Quantum Gravity, Statistical Physics, Chaos and Wormholes

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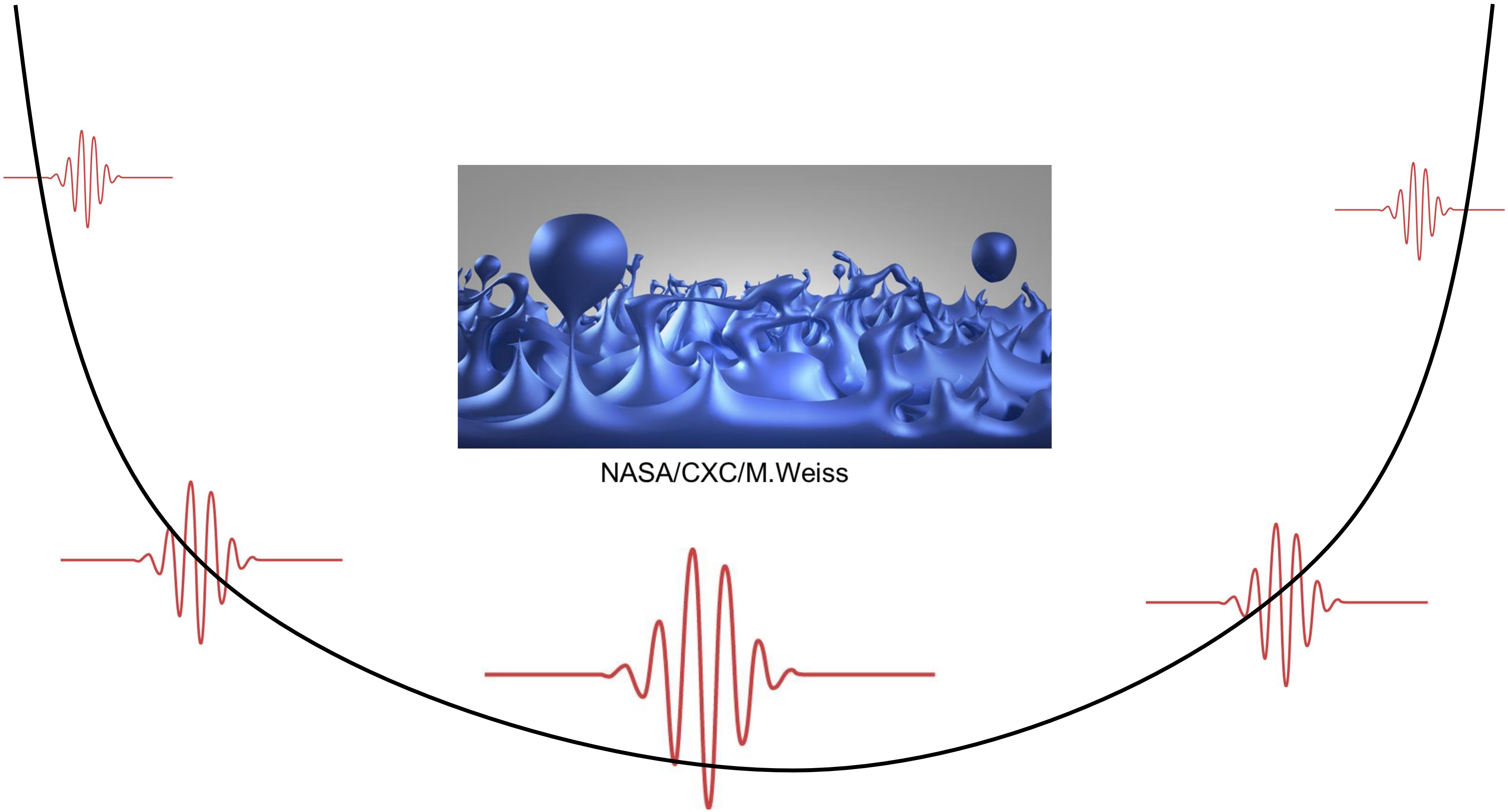
1. Why AdS/CFT?

