Strongly coupled chiral theories...

...from the point of view of symmetries and anomalies

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S. Bolognesi, K. Konishi and A.L. [arXiv:2206.00538 [hep-th]]
S. B., K. K. and A.L, Phys. Rev. D **103** (2021) no.9 [arXiv:2101.02601 [hep-th]]
S. B., K. K. and A.L., JHEP **08** (2021), 028 [arXiv:2105.03921 [hep-th]]

Why strongly coupled chiral gauge theories are interesting?

A very practical reason:

- Nature likes chiral theories (SM is an example).
- Also strongly coupled chiral theories might be out there.
- But, until we understand them, we can not use them in model building.

Why are they so difficult?

- No solved examples.
- No general theorems.
- No lattice experiments.

We shall look aside!

Our toolset...

- Good old 't Hooft anomaly matching + new generalizations...
  - gauging discrete groups,
  - gauging generalized symmetries.
- IR EFT consistency with the
  - spontaneously broken symmetries
  - anomalously broken symmetries
- Large-N limit (we will not use it).

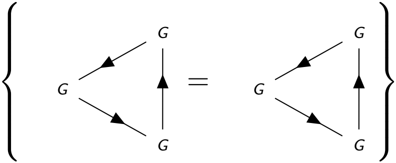
...is limited.

We will study some examples of asymptotically free $SU(N)$ gauge theories by applying mostly the first two tools.

't Hooft anomaly matching

- We have a theory with a **global symmetry group** G.
- We can **externally** gauge G (choose a **background** gauge field for G)
- The 't Hooft anomalies that arise are **RG-flow invariant**

UV theory



IR theory

- If

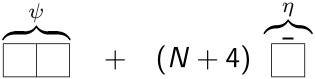
$$G \xrightarrow{\text{SSB}} H$$

you have to check only the anomalies of H.

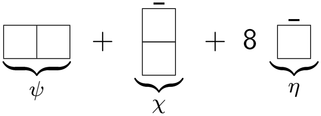
Our examples

Two $SU(N)$ chiral gauge theories:

- The $\psi\eta$ (Bars-Yankielowicz) model



- The $\psi\chi\eta$ model



We will study some of their possible IR descriptions, by using

't Hooft anomaly matching **IR effective theory consistency**

$\psi\eta$-model

$$SU(N)YM + \underbrace{\square\!\square}_{\psi^{ij}} + \underbrace{(N+4) \in \bar{\square}}_{\eta_i^A}$$

$$\beta = -\frac{g^3}{16\pi^2}\frac{9N-6}{3} << 0 \implies \text{ strongly coupled in IR}$$

The classical global symmetries

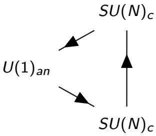
$$SU(N+4)_\eta \quad \times \quad \underbrace{U(1)_{\psi\eta}}_{\begin{cases}\psi \to e^{i\frac{N+4}{N^*}\alpha}\psi \\ \eta \to e^{-i\frac{N+2}{N^*}\alpha}\eta\end{cases}} \quad \times \quad \underbrace{U(1)_{\text{an}}}_{\begin{cases}\psi \to e^{i\delta}\psi \\ \eta \to \eta\end{cases}}$$

$$N^* = gcd(N,2) \qquad \alpha,\ \delta \in (0, 2\pi)$$

$\psi\eta$-model

$$SU(N)YM + \underbrace{\square\!\square}_{\psi^{ij}} + \underbrace{(N+4) \in \bar{\square}}_{\eta_i^A}$$

are reduced by the strong anomaly



to

$$SU(N+4)_\eta \quad \times \quad \underbrace{U(1)_{\psi\eta}}_{\begin{cases} \psi \to e^{i\frac{N+4}{N^*}\alpha}\psi \\ \eta \to e^{-i\frac{N+2}{N^*}\alpha}\eta \end{cases}} \quad \times \quad \underbrace{U(1)_{an}}_{\begin{cases} \psi \to \ldots \\ \eta \to \ldots \eta \end{cases}}$$

$$N^* = gcd(N,2) \qquad \alpha \in (0, 2\pi)$$

RG-flow: two possibilities

$\psi\eta$ model

RG − flow

No symmetry breaking:
Massless baryons match all the anomalies

Symmetry breaking:
some anomalies matched by NGB,
some anomalies by massless fermions

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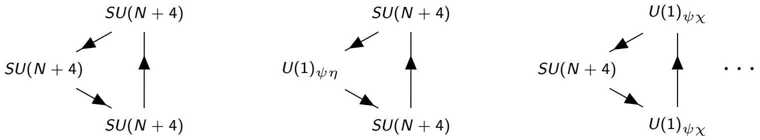
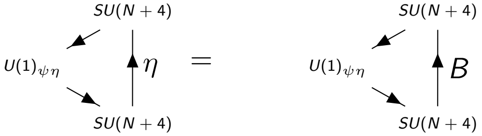
Many (standard) 't Hooft anomalies



but mass-less baryons...

$$B^{[AB]} \sim \psi^{ij}\eta_i^{[A}\eta_j^{B]}$$

...match all of them....



$$N_c \cdot \left(-\frac{N+2}{N^*}\right) \cdot d\left(\square\right) \qquad \left(-\frac{N+4}{N^*}\right) \cdot d\left(\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}\right)$$

...in a very nontrivial way!

