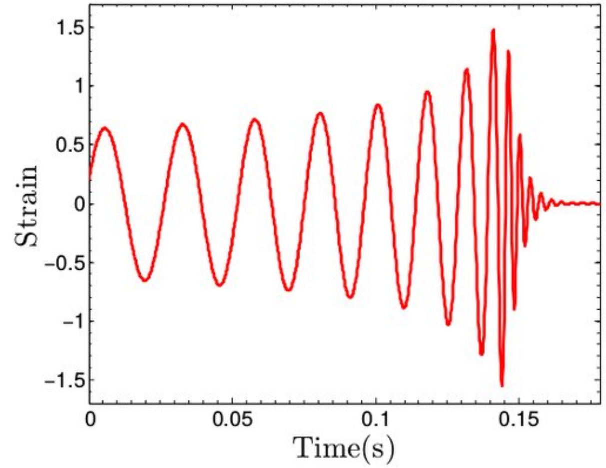
GW170104 from LIGO open data



$$h_{GW}^{\max} \sim 10^{-21} \sim 10^{-3}\, h_{LIGO}^{\text{broadband}}$$

$$\delta L/L = 10^{-21} \to \delta L \sim 10^{-9}\,\text{atom}\,!$$

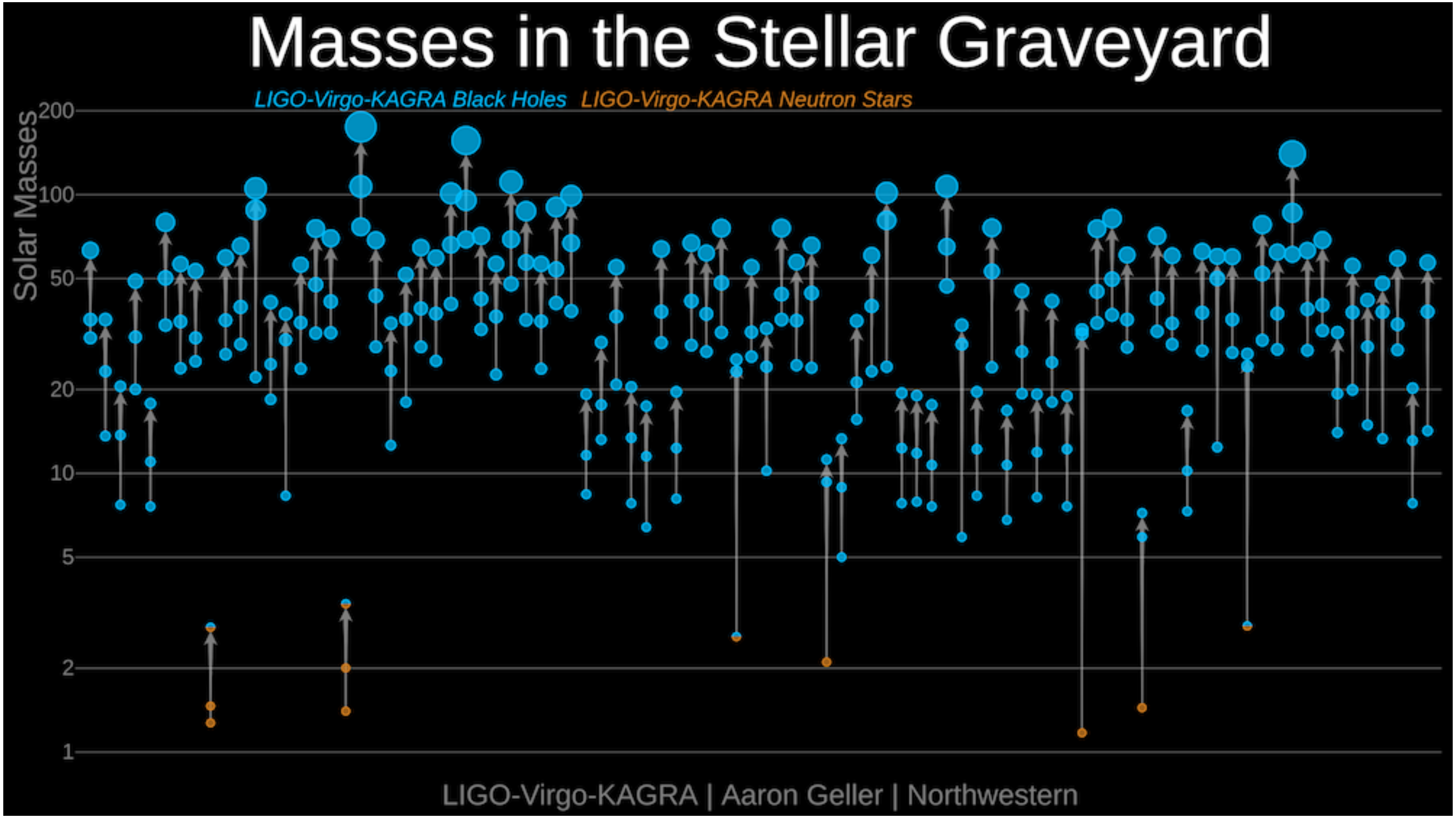
$$\frac{\delta L^{tot}}{\lambda} \sim \mathcal{F}\frac{L}{\lambda}\frac{\delta L}{L} \sim 10^{11} h \sim 10^{-10}\text{fringe}$$



LIGO-Virgo p>0.5 Events

(O1-O2-O3a-O3b; nov 2021)

90 events, incl.: 2 NS-NS; 3 NS-BH; 85 BH-BH

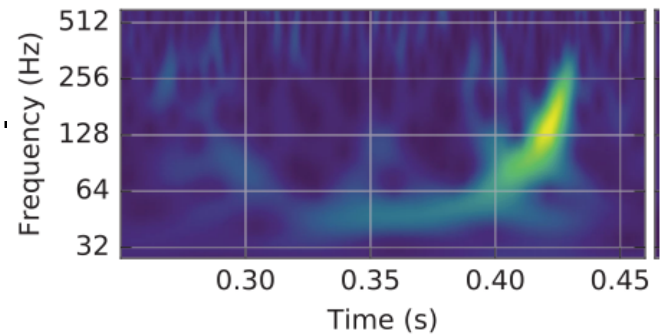


LIGO-Virgo data analysis

Various levels of search and analysis: online/offline, parameter estimation

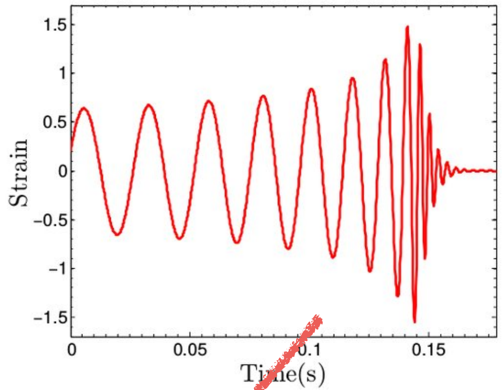
Online trigger searches:

CoherentWaveBurst **Time-frequency**
(Wilson, Meyer, Daubechies-Jaffard-Journe, Klimenko et al.
Omicron-LALInference sine-Gaussians
Gabor-type wavelet analysis **(Gabor,…,Lynch et al.)**
Matched-filter:
PyCBC (f-domain), gstLAL (t-domain)



Offline data analysis:

Generic transient searches
Binary coalescence searches



Here: focus on matched-filter definition

(crucial for high SNR, significance assessment, and parameter estimation)

Matched Filtering

$$\langle output | h_{\rm template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

Basics of Gravitational Waves

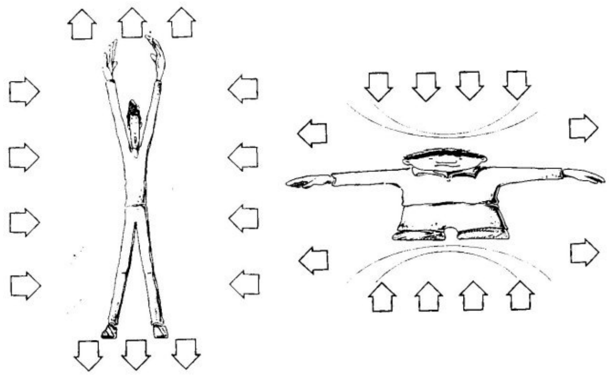
In linearized GR (Einstein 1916,1918): $g_{ij} = \delta_{ij} + h_{ij}$

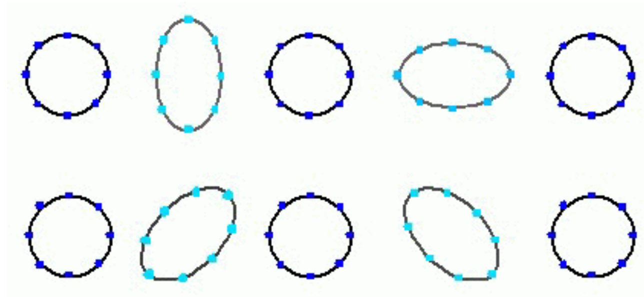
Two Transverse-Traceless (TT) tensor polarizations propagating at v=c

$$h_{ij} = h_{+}(x_i x_j - y_i y_j) + h_{\times}(x_i y_j + y_i x_j)$$

$$\frac{\delta L}{L} = \frac{1}{2} h_{ij} n^i n^j$$

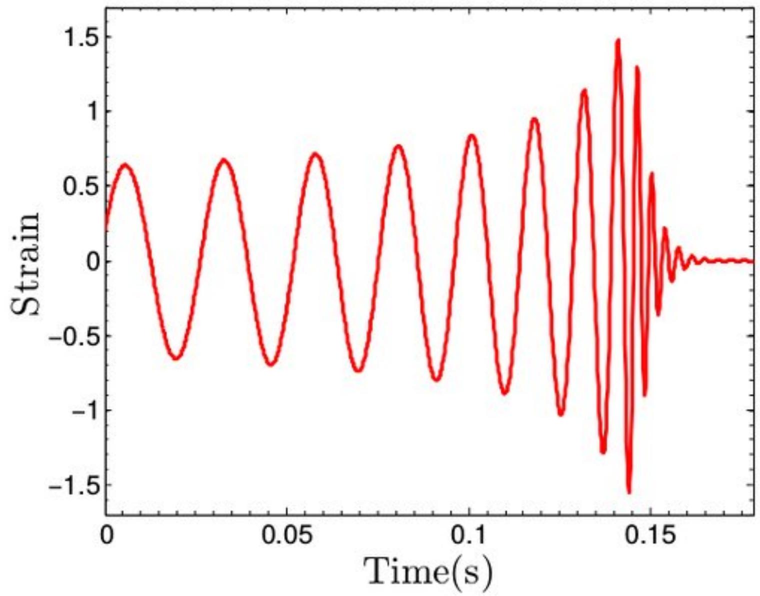
Weber, Pirani,…





Lowest-order generation:
quadrupole formula

$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$



BASICS OF BLACK HOLES

1916 Schwarzschild (non rotating) Black Hole (BH)

$$ds^2 = -(1 - \frac{2GM}{c^2 r})dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Schwarzschild radius (singularity ?): r_S= 2GM/c^2

radial potential

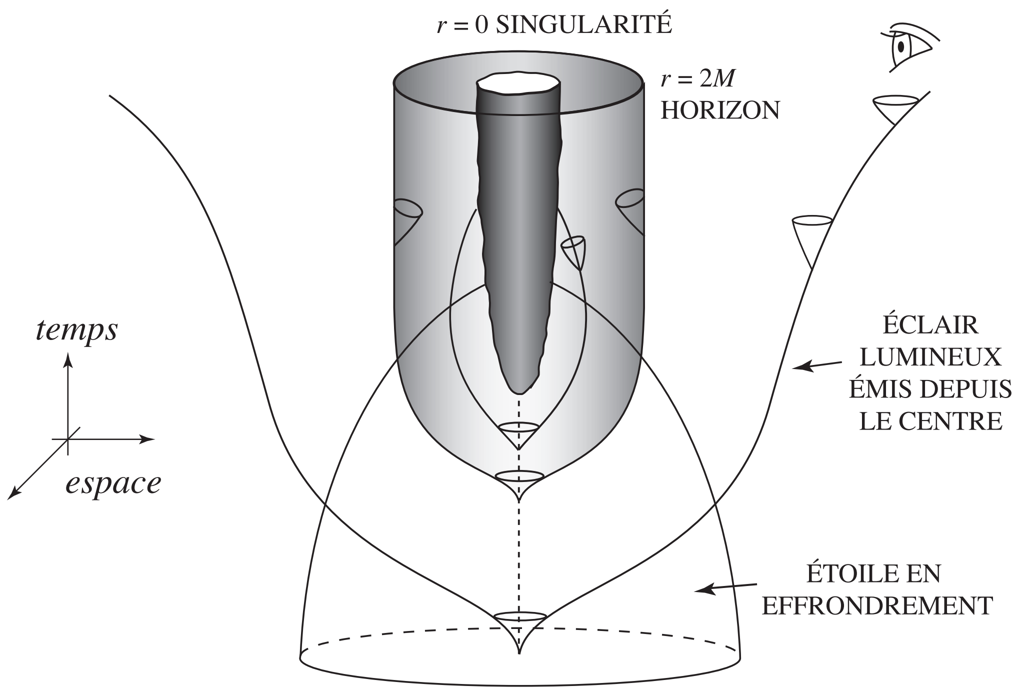
$$A_S(r) = 1 - \frac{2GM}{c^2 r}$$

1939 Oppenheimer-Snyder « continued collapse »

1963 Kerr Rotating BH: M, S

1965 Doroshkevich, Zel'dovich, Novikov

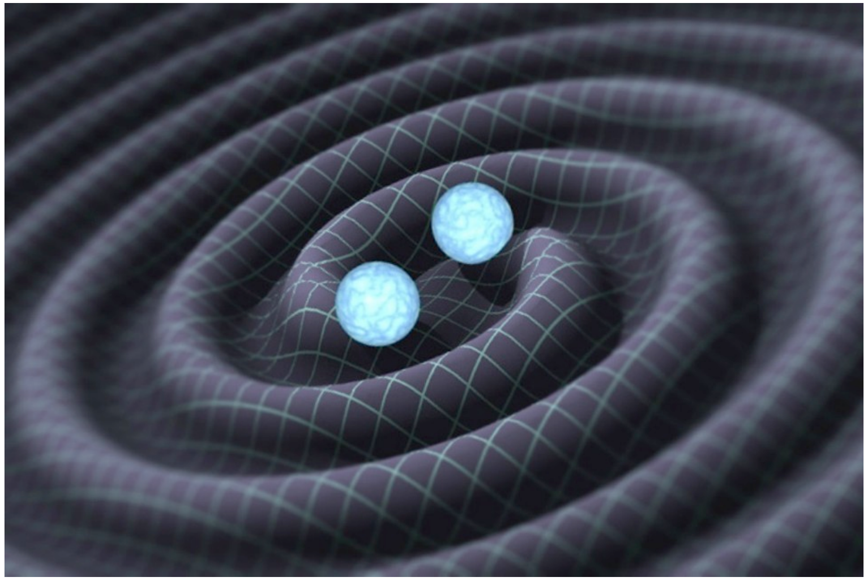
1969 Penrose



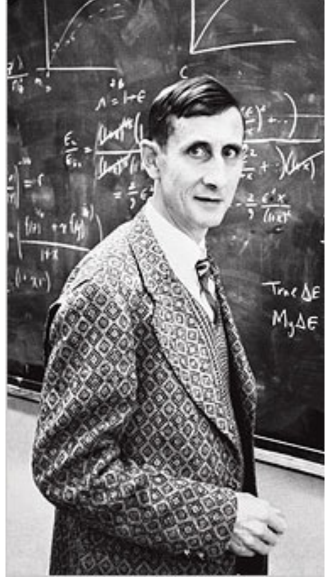
Horizon: cylindrical-like regular null hyper-surface whose sectional area is nearly constant, and actually slowly increasing (Christodoulou '70, Christodoulou-Ruffini '71, Hawking '71)

No hair property in D=4

Pioneering the GWs from coalescing compact binaries



Freeman Dyson 1963

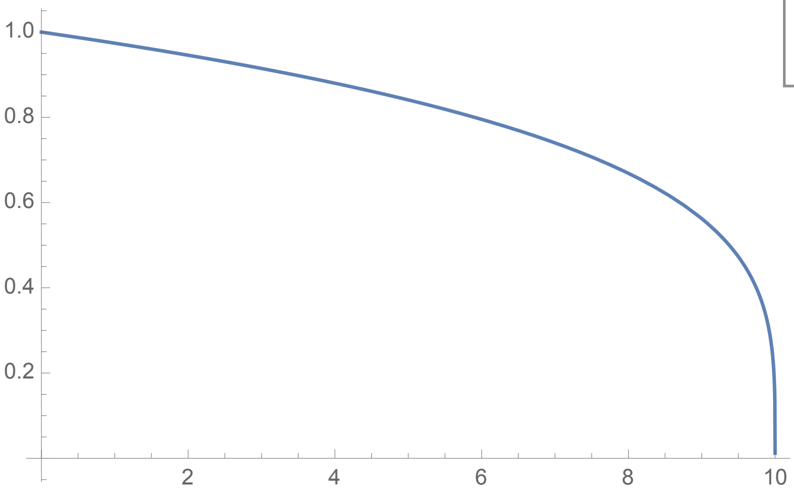


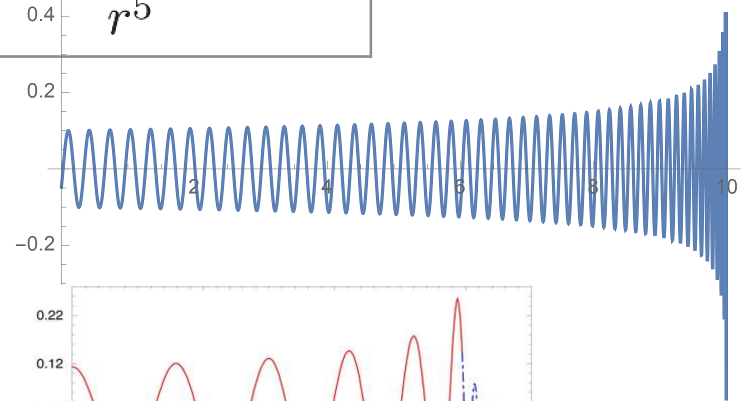
$$E = -\frac{G\, m_1\, m_2}{2r}$$

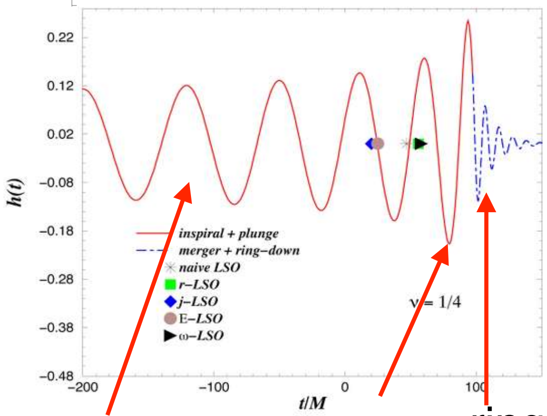
$$\frac{d}{dt} E = -F$$

Einstein 1918 + Landau-Lifshitz 1941

$$F = \frac{32\, G^4}{5\, c^5} \frac{m_1^2\, m_2^2 (m_1 + m_2)}{r^5}$$





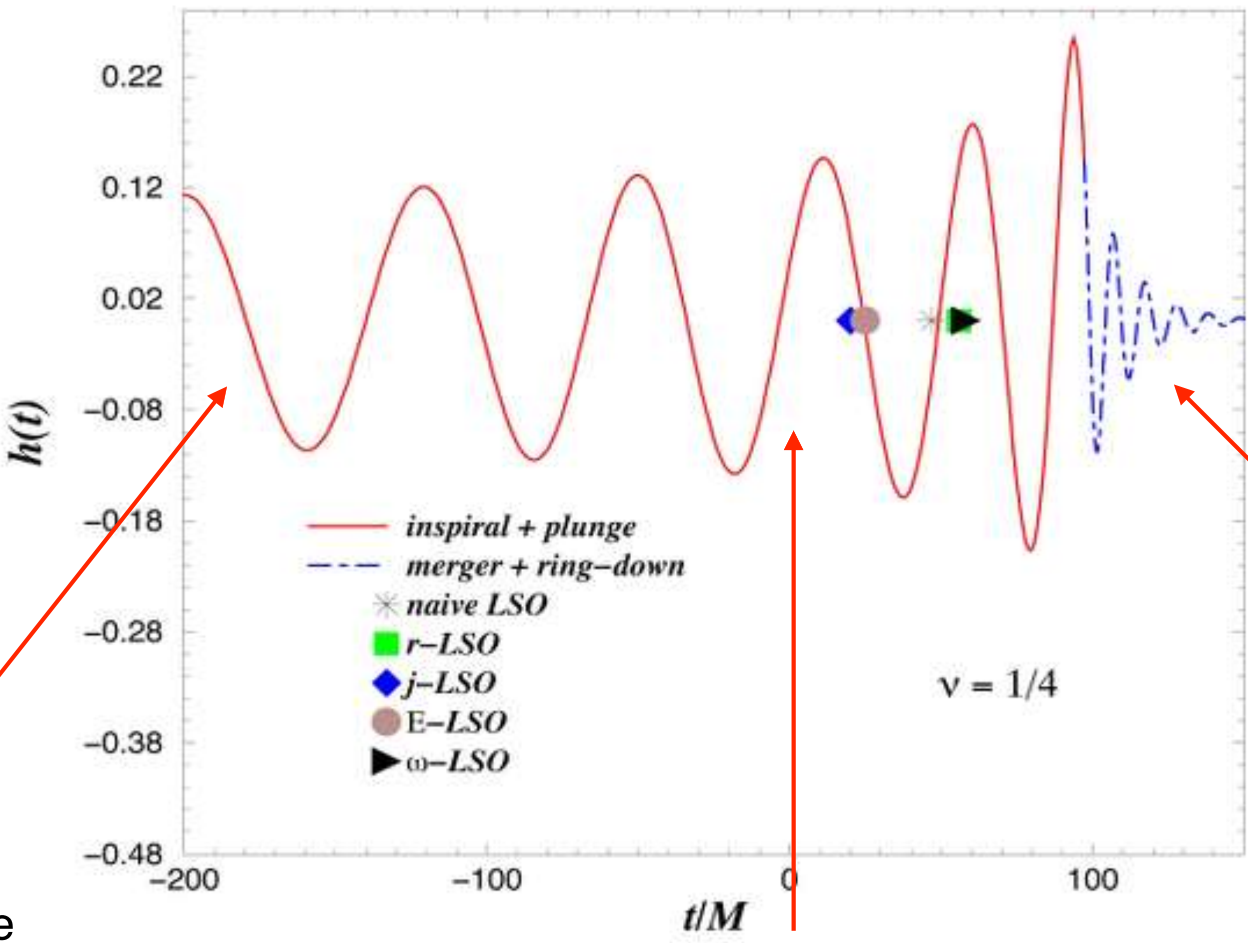


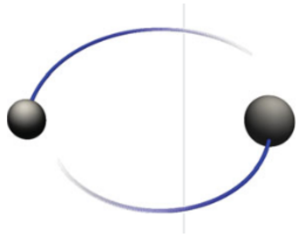
inspiral merger ringdown

Buonanno-Damour 2000

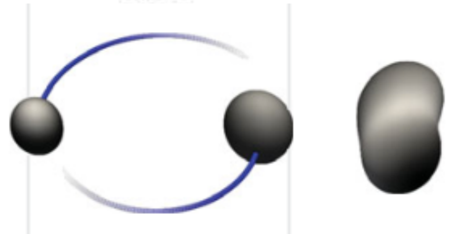
Freeman Dyson's challenge: describe the intense flash of GWs emitted by the last orbits and the merger of a binary BH, when v~c and r~GM/c^2

