From Interfaces to S-folds and Conformal Manifolds

Jesse van Muiden

Joint work with Nikolay Bobev, Fridrik Gautasson,
Kryztof Pilch, and Minwoo Suh

[1907.11132], [2003.09154], [2104.00977], [2111.11461] + ongoing work

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SISSA

Why Interfaces

- **Condensed matter:** impurities, conformal boundaries
- **String theory:** interfaces/defects $\sim$ branes, non-geometric backgrounds

Today we will take a **CFT perspective**

- **new (strongly coupled) 3d S-fold CFTs**

- **conformal manifolds with varying SUSY**

Conformal manifolds

Conformal manifolds are families of CFTs generated by **exactly marginal** couplings.

$$\mathcal{L} = \mathcal{L}_0 + h\mathcal{O}\,, \quad \Delta(\mathcal{O}) = d\,, \quad \text{and} \quad \beta(h) = 0\,.$$

Exactly marginal couplings have a vanishing β-function to all orders! Very stringent, e.g.

$$\mathcal{C}_{\mathcal{O}\mathcal{O}\tilde{\mathcal{O}}} = 0$$

Their existence is hard to show in $d > 2$, except when $\#Q \geq 4$ and one can utilize non-renormalization theorems.

Leigh, Strassler '95

CONFORMAL MANIFOLDS

- *Do conformal manifolds exist with $\#Q < 4$, in $d > 2$?*
- *What can we say about the spectral data of local operators along a conformal manifold?*

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- *What can we say about the spectral data of local operators along a conformal manifold?*

Recently the latter has been concretely verbalised as

Conjecture II: *All CFTs at infinite distance are HS points.*

Perlmutter, Rastelli, Vafa, Valenzuela '21

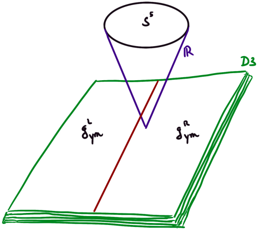
Where the distance is computed with respect to the Zamolodchikov metric

$$|x-y|^{2d}\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = g_{ij}$$

Holographically these questions are translated to the properties of continuous families of AdS$_{d+1}$ vacua.

Top-Down Holography

We use top-down holography to get a better understanding of the inherently strongly coupled CFTs that we will encounter.



- The CFTs arise from a stack of D3 branes at the tip of a Sasaki-Einstein (SE) cone.
- In this talk we specialize the SE_5 space to be S^5.

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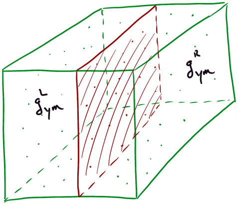
Interfaces and S-folds in $\mathcal{N} = 4\, SYM$

$\mathcal{N} = 4$ *SYM + an Interface*

We deform $\mathcal{N} = 4$ SYM with a spatial dependent coupling **(Janus interface)**.

$$\mathcal{L} = \mathcal{L}_{\text{SYM}} + \mathcal{L}_I(y)$$

Bak, Gutperle, Hirano '03
Clark, Freedman, Karch, Schnabl '05



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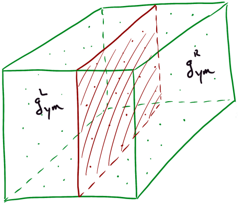

- The main difference between the two CFTs is the coupling!
- On the interface we preserve conformal symmetry.
- No new degrees of freedom are added.

$\mathcal{N} = 4 +$ *an Interface*

The interface operators schematically take the form

$$\mathcal{L}_I = \frac{\partial_y g_{YM}}{g_{YM}^3} \mathrm{Tr}\left[\psi^2 + \phi^3\right]$$

- Take g_{YM} to be a Heaviside function
- Details of the couplings determine the preserved SUSY

D'Hoker, Estes, Gutperle '06

$\mathcal{N}$	supergroup	R-symmetry	Flavour
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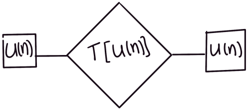
A more general class of spatially modulated couplings was recently studied as well.

Arav, Cheung, Gauntlett, Roberts, Rosen '20

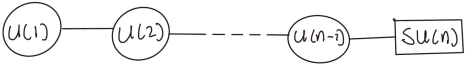
New 3d CFTs

Co-dimension one interfaces provide interesting pathways to the study of new three-dimensional strongly coupled physics.