RG-flow: two possibilities

$\psi\eta$ model

RG − flow

No symmetry breaking:
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Symmetry breaking:
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Generalized 't Hooft anomaly matching

More detail (for even N)

$$\begin{pmatrix} \psi \\ \eta \end{pmatrix} \xrightarrow{e^{i\frac{2\pi}{N}} \in SU(N)_c} \begin{pmatrix} e^{i\frac{4\pi}{N}}\psi \\ e^{-i\frac{2\pi}{N}}\eta \end{pmatrix} \xrightarrow{\mathbb{Z}_2^F \times U(1)_{\psi\eta}} \begin{pmatrix} \psi \\ \eta \end{pmatrix}$$

in other words

$$SU(N) \cap \left(U(1)_{\psi\eta} \times \mathbb{Z}_2^F\right) = \mathbb{Z}_N$$

similarly

$$SU(8) \cap \left(U(1)_{\psi\eta} \times \mathbb{Z}_2^F\right) = \mathbb{Z}_{N+4}$$

to perform the 't Hooft anomaly matching we shall gauge

$$\frac{SU(N+4) \times U(1)_{\psi\eta} \times \mathbb{Z}_2^F}{\mathbb{Z}_N \times \mathbb{Z}_{N+4}} \quad \text{not} \quad SU(N+4) \times U(1)_{\psi\eta}$$

Is that useful?

There are many $SU(N)_c \times \frac{SU(N+4)\times U(1)_{\psi\eta}\times \mathbb{Z}_2^F}{\mathbb{Z}_N\times\mathbb{Z}_{N+4}}$-bundles...

...that are not $SU(N)_c \times SU(N+4) \times U(1)_{\psi\chi}$-bundles.

(This can be understood as the gauging of a $\mathbb{Z}_N^{(1)}$ and a $\mathbb{Z}_{N+4}^{(1)}$ 1-form symmetries)

$SU(N)_c \times SU(N+4) \times U(1)_{\psi\chi}$	$SU(N)_c \times \frac{SU(N+4)\times U(1)_{\psi\eta}\times \mathbb{Z}_2^F}{\mathbb{Z}_N\times\mathbb{Z}_{N+4}}$
$\frac{1}{8\pi^2}\int tr(f_c \wedge f_c) = \mathcal{I}_c$	$\frac{1}{8\pi^2}\int tr(f_c \wedge f_c) = \mathcal{I}_c - \frac{\mathcal{B}_N}{N}$
$\frac{1}{8\pi^2}\int tr(F_{N+4} \wedge F_{N+4}) = \mathcal{I}_{N+4}$	$\frac{1}{8\pi^2}\int tr(F_{N+4} \wedge F_{N+4}) = \mathcal{I}_{N+4} - \frac{\mathcal{B}_{N+4}}{N+4}$
$\frac{1}{8\pi^2}\int dA_{\psi\eta} dA_{\psi\eta} = \mathcal{I}_{\psi\eta}$	$\frac{1}{8\pi^2}\int dA_{\psi\eta} dA_{\psi\eta} = \mathcal{I}_{\psi\eta} + \frac{\mathcal{B}_N}{N^2} + \frac{\mathcal{B}_{N+4}}{(N+4)^2}$

Topological number fractionalizes!

$SU(N)_c \times SU(N+4) \times U(1)_{\psi\chi}$	$SU(N)_c \times \frac{SU(N+4)\times U(1)_{\psi\eta}\times \mathbb{Z}_2^F}{\mathbb{Z}_N \times \mathbb{Z}_{N+4}}$
$\frac{1}{8\pi^2}\int tr(f_c \wedge f_c) = \mathcal{I}_c$	$\frac{1}{8\pi^2}\int tr(f_c \wedge f_c) = \mathcal{I}_c - \frac{\mathcal{B}_N}{N}$
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By acting with $(\mathbb{Z}_2^F)$

$$\mathcal{Z}_{UV} \to \mathcal{Z}_{UV}$$
$$\mathcal{Z}_{\text{baryons}} \to \mathcal{Z}_{\text{baryons}}$$