Physics entering the GWs emitted by coalescing BHs or NSs

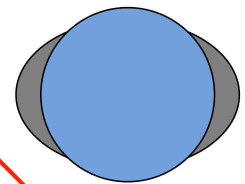




Inspiral:
perturbative
computation
of higher-order
contributions
to E=H and F
(expansion in v^2/c^2
tidal polarizability
of NS)



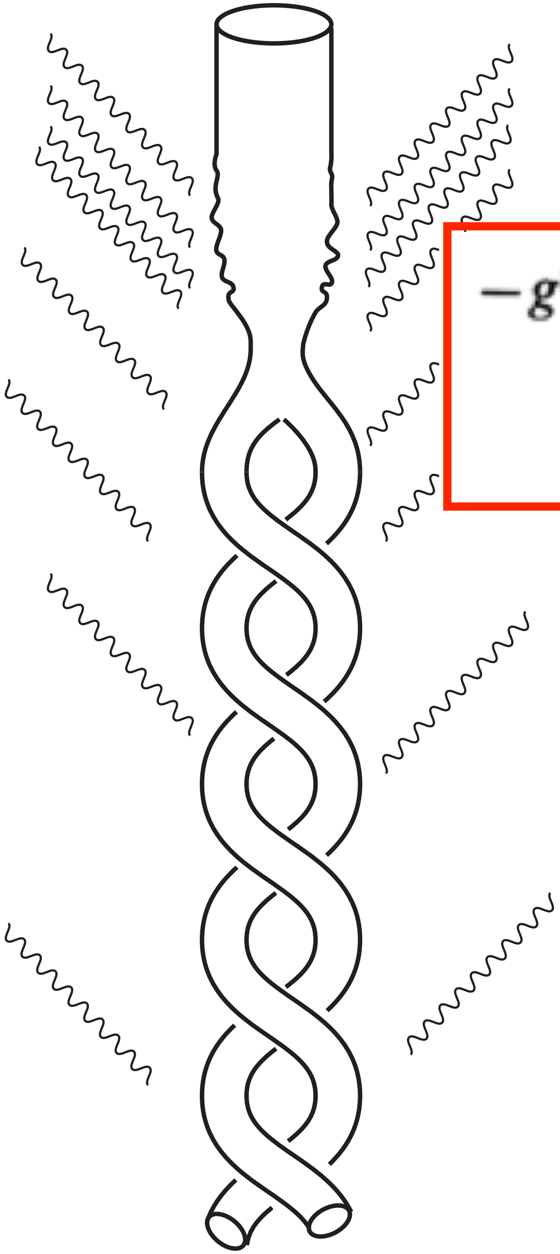
Late inspiral, « plunge » and merger:
needs either Numerical Relativity
or a resummation of perturbative results



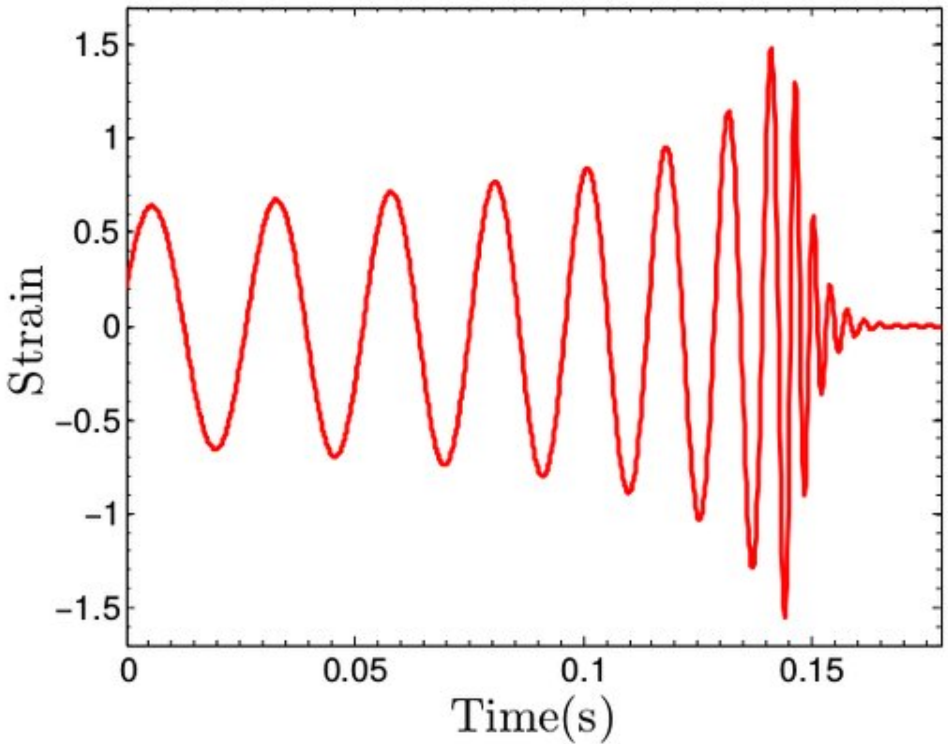
Ringdown (BBH):
« vibration modes »
of final BH (QNM);
perturbation
of BHs à la
Regge-Wheeler-Zerilli-
Teukolsky

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad R_{\mu\nu} = 0$$

$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$



$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} \big(g_{\alpha\mu,\,\rho}\, g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho}\, g_{\beta\sigma,\,\nu} + g_{\alpha\mu,\,\rho}\, g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho}\, g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha}\, g_{\nu\sigma,\,\beta} \big) = 0$$



Tools used for the GR 2-body pb

Post-Newtonian (PN) approximation (**expansion in 1/c; ie v^2/c^2 and GM/(c^2r)**)

Post-Minkowskian (PM) approximation (**expansion in G; ie in GM/(c^2b)**)
and its recent **Worldline EFT avatars**

Multipolar post-Minkowskian (MPM) approximation
theory to the GW emission of binary systems

Matched Asymptotic Expansions useful both for the motion of strongly self-gravitating bodies, and for the nearzone-wavezone matching

Gravitational Self-Force (SF): expansion in m1/m2, with « first law of BH mechanics » (LeTiec-Blanchet-Whiting'12,…)

Effective One-Body (EOB) Approach

Numerical Relativity (NR)

Effective Field Theory (EFT)

Quantum scattering_amplitude aided by Double-Copy, Generalized Unitarity,« Feynman-integral Calculus » (IBP, DE, regions, reverse unitarity,…), Kosower-Maybee-O'Connell
+ Worldline QFT

Tutti Frutti method

Quantum Scattering Amplitudes and 2-body Dynamics

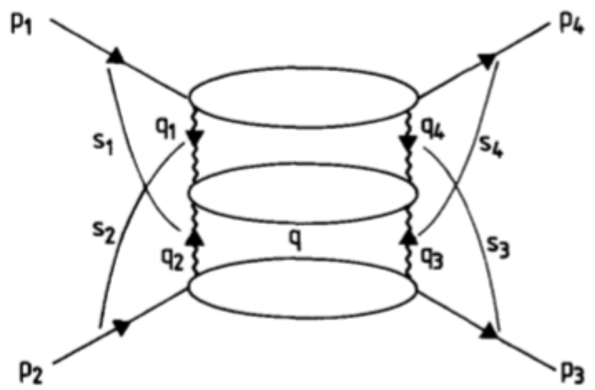


Fig. 3. The "H" diagram that provides the leading correction to the eikonal.

Amati-Ciafaloni-Veneziano 1987-2008
Ultra-High-Energy (s >> M_Planck^2)
Four-graviton Scattering at 2 loops

Eikonal phase \delta in D=4
with one- and two-loop corrections using the Regge-Gribov approach

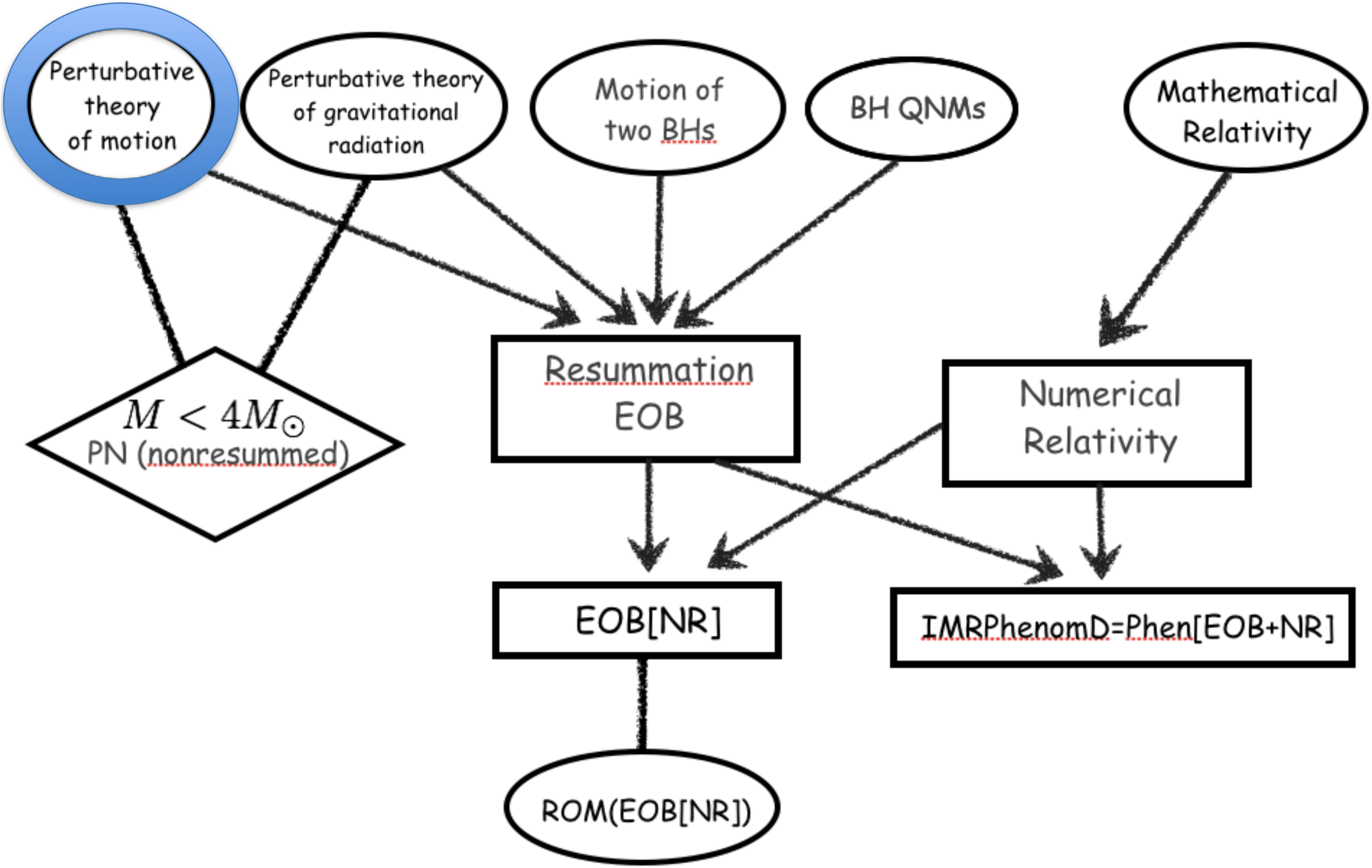
$$\delta = \frac{Gs}{\hbar}\left(\log\left(\frac{L_{IR}}{b}\right) + \frac{6\ell_s^2}{\pi b^2} + \frac{2G^2 s}{b^2}(1 + \frac{2i}{\pi}\log(\cdots))\right)$$

Methods for transforming scattering angle in PM-dynamics
(TD'16,18; Cheung-Rothstein-Solon'18; Kalin-Porto'19,…)

HE puzzle posed by 3PM=G^3=2-loop result of Bern et al.

G^3-puzzle resolved by taking into account radiative effects
(DiVecchia-Heissenberg-Russo-Veneziano'20,TD'20,….)-> **talk by Heissenberg**

Subtleties at G^4=3loop still to be clarified

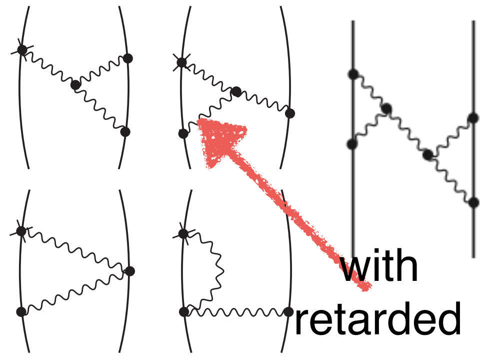

Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_{\odot}$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

State of the art for PN dynamics

- 1PN (including v^2/c^2) [Lorentz-Droste '17], Einstein-Infeld-Hoffmann '38
- 2PN (inc. v^4/c^4) Ohta-Okamura-Kimura-Hiida '74, Damour-Deruelle '81, Damour '82, Schäfer '85, Kopeikin '85
- 2.5 PN (inc. v^5/c^5) Damour-Deruelle '81, Damour '82, Schäfer '85, Kopeikin '85 **LO-radiation-reaction**

First complete 2PN and 2.5PN dynamics obtained by using 2PM (G^2) EOM of Bel et al.'81

- 3 PN (inc. v^6/c^6) Jaranowski-Schäfer '98, Blanchet-Faye '00, Damour-Jaranowski-Schäfer '01, Itoh-Futamase '03, Blanchet-Damour-Esposito-Farèse' 04, Foffa-Sturani '11
- 3.5 PN (inc. v^7/c^7) Iyer-Will '93, Jaranowski-Schäfer '97, Pati-Will '02, Königsdörffer-Faye-Schäfer '03, Nissanke-Blanchet '05, Itoh '09



with retarded propagator

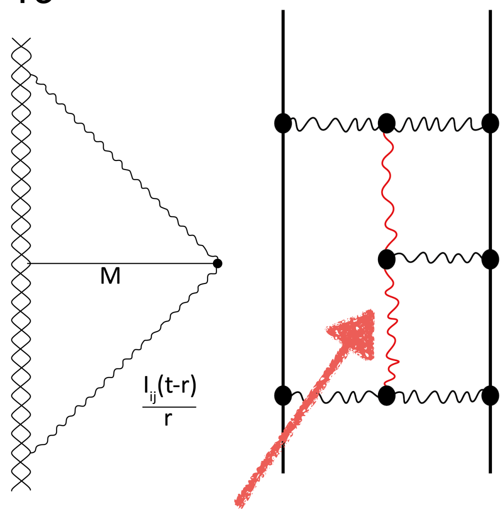
- **4PN** (inc. v^8/c^8) Jaranowski-Schäfer '13, Foffa-Sturani '13,'16 Bini-Damour '13, Damour-Jaranowski-Schäfer '14, Marchand+'18, Foffa+'19

New feature at G^4/c^8 (4PN and 4PM) : **non-locality in time** (linked to IR divergences of formal PN-expansion) (Blanchet,TD '88)

- **5PN** (inc. v^{10}/c^{10} and **G^6**) Bini-Damour-Geralico'19: complete **modulo two**
- **numerical** parameters; **Bluemlein et al'21**: potential-graviton contrib. and
- partial determination of radiation-graviton contrib. used QGRAF to generate

545812 4-loop diagrams, and 332020 5-loop diagrams

- **6PN** (inc. v^{12}/c^{12} and **G^7**) Bini-Damour-Geralico'20: complete **modulo four**
- additional parameters



soft (radiation) gravitons

Inclusion of **spin-dependent effects**: Barker-O' Connell'75, Faye-Blanchet-Buonanno'06, Damour-Jaranowski-Schaefer'08, Porto-Rothstein '06, Levi '10, Steinhoff-Hergt-Schaefer '10, Steinhoff'11, Levi-Steinhoff'15-18, Bini-TD, Vines , Guevara-Ochirov-Vines,….

