Stringy Quintessence Models

in the Swampland

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Coming soon to an arXiv near you!

Based on **"Stringy multifield quintessence and the swampland"**
with Michele Cicoli, Giuseppe Dibitetto and Francisco G. Pedro (to appear)

- Motivation: swampland vs. observation
- Around the swampland? Multifield quintessence!
- Stringy models I: universal moduli
- Stringy models II: non-universal moduli
- A potential problem: Q-balls

Motivation

A cosmological constant problem

- The CC is not a free parameter in quantum gravity.
- In EFT, it is just the vev of the scalar potential.
- A flat potential can sometimes have the effect of a CC (slow-roll).

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Observations

- CMB: explained by Inflation
- Today: accelerated expansion

$\Rightarrow$ Positive CC, $\Lambda > 0$

Theory

- No dS in parametric control
- Type IIA no-go theorem
- quantum break time
- ...

$\Rightarrow$ (no-)dS conjecture, $\Lambda \leq 0$

dS swampland conjecture and accelerating cosmologies

The (refined) dS conjecture

[Obied/Ooguri/Spodyneiko/Vafa '18; Garg/Krishnan '18; Ooguri/Palti/Shiu/Vafa '18]

The scalar potential V of an EFT coupled to quantum gravity in the UV must respect either

$$|\nabla V| \geq \frac{c}{M_p} \cdot V \qquad \text{or} \qquad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{{M_p}^2} \cdot V$$

with $c, c' > 0$ and $\mathcal{O}(1)$.

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with $c, c' > 0$ and $\mathcal{O}(1)$.

Implications for cosmology

- Clearly forbids (meta-)stable dS vacua $\nabla V = 0,\ V > 0$!
- Also flat slow-roll models $\epsilon_V \equiv \frac{1}{2}\left(\frac{\nabla V}{V}\right)^2 \ll 1$ are ruled out.

Multifield cosmology

Alternatives to dS or slow-roll

Accelerated expansion can be achieved with a steep potential, e.g. by **rotating in a curved field space.** [Brown '17; Achúcarro/Palma '18; Cicoli/Dibitetto/Pedro '20]

- Kinetic couplings: can't canonically normalize all fields at once
- Energy dissipates into rotation, slowing down the rolling field

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Multifield quintessence – from string theory?

- Hard to get many e-folds [Aragam/Chiovoloni/Paban/Rosati/Zavala '21]
- Focus on late-time cosmology instead
- Less e-folds needed, but observational constraints
- 2 field model has structure akin to string moduli

Can ST satisfy the dS conjecture **and** get late-time cosmology right?

[upcoming work with Cicoli, Dibitetto, Pedro]

Multifield quintessence

Accelerated Expansion

$$\text{Slow-roll:} \quad \epsilon_H = -\frac{\dot{H}}{H^2}\,, \quad 0 < \epsilon_H < 1$$

Single field case

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{\nabla V}{V}\right)^2 = \epsilon_H\,, \quad \epsilon_V \gtrsim 1 \quad \text{(dS conj.)}$$

The dS conjecture forbids flat potentials needed for slow roll.

Multifield quintessence

Accelerated Expansion

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Single field case

The dS conjecture forbids flat potentials needed for slow roll.

Multifield case

Rotation in moduli space slows down the rolling field.

$$\Rightarrow \quad \epsilon_V \equiv \frac{1}{2}\left(\frac{\nabla V}{V}\right)^2 \neq \epsilon_H \,!$$

Possible to have steeper potentials while accelerating.

The model

Action

Requirements: Gravity, 2 fields, kinetic coupling, scalar potential.

$$S = \int d^4x \sqrt{-g} \left(\frac{{M_p}^2}{2} R - \frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2} f(\phi_1)^2 (\partial\phi_2)^2 - V(\phi_1) \right)$$

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Natural situation in String theory

e.g. Kähler moduli in IIB flux vacua: $T = \tau + i\vartheta$, $K = -3\log(T + \overline{T})$

$$\mathcal{L}_{\text{kin}} \sim K_{T\overline{T}}\, \partial T \partial \overline{T} = \frac{3}{4\tau} \left((\partial\tau)^2 + (\partial\vartheta)^2 \right)$$

$$\Rightarrow \quad \phi_1 = \sqrt{3/2}\, \log(\tau)\,, \quad \phi_2 = \vartheta$$

$$f(\phi_1) = \sqrt{3/2}\, e^{\sqrt{3/2}\,\phi_1}$$

$$V(\phi_1) \sim \frac{1}{\tau} = e^{-\sqrt{3/2}\,\phi_1}$$

The model

Friedmann equations

$$H^2 = \frac{1}{6{M_p}^2}\left(\dot{\phi_1}^2 + f^2\dot{\phi_2}^2 + 2V + \rho_{\text{matter}}\right)$$