No new anomaly

$SU(N)_c \times SU(N+4) \times U(1)_{\psi\chi}$	$SU(N)_c \times \frac{SU(N+4)\times U(1)_{\psi\eta}\times \mathbb{Z}_2^F}{\mathbb{Z}_N\times\mathbb{Z}_{N+4}}$
$\frac{1}{8\pi^2}\int tr(f_c\wedge f_c) = \mathcal{I}_c$	$\frac{1}{8\pi^2}\int tr(f_c\wedge f_c) = \mathcal{I}_c - \frac{\mathcal{B}_N}{N}$
$\frac{1}{8\pi^2}\int tr(F_{N+4}\wedge F_{N+4}) = \mathcal{I}_{N+4}$	$\frac{1}{8\pi^2}\int tr(F_{N+4}\wedge F_{N+4}) = \mathcal{I}_{N+4} - \frac{\mathcal{B}_{N+4}}{N+4}$
$\frac{1}{8\pi^2}\int dA_{\psi\eta}dA_{\psi\eta} = \mathcal{I}_{\psi\eta}$	$\frac{1}{8\pi^2}\int dA_{\psi\eta}dA_{\psi\eta} = \mathcal{I}_{\psi\eta} + \frac{\mathcal{B}_N}{N^2} + \frac{\mathcal{B}_{N+4}}{(N+4)^2}$

By acting with $(\mathbb{Z}_2^F)$

$$\mathcal{Z}_{UV} \to \overbrace{(-1)\,\mathcal{Z}_{UV}}^{-1\sim e^{i\pi(\mathcal{B}_N+\mathcal{B}_{N+4})}}$$

$$\mathcal{Z}_{\text{baryons}} \to \mathcal{Z}_{\text{baryons}}$$

Anomaly mismatch!

On a **smooth manifold** there is a problem:

$$\mathcal{B}_N + \mathcal{B}_{N+4} \quad \text{is always } \textbf{even}.$$

(P. B. Smith, A. Karasik, N. Lohitsiri and D. Tong, JHEP **01** (2022), 112.)

This erases the new anomaly:

$$\mathcal{Z}_{UV} \to \underbrace{-\mathcal{Z}_{UV}}_{-1\sim e^{i\pi(B_N+B_{N+4})}} \xrightarrow{\text{simplifies}} \mathcal{Z}_{UV} \to \mathcal{Z}_{UV}$$

The new anomaly disappears!

By including $\mathbb{Z}_2^F$ defects the obstruction is lifted...

...but the defect can carry its own anomaly, **still work in progress**.

In the meantime, let us turn to some more heuristic arguments...

RG-flow: two possibilities

$\psi\eta$ model

RG − flow

No symmetry breaking:
Massless baryons match
all the anomalies
?

Symmetry breaking:
some anomalies matched
by NGB,
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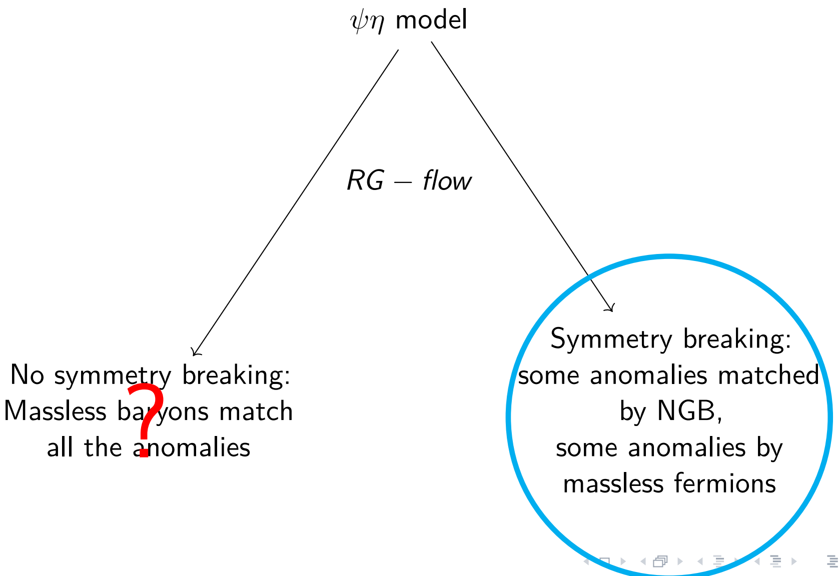
RG-flow: two possibilities



Color-Flavor-Locking (CFL)

$$\langle \psi^{\{ij\}} \eta_i^B \rangle \sim \Lambda^3 \underbrace{\left.\left(\begin{array}{cccc|cccc} 1 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & 0 & 0 \\ & & \ddots & & & & & \\ 0 & 0 & \dots & 1 & 0 & 0 & 0 & 0 \end{array} \right) \right\}}_{N+4 \text{ flavor}} N \text{ color}$$

$$SU(N)_c \times SU(N+4)_\eta \times U(1)_{\psi\eta} \to SU(N)_{cf} \times SU(4) \times U(1)'$$

- The entire $SU(N)_c$ is broken, all the gluons are gapped.
- $$U(1)_{\psi\eta} \times \underbrace{U(1)_D}_{\in SU(N+4)} \to U(1)' \;.$$
- $8N+1$ broken generators $\implies$ $8N+1$ NGBs.

