Brane evaporation in double holography and entanglement islands

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Based on work in progress with A. Bernamonti, R. Emparan, A. Frassino

Introduction

Quantum Information & AdS/CFT

Interplay between quantum information and holography has led to a fruitful bulk-boundary dialogue

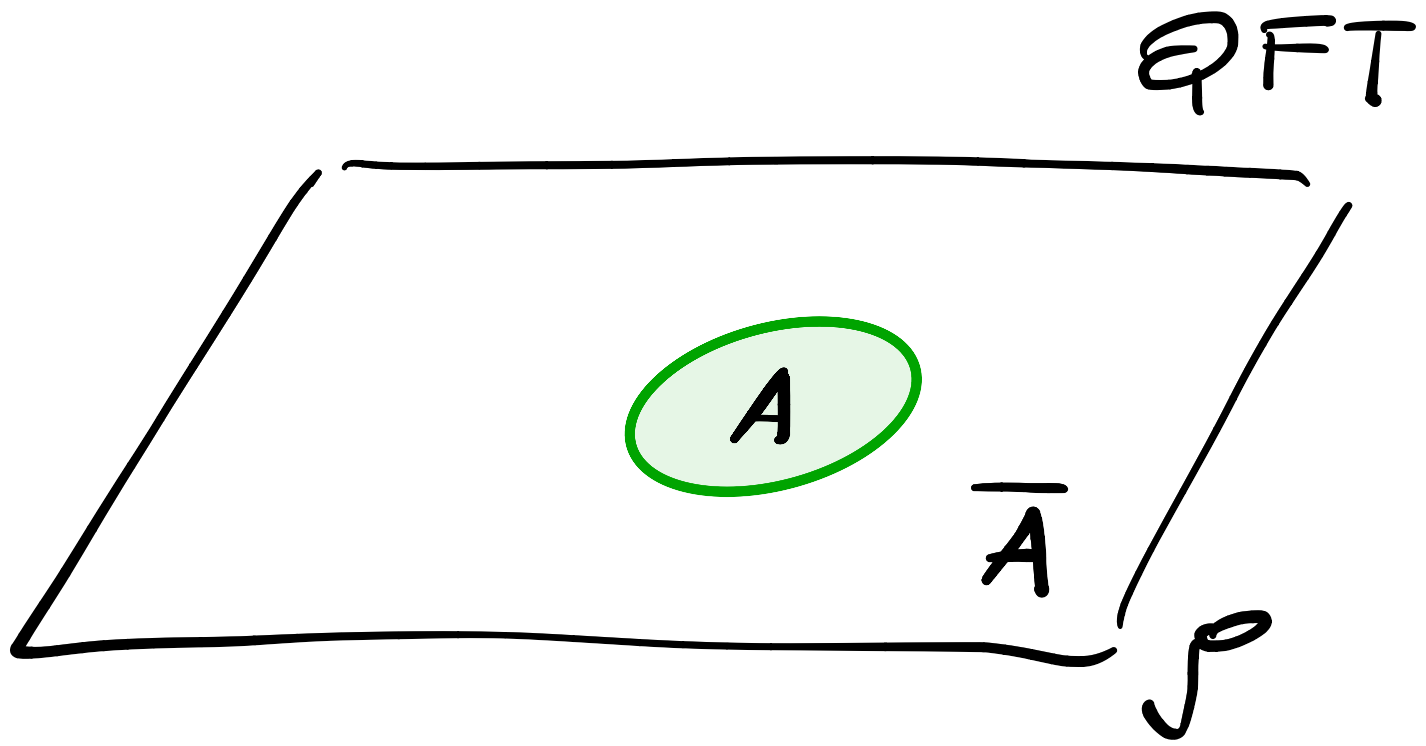
- New lessons for QFT: thermalization at strong coupling, entanglement scrambling, quantum chaos…
- New insight for quantum gravity: deep connection between gravity, geometry and the structure of quantum correlations in the dual field theory
- Renewed understanding of holographic dictionary: new fundamental entries, which parts of the bulk are encoded in the boundary…

New understanding for how entanglement entropy is evaluated in situations with dynamical gravity

Entanglement Entropy

Entanglement entropy of a subsystem A

$$S(A) = -\mathrm{Tr}_A\, \rho_A \log \rho_A \qquad\qquad \rho_A = \mathrm{Tr}_{\bar{A}}\, \rho$$



Quantifies the lack of information about the subsystem A

$S(A) = 0 \iff \rho_A$ pure state

$S(A) > 0$ for mixed states

Encodes information about the QFT state

$$S(A) = \frac{c}{6} \log \frac{L}{\delta} + \log g_b$$

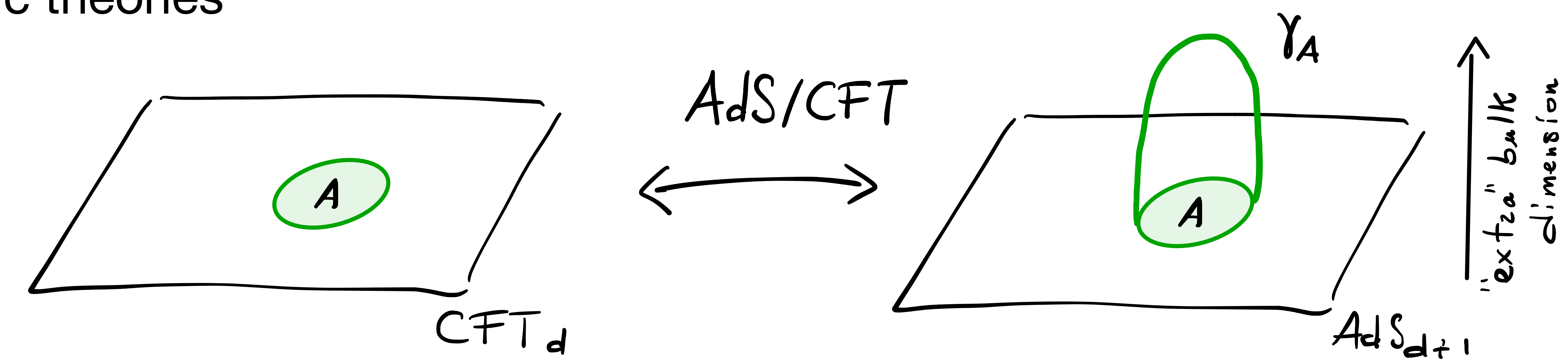
central charge

boundary entropy

$c_{\mathrm{boundary}} = \log g_b$

Holographic Entanglement Entropy

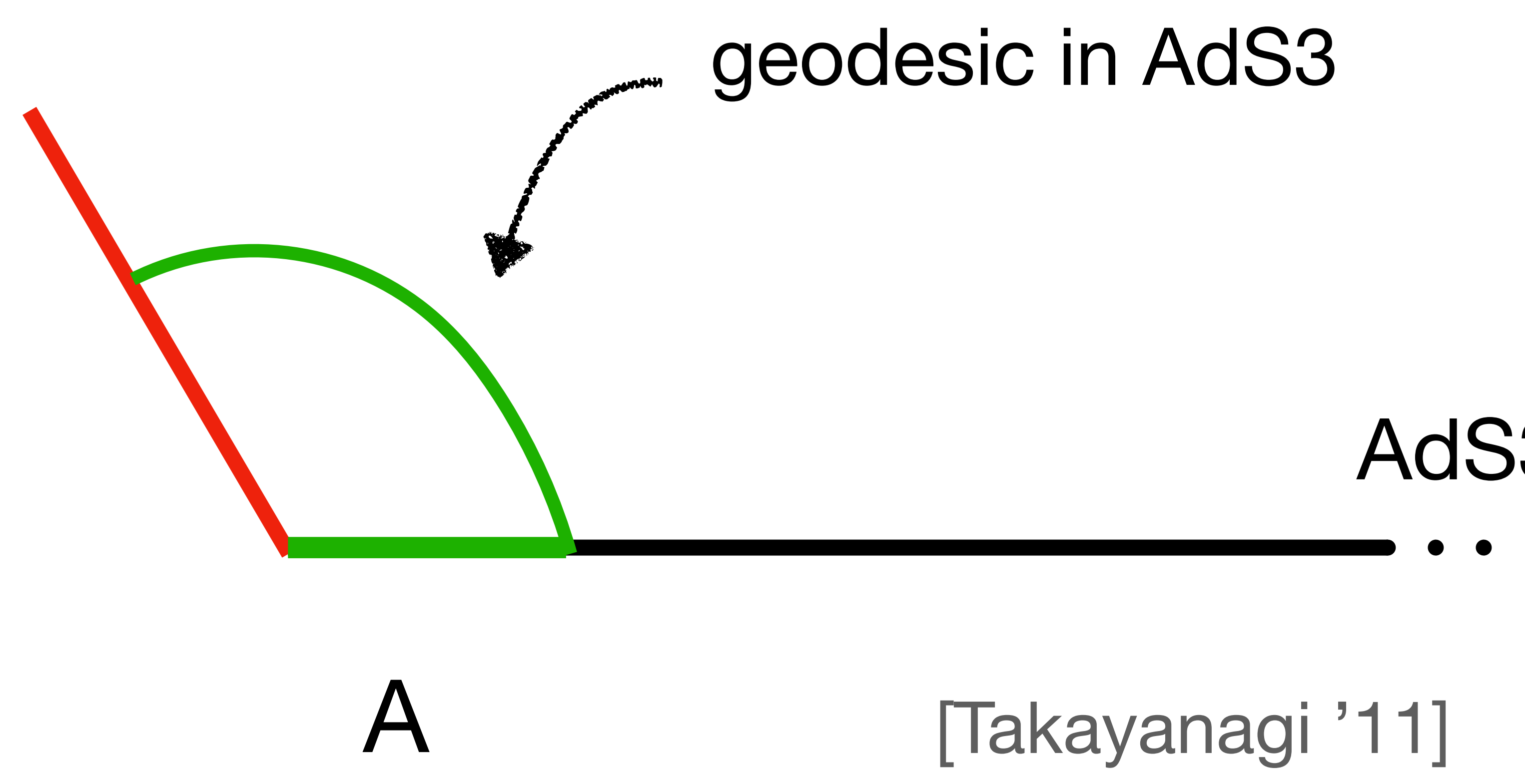
In holographic theories



Holographic entanglement entropy formula: evaluates the CFT entanglement entropy in geometric terms in AdS

$$S(A) = \mathrm{Min}\frac{\mathrm{Area}(\gamma_A)}{4G_N}$$

[Ryu, Takayanagi '06]



[Takayanagi '11]
[Fujita,Takayanagi,Tonni '11]