Defining new variables:

$$x_1 = \dot{\phi}_1 \;\; (\sqrt{6}H\,M_p)^{-1}$$
$$x_2 = f\,\dot{\phi}_2\,(\sqrt{6}H\,M_p)^{-1}$$
$$y_1 = \sqrt{V}\,(\sqrt{3}H\,M_p)^{-1}$$

Cosmological Observables:

$$\omega_\phi = \frac{{x_1}^2 + {x_2}^2 - {y_1}^2}{{x_1}^2 + {x_2}^2 + {y_1}^2}$$

$$\Omega_\phi = {x_1}^2 + {x_2}^2 + {y_1}^2$$

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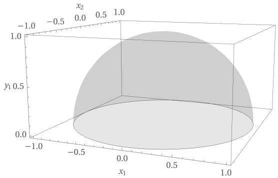

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Parameter space

- physical range: $x_1 \in [-1, 1]$, $x_2 \in [-1, 1]$, $y_1 \in [0, 1]$.
- In a flat Universe: $\Omega_{\text{matter}} = 1 - ({x_1}^2 + {x_2}^2 + {y_1}^2) > 0$.

$\Rightarrow$ Physical parameter space is upper half of a 3-ball.

Dynamics

Kinetic coupling

$$k_1 = -M_p \frac{\partial_{\phi_1} f}{f}\,, \qquad k_2 = -M_p \frac{\partial_{\phi_1} V}{V}\,.$$

In general, $k_i = k_i(\phi_i)$.

Evolution as autonomous system:

The dynamics is given in terms of e-folds ($' \equiv d/d(\ln a)$) by

$$x_1{}' = h_1(x_1, x_2, y_1)\,, \quad x_2{}' = h_2(x_1, x_2, y_1)\,, \quad y_1{}' = h_3(x_1, x_2, y_1)\,.$$

Dynamics are completely determined by initial conditions and k_i.

If $k_i = k_i(\phi_i)$, additionally require $\phi_1' = 6x_1$.

Fixed Points

Fixed points ${x_1}' = {x_2}' = {y_1}' = 0$ for constant k_i

	x_1	x_2	y_1	Ω_ϕ	ω_ϕ	stability
$\mathcal{K}_\pm$	± 1	0	0	1	1	unstable
$\mathcal{F}$	0	0	0	0	–	unstable
$\mathcal{S}$	$\frac{\sqrt{3/2}}{k_2}$	0	$\frac{\sqrt{3/2}}{k_2}$	$\frac{3}{k_2^2}$	0	$k_2^2 \geq 3$
$\mathcal{G}$	$\frac{k_2}{\sqrt{6}}$	0	$\sqrt{1 - \frac{k_2^2}{6}}$	1	$-1 + \frac{k_2^2}{3}$	$k_2 < \sqrt{6}$
$\mathcal{NG}$	$\frac{\sqrt{6}}{(2k_1+k_2)}$	$\frac{\pm\sqrt{k_2^2+2k_2k_1-6}}{2k_1+k_2}$	$\sqrt{\frac{2k_1}{2k_1+k_2}}$	1	$\frac{k_2-2k_1}{k_2+2k_1}$	$k_2 \geq \sqrt{6+k_1^2} - k_1$

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- The $\mathcal{NG}$ fixed points with $x_2 \neq 0$ exist only for multifield models
- Non-geodesic field trajectories, ϕ_2 dragged along by $\dot{\phi}_1$

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- The $\mathcal{NG}$ fixed points with $x_2 \neq 0$ exist only for multifield models
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- Only $\mathcal{G}$, $\mathcal{NG}$ can be accelerating $\omega_\phi < -1/3$
- But fixed points cannot fit $\Omega_\phi \sim 0.7$. Look for transients!