- The fermionic spectrum can be read from weak-coupling, by gapping the vector-like components:

	fields	$SU(N)_{\rm cf}$	$SU(4)_{\rm f}$	$U'(1)$
UV	ψ	$\boxtimes\boxtimes$	$\frac{N(N+1)}{2} \cdot (\cdot)$	1
	η^{A_1}	$\overline{\boxtimes\boxtimes} \oplus \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	$N^2 \cdot (\cdot)$	-1
	η^{A_2}	$4 \cdot \bar{\square}$	$N \cdot \square$	$-\frac{1}{2}$

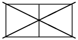
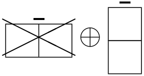
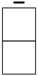
One can imagine...

$$(\psi^{ij}\eta_{jA})^\dagger(\psi^{ij}\eta_{jA}) \xrightarrow{<(\psi^{ij}\eta_{jA})^\dagger>\sim\delta^{iA}} M\ (\psi^{ij}\eta_{ji})$$

All the 't Hooft anomalies matches trivially.

Gauge invariant formulation

$$U^{iB} = \langle \psi^{\{ij\}} \eta_i^B \rangle = \ C\, \Lambda^3 \delta^{jB} \qquad \Longrightarrow \ \det U = \epsilon_{j_1 \dots j_N} \psi^{\{ij_1\}} \eta_i^{B_1} \dots \psi^{\{ij_N\}} \eta_i^{B_N}$$

	fields	$SU(N)_{\rm cf}$	$SU(4)_{\rm f}$	$U'(1)$
UV	ψ		$\frac{N(N+1)}{2} \cdot (\cdot)$	1
	η^{A_1}		$N^2 \cdot (\cdot)$	-1
	η^{A_2}	$4 \cdot$ 	$N \cdot$ 	$-\frac{1}{2}$
IR	$B^{[A_1 B_1]}$		$\frac{N(N-1)}{2} \cdot (\cdot)$	-1
	$B^{[A_1 B_2]}$	$4 \cdot$ 	$N \cdot$ 	$-\frac{1}{2}$

$$B^{A_1 B_1} \sim \psi^{ij} \eta_i^{A_1} \eta_j^{B_1} \qquad B^{A_1 B_2} \sim \psi^{ij} \eta_i^{A_1} \eta_j^{B_2}$$

We have to understand what happens to

$$U(1)_{an} : \begin{cases} \psi \to e^{i\alpha}\psi \\ \eta \to \eta \end{cases}$$

broken by the instanton   broken by the condensate

Exact global symmetries $\leftrightarrow$ 't Hooft anomalies

Anomalously broken symmetries $\leftrightarrow$?

Strong anomaly matching?

Let us see what happens in QCD (the η' problem!)

In QCD:

- $U(1)_A$ is broken by the instantons and broken by $U^i_j = <\psi^i \bar{\psi}_j>$.

- The effect of this anomaly is reproduced in the IR lagrangian

$$\mathcal{L}_{IR} \subset \frac{i}{2} q(x) log \frac{detU}{detU^\dagger} + \frac{N}{a_0 F_\pi^2} q^2(x) - \theta q(x) \qquad q(x) = \frac{1}{4\pi^2} F\tilde{F}$$

- Under $\psi \to e^{i\gamma^5 \alpha}\psi$

$$\mathcal{L}_{IR} \to \mathcal{L}_{IR} + 2N_f \alpha q(x) \quad \text{the anomaly is reproduced}$$

- Integrating $q(x)$ away... $\mathcal{L}_{eff} \supset -\frac{F_\pi^2 a_0}{4N}\left(\theta - \frac{i}{2} log \frac{detU}{detU^\dagger}\right)^2$

This make sense only if $< U > \neq 0$!

Can we do something similar in the $\psi\eta$ model?

$$\mathcal{L}_{IR} \subset \frac{i}{2} q(x) \log \frac{\det U}{\det U^\dagger} \quad \begin{cases} \checkmark & \text{gauge invariant} \\ \checkmark & U(1)_{an} - [SU(N)_c]^2 \text{ anomaly} \\ \text{✗} & \text{not invariant under } G_F! \end{cases}$$

Let us make it invariant under the full $G_F = SU(N) \times SU(N+4) \times U(1)_{\psi\eta}$

$$\mathcal{L} \subset \frac{i}{2} q(x) \left(\log \left(\epsilon BB \det U \right) - \log \left(\epsilon BB \det U \right)^\dagger \right)$$

$$\epsilon BB \det U = \epsilon_{i_1,\ldots,i_{N+4}} \epsilon_{j_1,\ldots,j_{N_c}} B^{N+1,N+2} B^{N+3,N+3} U^{i_1,j_1} \ldots U^{i_N,j_N}$$

✓ gauge invariant ✓ strong anomaly ✓ invariant under G_F

- To define $\frac{i}{2} q(x) \left(\log\left(\epsilon BB \det U\right) - \log\left(\epsilon BB \det U\right)^\dagger \right)$ we need

$$B^{A_2 B_2} \qquad A_2, B_2 = N+1, ..., N+4$$

	fields	$SU(N)_{\mathrm{cf}}$	$SU(4)_{\mathrm{f}}$	$U'(1)$
IR	$B^{[A_1 B_1]}$	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	$\frac{N(N-1)}{2} \cdot (\cdot)$	-1
	$B^{[A_1 B_2]}$	$4 \cdot \bar{\square}$	$N \cdot \square$	$-\frac{1}{2}$
	$B^{[A_2 B_2]}$	$6 \cdot (\cdot)$	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	0

- Both $< \det U > \neq 0$ and $< \epsilon BB > \neq 0$.