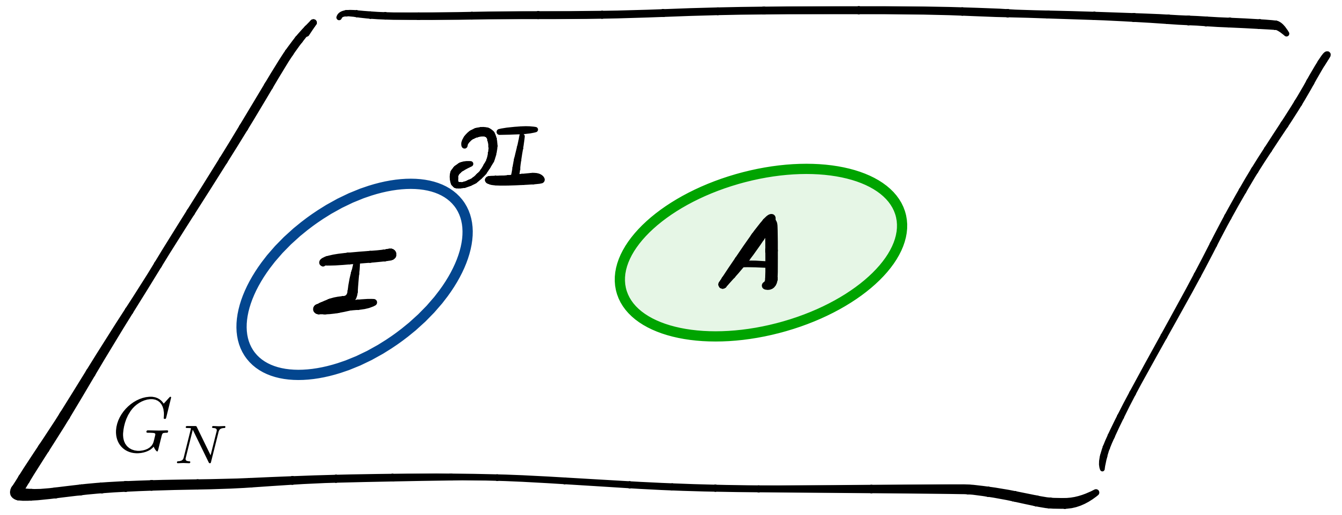
Entanglement Islands

In a theory with semi-classical gravity the entanglement entropy is computed by the Island Formula

$$S_{\mathrm{ISLAND}}(A) = \mathrm{Min}\left\{\mathrm{ext}_I\left[\frac{\mathrm{Area}(\partial I)}{4G_N} + S(A \cup I)\right]\right\}$$

geometric contribution
Bekenstein-Hawking-like

QFT entanglement
entropy



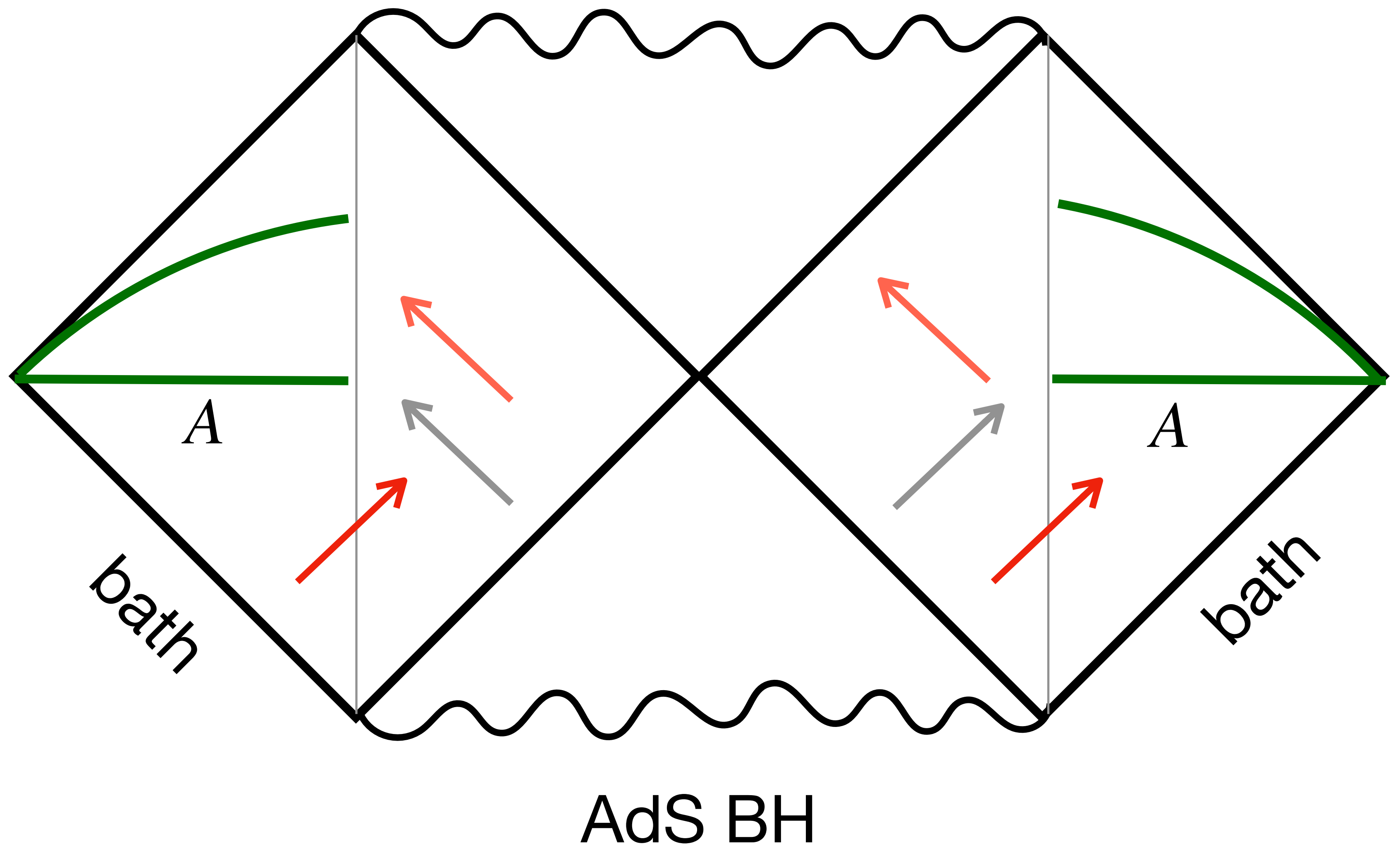
Allows for contributions from additional regions of spacetime: islands

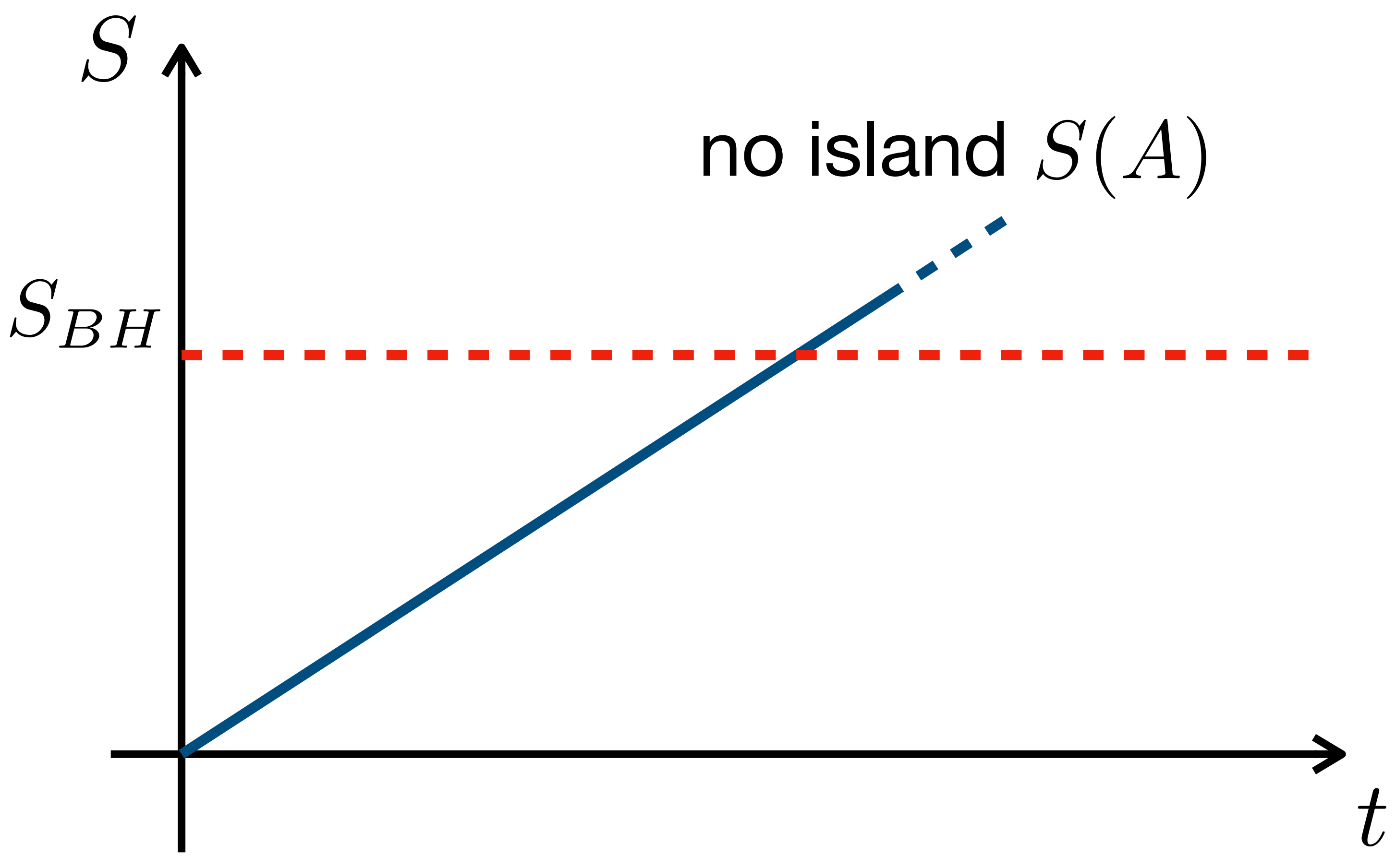
[Almheiri,Engelhardt,Marolf,Maxfield'19]
[Penington'19]
[Almheiri,Mahajan,Maldacena,Zhao'19]
[…]

Black Hole Information Paradox

Island formula yields, within semi-classical gravity, an evolution of the entanglement entropy compatible with unitarity

- Eternal AdS black hole in equilibrium with a thermal bath (pure state)





$S(A) \leq S_{BH}$

maximal amount of info that can be stored in the BH bounds the amount of EE the BH and bath can share

Black Hole Information Paradox

Island formula yields, within semi-classical gravity, an evolution of the entanglement entropy compatible with unitarity