Approaches to finding transients

Cosmological initial conditions

- Start in the past at phase of matter domination.
- $\Omega_\phi = 0$ at fluid domination fixed point $\mathcal{F}$: $x_1 = x_2 = y_1 = 0$.
- Search for evolution into dark energy domination today.

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- Search for evolution into dark energy domination today.

Observable initial conditions

- Start "today" with observable parameters $\omega_\phi \sim -1$, $\Omega_\phi \sim 0.7$.
- Seems underdetermined, but fixes i.c. to $x_1 = x_2 = 0$, $y_1 = \sqrt{0.7}$.
- Integrate backwards to determine past evolution.
- Trajectory is always viable today, but past has to be explained.

Stringy models I: universal moduli

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{{M_p}^2}{2} R - \frac{1}{2} (\partial \phi_1)^2 - \frac{1}{2} f(\phi_1)^2 (\partial \phi_2)^2 - V(\phi_1) \right)$$

Kähler moduli T in IIB flux vacua:

With $T = \tau + i\vartheta$, $K = -3 \log(T + \overline{T})$:

$$\phi_1 = \sqrt{3/2}\, \log(\tau)\,, \qquad \phi_2 = \vartheta\,;$$

$$f(\phi_1) = \sqrt{3/2}\, e^{\sqrt{3/2}\,\phi_1}\,, \quad V(\phi_1) \sim \frac{1}{\tau} = e^{-\sqrt{3/2}\,\phi_1}$$

$$\Rightarrow \quad k_1 = \sqrt{\frac{2}{3}}\,, \qquad k_2 = \sqrt{6}$$

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Kähler potential for chiral superfield X: $K = -p\log(X + \overline{X})$

$$k_1 = \sqrt{\frac{2}{p}}\,, \qquad k_2 = \sqrt{2p}$$

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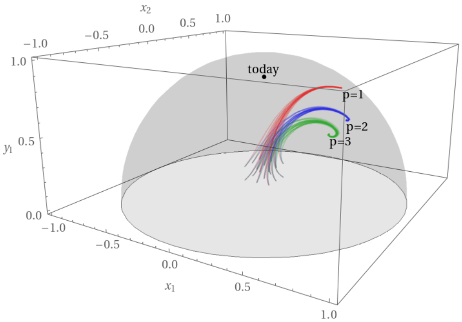
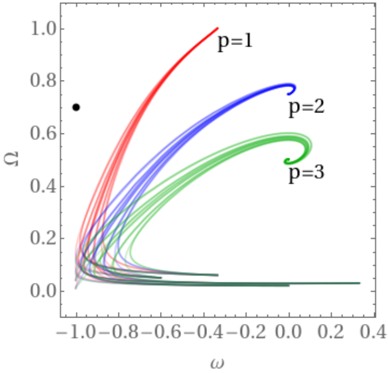
p	X	Theory	Sources	$\mathcal{M}_{\text{internal}}$
1	$S = e^{-\varphi} + \mathrm{i}\, a$	Heterotic	—	SU(3) str.
2	$T_2 = \mathrm{Vol}(\Sigma_4^{(2)}) + \mathrm{i} \int_{\Sigma_4^{(2)}} C_{(4)}$	Type IIB	D3/D7, O3/O7	K3-fibered CY_3
3	$T = \mathrm{Vol}(\Sigma_4) + \mathrm{i} \int_{\Sigma_4} C_{(4)}$	Type IIB	D3/O3	CY_3
...				
7	$Z = \mathrm{Vol}(\Sigma_3) + \mathrm{i} \int_{\Sigma_3} A_{(3)}$	M-theory	KK6/KKO6	G_2 str.

Stringy models I: universal moduli

Approach 1: Starting from matter domination

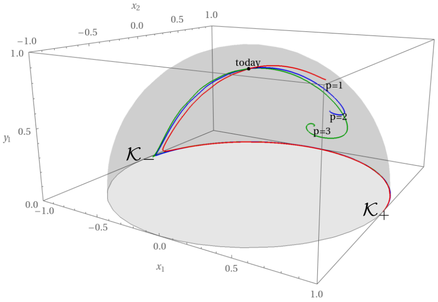


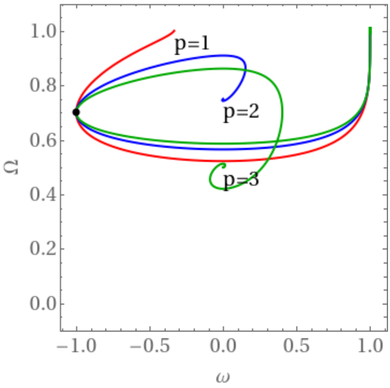


No viable trajectories starting from the matter dominated fixed point $\mathcal{F}$.