An important example is the discovery of the **T[U(N)]** theories living on the $\mathcal{N} = 4$ interface.



This quiver is the IR-fixed point of



undergoing a symmetry enhancement in the IR.

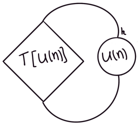
Gaiotto, Witten ('09, '10)

3d S-folds

We compactify the direction perpendicular to the interface, gauging the diagonal $\mathrm{U}(N) \subset \mathrm{U}(N) \times \mathrm{U}(N)$, with an $\mathcal{N} = 4$ vector, Chern-Simons level k, and superpotential

$$W_{UV} = -\frac{k}{4\pi}\mathrm{Tr}\,\Phi^2 + \mathrm{Tr}(\Phi(\mu_H + \mu_C)) \to W_{IR} = -\frac{2\pi}{k}\mathrm{Tr}\,\mu_H \mu_C$$

we find new 3d **S-folds**



Its S^3 free energy can be computed to all orders in N

$$F_{S^3} = \frac{N^2}{2}T + \sum_{j=1}^{N} \ln(1 - \mathrm{e}^{-jT}) = \frac{N^2}{2}T + f_0(T) - \sum_{k=1}^{\infty} \frac{\mathrm{e}^{-kTN}}{k(1 - \mathrm{e}^{kT})}\,.$$

Terashima, Yamazaki ’11; Ganor, Moore, Sun, Torres-Chicon ’14; Gang, Yamazaki ’18; Assel, Tomasiello ’18

New 3d CFTs

Similar setups can be constructed where the 3d CFT preserves less supersymmetry ($\mathcal{N} = 0, 1, 2$).

In what follows we focus on the $\mathcal{N} = 4$ S-fold and argue that it lies on a conformal manifold preserving $\mathcal{N} = 2$ **SUSY**.

Adding a mass term for Φ in the UV provides the marginal coupling in the IR

$$W_{IR} = -\frac{2\pi}{k}\mathrm{Tr}\,\mu_H \mu_C + \lambda \mathrm{Tr}\,\mu_H \mu_C \,.$$

Furthermore, our studies show that the conformal manifold is **non-compact and strongly coupled**.

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All this we do by constructing holographically dual AdS_4 solutions in type IIB string theory.

A Holographic Exploration

type IIB vs 5d SUGRA

Our starting point is a stack of N D3 branes at the tip of a cone over S^5 describing $\mathcal{N} = 4$ SYM.

- The interface breaks some D3 world volume isometries.
- Depending on the field theory couplings certain S^5 isometries are broken as well.

Solving PDEs in IIB is *hard*!

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Solving PDEs in IIB is *hard*!

Instead, we will consistently truncate the theory down to five-dimensional, $\mathcal{N} = 8$, SO(6) gauged, supergravity, reducing the problem to a set of ODEs (*easy*)!

The consistent truncation allows us to systematically uplift the lower-dimensional solutions back to type IIB!

Baguet, Hohm, Samtleben '15

The 5d Theory

The solutions of interest are described by a handful of scalars, preserving $\mathfrak{g} \subset \mathfrak{so}(6)$, dual to the field theory symmetries.

$\mathcal{N}$	supergroup	R-symmetry	Flavour
4	$\mathfrak{osp}(4\|4,\mathbb{R})$	$\mathfrak{so}(4)$	
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For the sake of concreteness we present the Janus interface that preserves $\mathfrak{so}(4) \subset \mathfrak{so}(6)$, and $\mathcal{N} = 4$ supersymmetry.

This truncation leaves five non-trivial scalars $\phi^a = (\alpha, \chi, \kappa, \varphi, c)$.