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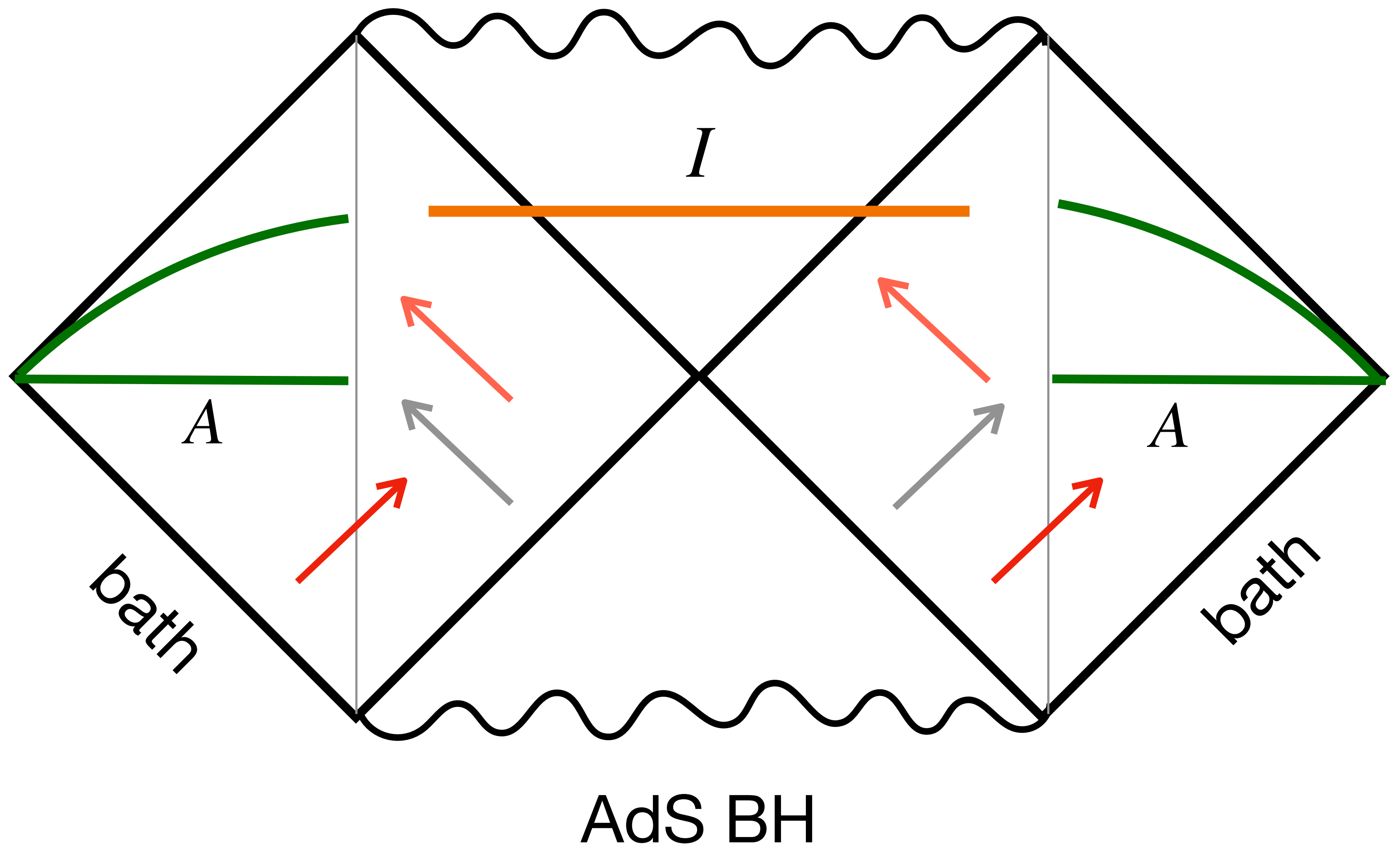
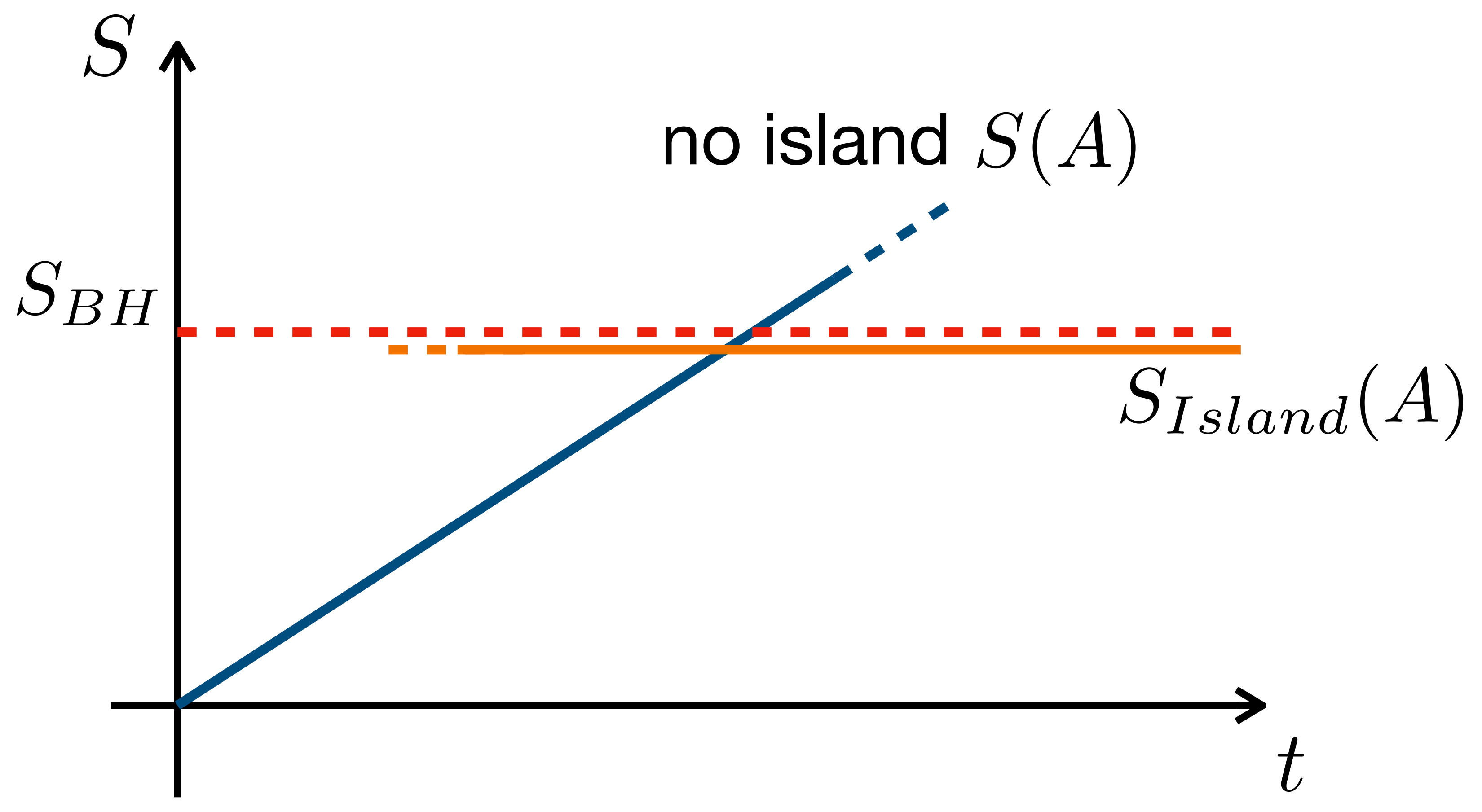




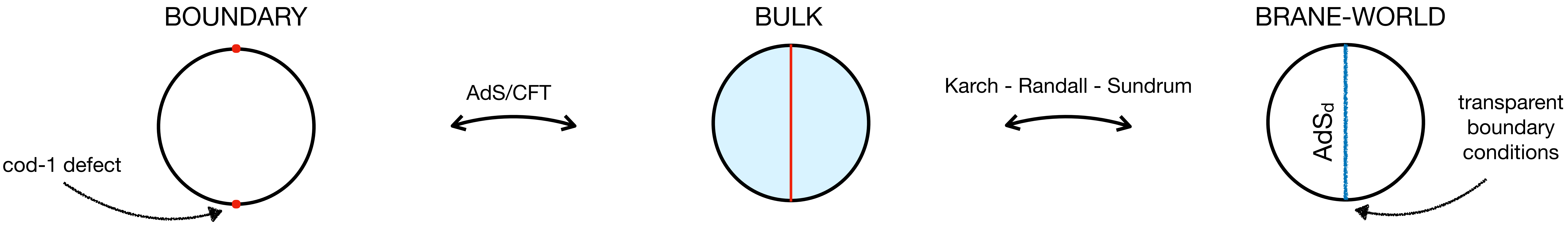
"Page curve"

$S(A) \leq S_{BH}$

maximal amount of info that can be stored in the BH bounds the amount of E the BH and bath can share

Double Holography

Island formula can be understood in more familiar terms and given a geometric interpretation in systems with a double holographic description:



Defect CFT$_d$

AdS$_{d+1}$ + brane

$$S_b = -T_0 \int d^dx\sqrt{h}$$

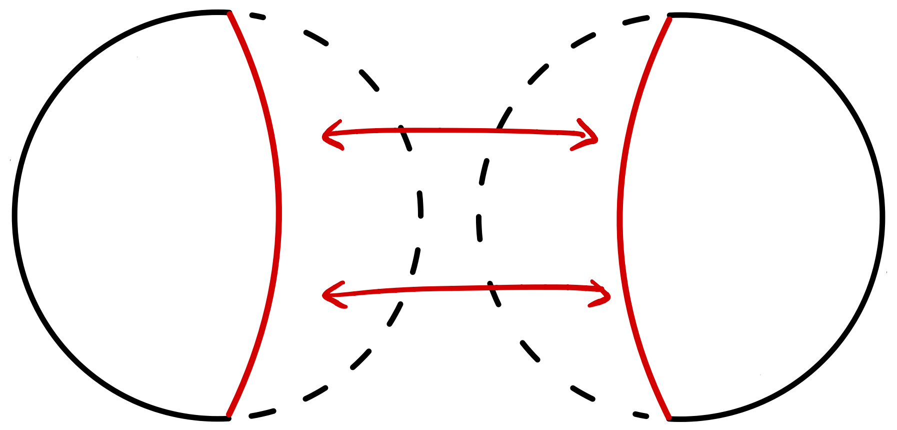
CFT$_d$ + gravity on the brane

$$S_g = \frac{1}{16\pi G_b}\int d^dx\sqrt{-h}\,(R - 2\Lambda_b + \dots)$$

higher curvature terms suppressed for $T_0 \simeq T_{\rm cr}$

Brane profile given by Israel's junction conditions:

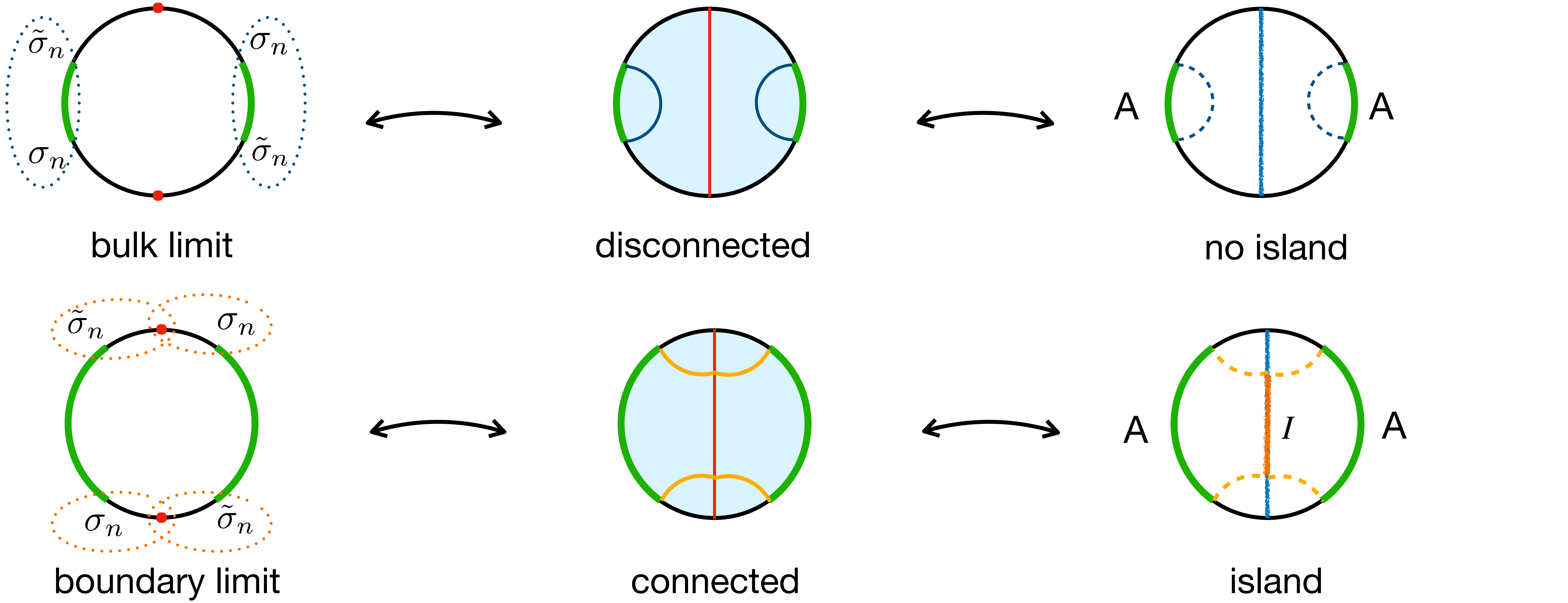
$$\Delta K_{ij} - h_{ij}\Delta K = -8\pi G_N T_0$$