- By expanding $\log \epsilon BB \det U$ one gets a mass for

$$\tilde{\pi} \propto \frac{\pi_1}{F_\pi^{(1)}} + \frac{\pi_2}{F_\pi^{(2)}} \qquad < detU > \sim e^{i\frac{\pi_1(x)}{F_\pi^{(1)}}} \qquad < \epsilon BB > \sim e^{i\frac{\pi_2(x)}{F_\pi^{(2)}}}$$

(the η' for the $\psi\eta$ model).

$\psi\chi\eta$-model

$$SU(N)\,YM + \underbrace{\square\!\square}_{\psi^{ij}} + \underbrace{\bar{\begin{array}{c}\square\\[-4pt]\square\end{array}}}_{\chi_{ij}} + \underbrace{8 \in \bar{\square}}_{\eta_i^A}$$

Still a strongly coupled system

$$\beta = -\frac{g^3}{16\pi^2}\frac{9N-8}{3} << 0 \implies \text{ strongly coupled in IR.}$$

Again the classical symmetries

$$SU(8)_\eta \quad \times \quad \underbrace{\tilde{U}(1)} \quad \times \quad \underbrace{U(1)_{\psi\chi}} \quad \times \quad \underbrace{U(1)_{\text{an}}}$$

$$\tilde{U}(1): \begin{cases} \psi \to e^{2i\alpha}\psi \\ \chi \to e^{-2i\alpha}\chi \\ \eta \to e^{-i\alpha}\eta \end{cases} \qquad U(1)_{\psi\chi}: \begin{cases} \psi \to e^{i\frac{N-2}{N^*}\beta}\psi \\ \chi \to e^{-i\frac{N+2}{N^*}\beta}\chi \\ \eta \to \eta \end{cases} \qquad U(1)_{\text{an}}: \begin{cases} \psi \to e^{i\delta}\psi \\ \chi \to e^{-i\delta}\chi \end{cases}$$

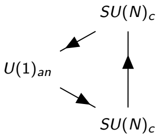
$\psi\chi\eta$-model

$$SU(N)\,YM + \underbrace{\square\!\square}_{\psi^{ij}} + \underbrace{\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}}_{\chi_{ij}} + \underbrace{8 \in \bar{\square}}_{\eta_i^A}$$

Still a strongly coupled system

$$\beta = -\frac{g^3}{16\pi^2}\frac{9N-8}{3} << 0 \implies \text{strongly coupled in IR.}$$

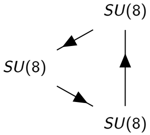
are broken by the strong anomaly



to

$$SU(8)_\eta \quad \times \quad \underbrace{\tilde{U}(1)}_{\begin{cases}\psi \to e^{2i\alpha}\psi \\ \chi \to e^{-2i\alpha}\chi \\ \eta \to e^{-i\alpha}\eta\end{cases}} \quad \times \quad \underbrace{U(1)_{\psi\chi}}_{\begin{cases}\psi \to e^{i\frac{N-2}{N^*}\beta}\psi \\ \chi \to e^{-i\frac{N+2}{N^*}\beta}\chi \\ \eta \to \eta\end{cases}} \quad \times \quad \underbrace{U(1)_{a}}_{\begin{cases}\psi \to \\ \chi \quad e^{-}\end{cases}}$$

There is no simple way to match the (standard) anomalies:



$\propto N \implies \mathcal{O}(N)$ IR fermions!

Dynamical Abelianization

$$<\psi^{ij}\chi_{jk}>\sim \begin{pmatrix} c_1 & 0 & \dots & 0 \\ 0 & c_2 & \dots & 0 \\ & & \ddots & \\ & & & c_N \end{pmatrix} \quad c_i \neq c_j$$

where

$$SU(N)_c \times \frac{SU(8) \times U(1)_{\psi\chi} \times \tilde{U}(1)}{\mathbb{Z}_N \times \mathbb{Z}_{8/N^*}} \to \prod_{\ell=1}^{N-1} U(1)_\ell \times \frac{SU(8) \times \tilde{U}(1)}{\mathbb{Z}_N \times \mathbb{Z}_2}$$

This produces naturally $\mathcal{O}(N)$, $SU(8)$ charged massless fermions.