2-body perturbative Hamiltonian: N + 1PN + 2PN

$$H_{\rm N}(\mathbf{x}_a, \mathbf{p}_a) = \frac{\mathbf{p}_1^2}{2m_1} - \frac{1}{2}\frac{Gm_1m_2}{r_{12}} + (1 \leftrightarrow 2)$$

$$c^2 H_{\rm 1PN}(\mathbf{x}_a, \mathbf{p}_a) = -\frac{1}{8}\frac{(\mathbf{p}_1^2)^2}{m_1^3} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(-12\frac{\mathbf{p}_1^2}{m_1^2} + 14\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2} + 2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)$$
$$+\frac{1}{4}\frac{Gm_1m_2}{r_{12}}\frac{G(m_1+m_2)}{r_{12}} + (1 \leftrightarrow 2),$$

$$c^4 H_{\rm 2PN}(\mathbf{x}_a, \mathbf{p}_a) = \frac{1}{16}\frac{(\mathbf{p}_1^2)^3}{m_1^5} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(5\frac{(\mathbf{p}_1^2)^2}{m_1^4} - \frac{11}{2}\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2} - \frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2m_2^2} + 5\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right.$$
$$\left. - 6\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2} - \frac{3}{2}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right)$$
$$+\frac{1}{4}\frac{G^2m_1m_2}{r_{12}^2}\left(m_2\left(10\frac{\mathbf{p}_1^2}{m_1^2} + 19\frac{\mathbf{p}_2^2}{m_2^2}\right) - \frac{1}{2}(m_1+m_2)\frac{27(\mathbf{p}_1\cdot\mathbf{p}_2) + 6(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)$$
$$-\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\frac{G^2(m_1^2 + 5m_1m_2 + m_2^2)}{r_{12}^2} + (1 \leftrightarrow 2),$$

2-body Taylor-expanded 3PN Hamiltonian [DJS 01]

$$
\begin{aligned}
c^6 H_{3\text{PN}}(\mathbf{x}_a, \mathbf{p}_a) = & -\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7} + \frac{1}{32}\frac{G m_1 m_2}{r_{12}}\left(-14\frac{(\mathbf{p}_1^2)^3}{m_1^6} + 4\frac{((\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 4\mathbf{p}_1^2\mathbf{p}_2^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4 m_2^2}\right. \\
& - 10\frac{(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2 + \mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4 m_2^2} \\
& + 2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} + \frac{(7\mathbf{p}_1^2\mathbf{p}_2^2 - 10(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2^3} \\
& + \frac{(\mathbf{p}_1^2\mathbf{p}_2^2 - 2(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2^3} + 15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} \\
& \left. - 18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3} + 5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3}\right) + \frac{G^2 m_1 m_2}{r_{12}^2}\left(\frac{1}{16}(m_1 - 27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4}\right. \\
& - \frac{115}{16} m_1 \frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} + \frac{1}{48} m_2 \frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 371\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} + \frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3} + \frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3} \\
& - \frac{1}{8} m_1 \frac{(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2) + 11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1))(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3 m_2} - \frac{3}{2} m_1 \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} \\
& + \frac{125}{12} m_2 \frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2 m_2^2} + \frac{10}{3} m_2 \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} \\
& \left. - \frac{1}{48}(220 m_1 + 193 m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2}\right) + \frac{G^3 m_1 m_2}{r_{12}^3}\left(-\frac{1}{48}\left(425 m_1^2 + \left(473 - \frac{3}{4}\pi^2\right) m_1 m_2 + 150 m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2}\right. \\
& + \frac{1}{16}\left(77(m_1^2 + m_2^2) + \left(143 - \frac{1}{4}\pi^2\right) m_1 m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \frac{1}{16}\left(20 m_1^2 - \left(43 + \frac{3}{4}\pi^2\right) m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} \\
& \left. + \frac{1}{16}\left(21(m_1^2 + m_2^2) + \left(119 + \frac{3}{4}\pi^2\right) m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2}\right) \\
& + \frac{1}{8}\frac{G^4 m_1 m_2^3}{r_{12}^4}\left(\left(\frac{227}{3} - \frac{21}{4}\pi^2\right) m_1 + m_2\right) + (1 \leftrightarrow 2).
\end{aligned}
$$

2-body Taylor-expanded 4PN Hamiltonian [DJS, 2014, JS 2015]

$$c^8 H^{\rm local}_{\rm 4PN}(\mathbf{x}_a,\mathbf{p}_a) = \frac{7(\mathbf{p}_1^2)^5}{256 m_1^9} + \frac{G m_1 m_2}{r_{12}} H_{48}(\mathbf{x}_a,\mathbf{p}_a) + \frac{G^2 m_1 m_2}{r_{12}^2} m_1 H_{46}(\mathbf{x}_a,\mathbf{p}_a)$$
$$+ \frac{G^3 m_1 m_2}{r_{12}^3}\left(m_1^2 H_{441}(\mathbf{x}_a,\mathbf{p}_a) + m_1 m_2 H_{442}(\mathbf{x}_a,\mathbf{p}_a)\right)$$
$$+ \frac{G^4 m_1 m_2}{r_{12}^4}\left(m_1^3 H_{421}(\mathbf{x}_a,\mathbf{p}_a) + m_1^2 m_2 H_{422}(\mathbf{x}_a,\mathbf{p}_a)\right)$$
$$+ \frac{G^5 m_1 m_2}{r_{12}^5} H_{40}(\mathbf{x}_a,\mathbf{p}_a) + (1 \leftrightarrow 2), \quad \text{(A3)}$$

$$H_{48}(\mathbf{x}_a,\mathbf{p}_a) = \frac{45(\mathbf{p}_1^2)^4}{128 m_1^8} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{64 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^3}{64 m_1^6 m_2^2} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{16 m_1^6 m_2^2}$$
$$- \frac{3(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{32 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{21(\mathbf{p}_1^2)^3\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{35(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{256 m_1^5 m_2^3}$$
$$+ \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{128 m_1^5 m_2^3} + \frac{33(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1^2)^2}{256 m_1^5 m_2^3} - \frac{85(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3}$$
$$- \frac{45(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2^3} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} + \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3}$$
$$+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{55(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3}$$
$$- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{23(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3}$$
$$+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{64 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4(\mathbf{p}_1^2)^2}{64 m_1^4 m_2^4}$$
$$- \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^4 m_2^4} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{64 m_1^4 m_2^4}$$
$$- \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{16 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{16 m_1^4 m_2^4}$$
$$- \frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_2^2)^2}{64 m_1^4 m_2^4} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{32 m_1^4 m_2^4} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_2^2)^2}{128 m_1^4 m_2^4}, \quad \text{(A4a)}$$

$$H_{46}(\mathbf{x}_a,\mathbf{p}_a) = \frac{369(\mathbf{n}_{12}\cdot\mathbf{p}_1)^6}{160 m_1^6} - \frac{889(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_1^2}{192 m_1^6} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2}{16 m_1^6} - \frac{63(\mathbf{p}_1^2)^3}{64 m_1^6} - \frac{549(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{128 m_1^5 m_2}$$
$$+ \frac{67(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{16 m_1^5 m_2} - \frac{167(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2}{128 m_1^5 m_2} + \frac{1547(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} - \frac{851(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2}$$
$$+ \frac{1099(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} + \frac{3263(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} + \frac{1067(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{480 m_1^4 m_2^2} - \frac{4567(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{3840 m_1^4 m_2^2}$$
$$- \frac{3571(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{320 m_1^4 m_2^2} + \frac{3073(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{480 m_1^4 m_2^2} + \frac{4349(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2}$$
$$- \frac{3461\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{3840 m_1^4 m_2^2} + \frac{1673(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_2^2}{1920 m_1^4 m_2^2} - \frac{1999(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} + \frac{2081(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} - \frac{13(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{8 m_1^3 m_2^3}$$
$$+ \frac{191(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{192 m_1^3 m_2^3} - \frac{19(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3}$$
$$+ \frac{11(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{192 m_1^3 m_2^3} + \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{96 m_1^3 m_2^3} + \frac{233(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{96 m_1^3 m_2^3} - \frac{47(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{32 m_1^3 m_2^3}$$
$$+ \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{185\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4}{4 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{4 m_1^2 m_2^4}$$
$$- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1\cdot\mathbf{p}_2)}{2 m_1^2 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{6 m_1^2 m_2^4} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{48 m_1^2 m_2^4}$$
$$- \frac{133(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{24 m_1^2 m_2^4} - \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{96 m_1^2 m_2^4} + \frac{197(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_2^2)^2}{96 m_1^2 m_2^4} - \frac{173\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{48 m_1^2 m_2^4} + \frac{13(\mathbf{p}_2^2)^3}{8 m_2^6}, \quad \text{(A4b)}$$

$$H_{441}(\mathbf{x}_a,\mathbf{p}_a) = \frac{5027(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{384 m_1^4} - \frac{22993(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{960 m_1^4} - \frac{6695(\mathbf{p}_1^2)^2}{1152 m_1^4} - \frac{3191(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{640 m_1^3 m_2}$$
$$+ \frac{28561(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{1920 m_1^3 m_2} + \frac{8777(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2} + \frac{752969\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{28800 m_1^3 m_2}$$
$$- \frac{16481(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{960 m_1^2 m_2^2} + \frac{94433(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{4800 m_1^2 m_2^2} - \frac{103957(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{2400 m_1^2 m_2^2}$$
$$+ \frac{791(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{400 m_1^2 m_2^2} + \frac{26627(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_2^2}{1600 m_1^2 m_2^2} - \frac{118261\mathbf{p}_1^2\mathbf{p}_2^2}{4800 m_1^2 m_2^2} + \frac{105(\mathbf{p}_2^2)^2}{32 m_2^4}, \quad \text{(A4c)}$$

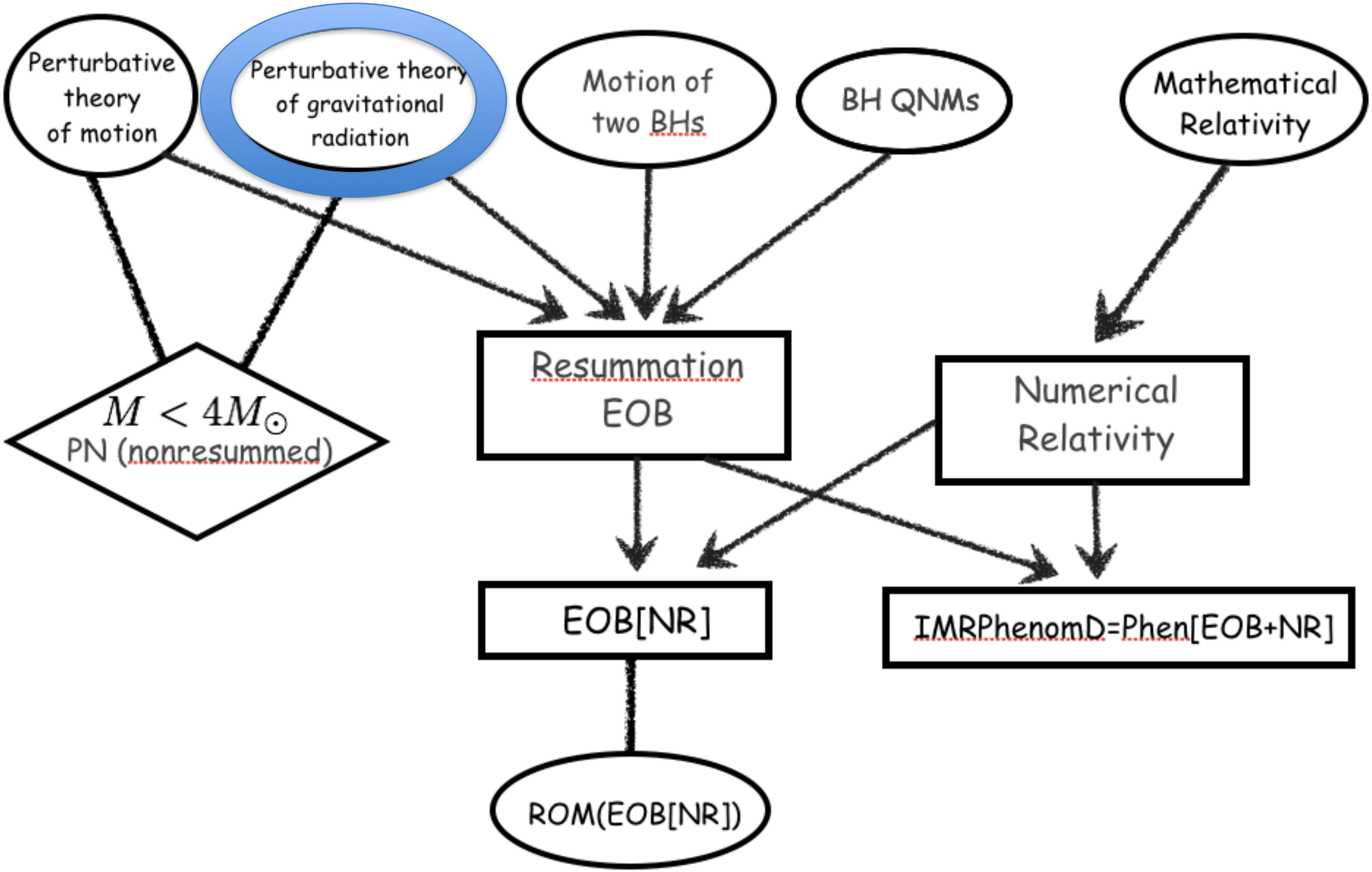
$$H_{442}(\mathbf{x}_a,\mathbf{p}_a) = \left(\frac{2749\pi^2}{8192} - \frac{211189}{19200}\right)\frac{(\mathbf{p}_1^2)^2}{m_1^4} + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{m_1^4} + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^4}$$
$$+ \left(\frac{10631\pi^2}{8192} - \frac{1918349}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{13723\pi^2}{16384} - \frac{2492417}{57600}\right)\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2}$$
$$+ \left(\frac{1411429}{19200} - \frac{1059\pi^2}{512}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{m_1^2 m_2^2} + \left(\frac{248991}{6400} - \frac{6153\pi^2}{2048}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^2 m_2^2}$$
$$- \left(\frac{30383}{960} + \frac{36405\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{1243717}{14400} - \frac{40483\pi^2}{16384}\right)\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2}$$
$$+ \left(\frac{2369}{60} + \frac{35655\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} + \left(\frac{43101\pi^2}{16384} - \frac{391711}{6400}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{m_1^3 m_2}$$
$$+ \left(\frac{56955\pi^2}{16384} - \frac{1646983}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2}, \quad \text{(A4d)}$$

$$H_{421}(\mathbf{x}_a,\mathbf{p}_a) = \frac{64861\mathbf{p}_1^2}{4800 m_1^2} - \frac{91(\mathbf{p}_1\cdot\mathbf{p}_2)}{8 m_1 m_2} + \frac{105\mathbf{p}_2^2}{32 m_2^2} - \frac{9841(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{1600 m_1^2} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{2 m_1 m_2}, \quad \text{(A4e)}$$

$$H_{422}(\mathbf{x}_a,\mathbf{p}_a) = \left(\frac{1937033}{57600} - \frac{199177\pi^2}{49152}\right)\frac{\mathbf{p}_1^2}{m_1^2} + \left(\frac{176033\pi^2}{24576} - \frac{2864917}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \left(\frac{282361}{19200} - \frac{21837\pi^2}{8192}\right)\frac{\mathbf{p}_2^2}{m_2^2}$$
$$+ \left(\frac{698723}{19200} + \frac{21745\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} + \left(\frac{63641\pi^2}{24576} - \frac{2712013}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2}$$
$$+ \left(\frac{3200179}{57600} - \frac{28691\pi^2}{24576}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_2^2}, \quad \text{(A4f)}$$

$$H_{40}(\mathbf{x}_a,\mathbf{p}_a) = -\frac{m_1^4}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169799}{2400}\right)m_1^3 m_2 + \left(\frac{44825\pi^2}{6144} - \frac{609427}{7200}\right)m_1^2 m_2^2. \quad \text{(A4g)}$$

$$H^{\rm nonloc}_{\rm 4PN}(t) = -\frac{1}{5}\frac{G^2 M}{c^8} I^{(3)}_{ij}(t) \times \mathrm{Pf}_{2r_{12}/c}\int_{-\infty}^{+\infty}\frac{\mathrm{d}v}{|v|} I^{(3)}_{ij}(t+v),$$



Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_{\odot}$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

Perturbative Theory of the Generation of Gravitational Radiation

Einstein ’16, ’18 (+ Landau-Lifshitz 41, and Fock ’55) : h_+, h_x and quadrupole formula

Relativistic, multipolar extensions of LO quadrupole radiation :

Sachs-Bergmann ’58, Sachs ’61, Mathews ’62, Peters-Mathews ’63, Pirani '64

Campbell-Morgan ’71,

Campbell et al ’75,

nonlinear effects:

Bonnor-Rotenberg ’66,

Epstein-Wagoner-Will ’75-76

Thorne ’80, .., Will et al 00

MPM Formalism:

Blanchet-Damour ’86,

Damour-Iyer ’91,

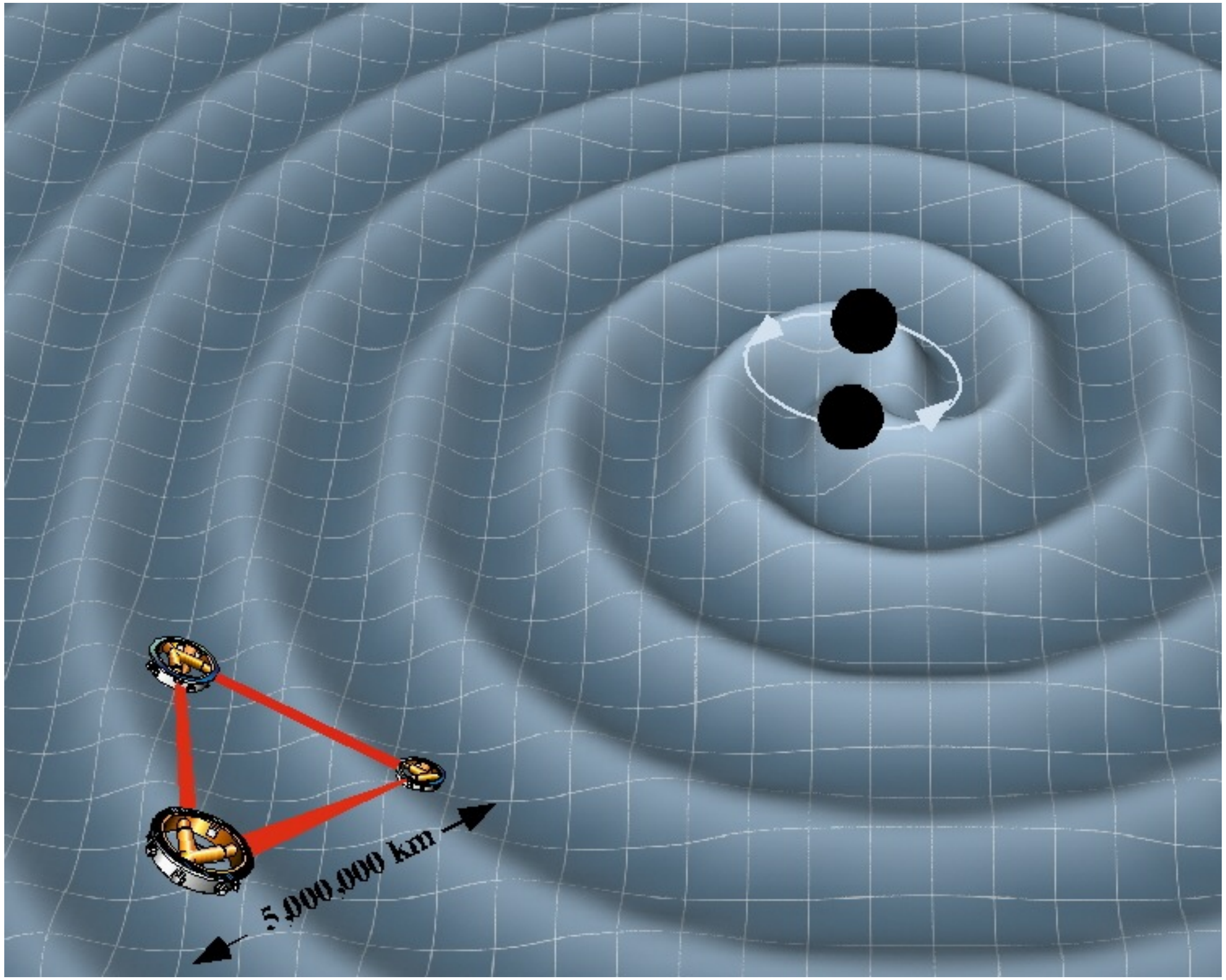
Blanchet ’95 ‘98

Combines multipole exp. ,

Post Minkowkian exp.,

analytic continuation,

and PN matching



Perturbative (3.5PN) GW flux from (circular) binary system

- lowest order : Einstein 1918 Peters-Mathews 63
- 1 + (v^2/c^2) : Wagoner-Will 76
- … + (v^3/c^3) : Blanchet-Damour 92, Wiseman 93
- … + (v^4/c^4) : Blanchet-Damour-Iyer Will-Wiseman 95
- … + (v^5/c^5) : Blanchet 96
- … + (v^6/c^6) : Blanchet-Damour-Esposito-Farèse-Iyer 2004
- … + (v^7/c^7) : Blanchet
- … + most of (v^8/c^8) : Blanchet et al

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$x = \left(\frac{v}{c}\right)^2 = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{\frac{2}{3}} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{2}{3}}$$

$$\begin{aligned}
\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ 1 &+ \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} \\
&+ \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} \\
&+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E - \frac{856}{105}\ln(16\,x) \right. \\
&\qquad \left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right] x^3 \\
&+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right) \Bigg\}.
\end{aligned}$$

LO quadrupole radiation

3.5PN

4PN still incomplete

Analytical GW Templates for BBH Coalescences ?