Islands in Double Holography

Emergence of islands can be understood in terms of the standard Ryu-Takayanagi prescription as the transition to a “connected” configuration

BOUNDARY	BULK	BRANE-WORLD
$S(A) = -\mathrm{Tr}_A\, \rho_A \log \rho_A$	$S(A) = \mathrm{Min} \frac{\mathrm{Area}(\gamma_A)}{4G_N}$	$S_{\mathrm{ISLAND}}(A) = \mathrm{Min}\left\{ \mathrm{ext}_I \left[\frac{\mathrm{Area}(\partial I)}{4G_N} + S(A \cup I) \right] \right\}$

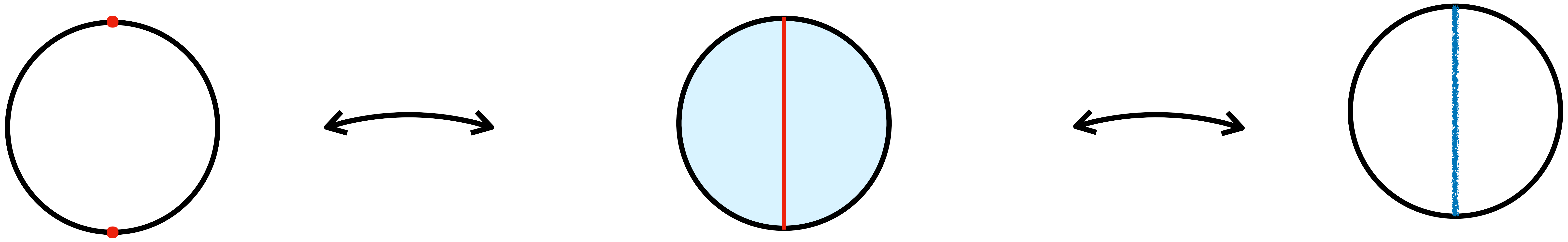


Takeaway message

- Island formula: new proposal for evaluating entanglement entropy in semi-classical gravity and provides interesting new insight into gravity

$$S_{\text{ISLAND}}(A) = \text{Min}\left\{\text{ext}_I\left[\frac{\text{Area}(\partial I)}{4G_N} + S(A \cup I)\right]\right\}$$

- Double holographic models: useful framework to study and better understand the island formula, and to refine the understanding of holographic dualities



Plan

Accelerated black holes in AdS3

Entanglement islands

Dynamical evaporation of the brane-world theory

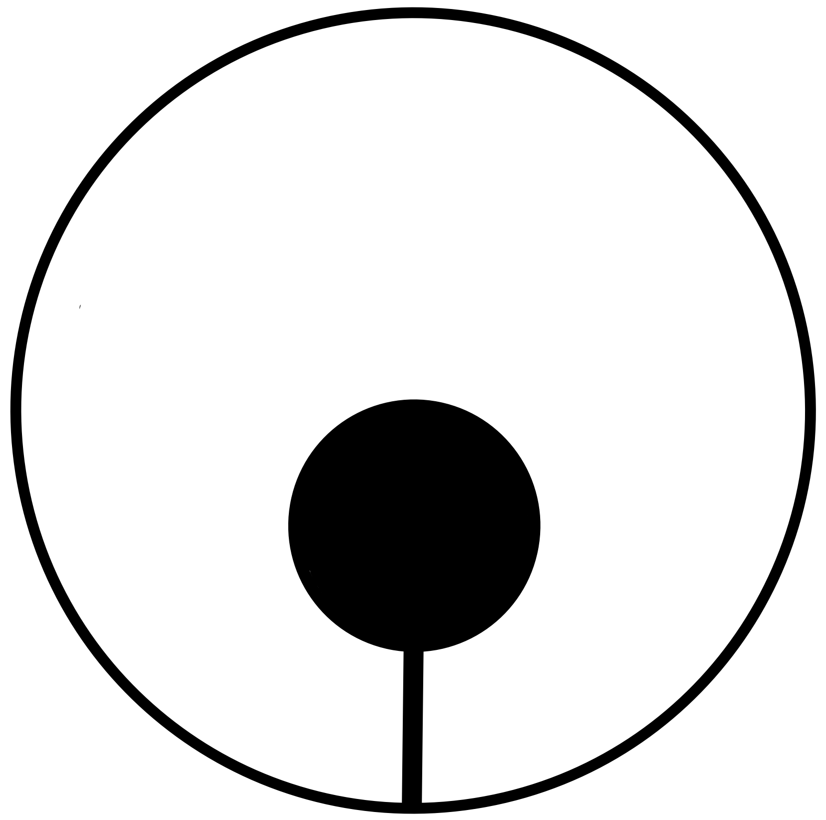
Accelerated black holes in AdS3

Accelerated black holes in AdS3

Belong to a class of solutions generalizing C-metrics to AdS3

[Arenas,Gregory,Scoins'22]
[Astorino'11]

One parameter extension of the BTZ geometry: black holes in AdS3 with a domain wall emerging from the horizon and pulling the black hole to the boundary



I.

$$ds^2 = \frac{1}{(1 - A\rho\cosh\psi/K)^2}\left[-f dt^2 + \frac{d\rho^2}{f} + \rho^2\frac{d\psi^2}{K^2}\right]$$

$f = -1 + \left(1 + A^2\right)\rho^2$ $\quad A \geq 0$ $\quad A = 0$ BTZ with $K = 1/r_h$

II.

$$ds^2 = \frac{1}{\left(1 - \tilde{A}\rho\cos\psi/K\right)^2}\left[-f dt^2 + \frac{d\rho^2}{f} + \rho^2\frac{d\psi^2}{K^2}\right]$$

$f = 1 - \left(1 - \tilde{A}^2\right)\rho^2$ $\quad \tilde{A} \geq 1$ $\quad$ no BTZ limit

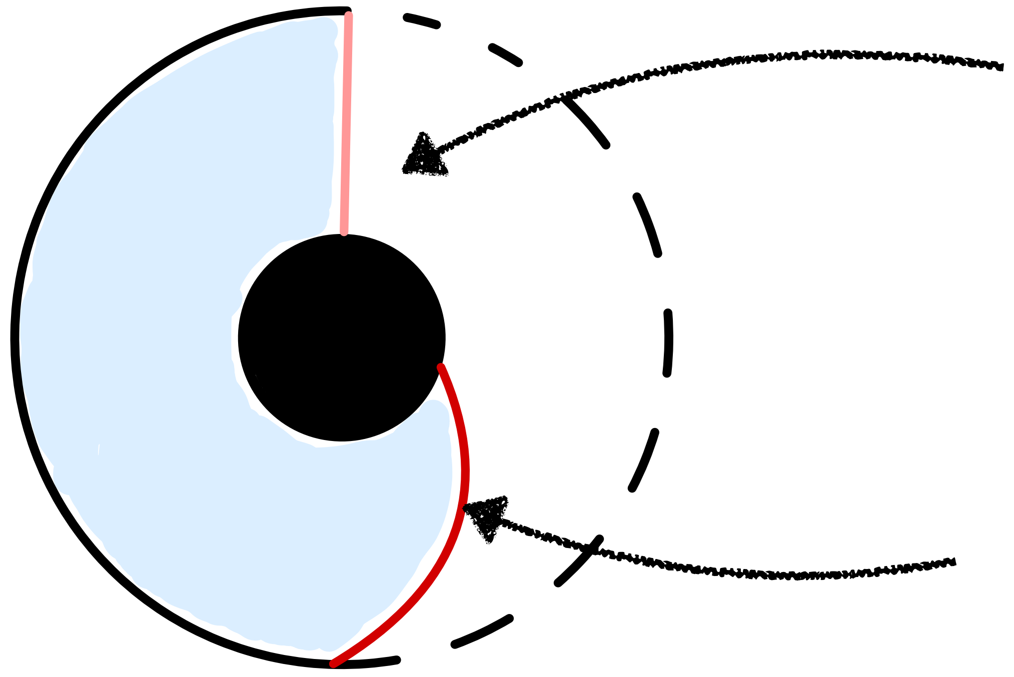
$A = |a| = \sqrt{-(\nabla_u u)^2}$ acceleration along the world-line of the origin

Accelerated black holes in AdS3

They can be understood as the back-reaction of BTZ geometry to a brane stretching from the boundary to the horizon