The IR-spectrum

- $SU(N)_c \xrightarrow{<\psi\chi>} U(1)^{N-1}$: there are $N-1$ "photons".

- $U(1)_{\psi\chi} \xrightarrow{<\psi\chi>\neq 0} \mathbb{Z}_{4/N^*} \implies \pi(x)$ NGB

$$< \psi\chi >= (const) \cdot e^{i\frac{\pi(x)}{F_\pi}} \ .$$

- ψ^{ij} and χ_{ij}, $i \neq j$, pair up and gap together:

$$\left(\psi^{i\ell}\chi_{\ell j}\right)^* \psi^{im}\chi_{mj} \xrightarrow{<\psi\chi>\neq 0} (c_i - c_j)\psi^{ij}\chi_{ij} \ .$$

- ψ^{ii} and η_i^A are chiral, and remain mass-less.

- All the standard 't Hooft anomalies are matched trivially.

Let's explore the

Consistency of the IR EFT

With the massles d.o.f. we can construct an IR theory...

$$\mathcal{L}^{(eff)} = \mathcal{L}(\psi, \eta, A_\mu^{(i)}) + \mathcal{L}(\pi) - \mathcal{V}(\pi, \psi, \eta) + \dots$$

$$\mathcal{L}(\psi, \eta, A_\mu^{(i)}) = \bar{\psi} i \bar{\sigma} D_A \psi + \bar{\chi} i \bar{\sigma} D_A \chi + \frac{1}{2e_\ell} F_\ell^{\mu\nu} F_\ell^{\mu\nu} \qquad \mathcal{L}(\pi) = \partial_\mu \pi \partial^\mu \pi + \dots$$

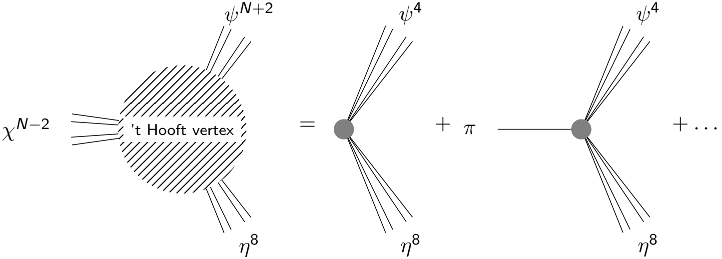
Now we shall use all the symmetries...

	unbroken by $< \psi\chi >$	unbroken by the instanton
$SU(8) \times \tilde{U}(1)$	✓	✓
$U(1)_{an}$	✓	✗
$U(1)_{\psi\chi}$	✗	✓

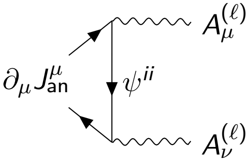
...to constraint $\mathcal{V}$.

$U(1)_{\text{an}} : \begin{cases} \psi \to e^{i\delta}\psi \\ \chi \to e^{-i\delta}\chi \end{cases}$	$<\psi\chi>$	instanton
	✓	✗

- "Small instantons" $\Longrightarrow$ 't Hooft vertices



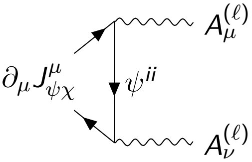
- "Large instantons" $\Longrightarrow$ leftover IR anomalous breaking



$$\Longrightarrow \partial_\mu J^\mu_{\text{an}} = \frac{1}{16\pi^2} \sum_{\ell=1}^{N-1} \ell(\ell+1) F^\ell_{\mu\nu} \tilde{F}^\ell_{\mu\nu}$$

	$<\psi\chi>$	instanton
$U(1)_{\psi\chi} : \begin{cases} \psi \to e^{i\frac{N-2}{N^*}\beta}\psi \\ \chi \to e^{-i\frac{N+2}{N^*}\beta}\chi \end{cases}$	✗	✓

$$U(1)_{\psi\chi} : \ \psi \to e^{i\frac{N-2}{N^*}\beta}\psi \qquad \eta \to \eta \qquad \pi \to \pi - \frac{4F_\pi}{N^*}\beta$$



$$\Longrightarrow \quad \partial_\mu J^\mu_{\psi\chi} = \frac{1}{16\pi^2}\frac{N-2}{N^*}\sum_{\ell=1}^{N-1}\ell(\ell+1)F^\ell_{\mu\nu}\tilde{F}^\ell_{\mu\nu}$$

...à la Fujikawa...

$$\int D\psi D\bar{\psi} \to \int D\psi D\bar{\psi}\ e^{i\beta\frac{1}{16\pi^2}\frac{N-2}{N^*}\sum_{\ell=1}^{N-1}\ell(\ell+1)F^\ell_{\mu\nu}\tilde{F}^\ell_{\mu\nu}} \ .$$

To restore $U(1)_{\psi\chi}$ we must add

$$\mathcal{V} \subset \frac{\pi}{16\pi^2}\frac{N-2}{4}\sum_{\ell=1}^{N-1}\ell(\ell+1)F^\ell_{\mu\nu}\tilde{F}^\ell_{\mu\nu}$$

axion like coupling!