The 5d Theory

We are interested in solutions of the form

$$\mathrm{d}s_5^2 = \mathrm{d}r^2 + \mathrm{e}^{2A(r)}\mathrm{d}s^2_{\mathrm{AdS}_4}\,, \quad \phi^a = \phi^a(r)\,.$$

The BPS equations solving this system are

$$\begin{aligned}
(\alpha' - \tfrac{\varphi'}{\cos\kappa})^2 &= \tfrac{|\partial_\alpha W|^2}{36}\,, & \kappa' &= (1+2\sinh\varphi)c'\,,\\
\chi'(\alpha' - \tfrac{\varphi'}{\cos\kappa}) &= \tfrac{\sinh 4\chi\,\mathrm{Re}(W\partial_\alpha W)}{24}\,, & c' &= -\,2\tfrac{\tan\kappa}{\sinh 2\varphi}\varphi'\,,\\
\varphi'(\alpha' - \tfrac{\varphi'}{\cos\kappa}) &= \tfrac{\tanh 4\chi\cos\kappa\,\mathrm{Im}(W\partial_\alpha W)}{24}\,, & (A')^2 &= \tfrac{1}{9}|W|^2 - \mathrm{e}^{-2A}\,,
\end{aligned}$$

where W is the superpotential

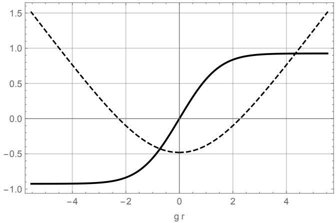
$$W = -\frac{3}{2}(\cosh 2\alpha\cosh 2\chi - \mathrm{i}\sinh 2\alpha\sinh 2\chi)\,.$$

This system can be solved analytically!

The 5d Janus Solution

The **full solution** is determined by in total four integration constants: $\mathcal{I}$, $\mathcal{J}$, F_0, and c_0:

$$\mathcal{I} \sim |g_{YM}^L - g_{YM}^R|\,, \qquad F_0 \sim g_{YM}^L + g_{YM}^R\,,$$
$$\mathcal{J} \sim |\theta_{YM}^L - \theta_{YM}^R|\,, \qquad c_0 \sim \theta_{YM}^L + \theta_{YM}^R\,.$$



$\mathcal{N} = 4$ *S-fold*

We can see that something special happens when

$$\mathcal{I} = 1\,, \quad \mathcal{J} = 0\,, \quad c_0 = 0\,,$$

All scalars become constant, except φ:

$$\mathrm{d}s_5^2 = \mathrm{d}r^2 + \mathrm{d}s^2_{\mathrm{AdS}_4}\,, \quad \varphi = \varphi_0 + r\,.$$

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We can **compactify** the radial direction, but additionally we have to **make φ periodic** using an $\mathfrak{sl}(2,\mathbb{R})_S$ transformation

$$\mathfrak{J}_k = \begin{bmatrix} k & 1 \\ -1 & 0 \end{bmatrix}\,, \qquad \mathfrak{J}_k^\dagger \mathcal{M}(r + r_0)\mathfrak{J}_k = \mathcal{M}(r)\,,$$

where $k = 2\cosh r_0$.

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where $k = 2\cosh r_0$.

The regularized on-shell action is $(T = \mathrm{arccosh}\, k/2)$

$$F_{S^3} = \frac{N^2}{2}T \sim \frac{N^2}{2}T + \sum_{j=1}^{N} \ln(1 - \mathrm{e}^{-jT})$$

matching the QFT computation.

$\mathcal{N} < 4$ *S-folds and Conformal Manifolds*

The 3d $\mathcal{N} < 4$ S-folds

Less supersymmetric S-folds are conceptually very similar.

$\mathcal{N}$	S-twist	G_R	G_F	$\mathcal{F}_{S^3}$
4	hyperbolic	$\mathfrak{so}(4)$		$\frac{N^2}{2}\text{arccosh}(n/2)$
2	hyperbolic	$\mathfrak{u}(1)$	$\mathfrak{su}(2)/\mathfrak{u}(1)$	$\frac{N^2}{2}\text{arccosh}(n/2)$
1	hyperbolic		$\mathfrak{su}(3)/\mathfrak{u}(1)^2$	$\sqrt{\frac{5^5}{3^6}}\frac{N^2}{4}\text{arccosh}(n/2)$
1	elliptic		$\mathfrak{so}(3)/\mathfrak{u}(1)$	$\sqrt{\frac{5^5}{3^9}}\frac{N^2}{2}2\pi\left(k+\frac{1}{n}\right)$
1	elliptic		$\mathfrak{u}(1)$	$\frac{81N^2}{32\sqrt{70+26\sqrt{13}}}2\pi\left(k+\frac{1}{n}\right)$

Arav, Gauntlett, Roberts, Rosen, Giambrone, Guarino, Malek, Samtleben, Sterckx, Trigiante, Cesaro, Larios, Varela, Berman, Fischbacher, Inverso, Bobev, Gautason, Pilch, Suh, JvM, … '19-21

Because there is no localization computation available for the $\mathcal{N} = 1$ S-folds it is hard to check their QFT constructions.