$$ds^2 = -(r^2 - r_h^2)dt^2 + \frac{dr^2}{r^2 - r_h^2} + r^2 d\phi^2$$

$$S_b = -T_0 \int d^2x \sqrt{-h}$$



$K_{ij} = 0$

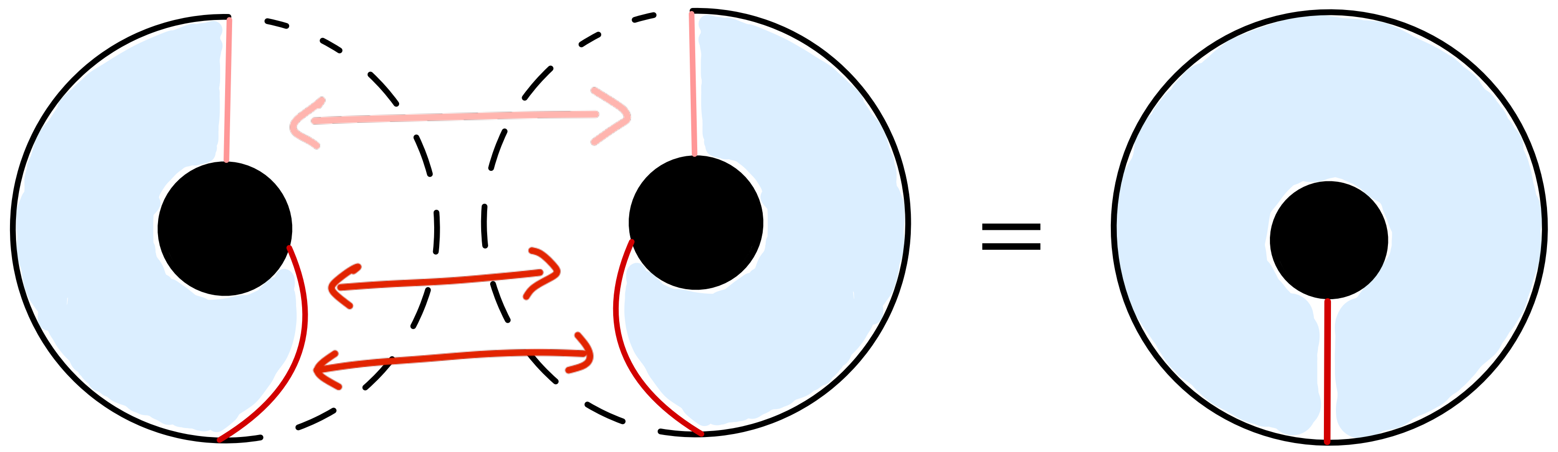
$\phi = const.$

no brane here

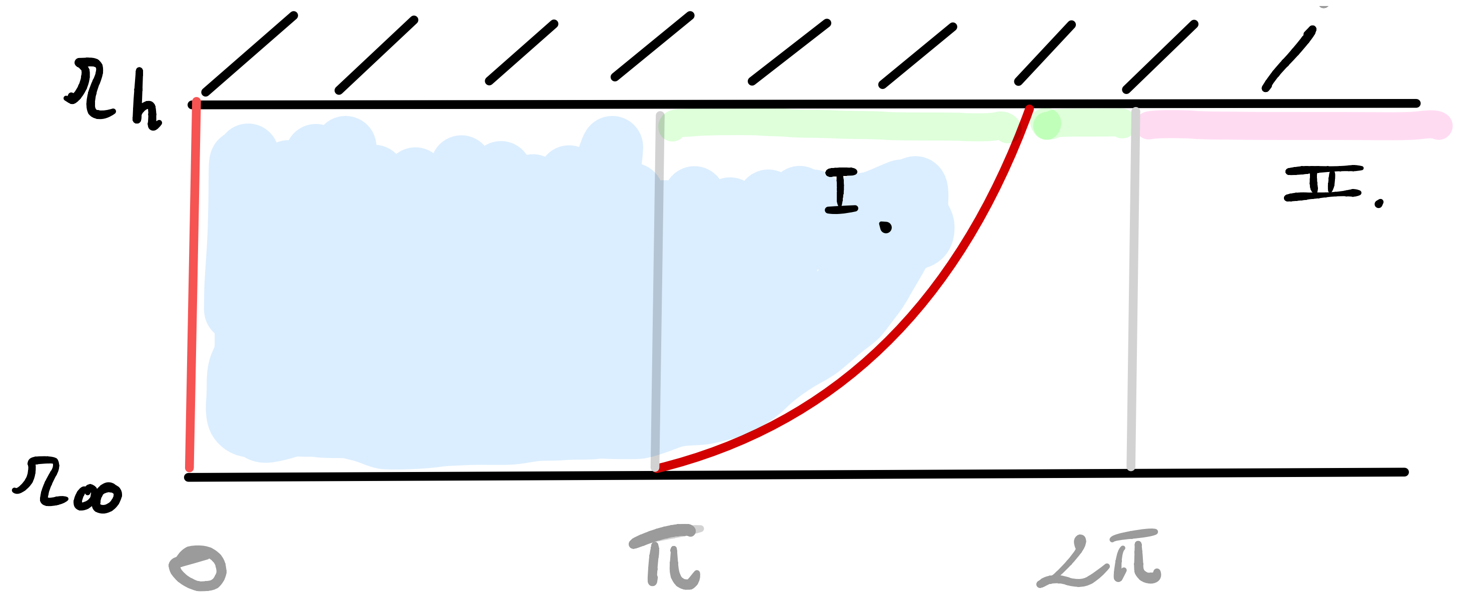
Israel's junction conditions

$$2K_{ij} - 2h_{ij}K = -8\pi G_N T_0$$

$$\sinh r_h \phi = \frac{k r_h}{r} \qquad k = \frac{4\pi G_N T_0}{\sqrt{1 - 16\pi^2 G_N^2 T_0^2}}$$



$$T_0 = \frac{A}{4\pi G_N} \sinh \frac{\pi}{K} \qquad \coth \pi/K = \sqrt{1 + A^2} \coth(\pi r_h)$$

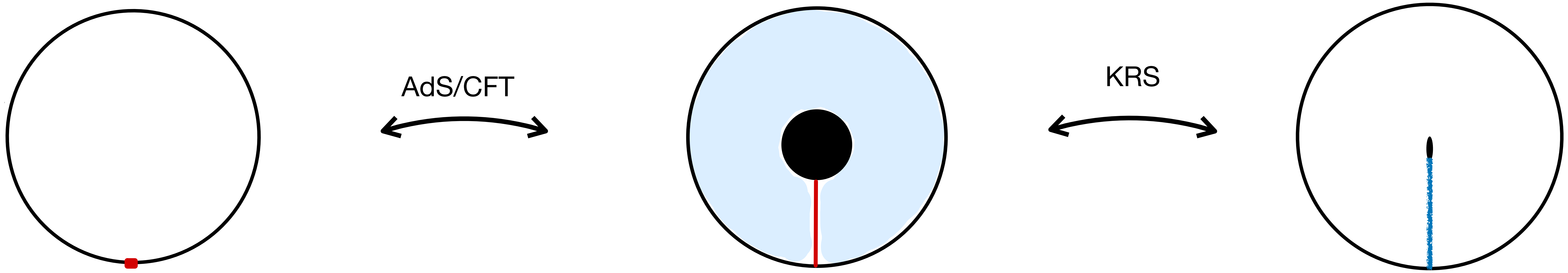


More geometry in the bulk spacetime

Elongated horizon $2\pi + 2/rh \, \mathrm{arcsinh}\, \mathrm{k}$

Double Holography Description

BOUNDARY | BULK | BRANE-WORLD



BOUNDARY: 2d CFT with a defect

$$c_{def} = 2\log g = \frac{c}{3}\,\mathrm{arcsinh}\,k$$

BULK: Accelerated AdS3 BH = BTZ + brane

$$T = \frac{r_h}{2\pi R}$$

$$S = \frac{2\pi^2 c}{3} R T + \frac{c}{3}\,\mathrm{arcsinh}\,k$$

BRANE-WORLD: AdS2 BH + 2d CFT bath

$$ds_b^2 = -\left(\frac{\rho^2}{\ell_b^2} - r_h^2\right) dt^2 + \frac{d\rho^2}{\frac{\rho^2}{\ell_b^2} - r_h^2}$$

$$\frac{\ell_b^2}{L^2} = k^2 + 1 = \frac{1}{1 - 16\pi^2 G_N^2 L^2\, T_0^2}$$

$$S_g = \frac{1}{16\pi G_b}\int d^2x\sqrt{-h}\left(\frac{2}{\ell_b^2} - \tilde{R}\,\log\left(-\frac{L^2}{8}\tilde{R}\right) + \tilde{R} + \frac{L^2}{8}\tilde{R}^2 + \cdots\right)$$

$$T_0 \approx T_{\text{crit.}}$$

[Grimaldi,Hernandez,Myers'22]

$$c_{def} \quad \longleftrightarrow \quad k = \frac{4\pi G_N T_0}{\sqrt{1 - 16\pi^2 G_N^2 T_0^2}} \quad \longleftrightarrow \quad \Lambda_b,\, \rho_h$$

Evaporating the brane

Reducing the tension T_0:

In the bulk picture the BTZ black hole decelerates, moves to the center of AdS3 and the extra geometry created by the brane back-reaction disappears

In the boundary picture the degrees of freedom associated with the defect are decreased $c_{def.} \to 0$

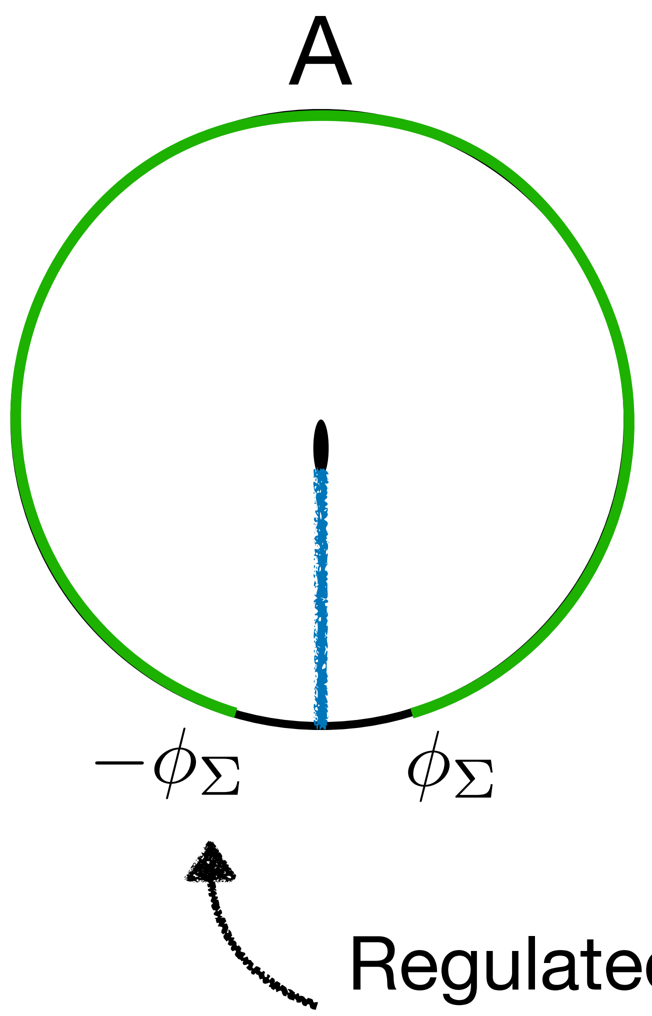
In the brane picture the AdS2 black hole horizon gets smaller as the 2d brane-world disappears into the AdS3 spacetime $\Lambda_b \to \Lambda_3$