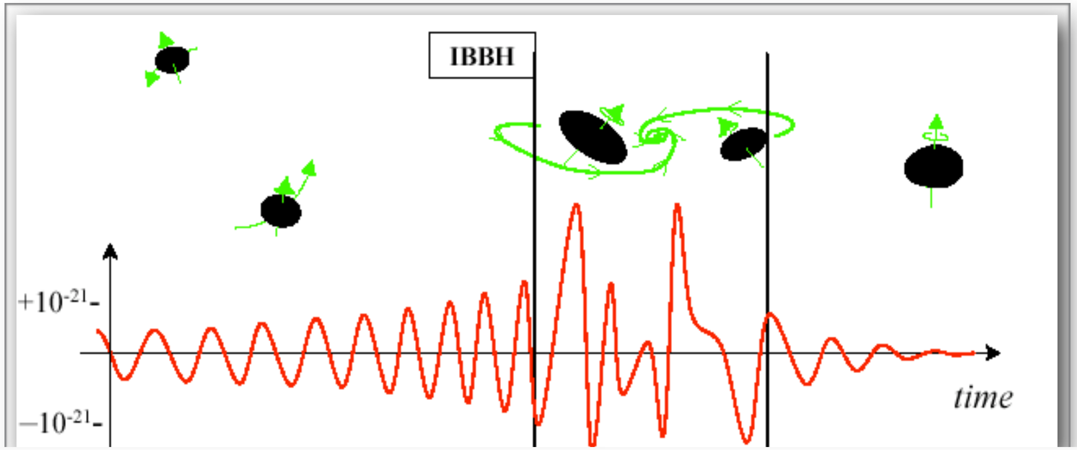
PN corrections to Einstein's quadrupole frequency « chirping »
from PN-improved balance equation dE(f)/dt = - F(f)

$$\frac{d\phi}{d\ln f} = \frac{\omega^2}{d\omega/dt} = Q_\omega^N \widehat{Q}_\omega \qquad \frac{v}{c} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{1}{3}}$$

$$Q_\omega^N = \frac{5\,c^5}{48\,\nu\,v^5}\,;\ \widehat{Q}_\omega = 1 + c_2\left(\frac{v}{c}\right)^2 + c_3\left(\frac{v}{c}\right)^3 + \cdots \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

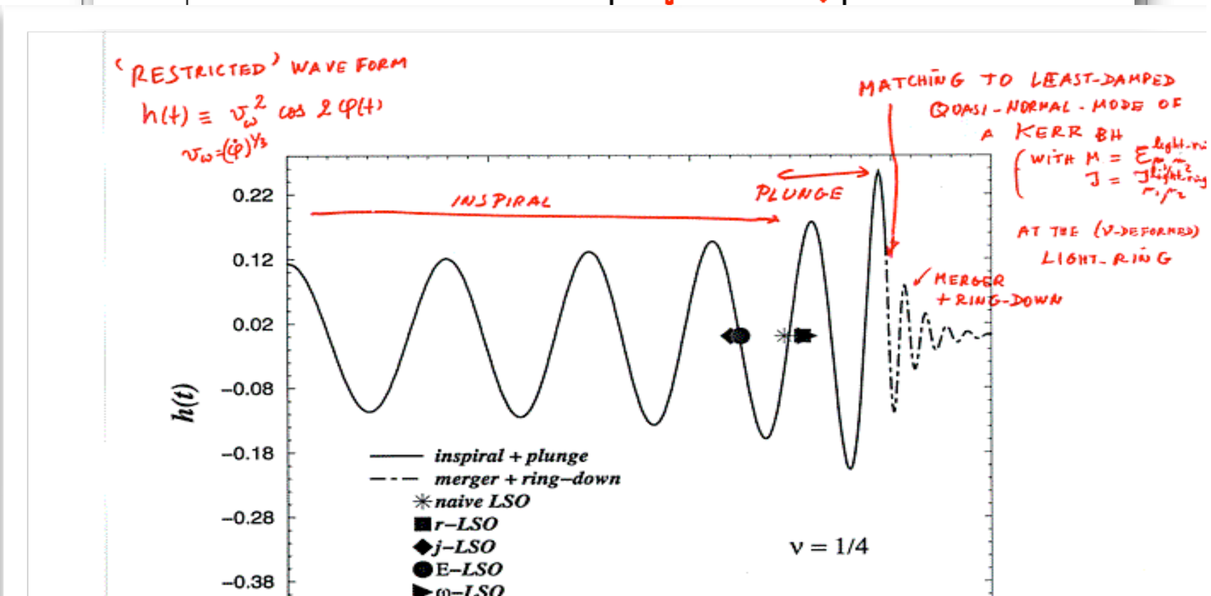
Cutler et al. '93:
« slow convergence of PN »

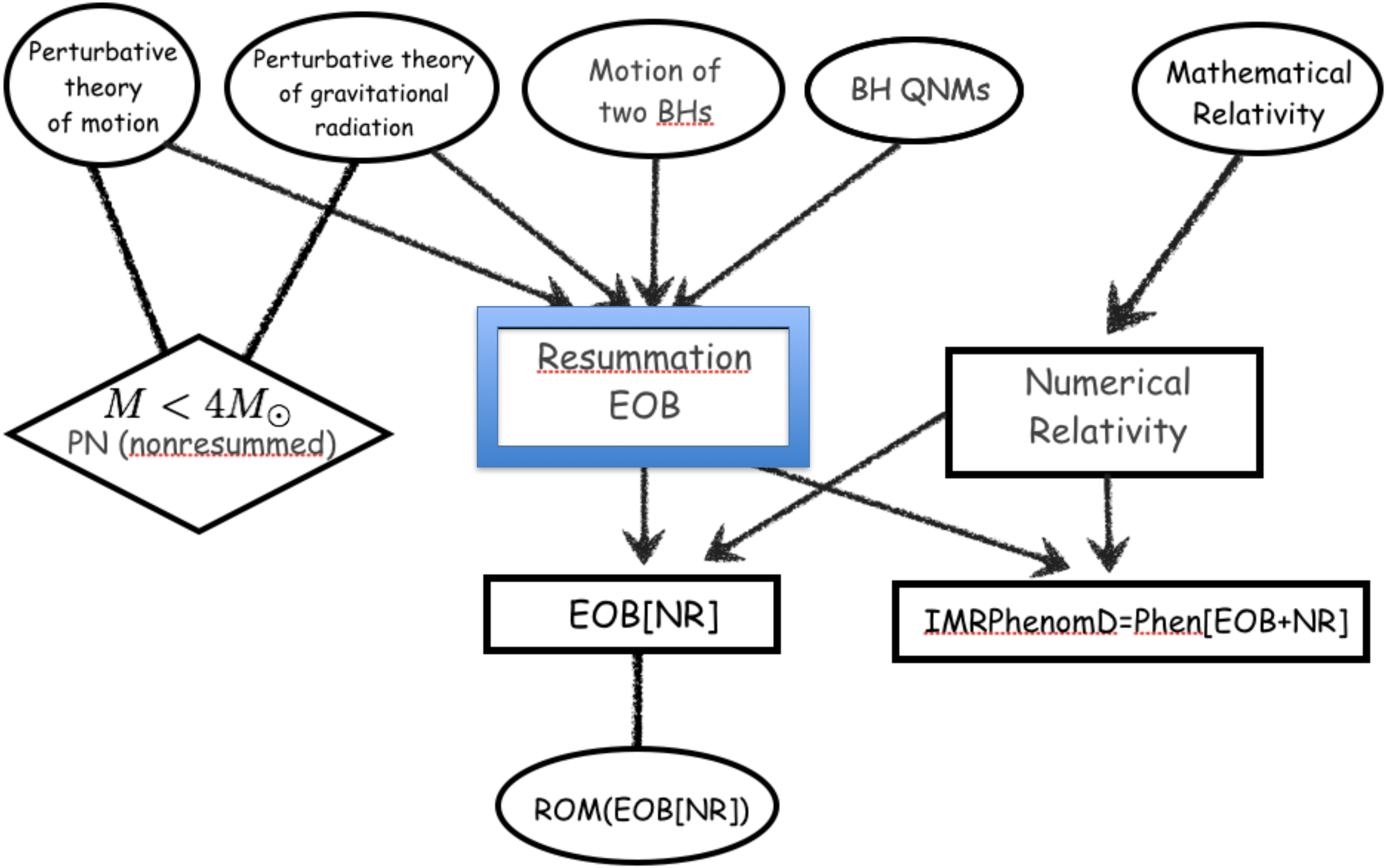
Brady-Creighton-Thorne'98:
« inability of current computational techniques to evolve a BBH through its last ~10 orbits of inspiral » and to compute the merger



Damour-Iyer-Sathyaprakash'98:
use resummation methods for E and F

Buonanno-Damour '99-00:
novel, resummed approach:
Effective-One-Body
analytical formalism





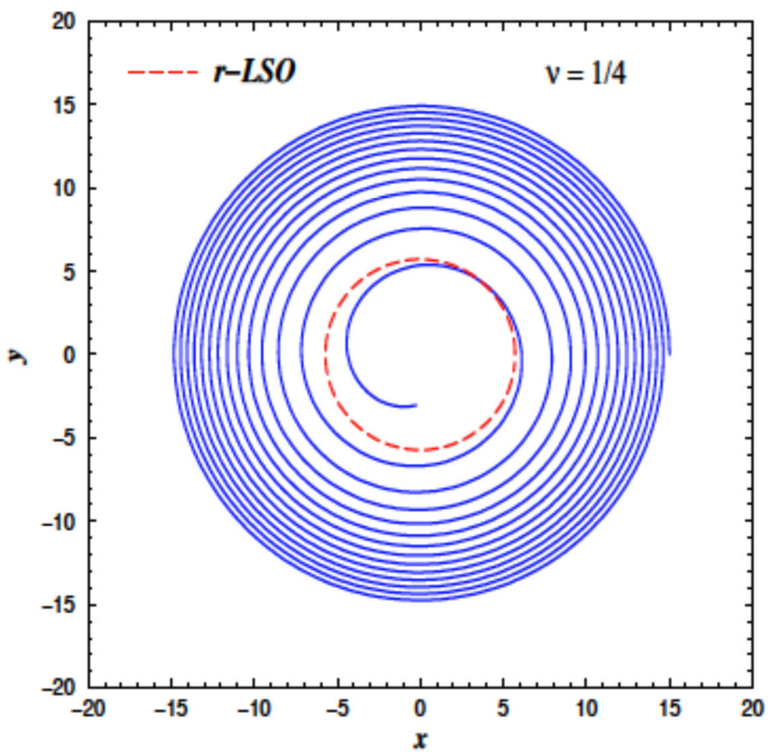
Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_\odot$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

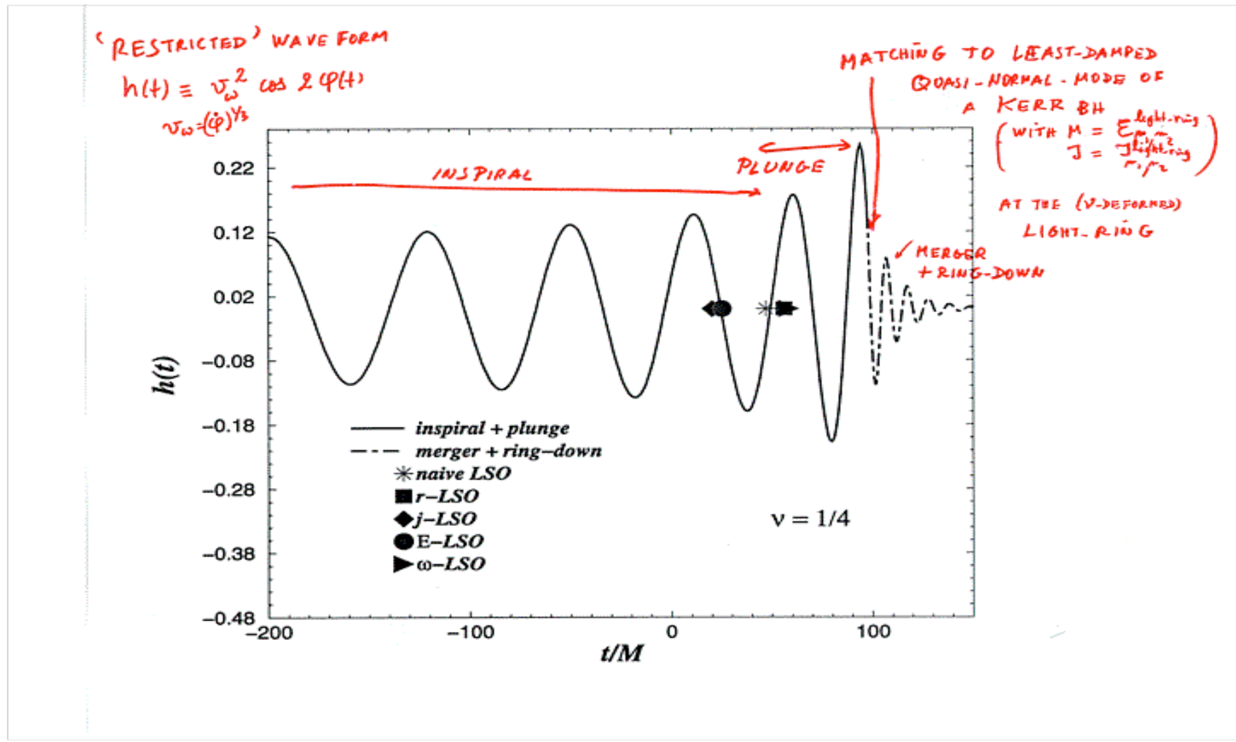
Effective One Body (EOB) Method

Buonanno-Damour 1999, 2000; Damour-Jaranowski-Schaefer 2000; Damour 2001 (SEOB)
[developped by: Barausse, Bini, Buonanno, Damour, Jaranowski, Nagar, Pan, Schaefer, Taracchini, …]

Resummation of perturbative PN results ⟶ description of the coalescence
+ addition of ringdown (Vishveshwara 70, Davis-Ruffini-Tiomno 1972) [+ CLAP (Price-Pullin'94)]

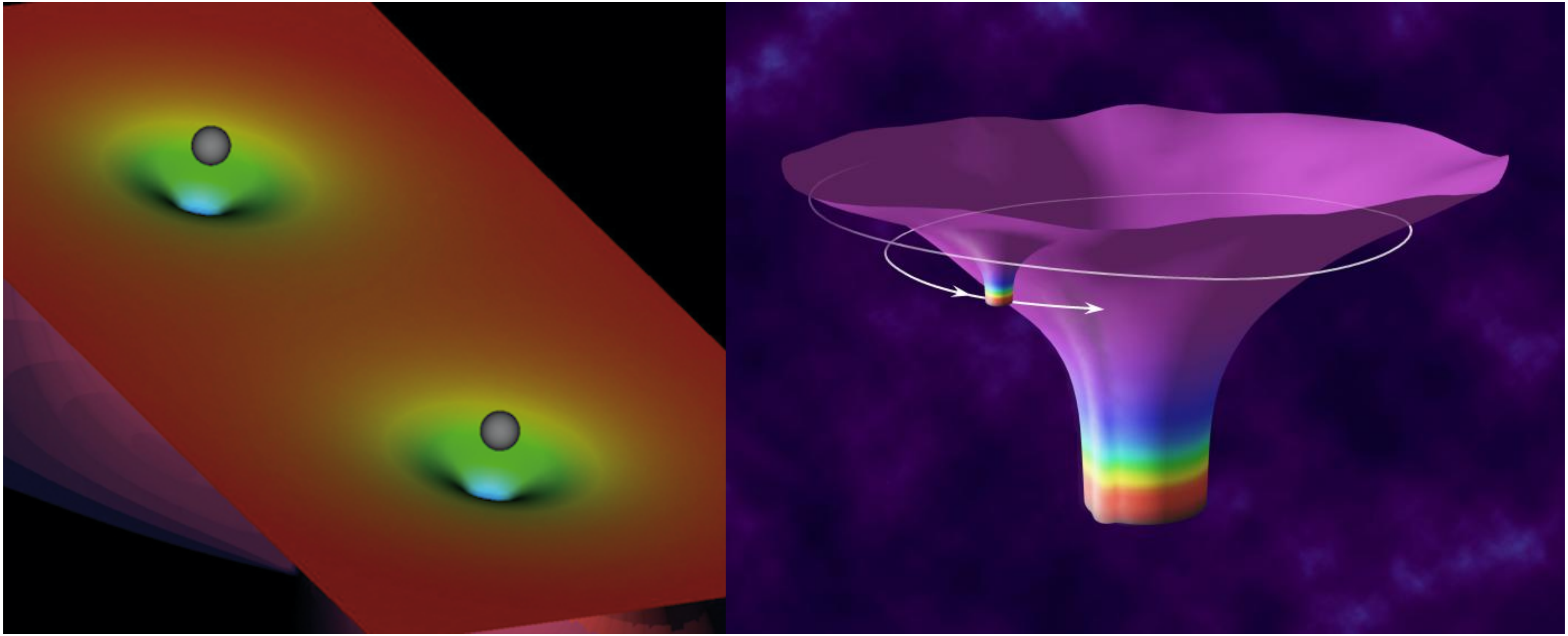
Buonanno-Damour 2000





Predictions as early as 2000 :
continued transition, non adiabaticity, first complete waveform, final spin (OK within 10%), final mass

EOB: resumming the dynamics of a two-body system (m_1,m_2,S_1,S_2) in terms of the dynamics of a particle of mass mu and spin S* moving in some effective metric g(M,S)



Effective metric for non-spinning bodies: a nu-deformation of Schwarzschild

$$M = m_1 + m_2 \qquad \mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \nu = \frac{\mu}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\boxed{ds^2_{\rm eff} = -A(r;\nu)\, dt^2 + B(r;\nu)\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)}$$

2-body Taylor-expanded N + 1PN + 2PN+ 3PN Hamiltonian

$$H_{\mathrm{N}}(\mathbf{x}_a,\mathbf{p}_a)=\frac{\mathbf{p}_1^2}{2m_1}-\frac{1}{2}\frac{Gm_1m_2}{r_{12}}+(1\leftrightarrow 2)$$

$$c^2H_{\mathrm{1PN}}(\mathbf{x}_a,\mathbf{p}_a)=-\frac{1}{8}\frac{(\mathbf{p}_1^2)^2}{m_1^3}+\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(-12\frac{\mathbf{p}_1^2}{m_1^2}+14\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2}+2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)$$
$$+\frac{1}{4}\frac{Gm_1m_2}{r_{12}}\frac{G(m_1+m_2)}{r_{12}}+(1\leftrightarrow 2),$$

$$c^4H_{\mathrm{2PN}}(\mathbf{x}_a,\mathbf{p}_a)=\frac{1}{16}\frac{(\mathbf{p}_1^2)^3}{m_1^5}+\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(5\frac{(\mathbf{p}_1^2)^2}{m_1^4}-\frac{11}{2}\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2}-\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}+5\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right.$$
$$\left.-6\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2}-\frac{3}{2}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right)$$
$$+\frac{1}{4}\frac{G^2m_1m_2}{r_{12}^2}\left(m_2\left(10\frac{\mathbf{p}_1^2}{m_1^2}+19\frac{\mathbf{p}_2^2}{m_2^2}\right)-\frac{1}{2}(m_1+m_2)\frac{27(\mathbf{p}_1\cdot\mathbf{p}_2)+6(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)$$
$$-\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\frac{G^2(m_1^2+5m_1m_2+m_2^2)}{r_{12}^2}+(1\leftrightarrow 2),$$

$$c^6H_{\mathrm{3PN}}(\mathbf{x}_a,\mathbf{p}_a)=-\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7}+\frac{1}{32}\frac{Gm_1m_2}{r_{12}}\left(-14\frac{(\mathbf{p}_1^2)^3}{m_1^6}+4\frac{((\mathbf{p}_1\cdot\mathbf{p}_2)^2+4\mathbf{p}_1^2\mathbf{p}_2^2)\mathbf{p}_1^2}{m_1^4m_2^2}+6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4m_2^2}\right.$$
$$-10\frac{(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2+\mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2)\mathbf{p}_1^2}{m_1^4m_2^2}+24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4m_2^2}$$
$$+2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3m_2^3}+\frac{(7\mathbf{p}_1^2\mathbf{p}_2^2-10(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3m_2^3}$$
$$+\frac{(\mathbf{p}_1^2\mathbf{p}_2^2-2(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3m_2^3}+15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3m_2^3}$$
$$\left.-18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3m_2^3}+5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3m_2^3}\right)+\frac{G^2m_1m_2}{r_{12}^2}\left(\frac{1}{16}(m_1-27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4}\right.$$
$$-\frac{115}{16}m_1\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3m_2}+\frac{1}{48}m_2\frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2+371\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2}+\frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3}+\frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3}$$
$$-\frac{1}{8}m_1\frac{(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)+11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1))(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3m_2}-\frac{3}{2}m_1\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3m_2}$$
$$+\frac{125}{12}m_2\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2}+\frac{10}{3}m_2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}$$
$$\left.-\frac{1}{48}(220m_1+193m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right)+\frac{G^3m_1m_2}{r_{12}^3}\left(-\frac{1}{48}\left(425m_1^2+\left(473-\frac{3}{4}\pi^2\right)m_1m_2+150m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2}\right.$$
$$+\frac{1}{16}\left(77(m_1^2+m_2^2)+\left(143-\frac{1}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2}+\frac{1}{16}\left(20m_1^2-\left(43+\frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2}$$
$$\left.+\frac{1}{16}\left(21(m_1^2+m_2^2)+\left(119+\frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)$$
$$+\frac{1}{8}\frac{G^4m_1m_2^3}{r_{12}^4}\left(\left(\frac{227}{3}-\frac{21}{4}\pi^2\right)m_1+m_2\right)+(1\leftrightarrow 2).$$