Generalized 't Hooft anomaly matching

Again, the fact that the symmetry group is

$$G = SU(8) \times U(1)_{\psi\chi} \times \tilde{U(1)}$$

is a lie...it is

$$G = \frac{SU(8) \times U(1)_{\psi\chi} \times \tilde{U}(1)}{\mathbb{Z}_N \times \mathbb{Z}_8}$$

as

$$SU(N)_c \cap \tilde{U}(1) = \mathbb{Z}_N \quad SU(8) \cap \Big(U(1)_{\psi\chi} \times \tilde{U}(1) \Big) = \mathbb{Z}_8 \ .$$

To get the strongest constraint, we should gauge the correct symmetry group.

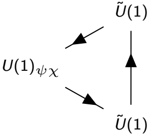
Again, flux fractionalization

$$\frac{1}{8\pi^2}\int tr(f_c \wedge f_c) = \mathcal{I}_1 \qquad \frac{1}{8\pi^2}\int d\tilde{A}\wedge d\tilde{A} = \mathcal{I}_2$$

$$\downarrow$$

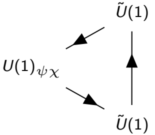
$$\frac{1}{8\pi^2}\int tr(f_c \wedge f_c) = \mathcal{I}_1 - \frac{1}{N}\mathcal{B}_N \qquad \frac{1}{8\pi^2}\int d\tilde{A}\wedge d\tilde{A} = \mathcal{I}_2 + \frac{1}{N^2}\mathcal{B}_N$$

The relevant anomaly is from $\tilde{U}(1)$



$$= -\frac{4N^2}{N^*}$$

$$\xrightarrow{\text{flux fractionalization}}$$



$$= -\frac{4}{N^*}$$

$$\mathcal{Z} \to e^{-i\frac{4N^2}{N^*}\alpha}\mathcal{Z} \qquad\qquad \mathcal{Z} \to e^{-i\frac{4}{N^*}\alpha}\mathcal{Z}$$

$$U(1)_{\psi\chi} \to \mathbb{Z}_{4N^2/N^*} \qquad\qquad U(1)_{\psi\chi} \to \mathbb{Z}_{4/N^*}$$

Exactly reproduced by $< \psi\chi > \neq 0 \implies U(1)_{\psi\chi} \to \mathbb{Z}_{4/N^*}$.

Few lessons and few perspectives:

- Generalized 't Hooft anomalies
  theoretically solid ✓
  top. obs. can ruin the result ✗

  → *Can the inclusion of defects lift some of these topological obstructions, without canceling the anomaly?*

- The "matching" of strong anomaly is an heuristic argument.

  → *Is it possible to render it more rigorous?*

- Some interesting features can be extracted form the symmetries alone, with some ad hoc arguments.

  → *Is it possible to re-organize them systematically?*

Thanks for the attention!

Backup slides

generalizing...

Bars-Yankielowicz models

$$SU(N)\ YM \ + \ \square\!\square \ + \ (N+4)\,\bar{\square} \quad (+\ p\,\square \ + \ p\,\bar{\square})$$

$$\beta = -\frac{g^3}{16\pi^2}\frac{9N-6-2p}{3} \implies \text{AF until } p \sim \frac{9/2}{N} - 3$$

Symmetries:

$$SU(N)_c \times U(N+4+p)_\eta \times U(p)_\zeta \times U(1)_\psi$$

$$\downarrow \text{strong anomaly}$$

$$SU(N)_c \times SU(N+4+p)_\eta \times SU(p)_{\bar{\eta}} \times U(1)_{\psi\eta} \times U(1)_{\psi\zeta}$$

$$U(1)_{\psi\eta} : \begin{cases} \psi \to e^{i(N+4+p)\alpha}\psi \\ \eta \to e^{-i(N+2)\alpha}\eta \end{cases} \qquad U(1)_{\psi\zeta} : \begin{cases} \psi \to e^{i(p)\beta}\psi \\ \zeta \to e^{-i(N+2)\alpha}\zeta \end{cases}$$

Massless baryons in BY

Even more surprisingly in the $p \neq 0$ case...