Reducing the tension geometrically evaporates the brane

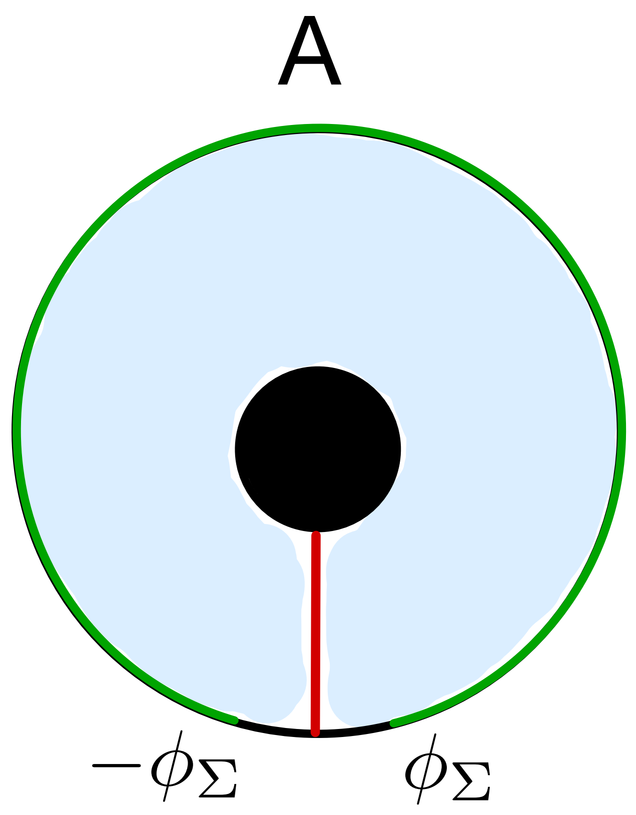
Entanglement Islands

Bath Entanglement Entropy

Entanglement entropy between the CFT bath and the evaporating brane-world



Regulated surface
Exclude a small region around the defect

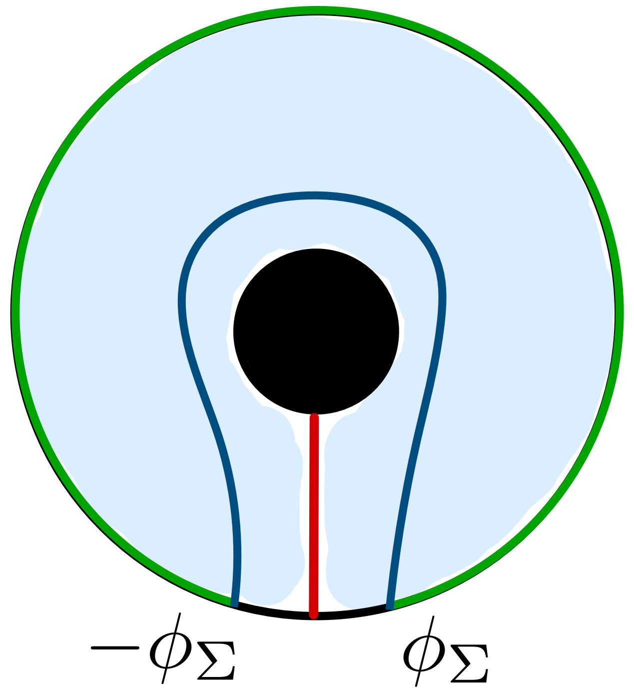


$$S_{\text{ISLAND}}(A) = \text{Min}\left\{\text{ext}_I \left[\frac{\text{Area}(\partial I)}{4G_N} + S(A \cup I)\right]\right\}$$

$$S(A) = \text{Min}\frac{\text{Area}(\gamma_A)}{4G_N}$$

Bath Entanglement Entropy

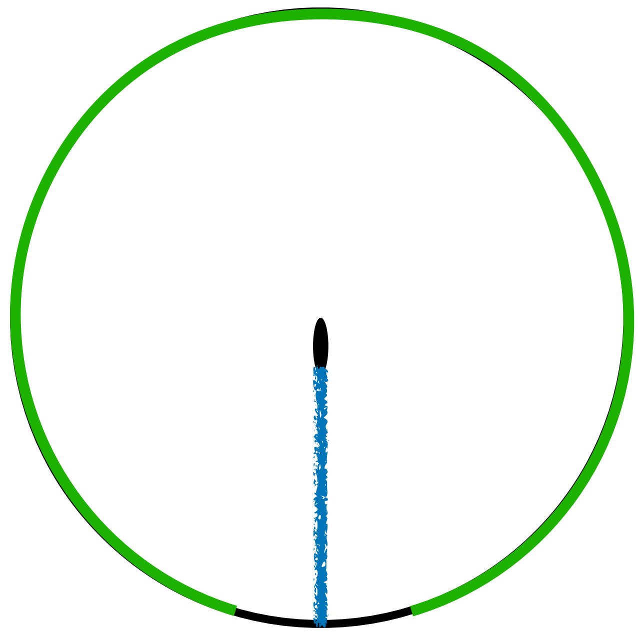
In the bulk description: geodesics in the accelerated black hole geometry



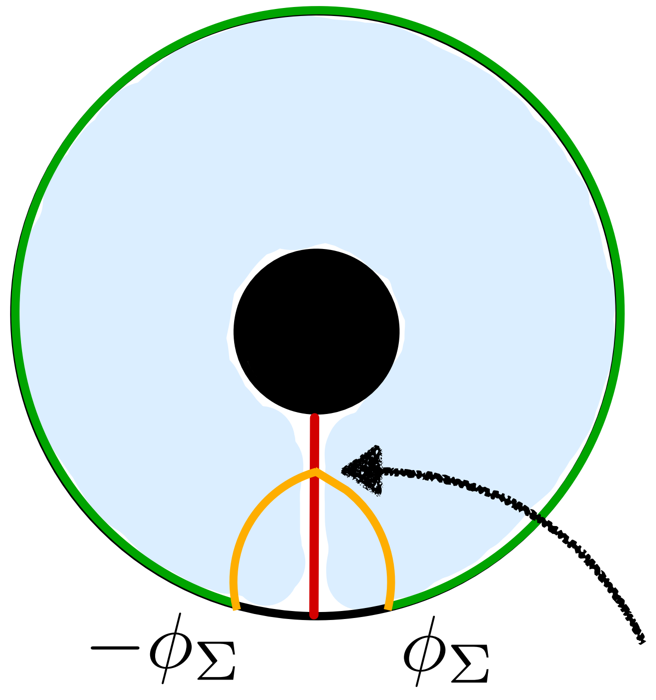
$$S_{\rm th} = \frac{c}{3} \log \frac{\sinh(2\pi R T(\pi - \phi_\Sigma))}{\pi T\, \delta}$$

UV-cutoff

no island - same as the EE computed in the thermal 2d CFT

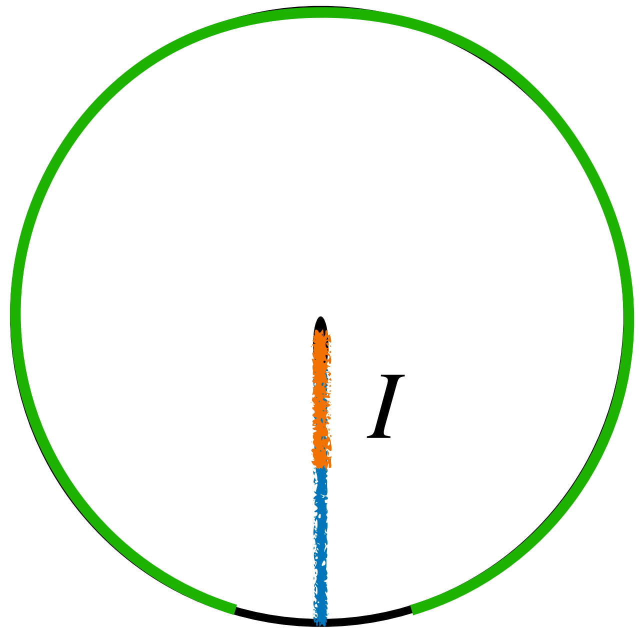


thermal contribution associated with $(-\phi_\Sigma, \phi_\Sigma)$



$$S_{\rm Island} = \frac{c}{3} \log \left(k + \sqrt{1+k^2} \right) \frac{\sinh(2\pi R T\, \phi_\Sigma)}{\pi T\, \delta}$$

yields an island

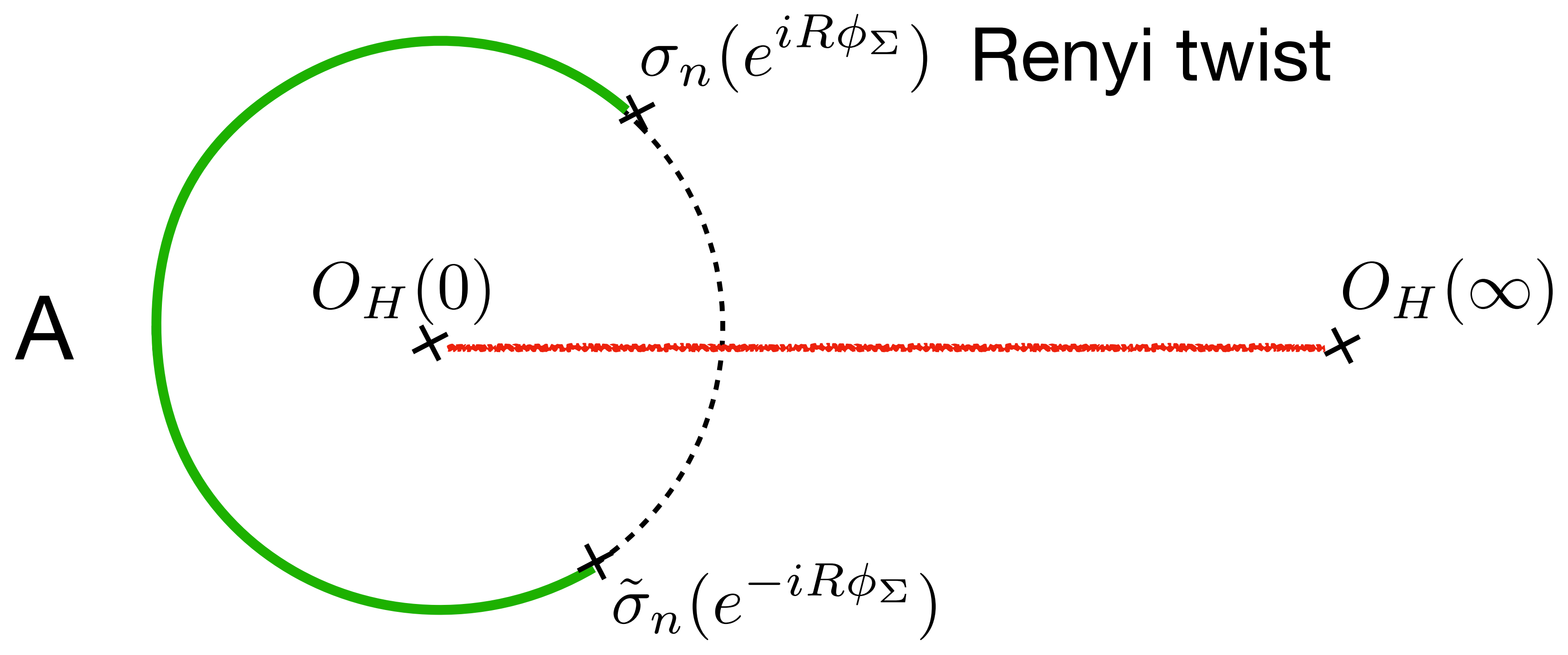


meeting point on the brane determined minimizing the geodesics length

accounts for the extra spacetime that the geodesics need to travel as compared to BTZ

Boundary Interpretation

BTZ black hole geometry as an heavy states in the holographic CFT



O_H primary with weight $h_H = \bar{h}_H \sim c$

$$\beta = \frac{2\pi R}{\sqrt{24 h_H/c - 1}}$$

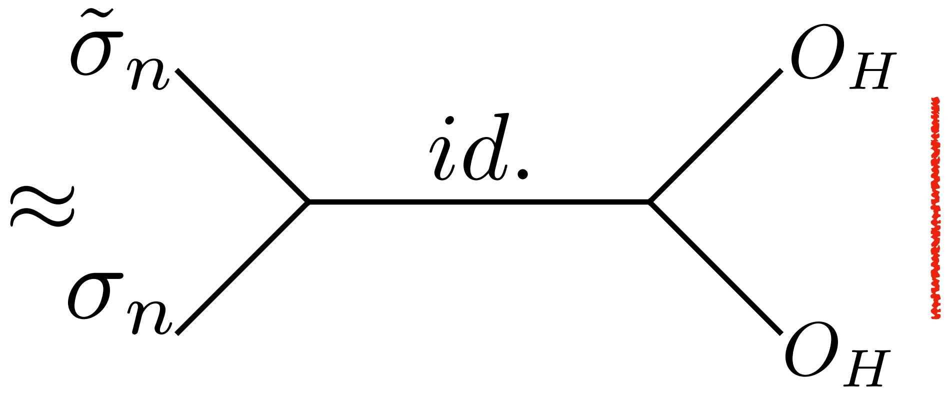
Replica trick: $S(A) = \lim_{n\to 1} \frac{1}{1-n} \langle O_H \sigma_n \tilde{\sigma}_n O_H \rangle$ in the defect geometry