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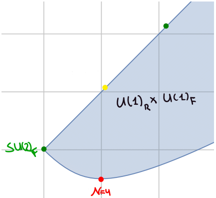
Because there is no localization computation available for the $\mathcal{N}=1$ S-folds it is hard to check their QFT constructions.

- **The $\mathcal{N}=4$ and $\mathcal{N}=2$ S-folds on a conformal manifold!**

A conformal manifold of $\mathcal{N} = 2$ S-folds

Holographically we constructed a two-parameter family of AdS_4 solutions dual to the conformal manifold.

Bobev, Gautason, JvM '21
Guarino, Sterckx, Trigiante '20
Arav, Cheung, Gauntlett, Roberts, Rosen '21



As mentioned at the start, the field theory description is given by the superpotential

$$W_{IR} = \lambda \mathrm{Tr}\, \mu_H \mu_C - \frac{2\pi}{k} \mathrm{Tr}\, \mu_H \mu_C \,.$$

A conformal manifold of $\mathcal{N} = 2$ S-folds

The Zamolodchikov metric is

$$\mathrm{d}s_Z^2 = \frac{(1+2x^2)(\mathrm{d}x^2 + 2(1+x^2)\mathrm{d}y^2)}{2(1+x^2)^2}\,, \quad R_Z = \frac{4(4x^4+2x^2-1)}{(1+2x^2)^3}\,.$$

The conformal manifold appears to be **non-compact**.

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The lowest lying spectrum of operators equals ($X\overline{Y}[\Delta; j; r; F]$)

$$\begin{array}{llll} A_1\overline{A}_1[2;1;0;0]\,, & L\overline{A}_1[\tfrac{5}{2};\tfrac{1}{2};\pm1;0]\,, & L\overline{L}[\tfrac{1}{2}+\alpha;0;0;\pm2]\,, & \\ & & & L\overline{B}_2[2;0;\pm2;0]\,, \\ A_2\overline{A}_2[1;0;0;0]\,, & L\overline{L}[\tfrac{1}{2}+\beta_\pm;0;0;0]\,, & L\overline{L}[\tfrac{1}{2}+\gamma_\pm;\tfrac{1}{2};0;\pm1]\,, & \end{array}$$

where

$$\alpha^2 = \tfrac{1}{4} + 2x^2 + \tfrac{4y^2}{1+x^2}\,, \quad \beta_\pm^2 = \tfrac{17+(17\pm16)x^2}{4(1-x^2)}\,, \quad \gamma_\pm^2 = \tfrac{x^2+(x^2+2)^2+2y^2\pm2x\sqrt{(x^2+2)^2+2y^2}}{2(1+x^2)}\,.$$

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Non-compact CM without a free point!

A conformal manifold of $\mathcal{N} = 2$ S-folds

The χ-direction has been shown to be periodic through a 10d analysis (KK-spectrum).

Giambrone, Malek, Samtleben, Trigiante '21

The limit $x \to \infty$ was shown to be singular in 5d supergravity.

$$\mathrm{d}s_5 \sim x^{-2/3}\mathrm{d}s^2_{\mathrm{AdS}_4} + x^{4/3}\mathrm{d}r^2$$

Arav, Gauntlett, Roberts, Rosen '21

The 10d solution is a mess.. Here we show simply the metric in the $x \to \infty$ limit:

$$\mathrm{d}s^2_{10} \sim f_1(\theta_i)\left[x\,\mathrm{d}s^2_{S^1} + \mathrm{d}s^2_{\mathrm{AdS}_4} + \mathrm{d}s^2_{\tilde{S}^5}\right] + \mathcal{O}(x^{-1})\,,$$

The remaining fields go as

$$(B_2 + \mathrm{i}C_2, C_4, \tau_{\mathrm{IIB}}) \sim \mathcal{O}(x^0)\,.$$

A conformal manifold of $\mathcal{N} = 2$ S-folds

In the $x \to \infty$ limit we find that x only influences the radius of the S-fold circle, which holographically controls the CS-level.