$$SU(N)_c \times SU(N+4+p)_\eta \times SU(p)_{\bar{\eta}} \times U(1)_{\psi\eta} \times U(1)_{\psi\zeta}$$

the baryons

$$(B_1)^{[AB]} = \psi^{ij}\eta_i^A\eta_j^B \ , \qquad (B_2)_A^a = \bar{\psi}_{ij}\bar{\eta}_A^i\xi^{j,a} \ , \qquad (B_3)_{\{ab\}} = \psi^{ij}\bar{\xi}_{i,a}\bar{\xi}_{j,b} \ ,$$

where

	$SU(N)_{\rm c}$	$SU(N+4+p)$	$SU(p)$	$U(1)_{\psi\eta}$	$U(1)_{\psi\xi}$
B_1	$\frac{(N+4+p)(N+3+p)}{2} \cdot (\cdot)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\frac{(N+4+p)(N+3+p)}{2} \cdot (\cdot)$	$-N+p$	p
B_2	$(N+4+p)p \cdot (\cdot)$	$p \cdot \bar{\square}$	$(N+4+p) \cdot \square$	$-(p+2)$	$-(N+p+2)$
B_3	$\frac{p(p+1)}{2} \cdot (\cdot)$	$\frac{p(p+1)}{2} \cdot (\cdot)$	$\overline{\square\square}$	$N+4+p$	$2N+4+p$

match all the standard anomalies!

Generalized 1-form global symmetries

0-form global symmetries act on 0-dimensional objects

$$\psi \to e^{i\alpha}\psi \qquad \alpha \in (0, 2\pi) .$$

1-form symmetries act on 1-dimensional objects

$$W[L] \to e^{-2\pi\epsilon[L]/N} W[L] \quad \epsilon \in H^1(\mathcal{M}, U(1)) .$$

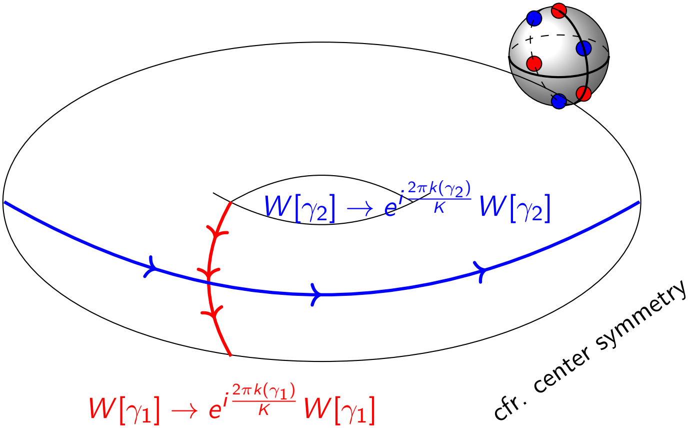
An example 1-form center symmetry

$\sim$ gauge symmetry with a wrong periodicity.

1-form symmetries can be gauged with a 2-form gauge field.

1-form center symmetry

What if I perform a "gauge transformation" with the wrong periodicity?



$$g(\theta) : S_1 \to R \qquad g(\theta = 0) = \mathbb{1} \qquad g(\theta = 2\pi) = e^{i\frac{2\pi}{K}k}\mathbb{1}$$

1-form global CFL symmetry

In a $SU(N) \times \tilde{U}(1)$ bundle

$$\text{Cocycle condition} \qquad g_{ij} g_{jk} g_{ki} = \mathbb{1} \,, \qquad u_{ij} u_{jk} u_{ki} = 1 \,,$$

so, if $z_{ij} \in \mathbb{Z}_N$ and

$$\boxed{z_{ij} z_{jk} z_{ki} = 1}$$

$$g_{ij} \to z_{ij} g_{ij} \qquad u_{ij} \to z_{ij}^{-1} u_{ij}$$

is a 1-form $\mathbb{Z}_N$ **global** symmetry.

Gauging the 1-form global CFL symmetry

Let's introduce the "2-form" gauge field...

Cocycle condition $\qquad g_{ij}g_{jk}g_{ki} = B_{ijk} \qquad u_{ij}u_{jk}u_{ki} = B_{ijk}^{-1}$

so, if $z_{ij} \in \mathbb{Z}_N$ and

$$\boxed{z_{ij}z_{jk}z_{ki} \neq 1}$$

$$g_{ij} \to z_{ij}g_{ij} \qquad u_{ij} \to z_{ij}^{-1}u_{ij}$$

is a 1-form $\mathbb{Z}_N$ **gauge** symmetry.

This is nothing more that an $U(N)$ gauge field...

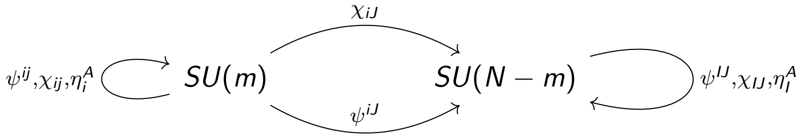
$$SU(N) \times U(1) \xrightarrow{\text{gauge } Z_N^{(1)}} U(N)$$

Tumbling $\psi\chi\eta$

Dynamical Abelianization in the $\psi\chi\eta$ model can be thought as the end point of a tumbling process...

$$\psi\chi\eta - \text{model}$$

$$\downarrow \psi\chi =\sim \begin{pmatrix} c_1\delta_{m\times m} & 0 \\ 0 & c_2\delta_{(N-m)\times(N-m)} \end{pmatrix} \quad c_1 \neq c_2$$



$$\downarrow$$

$$\downarrow$$