Explicit 3PN EOB dynamics (Damour-Jaranowski-Schaefer '01)

post-geodesic effective mass-shell:

$$g_{\rm eff}^{\mu\nu}\, P'_\mu\, P'_\nu + \mu^2\, c^2 + Q(P'_\mu) = 0\,,$$

$$ds^2_{\rm eff} = -A(R;\nu)dt^2 + B(R;\nu)dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$u \equiv \frac{GM}{R\,c^2}$$

$$M = m_1 + m_2\,, \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}\,, \qquad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2} = \frac{\mu}{M}$$

$$-P'_0 = \mathcal{E}_{\rm eff}$$

$$A^{\rm 3PN}(u) = 1 - 2u + 2\nu\, u^3 + \left(\frac{94}{3} - \frac{41}{32}\,\pi^2\right)\nu u^4,$$

$$\overline{D}^{\rm 3PN}(u) = 1 + 6\nu\, u^2 + (52\nu - 6\nu^2)\, u^3,$$

$$\widehat{Q}^{\rm 3PN} \equiv \frac{Q}{\mu^2 c^2} = (8\nu - 6\nu^2)\, u^2\, \frac{p_r^4}{c^4}.$$

$$P'_\varphi = J_{\rm eff} = J_{\rm real}$$

$$\mathcal{E}_{\rm eff} = \frac{(\mathcal{E}_{\rm real})^2 - m_1^2 - m_2^2}{2(m_1 + m_2)}.$$

Spinning EOB effective Hamiltonian

$$H_{\rm eff} = H_{\rm orb} + H_{\rm so} \quad \rightarrow H_{\rm EOB} = Mc^2\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu c^2}-1\right)}$$

$$\hat{H}^{\rm eff}_{\rm orb} = \sqrt{A\left(1 + B_p \boldsymbol{p}^2 + B_{np}(\boldsymbol{n}\cdot\boldsymbol{p})^2 - \frac{1}{1+\frac{(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}{r^2}}\frac{(r^2+2r+(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2)}{\mathcal{R}^4+\Delta(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}((\boldsymbol{n}\times\boldsymbol{p})\cdot\boldsymbol{\chi}_0)^2 + Q_4\right)}.$$

$$H_{\rm so} = G_S \boldsymbol{L}\cdot\boldsymbol{S} + G_{S^*}\boldsymbol{L}\cdot\boldsymbol{S}^*,$$

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}\,;\; \mathbf{S}_* = \frac{m_2}{m_1}\mathbf{S_1} + \frac{m_1}{m_2}\mathbf{S_2}\,,$$

Gyrogravitomagnetic ratios (when neglecting spin^2 effects)

$$r^3 G_S^{\rm PN} = 2 - \frac{5}{8}\nu u - \frac{27}{8}\nu p_r^2 + \nu\left(-\frac{51}{4}u^2 - \frac{21}{2}up_r^2 + \frac{5}{8}p_r^4\right) + \nu^2\left(-\frac{1}{8}u^2 + \frac{23}{8}up_r^2 + \frac{35}{8}p_r^4\right)$$

$$r^3 G_{S_*}^{\rm PN} = \frac{3}{2} - \frac{9}{8}u - \frac{15}{8}p_r^2 + \nu\left(-\frac{3}{4}u - \frac{9}{4}p_r^2\right) - \frac{27}{16}u^2 + \frac{69}{16}up_r^2 + \frac{35}{16}p_r^4 + \nu\left(-\frac{39}{4}u^2 - \frac{9}{4}up_r^2 + \frac{5}{2}p_r^4\right) + \nu^2\left(-\frac{3}{16}u^2 + \frac{57}{16}up_r^2 + \frac{45}{16}p_r^4\right)$$

Resummed EOB waveform

(Damour-Iyer-Sathyaprakash '98) Damour-Nagar '07, Damour-Iyer -Nagar '08, Pan et al. '10

$$h_{\ell m} \equiv h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \hat{h}^{\rm NQC}_{\ell m}$$

$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell + 1)} e^{\pi \hat{\hat{k}}} e^{2i\hat{\hat{k}} \ln(2kr_0)}$$

NB: T_lm resums an infinite number of terms and already contains, eg, 4.5PN tail^3 terms (Messina-Nagar17)

$$\rho_{22}(x;\nu) = 1 + \left(\frac{55\nu}{84} - \frac{43}{42}\right) x + \left(\frac{19583\nu^2}{42336} - \frac{33025\nu}{21168} - \frac{20555}{10584}\right) x^2$$
$$+ \left(\frac{10620745\nu^3}{39118464} - \frac{6292061\nu^2}{3259872} + \frac{41\pi^2\nu}{192} - \frac{48993925\nu}{9779616} - \frac{428}{105}\mathrm{eulerlog}_2(x) + \frac{1556919113}{122245200}\right) x^3$$
$$+ \left(\frac{9202}{2205}\mathrm{eulerlog}_2(x) - \frac{387216563023}{160190110080}\right) x^4 + \left(\frac{439877}{55566}\mathrm{eulerlog}_2(x) - \frac{16094530514677}{533967033600}\right) x^5 + \mathcal{O}(x^6),$$

$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\rm max}} \sum_{m=1}^{\ell} (m\Omega)^2 |R h^{(\epsilon)}_{\ell m}|^2$$

EOB

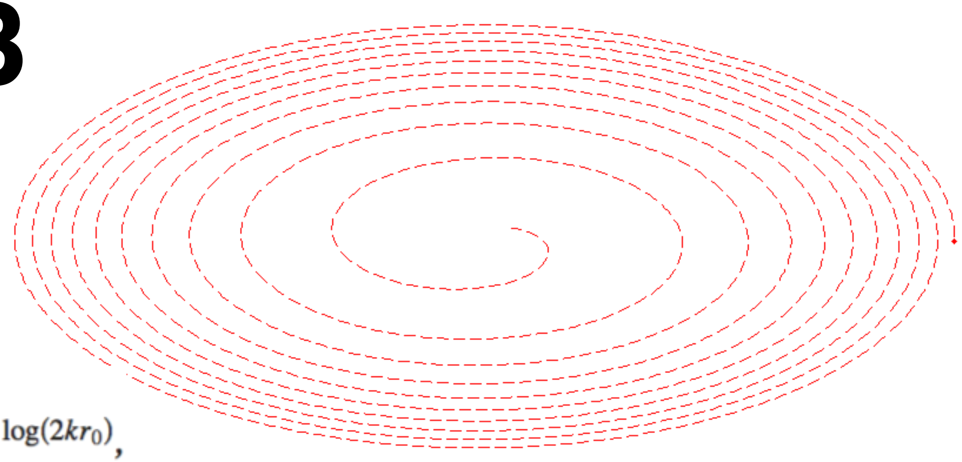
$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}} \,,$$

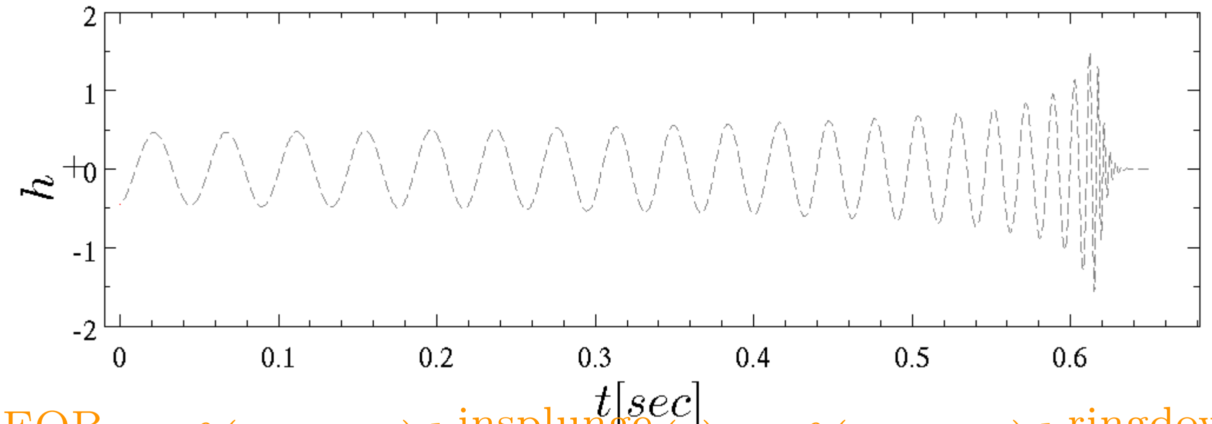
$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} \,,$$

$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi}$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi \,.$$

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi \hat{\hat{k}}} e^{2i\hat{\hat{k}} \log(2kr_0)},$$





$$h_{\ell m} \equiv h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \hat{h}^{\rm NQC}_{\ell m}$$

$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\rm max}} \sum_{m=1}^{\ell} (m\Omega)^2 |Rh^{(\epsilon)}_{\ell m}|^2$$

$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}$$

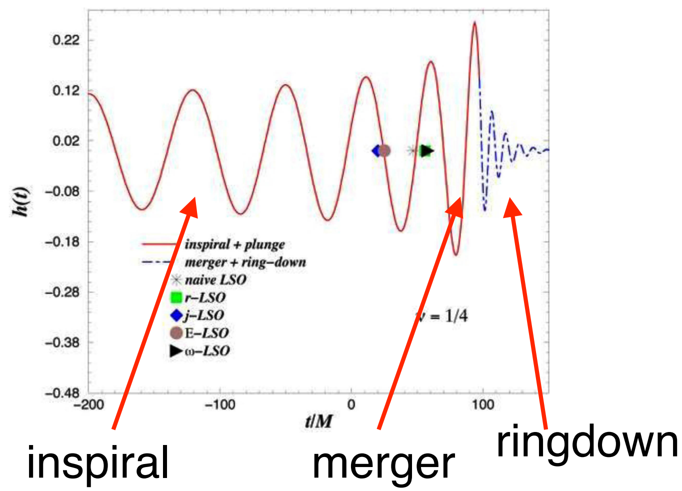
$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N (t - t_m)}$$

First complete waveforms for BBH coalescences:

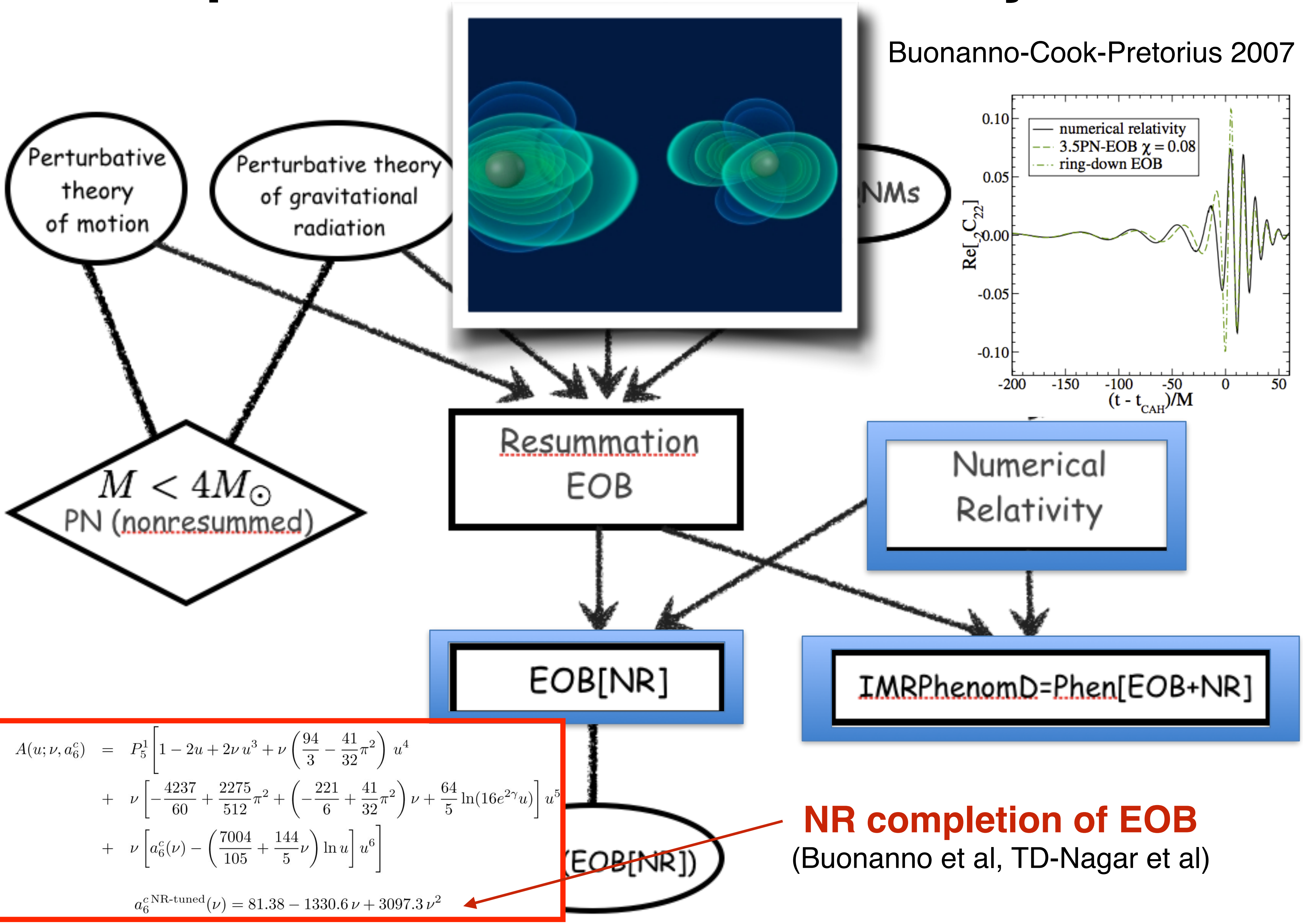
analytical EOB (Buonanno-Damour'00,Buonanno-Chen-Damour'05)

After the 2005 NR breakthrough (Pretorius,...)

development of the NR-completed EOB waveforms



The impact of Numerical Relativity (Pretorius 2005,…)



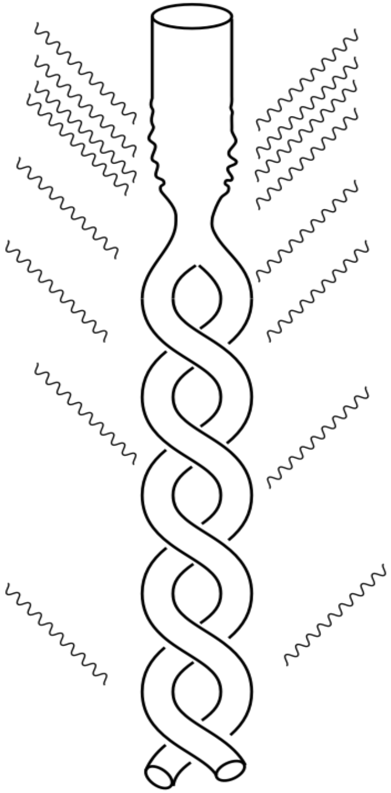
$$
\begin{aligned}
A(u;\nu,a_6^c) &= P_5^1\Big[1-2u+2\nu\,u^3+\nu\left(\frac{94}{3}-\frac{41}{32}\pi^2\right)u^4 \\
&+ \nu\left[-\frac{4237}{60}+\frac{2275}{512}\pi^2+\left(-\frac{221}{6}+\frac{41}{32}\pi^2\right)\nu+\frac{64}{5}\ln(16e^{2\gamma}u)\right]u^5 \\
&+ \nu\left[a_6^c(\nu)-\left(\frac{7004}{105}+\frac{144}{5}\nu\right)\ln u\right]u^6\Big]
\end{aligned}
$$

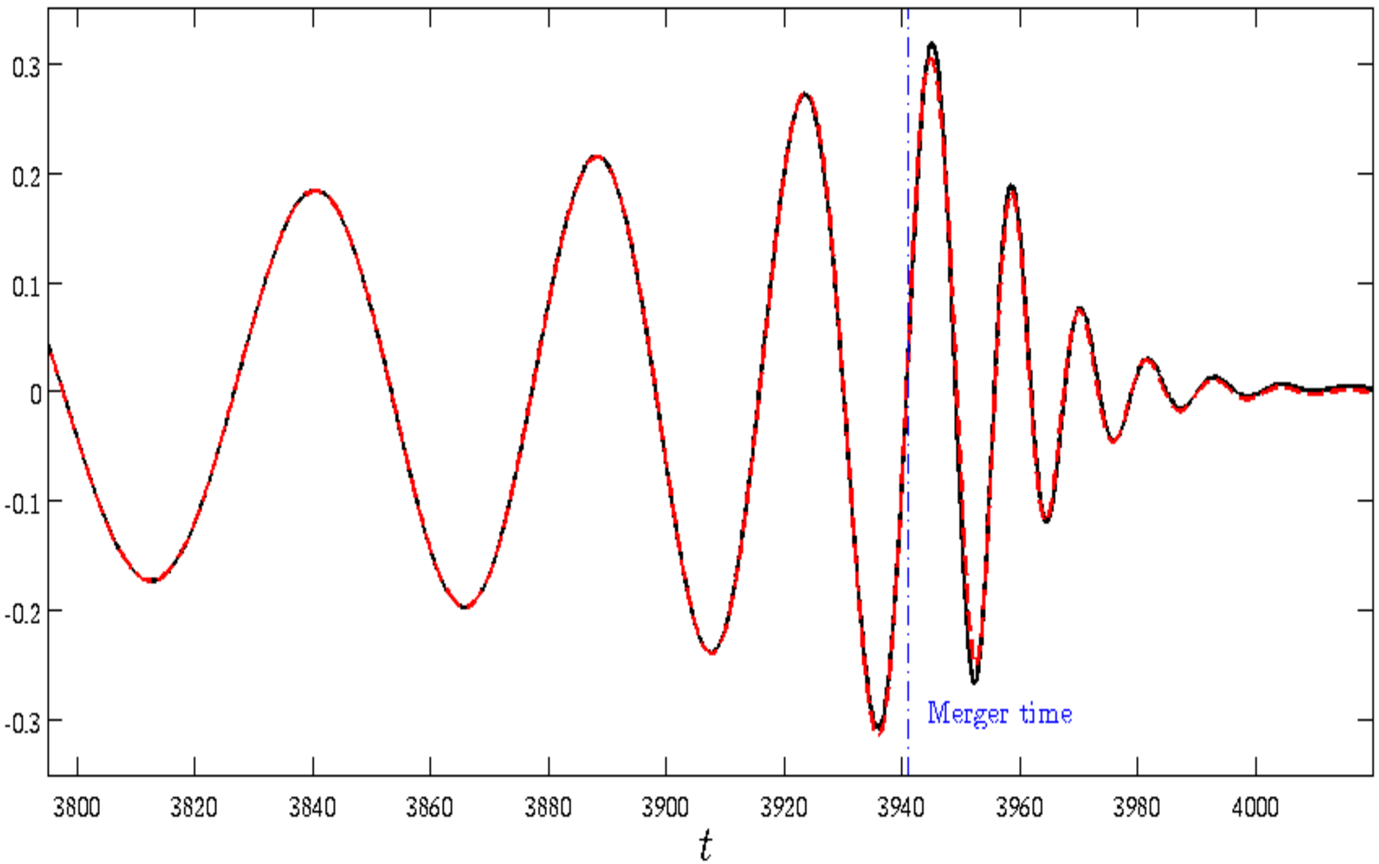
$$a_6^{c\,\text{NR-tuned}}(\nu) = 81.38 - 1330.6\,\nu + 3097.3\,\nu^2$$

NR completion of EOB

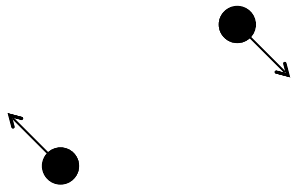
(Buonanno et al, TD-Nagar et al)

EOB[NR] / NR Comparison



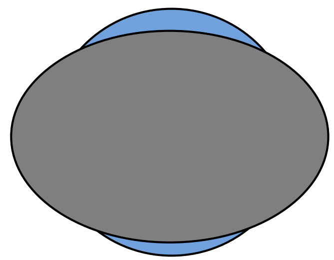


Inspiral + « plunge »