Remember the field theory superpotential

$$W_{IR} = -\frac{2\pi}{k}\mathrm{Tr}\,\mu_H\mu_C + \lambda\mathrm{Tr}\,\mu_H\mu_C\,.$$

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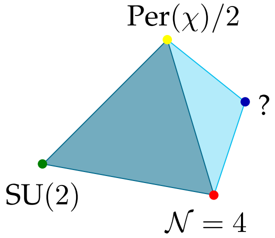
Does (a sector) become free/topological in this limit?

Duality on the CM involving the CS-level, N, and possibly $SL(2,\mathbb{Z})$?



Final Remarks

We showed how 3d S-fold CFTs can be constructed starting from 4d Janus interfaces.

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Can we conclusively determine the topology of the conformal manifold?

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Can we conclusively determine the topology of the conformal manifold?

Supersymmetric localization can compute the free energy analytic in N

$$F_{S^3} = \frac{N^2}{2}T + f_0(T) - \sum_{k=1}^{\infty} \frac{\mathrm{e}^{-kTN}}{k(1-\mathrm{e}^{kT})}\,.$$

Are α' and g_s corrections in type IIB supergravity able to reproduce more than the N^2 term?

Thank you

Extra slides

We worked in five-dimensional maximal $\mathrm{SO}(6)$ gauged supergravity, and four-dimensional $\mathrm{SO}(6) \times \mathrm{SO}(1,1) \times \mathbb{R}^{12}$ supergravity to find our results.

$$content... \tag{1}$$

What about $\mathcal{N} = 1$ S-folds?

The lack of QFT tools makes it difficult to explain the existence of the $\mathcal{N} = 1$ S-folds.

$\mathcal{N}$	S-twist	G_F	$\mathcal{F}_{S^3}$
1	hyperbolic	$\mathfrak{su}(3)/\mathfrak{u}(1)^2$	$\sqrt{\frac{5^5}{3^6}}\frac{N^2}{4}\mathrm{arccosh}(k/2)$
1	elliptic	$\mathfrak{so}(3)/\mathfrak{u}(1)$	$\sqrt{\frac{5^5}{3^9}}\frac{N^2}{2}2\pi\left(k+\frac{1}{n}\right)$
1	elliptic	$\mathfrak{u}(1)$	$\frac{81N^2}{32\sqrt{70+26\sqrt{13}}}2\pi\left(k+\frac{1}{n}\right)$

What about $\mathcal{N} = 1$ S-folds?

The lack of QFT tools makes it difficult to explain the existence of the $\mathcal{N} = 1$ S-folds.

$\mathcal{N}$	S-twist	G_F	$\mathcal{F}_{S^3}$
1	hyperbolic	$\mathfrak{su}(3)/\mathfrak{u}(1)^2$	$\sqrt{\frac{5^5}{3^6}\frac{N^2}{4}\mathrm{arccosh}(k/2)}$
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1	elliptic	$\mathfrak{u}(1)$	$\frac{81N^2}{32\sqrt{70+26\sqrt{13}}}2\pi\left(k+\frac{1}{n}\right)$

Interestingly it turns out that all known $\mathcal{N} = 1$ S-folds lie on a conformal manifold.

Guarino, Sterckx '21
Berman, Fischbacher, Inverso '21
Bobev, Gautason, JvM '21

The 3d exactly marginal couplings arise from turning on a 4d Wilson line on the S-fold circle, breaking the flavor symmetry down to its Cartan.

BRINGING THE JANUS BACK TO 10D

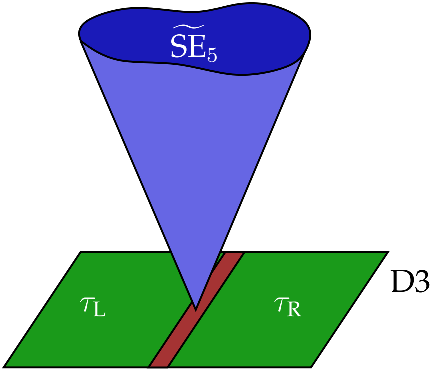
Although working in 5D has its merits, we can **go back to 10D**.