Holographic CFT: $\langle O_H \sigma_n \tilde{\sigma}_n O_H \rangle$ is dominated by the Identity contribution in the Virasoro blocks expansion

Boundary Interpretation

Bulk channel

$\langle O_H \sigma_n \tilde{\sigma}_n O_H \rangle \approx$



$= \mathcal{F}_0(z)\,\bar{\mathcal{F}}_0(\bar{z})$

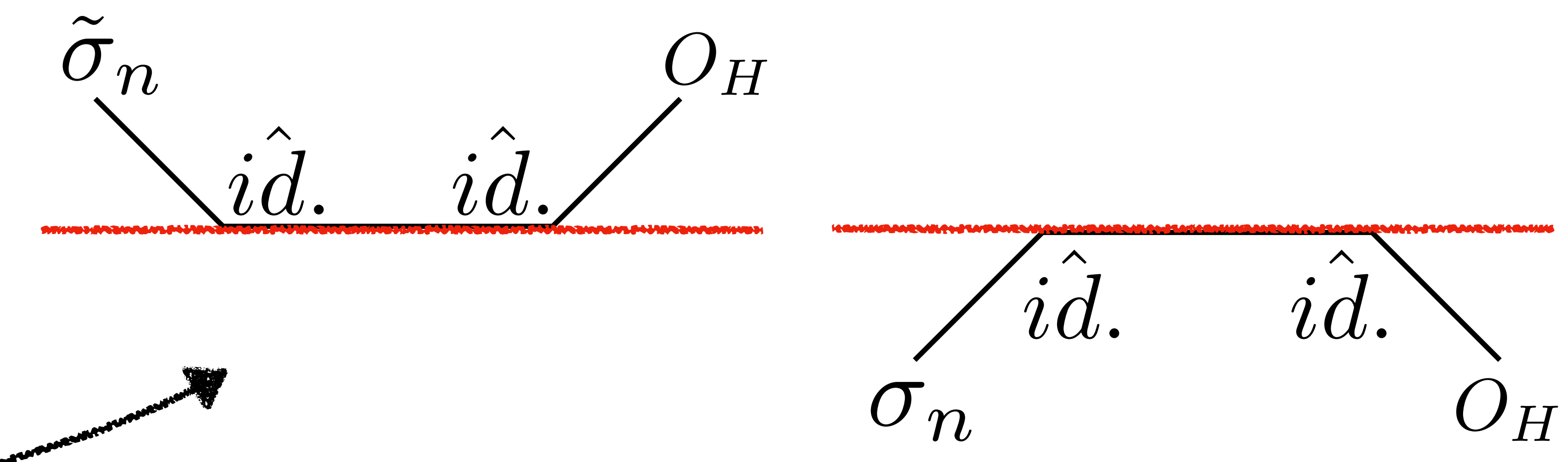
$S = \lim_{n\to 1} \frac{1}{1-n} \langle O_H \sigma_n \tilde{\sigma}_n O_H \rangle$

with a careful choice of branch

$$S_{\text{th}} = \frac{c}{3} \log \frac{\sinh(2\pi RT(\pi - \phi_\Sigma))}{\pi T\,\delta}$$

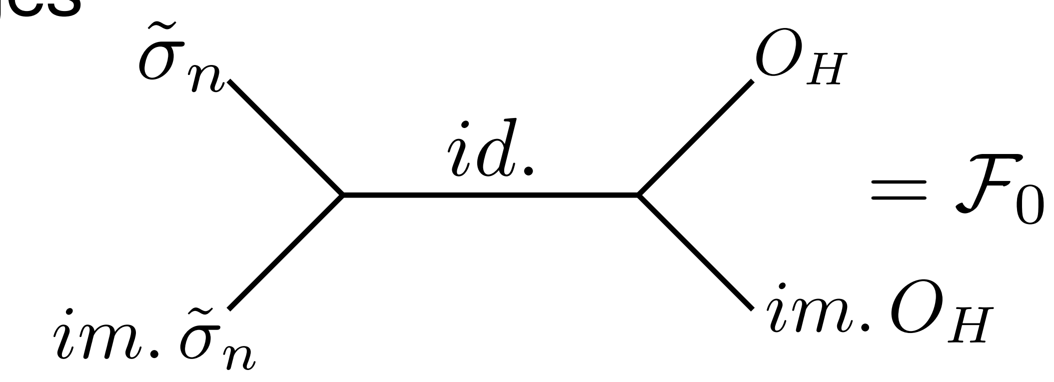
Defect channel

$\langle O_H \sigma_n \tilde{\sigma}_n O_H \rangle \approx$



$= a_0^2\,\mathcal{F}_0(z)\,\bar{\mathcal{F}}_0(\bar{z})$

holomorphic block obtained with images



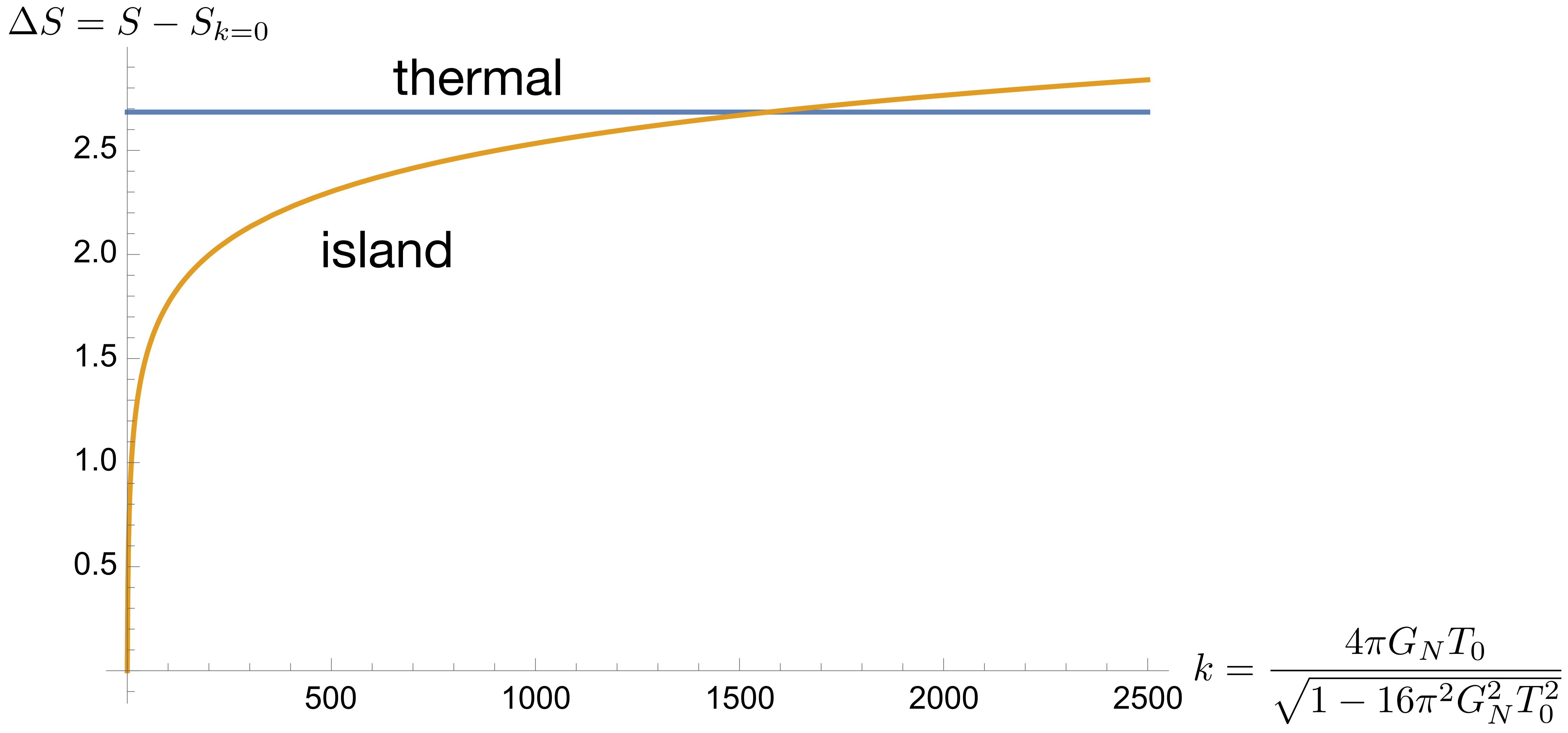
$= \mathcal{F}_0(z)$

$$\lim_{n\to 1} \frac{\log a_o}{1-n} = \frac{c}{6} \log k + \sqrt{1+k^2}$$

$$S_{\text{Island}} = \frac{c}{3} \log \left(k + \sqrt{1+k^2}\right) \frac{\sinh(2\pi RT\,\phi_\Sigma)}{\pi T\,\delta}$$

Adiabatic Evaporation

Entanglement entropy of the bath: 2 competing configurations



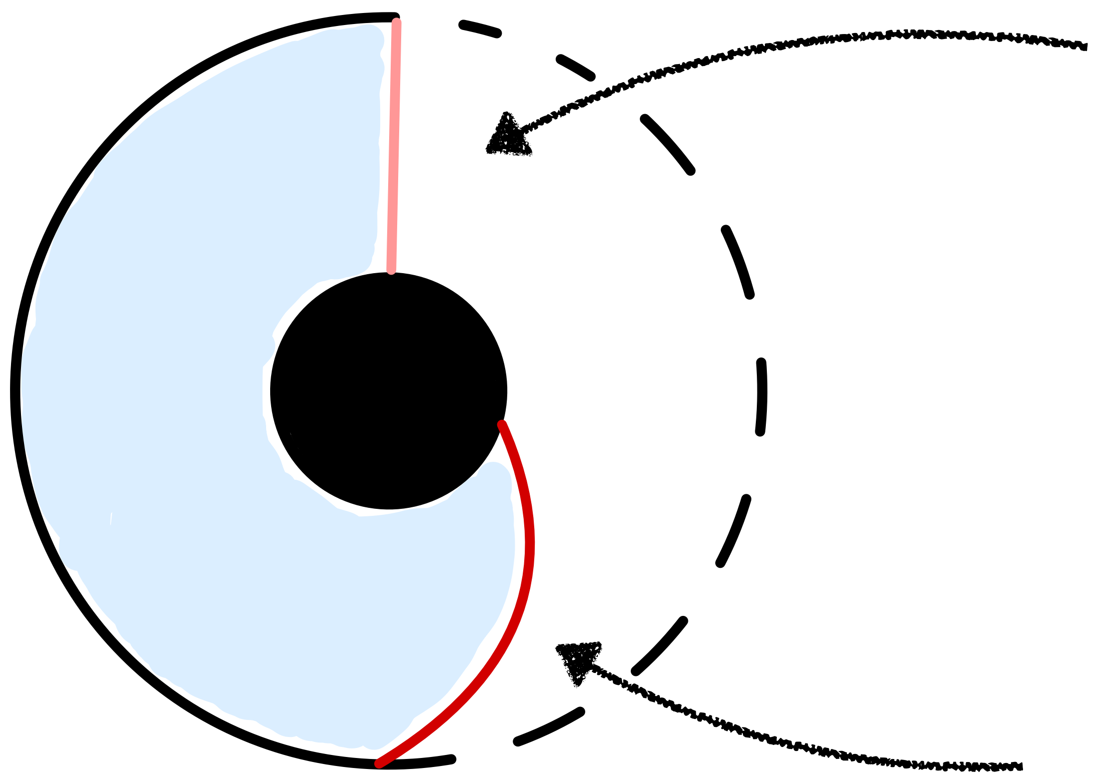
As a function of the tension, no energy or d.o.f. transferred to the bath