Two orbiting point-masses:
Resummed dynamics

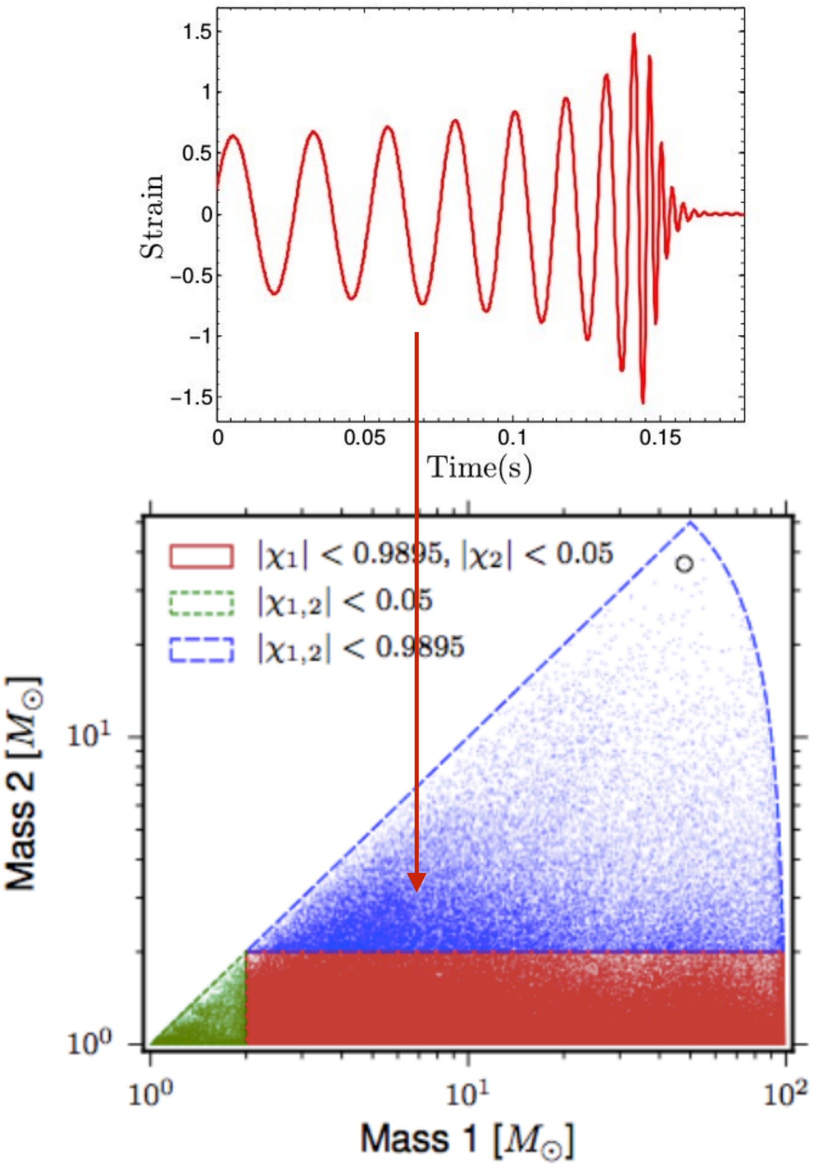
Ringing BH



Instantaneous GW power at coalescence ~ 10^56 erg/s ~ 10^-3 c^5/G

MATCHED FILTERING SEARCH AND DATA ANALYSIS

Banks of templates (e.g. 250 000 EOBNR templates in O1) for search inspiralling and coalescing BBH GW waveforms: m1, m2, chi1=S1/m1^2, chi2=S2/m2^2 for m1+m2> 4Msun; + ~ 50 000 PN inspiralling templates for m1+m2< 4 Msun;
O2: ~ 325 000 EOB templates + 75 000 PN templates



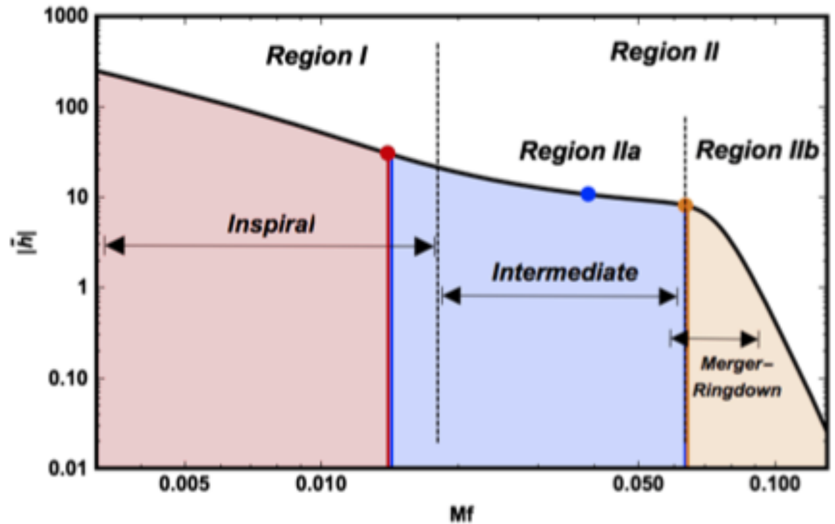
Two types of templates:

Bank of spinning EOB[NR] templates
(Taracchini et al. 14, Bohé et al'17) in ROM form (Puerrer et al.'14); Nagar et al…

Bank of Phenom[EOB+NR] templates
(Ajith…'07, Hannam…'14, Husa…'16, Khan…'16)

$$h(f) = A(f)e^{i\Psi(f)}$$

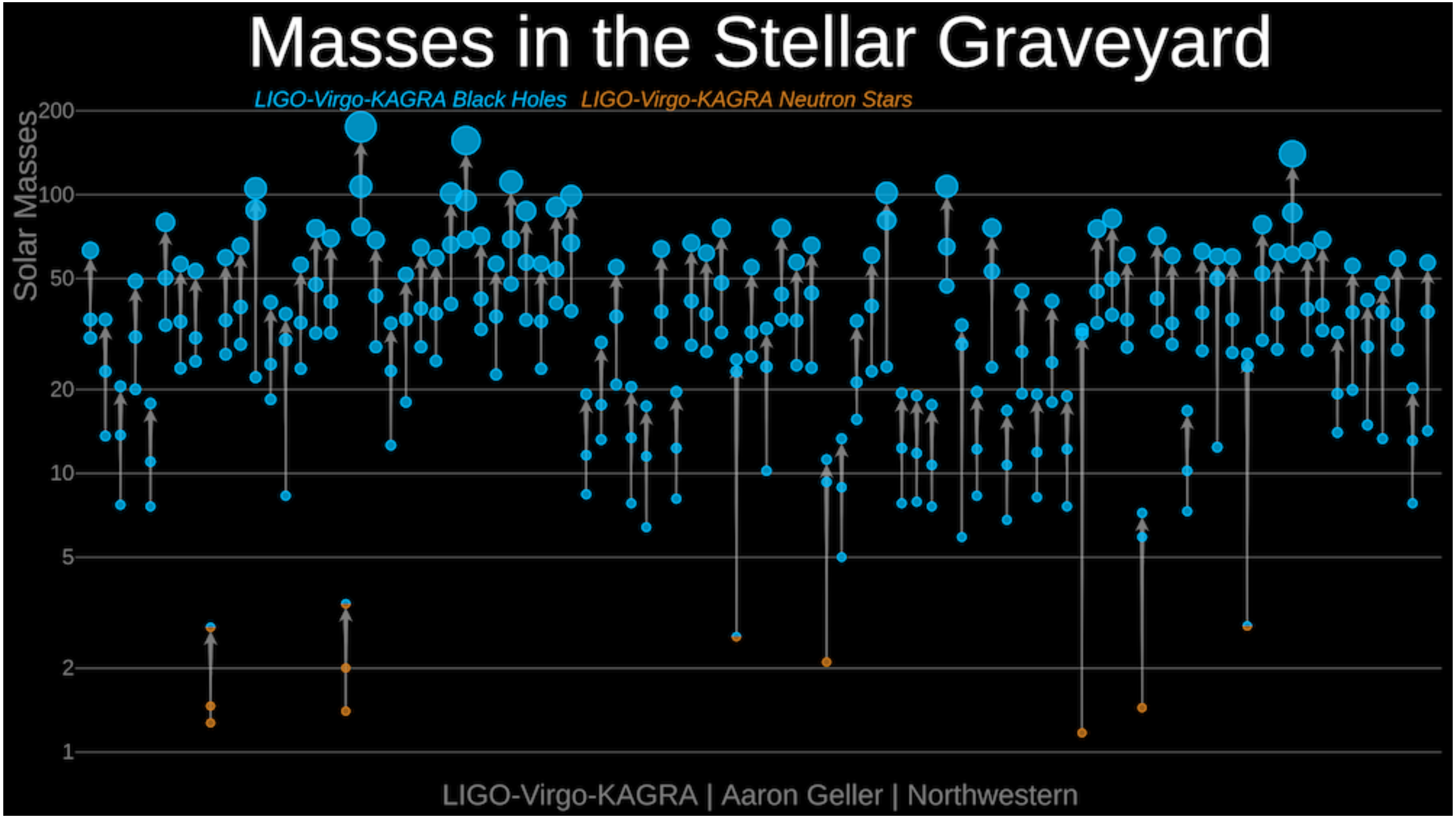
$$\Psi(f) = \sum_n c_n v^n(f)\ ;\ v(f) \equiv (\pi M f)^{\frac{1}{3}}$$



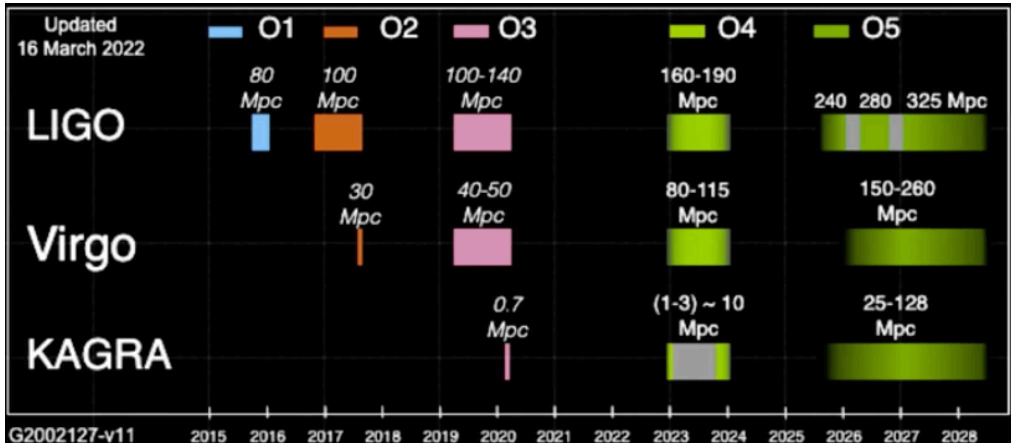
LIGO-Virgo p>0.5 Events

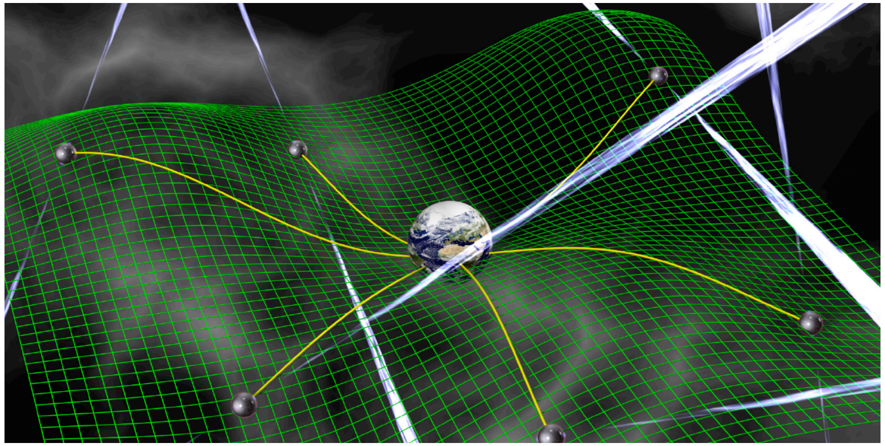
(O1-O2-O3a-O3b; nov 2021)

90 events, incl.: 2 NS-NS; 3 NS-BH; 85 BH-BH



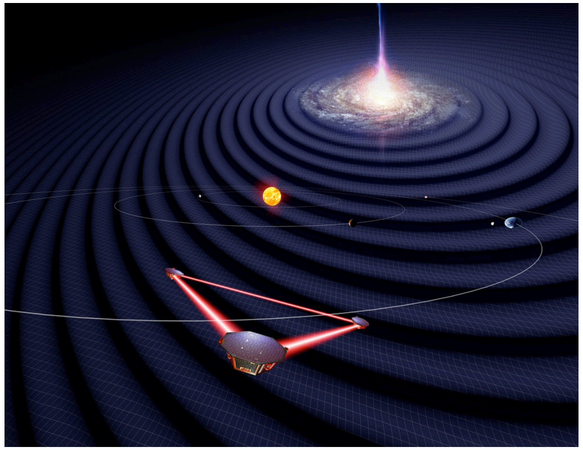
Towards the Future



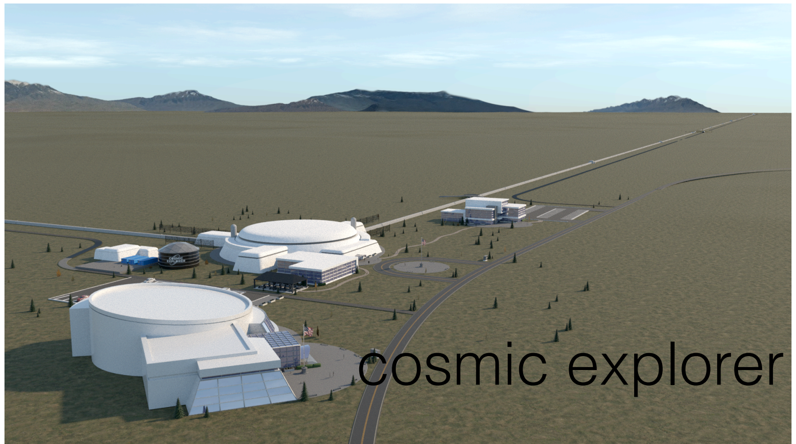


pulsar timing array

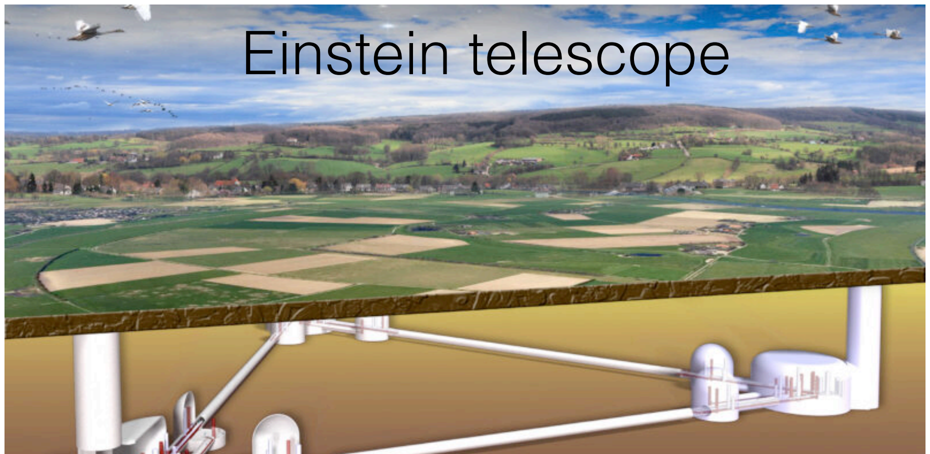
ligo india



lisa



cosmic explorer

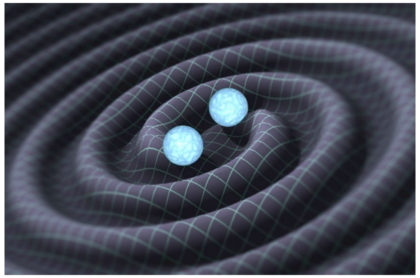


Einstein telescope

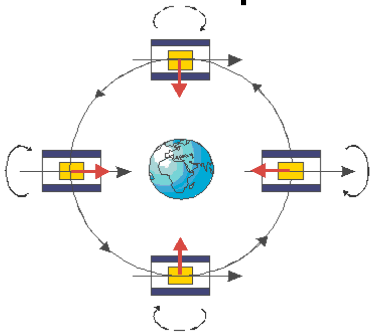
Probing Fundamental Physics with Gravitational Data (GR and Beyond)

(review: particle data group)

grav. waves

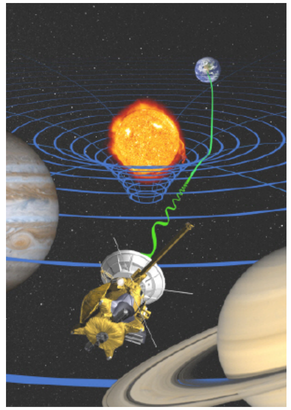


Microscope

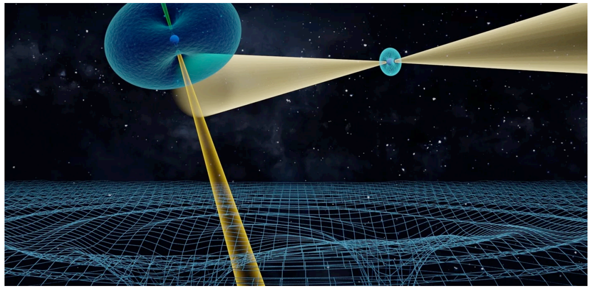


Lunar laser ranging

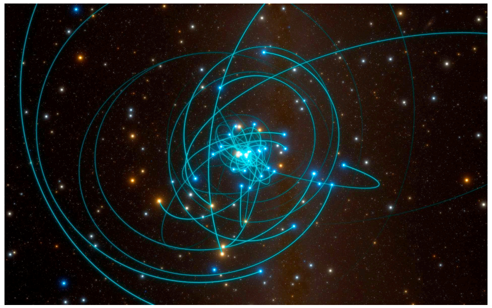
Cassini



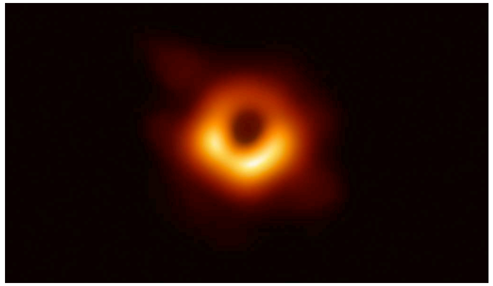
Binary pulsars



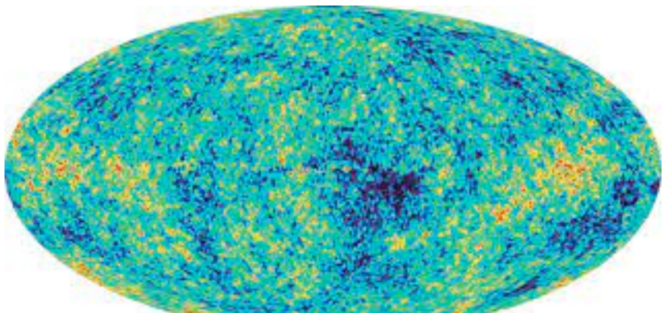
Galactic center



EHT



Cosmology



Phenomenological approaches to GR deviations

Equivalence Principle:

$$\frac{a_A - a_B}{\bar{a}} = 0\ ? \quad \frac{d\alpha/dt}{\alpha} = 0\ ?$$

Post-Einsteinian

Weak-field Gravity (PPN):

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2\beta}{c^4}V^2 + O\left(\frac{1}{c^6}\right)$$

$$g_{0i} = -\frac{2(\gamma+1)}{c^3}V_i + O\left(\frac{1}{c^5}\right),$$

$$g_{ij} = \delta_{ij}\left[1 + \frac{2\gamma}{c^2}V\right] + O\left(\frac{1}{c^4}\right),$$

$$\beta = 1\ ?\ \gamma = 1$$

Keplerian

Post-Keplerian

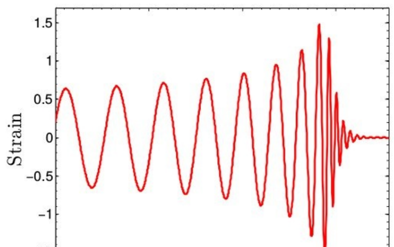
Binary-pulsar timing (PPK) strong-field +radiative gravity:

$$t_N - t_0 = F[T_N(\nu_p, \dot{\nu}_p, \ddot{\nu}_p)\,;\, \{p^K\}\,;\, \{p^{PK}\}]$$

$$\begin{aligned}
k^{\mathrm{GR}}(m_1, m_2) &= 3(1-e^2)^{-1}(GMn/c^3)^{2/3}\,, \\
\gamma^{\mathrm{GR}}_{\mathrm{timing}}(m_1, m_2) &= en^{-1}(GMn/c^3)^{2/3} m_2(m_1 + 2m_2)/M^2\,, \\
\dot{P}^{\mathrm{GR}}_b(m_1, m_2) &= -(192\pi/5)(1-e^2)^{-7/2}\left(1 + \tfrac{73}{24}e^2 + \tfrac{37}{96}e^4\right) \\
&\quad \times (GMn/c^3)^{5/3} m_1 m_2/M^2\,, \\
r^{\mathrm{GR}}(m_1, m_2) &= Gm_2/c^3\,, \\
s^{\mathrm{GR}}(m_1, m_2) &= nx(GMn/c^3)^{-1/3} M/m_2\,.
\end{aligned}$$

Phenomenological tests of GW data

$$h_{obs}(t) = h_{GR}(t; p_i)\ ?$$



$$\psi(f) = \sum_i \left[p_i^{\mathrm{GR,NS}}(m_1, m_2)(1 + \delta\hat{p}_i) + p_i^{\mathrm{GR,S}}(m_1, m_2, S_1, S_2)\right] u_i(f).$$

$$\omega_a = (c^3/GM_f)[2\pi \hat{f}_a^{\mathrm{QNM}}(a_f) - i/\hat{\tau}_a^{\mathrm{QNM}}(a_f)]$$

Tests of the Equivalence Principle

Variation of « constants »

$$\alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137.036}$$

$$\omega_a = (c^3/GM_f)[2\pi \hat{f}_a^{\rm QNM}(a_f) - i/\hat{\tau}_a^{\rm QNM}(a_f)]$$