$$
\begin{aligned}
\mathrm{d}s_{10}^2 &= \cosh\chi\left[\mathrm{d}s_5^2 + \frac{4}{g^2}\left(\zeta^2 + \frac{\mathrm{d}s_4^2}{\cosh^2\chi}\right)\right]\,,\\
\tau_{\text{IIB}} = C_0 + \mathrm{i}\mathrm{e}^{-\Phi} &= \frac{\sinh 2\varphi\cos c + \mathrm{i}}{\cosh 2\varphi - \sinh 2\varphi \sin c}\\
C_2 - \tau_{\text{IIB}} B_2 &= -\frac{4\mathrm{i}}{g^2}\frac{\mathrm{e}^{-\mathrm{i}\omega}\tanh\chi}{\cosh\varphi + \mathrm{i}\mathrm{e}^{\mathrm{i}c}\sinh\varphi}e^{3\mathrm{i}\phi}\Omega\,,\\
C_4 &= \frac{16}{g^4}\mathrm{d}\phi\wedge\sigma\wedge J\,,
\end{aligned}
$$

where $\mathrm{d}s_4^2$ is a Kähler-Einstein metric with Kähler form J, and a holomrphic $(2,0)$ form Ω such that

$$
2J = \mathrm{d}\sigma, \quad \Omega\wedge\overline{\Omega} = 2J\wedge J, \quad \mathrm{d}\Omega = 3\mathrm{i}\sigma\wedge\Omega, \quad \text{and} \quad \zeta = \mathrm{d}\phi + \sigma\,.
$$

The Picture Again



Bringing the J-fold Back to 10D

We can do the same for the J-folds

$$
\begin{aligned}
\mathrm{d}s_{10}^2 =& \sqrt{\frac{5}{6}}\frac{2}{3g^2}\left(4\mathrm{d}r^2 + 5\mathrm{d}s^2_{\mathrm{AdS}_4} + 6\mathrm{d}s_4^2 + \tfrac{36}{5}\zeta^2\right)\,,\\
\tau_{\text{IIB}} =& \frac{\cosh(2\varphi + r_0) + \mathrm{i}\sinh r_0}{\cosh 2\varphi}\,,\\
C_2 - \tau_{\text{IIB}}B_2 =& -\frac{2\mathrm{i}}{g^2}\frac{\sqrt{\frac{2}{3}\sinh r_0}}{\cosh\varphi + \mathrm{i}\sinh\varphi}e^{3\mathrm{i}\phi}\Omega\,,\\
C_4 =& \frac{16}{g^4}\mathrm{d}\phi\wedge\sigma\wedge J\,,
\end{aligned}
$$

where $\varphi = \varphi_0 + r$.

$\mathcal{N} = 4$ *SYM*

In the $\mathcal{N} = 4$ theory we have the following field content

$$\left(A_\mu, \psi^i, \phi_{[ij]}\right), \qquad i, j \in \mathrm{SU}(4)_R, \quad \text{adjoint in } \mathrm{SU}(N)\,.$$

The Lagrangian can be written as

$$\mathcal{L} = \mathrm{Tr}\left[\frac{1}{g_{YM}^2}\Big(-\frac{1}{4}F_{\mu\nu}^2 + \left(D_\mu\phi^{ij}\right)^2 - \frac{1}{2}\overline{\psi}_i \not{D}\psi^i \right.$$
$$\left. - \overline{\psi}^i\left[\phi_{ij}, \psi^j\right] + \left[\phi^{ij}, \phi^{kl}\right]^2\Big) + \frac{\theta_{YM}}{8\pi^2}\left(F \wedge F\right)\right].$$

$\mathcal{N} = 4 \; SYM$

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The theory is conformal. We break this symmetry partially by adding a spatial dependent coupling.

Final remarks

Lastly, we studied different $\mathcal{N} = 1$ S-folds and show that they lie on a conformal manifold.

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Due to the lack of exact QFT tools for such theories it is hard to explain their origin.

The holographic construction suggests that they should arise from an $\mathbb{R}^{1,2} \times S^1$ compactification of $\mathcal{N} = 4$ SYM, with an S-duality twist.

Final remarks

Lastly, we studied different $\mathcal{N} = 1$ S-folds and show that they lie on a conformal manifold.

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The 3d $\mathcal{N} = 1$ conformal manifolds are rare, and no QFT theorems protect the Kahler potential from corrections.

It will be interesting to see if and how string theory corrections lift the continuous families of $\mathcal{N} = 1$ AdS$_4$ vacua.