Stringy models I: universal moduli

Approach 2: Starting from observed parameters





Trajectories passing through the observed point.

Stringy models I: universal moduli

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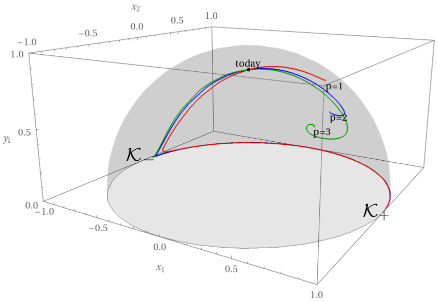

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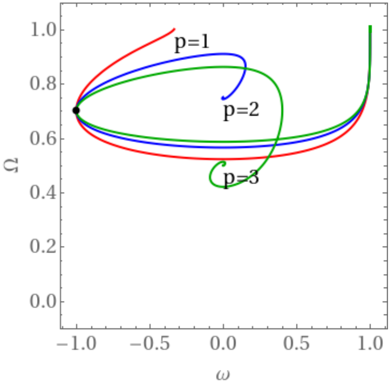
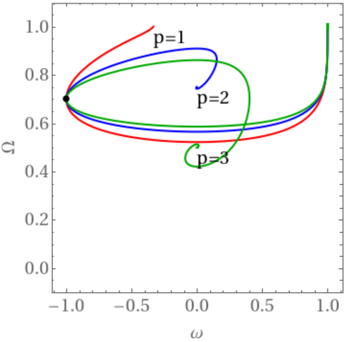


- Universal "spine" asymptoting to $\mathcal{K}_\pm$ fixed points (in the past).
- No trajectories come close to matter domination $\Omega_\phi = 0$.

Stringy models I: universal moduli

What is matter domination really?

- Def. matter: $\omega_m = 0$
- Is $\Omega_\phi \neq 0$ matter dominated, as long as $\omega_\phi = 0$?
- Trajectories have $\omega_\phi = 0$ in past.



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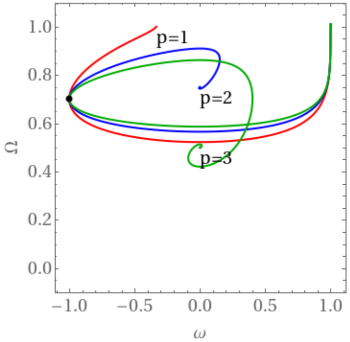

Initial conditions are harder to justify, but maybe not impossible!

Stringy models II: blow-up modes

Non-universal blow-up modes

- Govern the size of a blow-up (singularity resolution)
- Weak swiss cheese type: $\mathcal{V} = \alpha \, {\tau_b}^{3/2} - \lambda \, {\tau_s}^{3/2}$
- For $\tau_b \gg \tau_s \gg 1$: Kähler potential is power law, not logarithm!

$$K = -3 \ln \tau_b + 2 \left(\frac{\tau_s}{\tau_b} \right)^{\frac{3}{2}}$$

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Kinetic terms for canonically normalized blow-up mode

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}\sqrt{-g}\left((\partial\phi_1)^2 + \left(\frac{M_p}{\phi_1}\right)^{2/3}(\partial\phi_2)^2\right)$$

with $\phi_1 \sim (\frac{\tau_s}{\tau_b})^{\frac{3}{4}}$ and $\phi_2 \sim (\frac{\vartheta_s}{i\tau_b})^{\frac{3}{4}}$ the corresponding axion.

$$f(\phi_1) = \left(\frac{M_p}{\phi_1}\right)^{1/3} \quad \Rightarrow \quad k_1 = \frac{1}{3}\frac{M_p}{\phi_1}\,.$$

Stringy models II: blow-up modes

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Potential terms

- Perturbative corrections (string loops, higher derivative effects).
- Depends on the explicit setup, but is always power-law:

$$V(\phi_1) = V_0 \left(\frac{M_p}{\phi_1}\right)^{\pm 2/3} \quad \text{or} \quad V(\phi_1) = \frac{V_0}{C - (\phi_1/M_p)^{2/3}}\,.$$

$$\Rightarrow \quad k_2 \sim \frac{2}{3}\frac{M_p}{\phi_1}\,.$$

Stringy models II: blow-up modes

General power-law models [Cicoli/Dibitetto/Pedro '20]