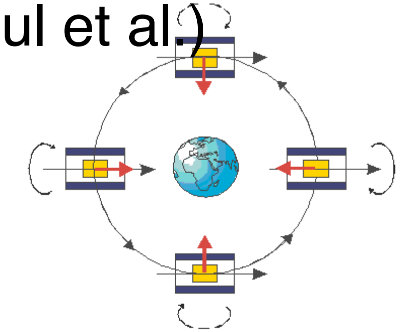
$$d\ln(\alpha_{\rm em})/dt = (-2.5 \pm 2.6) \times 10^{-17}{\rm yr}^{-1},$$
$$d\ln(\mu)/dt = (-1.5 \pm 3.0) \times 10^{-16}{\rm yr}^{-1},$$
$$d\ln(m_q/\Lambda_{\rm QCD})/dt = (7.1 \pm 4.4) \times 10^{-15}{\rm yr}^{-1}.$$

cosmological
Oklo
Atomic-clocks

Eotvos (Adelberger et al)

$$(\Delta a/a)_{\rm BeTi} = (0.3 \pm 1.8) \times 10^{-13};$$
$$(\Delta a/a)_{\rm BeAl} = (-0.7 \pm 1.3) \times 10^{-13};$$
$$(\Delta a/a)_{\rm TiPt} = (-1 \pm 9({\rm stat}) \pm 9({\rm syst})) \times 10^{-15}$$

Microscope
(Touboul et al.)



$$(\Delta a/a)_{\rm EarthMoon} = (-3 \pm 5) \times 10^{-14}.$$

Lunar-Laser ranging

Tests of the 1/r^2 law

$$V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha \exp(-r/\lambda)],$$
$$|\alpha| < 1 \text{ down to } 56\mu$$

(Kapner et al)

Phenomenological tests of post-Newtonian gravity (solar system)

Two main post-Newtonian parameters

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2\beta}{c^4}V^2 + O\left(\frac{1}{c^6}\right)$$

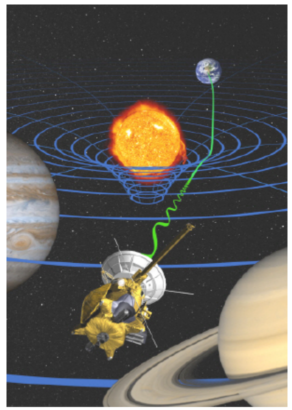
$$g_{0i} = -\frac{2(\gamma+1)}{c^3}V_i + O\left(\frac{1}{c^5}\right),$$

$$g_{ij} = \delta_{ij}\left[1 + \frac{2\gamma}{c^2}V\right] + O\left(\frac{1}{c^4}\right),$$

$$\bar{\gamma} = \gamma - 1$$

$$\bar{\beta} = \beta - 1$$

Cassini Mission



$$\overline{\gamma} = (2.1 \pm 2.3) \times 10^{-5}$$

$$|\overline{\beta}| < 7 \times 10^{-5}$$

Phenomenological Binary Pulsar Tests: strong and radiative fields

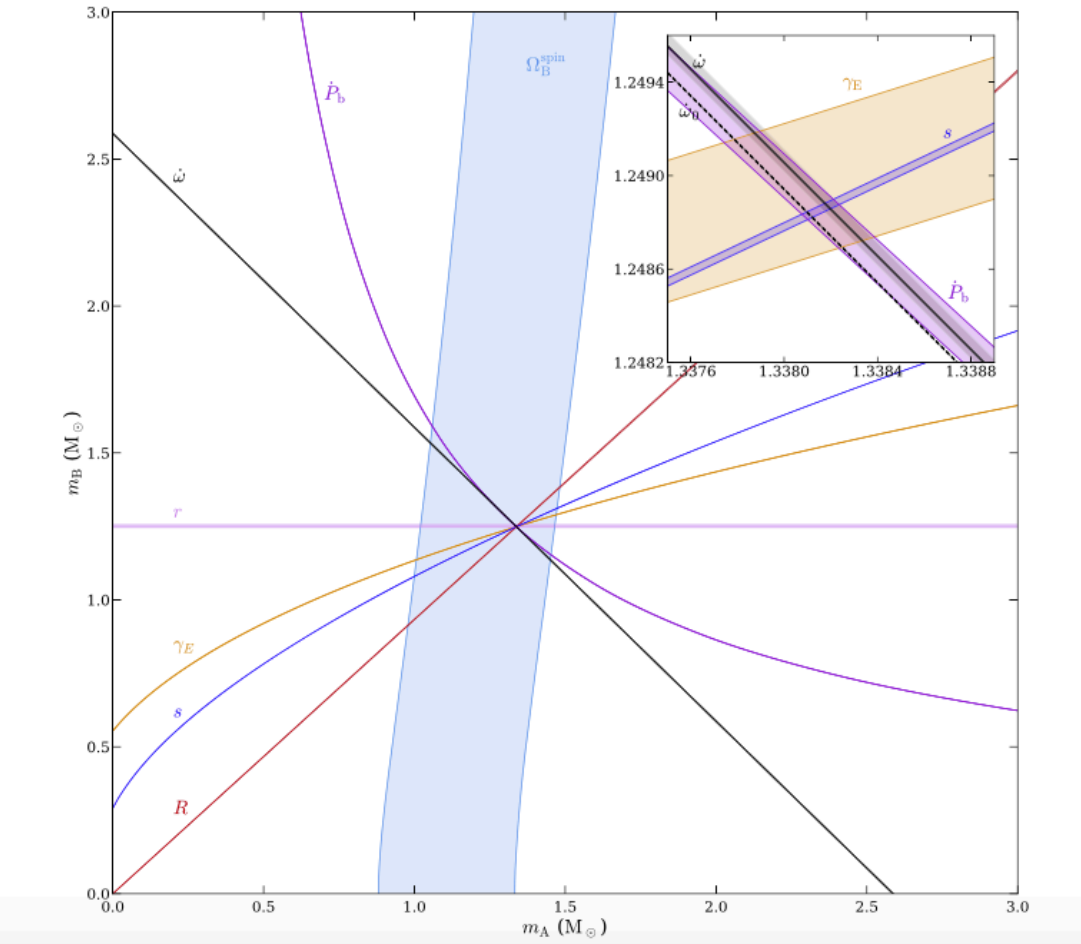
PSR1913+16 (Hulse-Taylor)

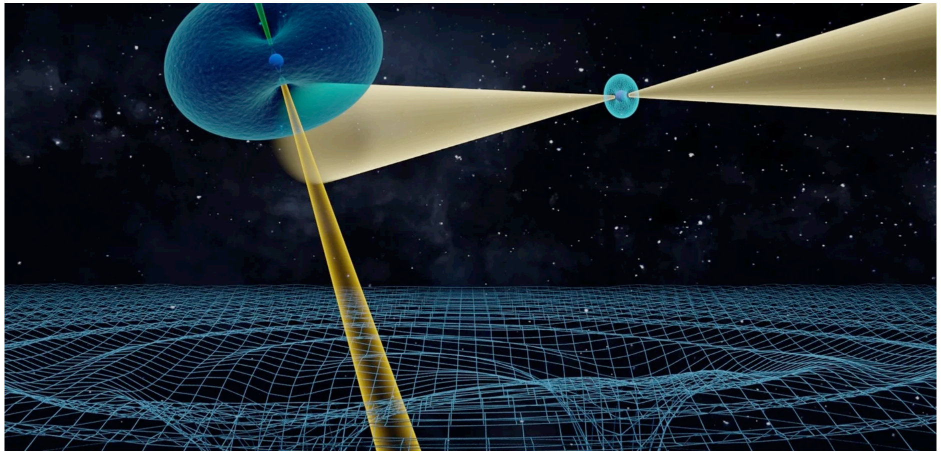
$$\left[\frac{\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal}}{\dot{P}_b^{\rm GR}[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]}\right]_{1913+16} = 0.9983 \pm 0.0016$$

Double Pulsar(Kramer et al)

5 precision tests of GR

M. KRAMER *et al.* PHYS. REV. X **11,** 041050 (2021)





$$\dot{P}_{\rm b}^{\rm GW} / \dot{P}_{\rm b}^{\rm GW,GR} = 0.999963(63)$$

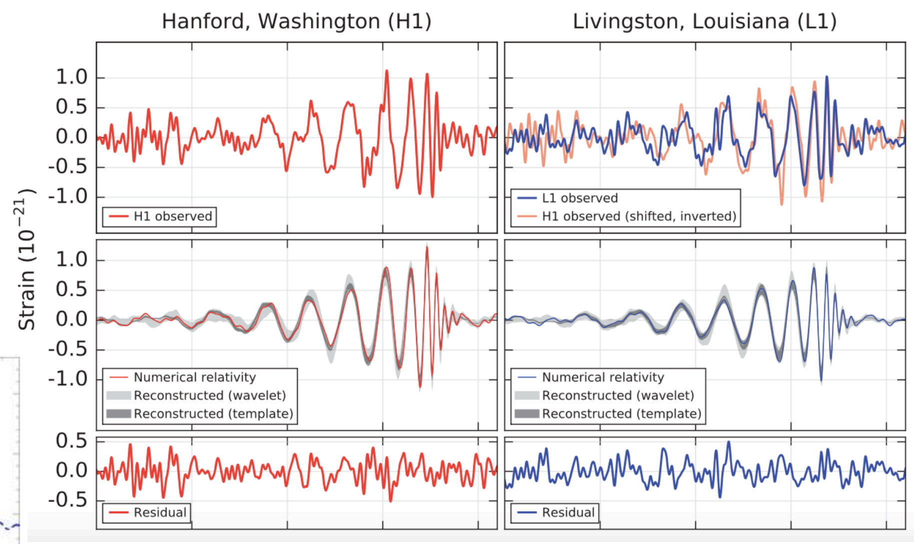
$$s^{\rm obs} / s^{\rm GR} = 1.000\,09(18)$$

Triple Pulsar (SEP)

$$|\Delta a/a| < 2.05 \times 10^{-6} \ (95\% \ {\rm C.L.})$$

Phenomenological GR tests from LIGO-Virgo

The most direct evidence that the BHs predicted by GR exist and have the expected structure

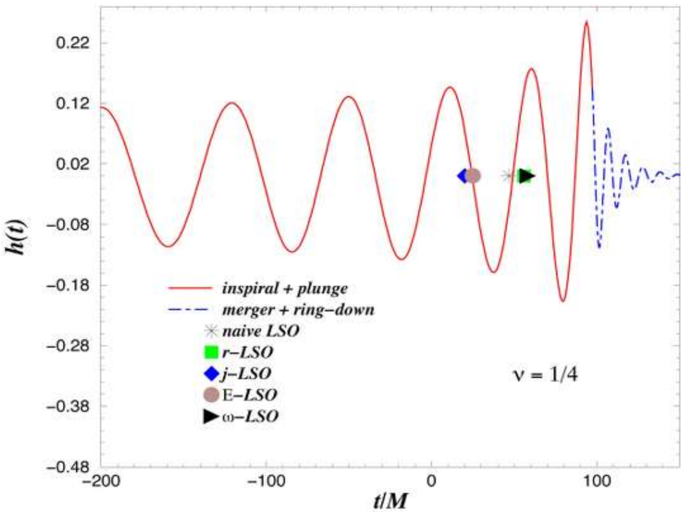


Global Fitting factor observed/predicted signal

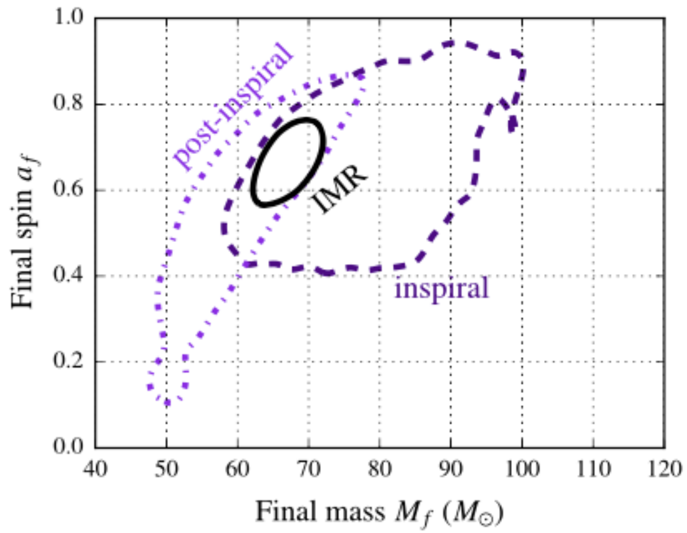
$$\frac{SNR_{GR}}{\sqrt{SNR_{GR}^2 + SNR_{res,90}^2}} = 0.97$$

Dividing in inspiral and post-inspiral

Confirmation of final damped vibration modes



PRL **116,** 221101 (2016)



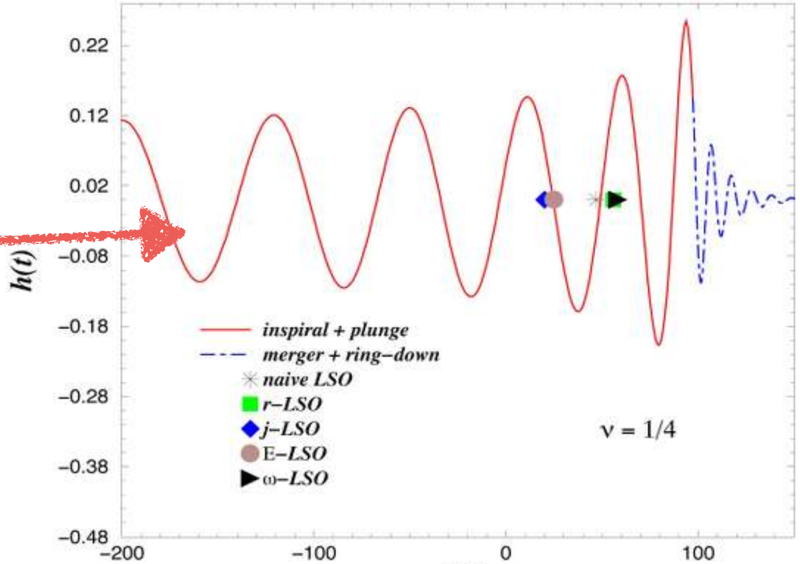
recent pSEOBNRv4HM analysis of GWTC3

$$\delta \hat{f}_{220} = 0.02^{+0.03}_{-0.03}; \delta\hat{\tau}_{220} = 0.13^{+0.11}_{-0.11}$$

Phenomenological GR tests LIGO-Virgo (2)

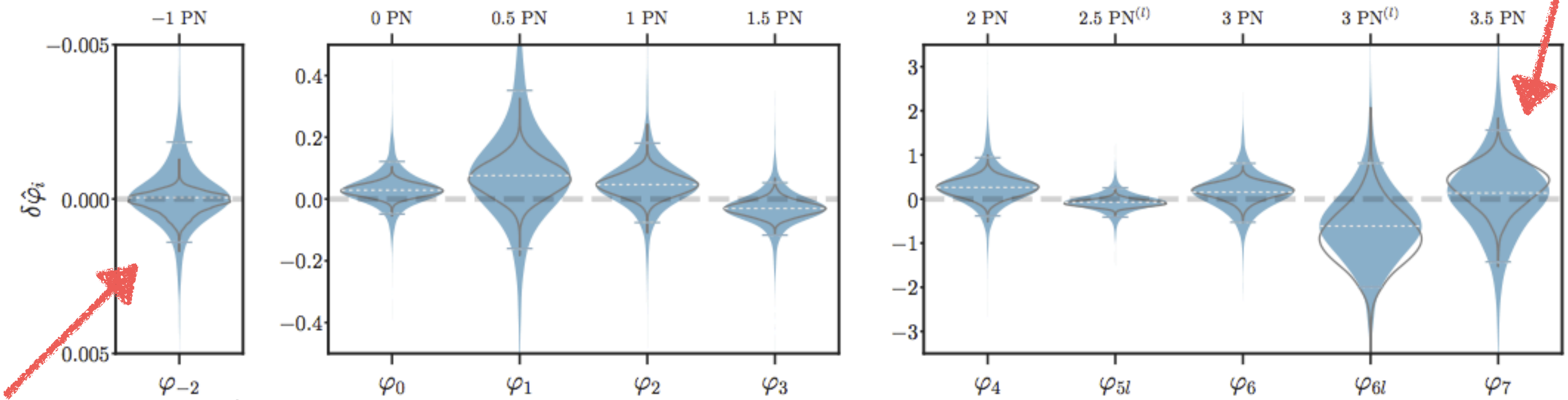
Phenom. tests of inspiral signal

$$h(f) = A(f)e^{i\Psi(f)}$$



$$\psi(f) = \sum \left[p_i^{\mathrm{GR,NS}}(m_1, m_2)(1 + \delta\hat{p}_i) + p_i^{\mathrm{GR,S}}(m_1, m_2, S_1, S_2)\right] u_i(f).$$

O(1) limit



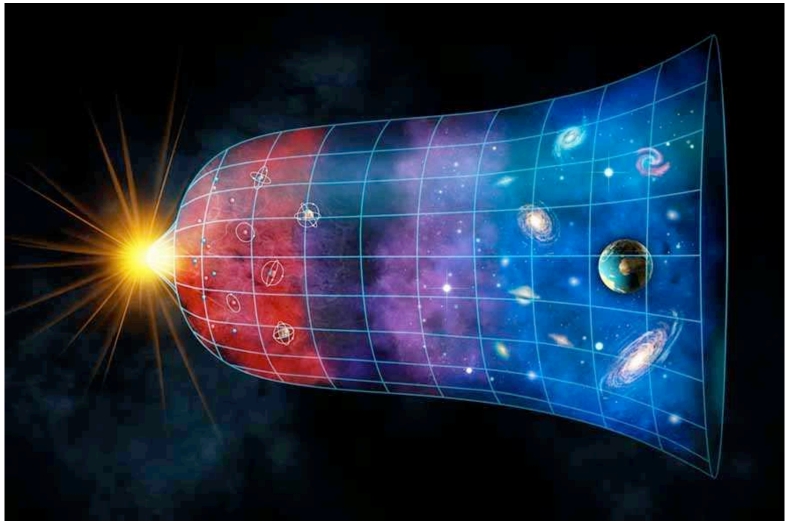
Best limit for dipole rad:
10^-3 level (pulsars -> 10^-9)

Speed of GWs vs light (GW170817)

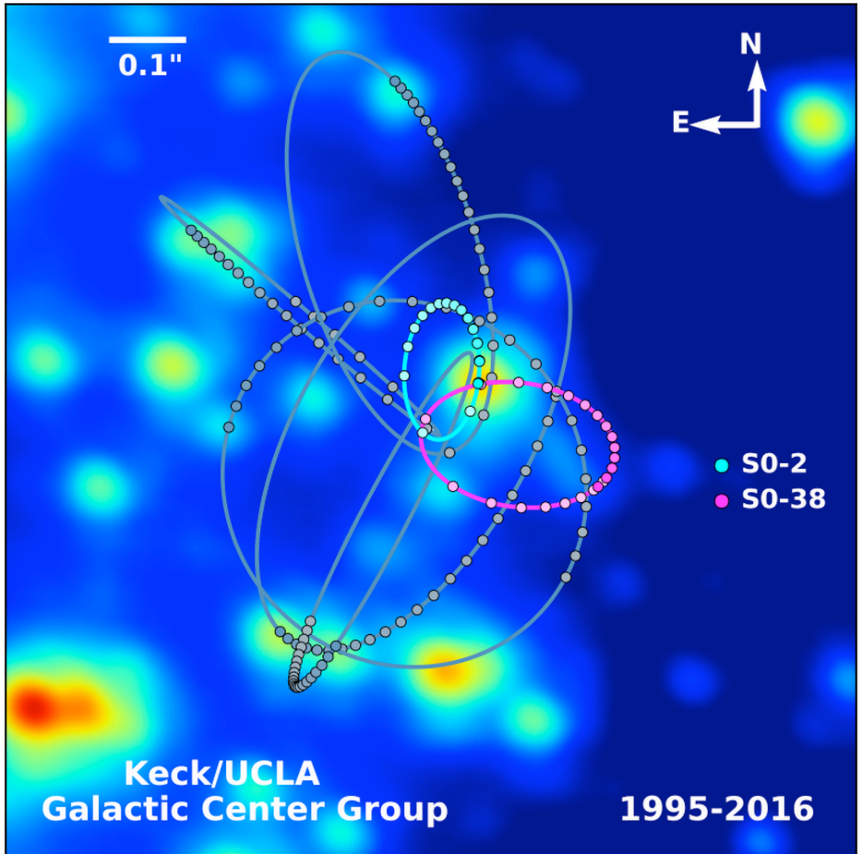
$$-3 \times 10^{-15} < \frac{c_{\mathrm{GW}} - c}{c} < +7 \times 10^{-16}.$$

Other tests of GR

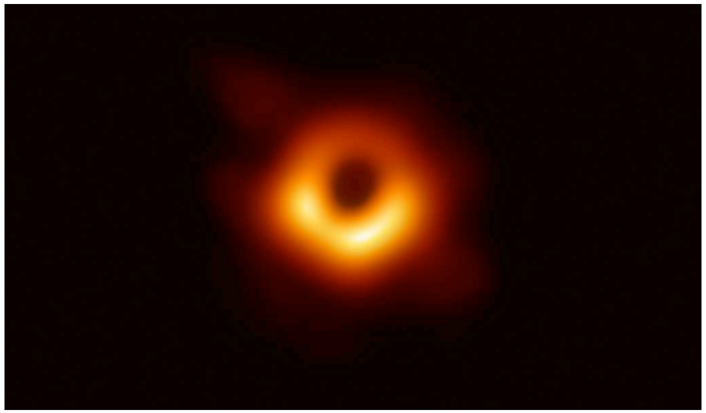
cosmology



centre of our Galaxy SgrA*: notably S2



EHT

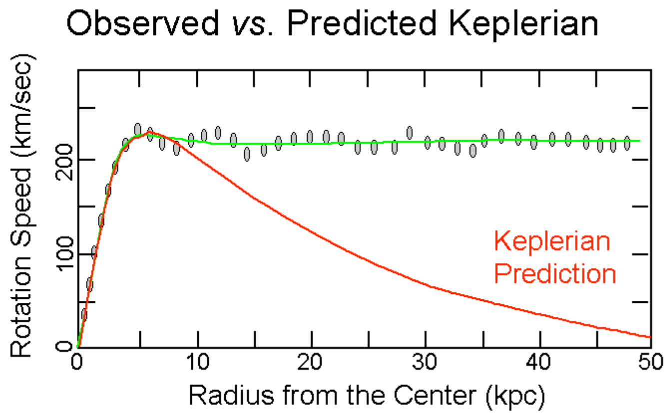


Theory-based approaches to GR deviations ?