The island contribution becomes dominant since the amount of entanglement between the bath and the AdS2 BH is limited by amount of information that can be contained in each partition

Dynamical Evaporation of the Brane-World

Dynamical Brane Evaporation

Reconsider the construction of the AdS3 accelerating black hole:



$$K_{ij} = 0$$

$$\phi = const.$$

no brane here

$$2K_{ij} - 2h_{ij}K = -8\pi G_N T_0$$

$$\sinh r_h \phi = \frac{k r_h}{r} \quad k = \frac{4\pi G_n T_0}{\sqrt{1 - 16\pi^2 G_N^2 T_0^2}}$$

Making the tension and the brane profile time dependent: too simplistic

Slowly Varying Tension

$\phi - f(r,t) = 0$ needs a more general and non-conserved stress-tensor associated to the brane

$$S_{ij} = -T_0(t)h_{ij} + \delta S_{ij}(t,r)$$

$$\nabla_i S^{ij} = J^j(t,r)$$

Israel's junction conditions at the brane

$$2K_{ij} - h_{ij}2K_l^l = 8\pi G_N S_{ij}$$

Work in a time derivative expansion, i.e. slowly varying tension

$$T_0(t) = T_0(\mu t) \qquad \delta S_{ij}(t,r) = \mu \delta S_{ij}(\mu t, r)$$

$$J^j(t) = \mu J^j(\mu t) \qquad f(t,r) = \sum_{n=0}^{\infty} \mu^n f_n(\mu t, r)$$

Up to first order $J^r, \delta S_{tt}, \delta S_{rr}$ decouple from the rest: set them to zero

Slowly Varying Tension

General solution up to first order is completely specified in terms of a time dependent function:

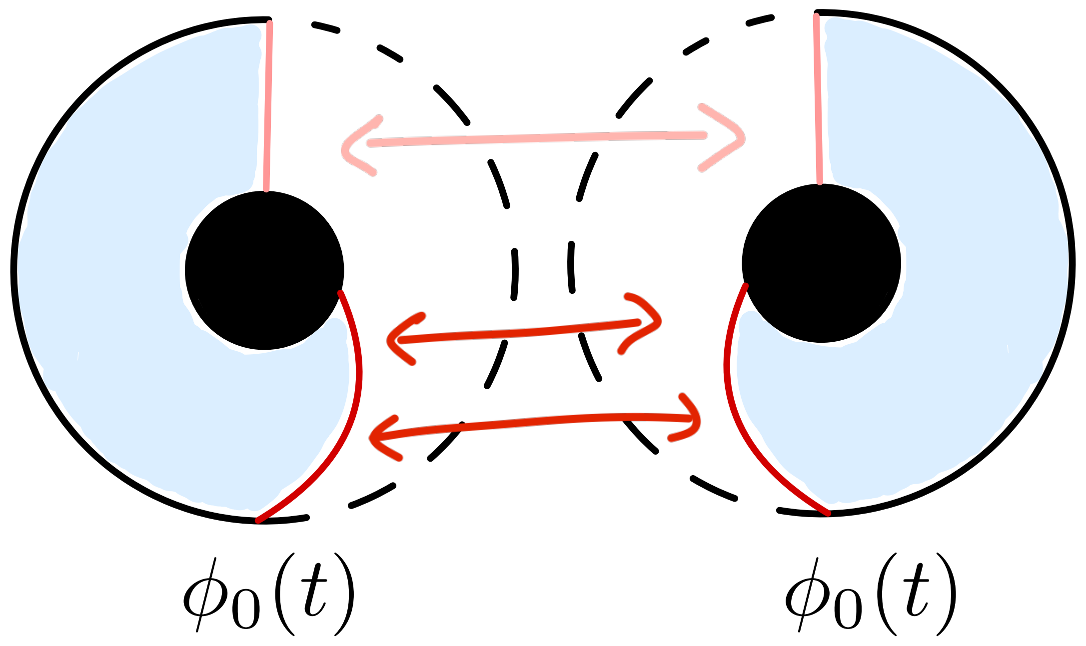
$$\phi = \phi_0(t) + \frac{1}{r_h} \log \frac{r_h k(t) + \sqrt{r^2 + r_h^2 k^2(t)}}{r} \qquad k(t) = \frac{4\pi G_N T_0(t)}{\sqrt{1 - 16\pi^2 G_N^2 T_0(t)^2}}$$

$$\delta S_{tr}(t,r) = \frac{\dot\phi_0(t) r r_h^2 \sqrt{1+k^2(t)}}{(r^2 - r_h^2)\sqrt{r^2 + r_h^2 k^2(t)}} + \frac{r \dot k(t)}{(r^2 - r_h^2)\sqrt{1+k^2(t)}}$$

$$J^t = -\frac{2\dot\phi_0 r r_h^2}{(r^2 - r_h^2)^2}\sqrt{\frac{r^2 + r_h^2 k^2}{(1+k^2)}}\left(1 + \frac{(r^2 - r_h^2)^2 (r^2 + r_h^2 k^2)}{r^2(1+k^2)}\right) + \frac{\dot k}{(r^2 - r_h^2)(1+k^2)^{3/2}}\left[1 - \frac{(r^2 - r_h^2)^2(r^2 + r_h^2 k^2)}{r^2(1+k^2)} - 2\frac{r^2 + r_h^2 k^2}{r^2 - r_h^2}\left(1 + \frac{(r^2 - r_h^2)^2 (r^2 + r_h^2 k^2)}{r^2(1+k^2)}\right)\right]$$

The point where the brane is anchored at the boundary, $\phi_0(t)$, can change in time

Length of the boundary circle can change in time



Slowly Varying Tension

In the brane-world description: evaporate the brane-world moving the degrees of freedom from the brane to the bath

Holding the entropy of the full system fixed:

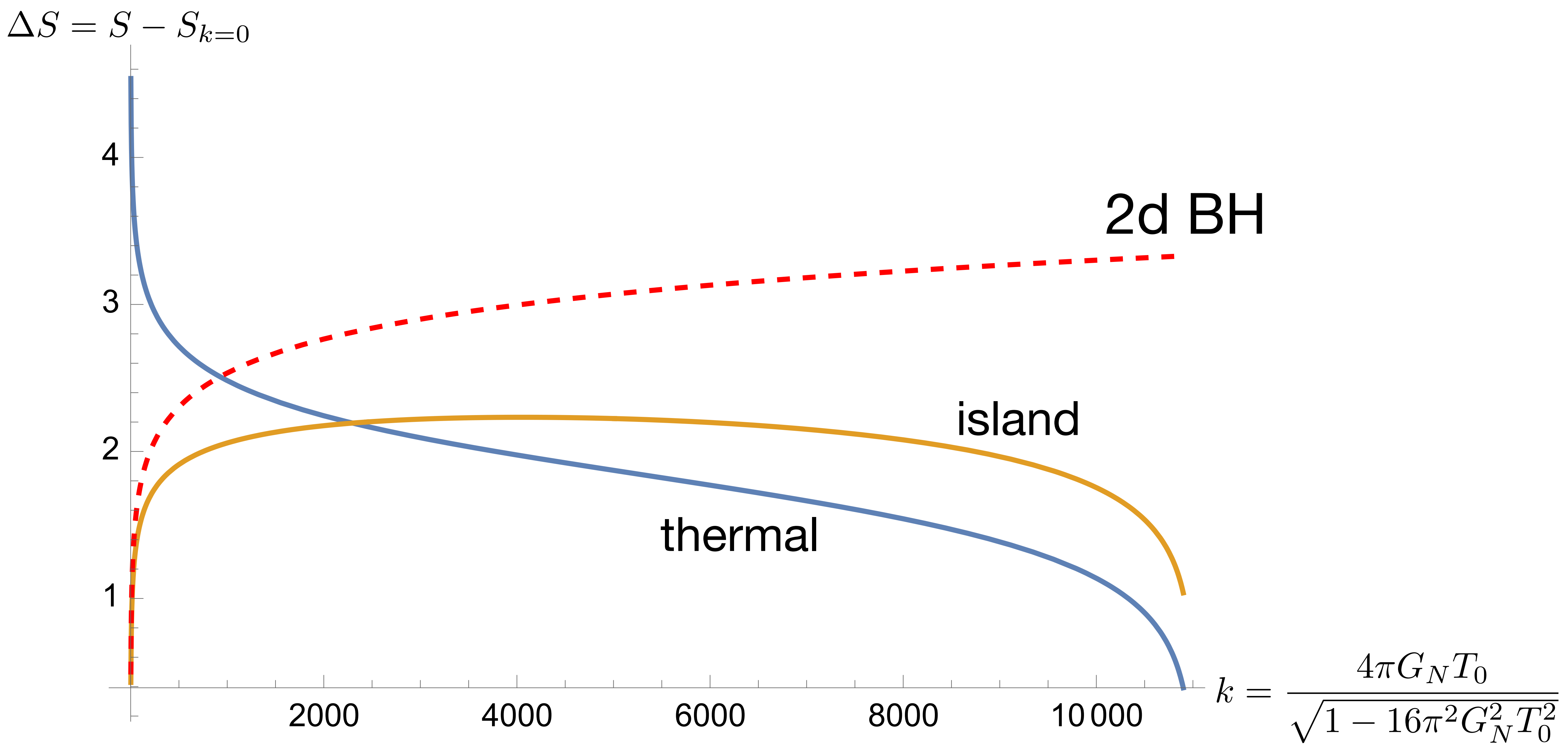
$$S = \frac{2\pi^2 c}{3} RT + \frac{c}{3} \operatorname{arcsinh} k$$

gives (at fixed T) $\dot{\phi}_0(t) = -\frac{\dot{k}(t)}{2\pi T \sqrt{1+k^2(t)}}$

that is the boundary CFT circle enlarges to accommodate the redistribution of d.o.f.

Slowly Varying Tension

In the brane picture, the evolution of the bath entanglement entropy as the tension decreases is then



The entanglement entropy of the radiation collected in the bath cannot be bigger than the thermodynamic entropy of the evaporating BH. The island contribution makes sure this is the case.

Conclusions

- Double holography is a powerful construction to investigate aspects of gravitational theories, such as entanglement islands
- Construction that geometrizes the evaporation of the black hole in the brane perspective, and of the brane-world background itself
- Entanglement islands reproduce the expected “Page curve”
- Extensions to the broader class of C-metrics (higher-dimensional, $\Lambda \lesseqgtr 0$)
- Holographic complexity: the fully unentangled reference state of circuit complexity is associated to the absence of a spacetime description. Here this represents the endpoint of the dynamical braneworld evaporation

Thank you!