Power-law Kähler and scalar potentials fall into class of models

$$f(\phi_1) = \left(\frac{M_p}{\phi_1}\right)^{p_1} \quad \text{and} \quad V(\phi_1) = V_0 \left(\frac{M_p}{\phi_1}\right)^{p_2}$$

$$\text{with} \qquad k_1 = p_1 \frac{M_p}{\phi_1}, \quad k_2 = p_2 \frac{M_p}{\phi_1}.$$

Hierarchy unnatural?

- Viable trajectories need at least $\mathcal{O}(10)$ hierarchy between p_1, p_2.
- In blow-up example, $p_1/p_2 = 1/2 \ll \mathcal{O}(10)$
- Other string examples also fail to produce hierarchy.
- But hard to rule out conclusively.

Conserved currents and Q-balls

Dynamic Q-ball formation

- Our models have a conserved current $J^\mu = \sqrt{-g} f^2 \partial^\mu \phi_2$.
- In such models Q-balls may appear. [Coleman '85; Krippendorf/Muia/Quevedo '18]
- Production of Q-balls screens the dark energy. [Kasuya '01; Li/Hao/Liu '01]
- Must ensure that our models avoid producing Q-balls!

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Jeans length

Q-balls can only form if

$$0 < \frac{k^2}{a^2} < 3H^2 M_p^2 \left((4k_1^2 - 2k_1')\, x_2^2 - (k_2^2 - k_2')\, y_1^2 \right) ,$$

leaving a safe wedge in the center of parameter space.

Conserved currents and Q-balls

Jeans length for constant k_i

Q-balls can only form if

$$0 < \frac{k^2}{a^2} < 3H^2 M_p^2 \left(4k_1^2\, x_2^2 - k_2^2\, y_1^2\right) ,$$

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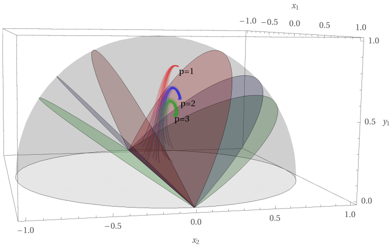

Implications

- No problem for constant k_i or power-law models.
- May become important if a model has $k1/k2$ hierarchy.

Conclusions

- Many accelerating models satisfy the dS conjecture.
- But can they model our late time cosmology?
- Multi-field approach natural in string theory.
- Kinetic coupling allows for a steep potential.
- Fixed points don't include our universe. Transients!

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- Non-universal moduli can't generate necessary $p1/p2$ hierarchy.

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- Universal moduli have viable transients with questionable past.
- Non-universal moduli can't generate necessary $p1/p2$ hierarchy.

- Q-balls could ruin the solution. Not a problem in most cases.

Thank you for your attention!

Autonomous system

The autonomous system:

$$x_1' = 3x_1(x_1^2 + x_2^2 - 1) + \sqrt{\frac{3}{2}}(-2k_1 x_2^2 + k_2 y_1^2) - \frac{3}{2}\gamma x_1(x_1^2 + x_2^2 + y_1^2 - 1)\,,$$

$$x_2' = 3x_2\left(x_1^2 + x_2^2 - 1\right) + \sqrt{6}k_1 x_1 x_2 - \frac{3}{2}\gamma x_2\left(x_1^2 + x_2^2 + y_1^2 - 1\right),$$

$$y_1' = -\sqrt{\frac{3}{2}}k_2 x_1 y_1 - \frac{3}{2}\gamma y_1\left(x_1^2 + x_2^2 + y_1^2 - 1\right) + 3y_1\left(x_1^2 + x_2^2\right),$$

and the cosmological parameters:

$$\Omega_\phi' = -3\left(\Omega_\phi - 1\right)\Omega_\phi(\omega_b - \omega_\phi)\,,$$

$$\omega_\phi' = (\omega_\phi - 1)\left(-k_2\sqrt{3(\omega_\phi + 1)\Omega_\phi - 6x_2^2} + 3(1 + \omega_\phi)\right).$$