Puzzles posed by GR

short-distance incompleteness

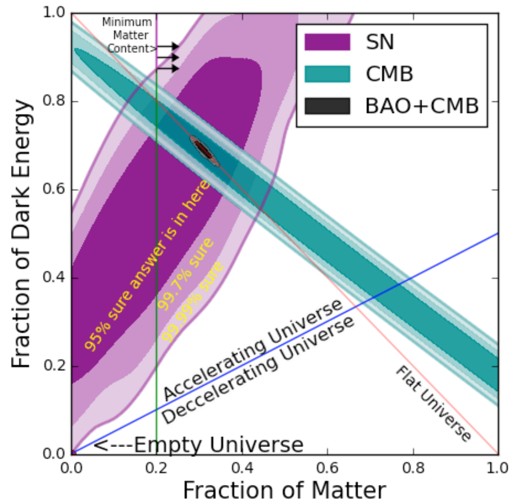
UV completion at L > L_Planck ?



dark matter

long-distance « black clouds »

dark energy



Theory-based approaches to GR deviations

Completions/extensions of GR

« Historical » extensions:

Kaluza-Klein, Jordan-Fierz: dilaton-like scalar field

Cartan: torsion

Einstein: $G_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu}$

String theory: tree-level massless sector

$$G_{\mu\nu}; \Phi; B_{\mu\nu}$$

$$+ \text{ moduli from compactified dim}$$

$$+ \ell_s - \text{corrections}$$

Consequences of adding a scalar dof

Generic EP violations from dilaton-like coupling

$$\mathcal{L}_{\mathrm{int}\phi} = \kappa\phi\Big[+\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=e,u,d}(d_{m_i}+\gamma_{m_i}d_g)m_i\bar{\psi}_i\psi_i\Big].$$

Weak-field deviations from composition-independent coupling to T=T^mu_mu

$$\mathcal{L}_{\mathrm{tot}}[g_{\mu\nu},\varphi,\psi,A_\mu,H] = \frac{c^4}{16\pi G_*}\sqrt{g}(R(g_{\mu\nu}) - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) - \sqrt{g}V(\varphi) + \mathcal{L}_{\mathrm{SM}}[\psi,A_\mu,H,\tilde{g}_{\mu\nu}]\,.$$

two functions: potential V(phi), coupling a(phi)

$$\tilde{g}_{\mu\nu} = \exp(2a(\varphi))g_{\mu\nu}$$

field-dependent coupling

$$\alpha(\varphi) \equiv \partial a(\varphi)/\partial\varphi$$

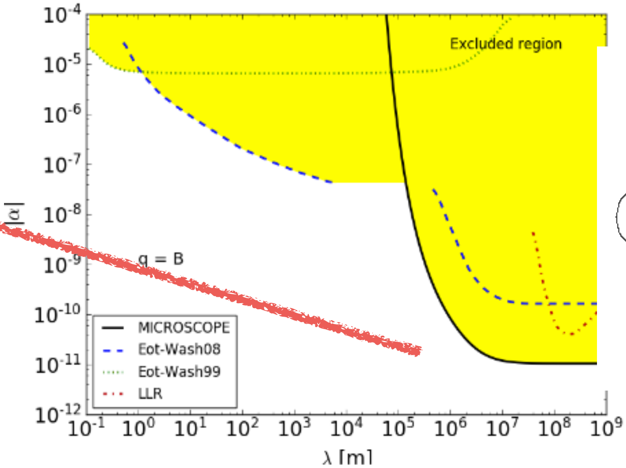
$$\overline{\gamma} = -2\frac{\alpha_0^2}{1+\alpha_0^2}\,;\qquad \overline{\beta} = +\tfrac{1}{2}\frac{\beta_0\alpha_0^2}{(1+\alpha_0^2)^2}$$

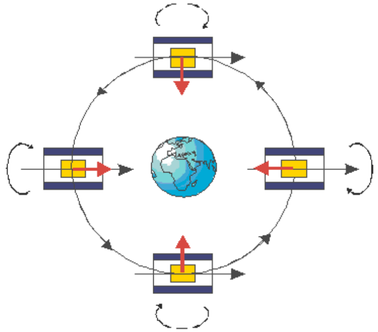
$$\alpha_0 \equiv \alpha(\varphi_0),\ \text{and}\ \beta_0 \equiv \partial\alpha(\varphi_0)/\partial\varphi_0$$

Stringent constraints on light scalar dof

From EP tests (Bergé et al'18)

$$\alpha_0^2 = \alpha < 10^{-11}$$

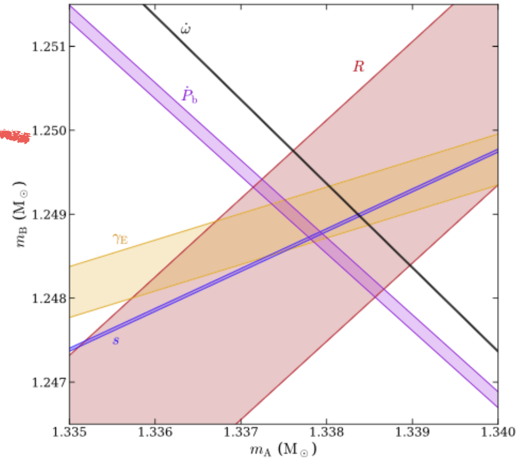




From solar-system tests

$$\alpha_0^2 < 10^{-5}$$

More stringent constraints from binary-pulsar strong-field+radiative tests



More general 2-derivative scalar-tensor (Horndeski) $X \equiv -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$

$$\begin{aligned} L_{\rm tot}[g_{\mu\nu},\varphi,\psi] =& G_2(\varphi,X) - G_3(\varphi,X)\Box_g\varphi + G_4(\varphi,X)R \\ &+ G_{4X}(\varphi,X)[(\Box_g\varphi)^2 - \varphi^{\mu\nu}\varphi_{\mu\nu}] \\ &+ G_5(\varphi,X)G^{\mu\nu}\varphi_{\mu\nu} - \frac{1}{6}G_{5X}(\varphi,X)[(\Box_g\varphi)^3 \\ &- 3\Box_g\varphi\varphi^{\mu\nu} + 2\varphi_{\mu\nu}\varphi^{\mu\lambda}\varphi^{\nu}_{\lambda}] + L_{\rm matter}[g_{\mu\nu},\psi] \end{aligned}$$

But $$\frac{c_{\rm GW}^2}{c^2} = \frac{G_4 - X(\ddot\varphi G_{5X} + G_{5\varphi})}{G_4 - 2XG_{4X} - X(H\dot\varphi G_{5X} - G_{5\varphi})}$$

$$-3\times 10^{-15} < \frac{c_{\rm GW} - c}{c} < +7\times 10^{-16}$$

Naturalness of phenom-relevant GR deviations ???

Cosmological attractor
(TD-Polyakov, TD-Piazza-Veneziano)

$$\alpha_0^2 \lll 1$$

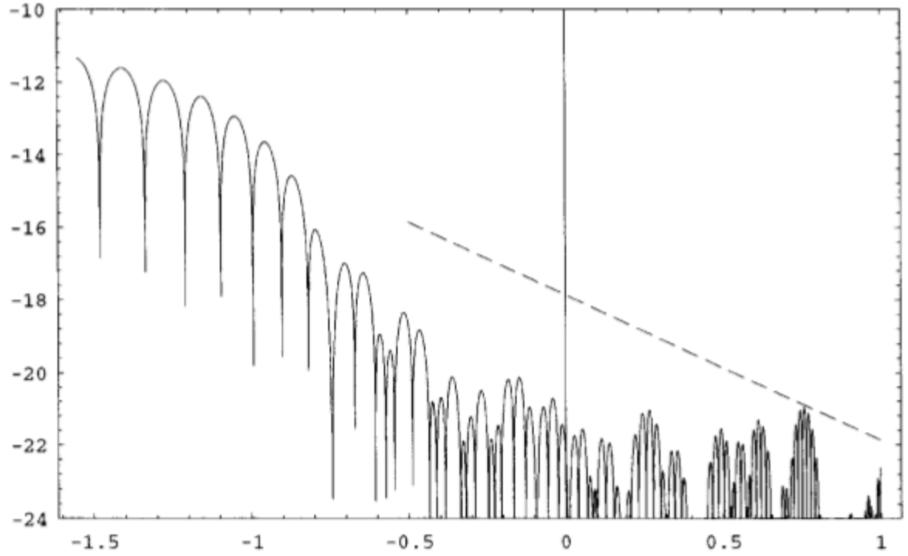


Fig. 3. The solid line represents $\log_{10}(\Delta a / a)_{\max}$ as a function of $\log_{10} \kappa$, i.e. the expected present level of violation of the equivalence principle (when comparing uranium with a light element) as a function of the curvature κ of the (string-loop induced) function $\ln B^{-1}(\phi)$ near a minimum ϕ_m. The dashed

Chameleon effect of V(phi)
(Khoury-Weltman)

O(10^-18) EP tests would be the best probes

Can one expect to see GR deviations in GW observations of BH coalescences ???

No-hair theorems in D=4 very much restrict possibilities

Few theories can predict hairy BHs. Interesting exceptions:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} (\nabla_\mu \phi)^2 + M_{\rm Pl} \alpha \phi \mathcal{R}^2_{\rm GB} + M_{\rm Pl} \tilde{\alpha} \phi R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} \right)$$

Length-squared coupling

$$\alpha = \ell^2$$

Gauss-Bonnet

$$\mathcal{R}^2_{\rm GB} \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2$$

Pontryagin

Need $\ell \sim 10 \text{ km}$

$(|\alpha| < 1 \text{ down to } 56\mu)$

for LIGO-observable deviations:
both through \alpha_eff=O(1)
and through QNM modifications
(Sotiriou…,Yagi…,Yunes…, Julié-Berti,…)

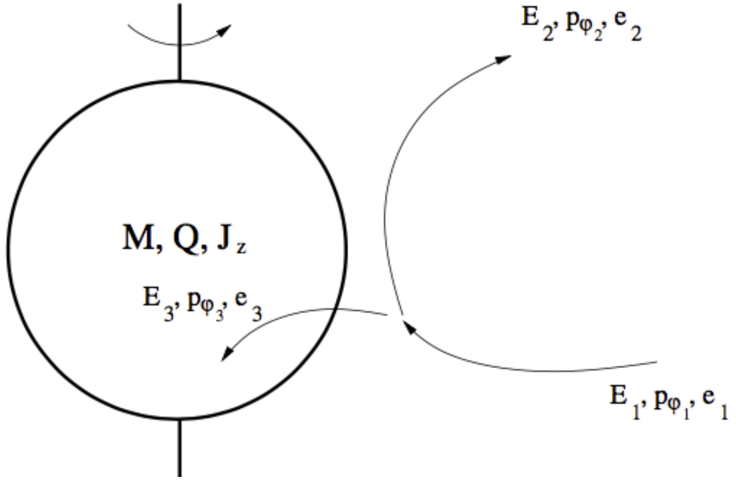
However:
classical causality PDE problems in strong fields (Pretorius…)
quantum causality constraints (Serra^2-Trincherini-Trombetta'22
à la Camanho-Edelstein-Maldacena-Zhiboedov + Caron-Huot, Arkani-Hamed…,Bern…,Bellazini,…)

$$\ell \lesssim \ell_{EFT}$$

Phenom. tests still interesting notably BH Love #

Other possible new signals

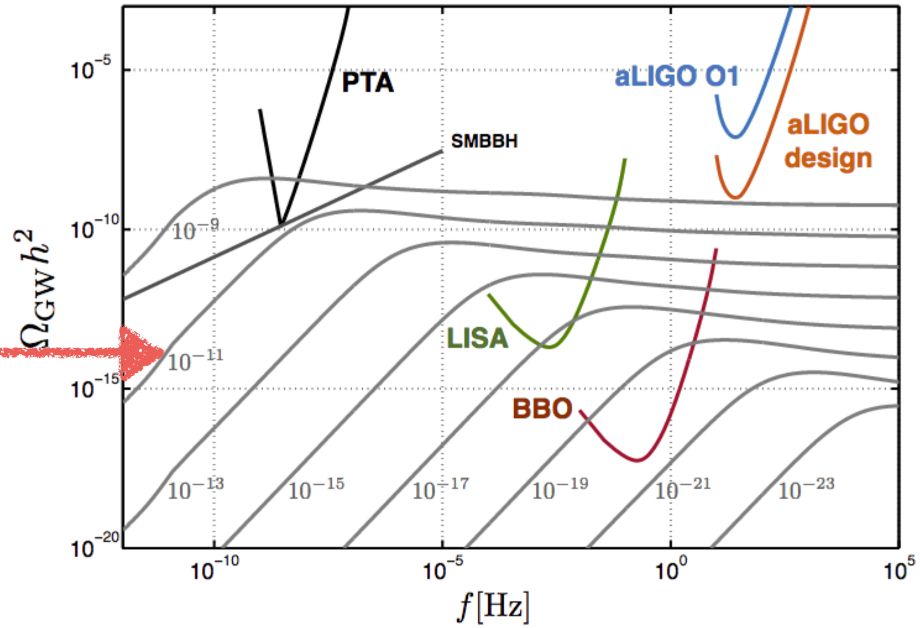
Ultra-light bosonic fields
(e.g. Axion-like particles)
and BH superradiance
Penrose process
(review Berti-Cardoso-Pani'21)



Cosmic (super-)stringsl

(TD-Vilenkin'00,…BlancoPillado-Olum-Siemens'17)

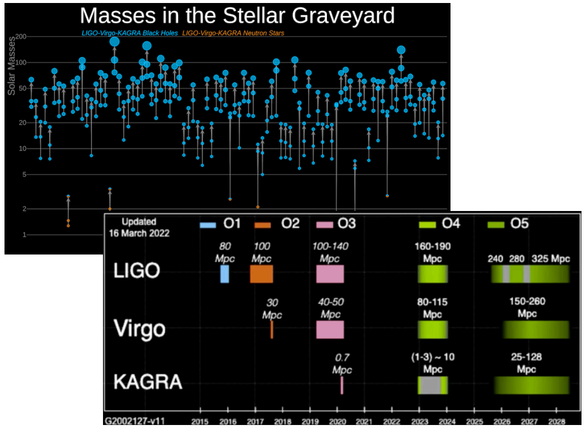
$G\mu$

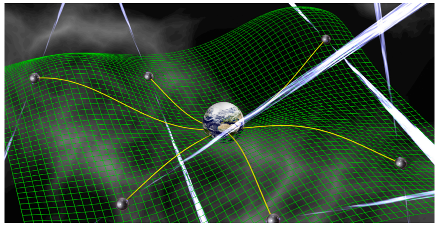


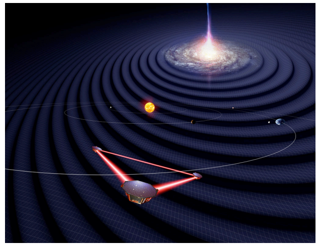
Quantum-generated GWs from inflation in CMB (B modes)

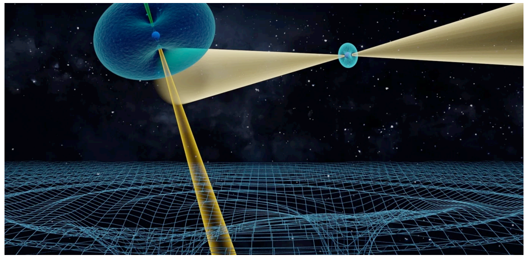
GWs from phase transitions

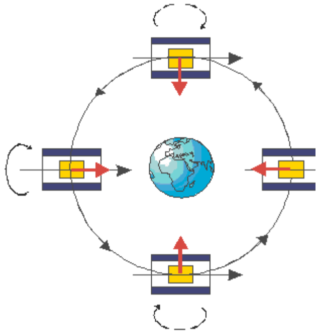
TAKE HOME IMAGES

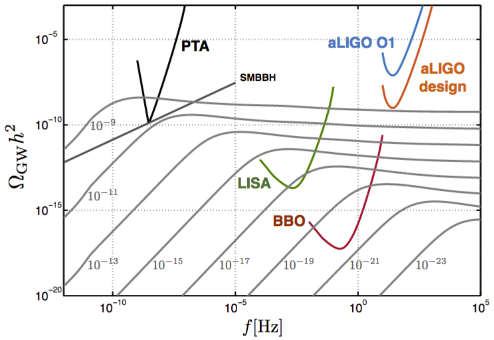


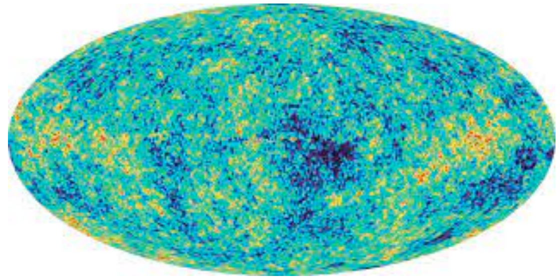


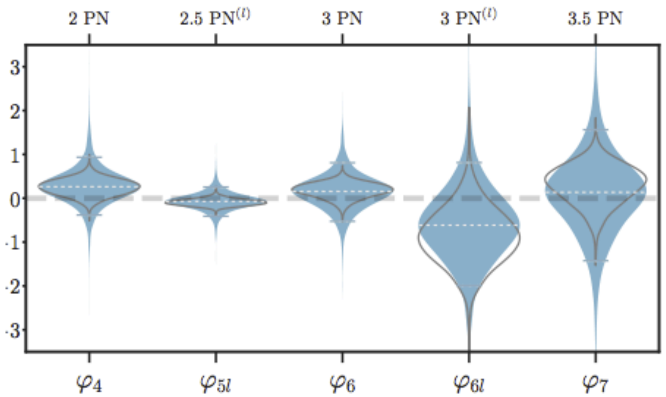


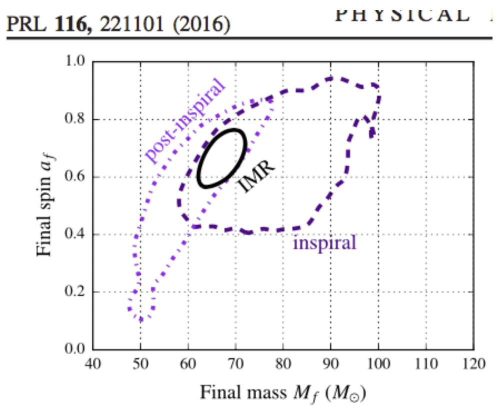












$$-3\times 10^{-15} < \frac{c_{\rm GW}-c}{c} < +7\times 10^{-16}\,.$$

$$\mathcal{L}_{\rm tot}[g_{\mu\nu},\varphi,\psi,A_\mu,H] = \frac{c^4}{16\pi G_*}\sqrt{g}\left(R(g_{\mu\nu})-2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) - \sqrt{g}V(\varphi)+\mathcal{L}_{\rm SM}[\psi,A_\mu,H,\tilde{g}_{\mu\nu}]\,.$$

$$\mathcal{L}_{{\rm int}\phi} = \kappa\phi\left[+\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu}-\frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=e,u,d}(d_{m_i}+\gamma_{m_i}d_g)m_i\bar{\psi}_i\psi_i\right].$$

$$S=\int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R-\frac{1}{2}(\nabla_\mu\phi)^2+M_{\rm Pl}\alpha\phi\mathcal{R}^2_{\rm GB}+M_{\rm Pl}\tilde{\alpha}\phi R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\right)$$