Quantum gravity is tricky subject. Spacetime fluctuates and it is a priori not even clear how to properly define obervables.

Idea: put gravity in a box

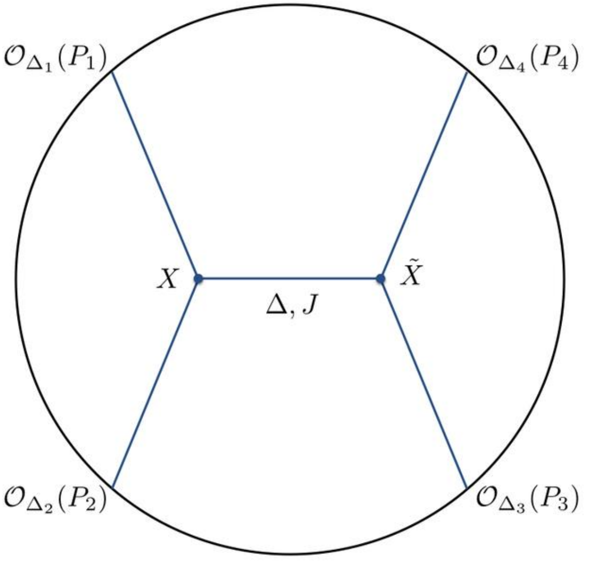


NASA/CXC/M.Weiss

A great box is Anti de-Sitter Space (AdS)

$$ds^2 = dr^2 + e^{2r}(-dt^2 + d\vec{x}^2)$$

which allows us to have well-defined observables at the boundary.



To study spacetime is somewhat similar to medical imaging

Based on general arguments (not string theory) one can argue that some quantum theory (QT) lives at the boundary with the following properties:

- QT must be strongly coupled.
- QT must have a large number of degrees of freedom (“large N theories”)
- QT must not have simple low-energy operators of spin larger than two.
- QT must have only a few simple low-energy operators.
- QT must be scale invariant.

Do such QTs exist? Yes, many examples have been identified in string theory where QT is some CFT starting with the famous paper Maldacena 1997

In AdS/CFT, the precise statement is that UV complete non-perturbative quantum gravity in AdS is exactly the same as a CFT on the boundary.

No similar statements are currently available on other space-times like flat space, de Sitter space, etc.

In the remainder the focus will therefore mostly be on AdS/CFT, though lessons learned may well apply to other spacetimes.

Take home message:

- AdS provides an IR regulator of gravity which allows for the precise definition of observables.
- String theory provides a UV complete description (CFT).
- Many features of AdS/CFT are generic and do not rely on string theory.

2. Black holes in AdS

Black Hole in AdS = CFT at finite temperature

At low temperatures, a thermal gas of particles in AdS corresponds to a thermal gas of (confined) excitations in the CFT.

At higher temperatures, the thermal gas collapses into a black hole. In the CFT, the theory deconfines and one obtains a deconfined plasma.

Black Hole in AdS = CFT at finite temperature

Subset of Einstein Field Equations = equations of hydrodynamics for CFT plasma (gravity somehow knows about the right variables for hydrodynamics)

Bhattacharyya, Hubeny, Minwalla, Rangamani, ‘08

Falling into the black hole = dissipation

Black hole creation = thermalization

Gravitational predictions:

- Hydrodynamics has very low viscosity Kovtun, Son, Starinets,’01
- Thermalization proceeds maximally fast

Balasubramanian, Bernamonti, JdB et al, ‘11

Two related interesting observations:

Black holes are very fast scramblers of information (and so is the CFT plasma) Sekino, Susskind '08

Black holes are maximally chaotic (and so is the CFT plasma) Maldacena, Shenker, Stanford, '15

$$\langle V(0)W(t)V(0)W(t)\rangle_\beta \sim c_0 - c_1 N^{-2} e^{\lambda t}$$

$$\lambda \leq \frac{2\pi}{\beta}$$

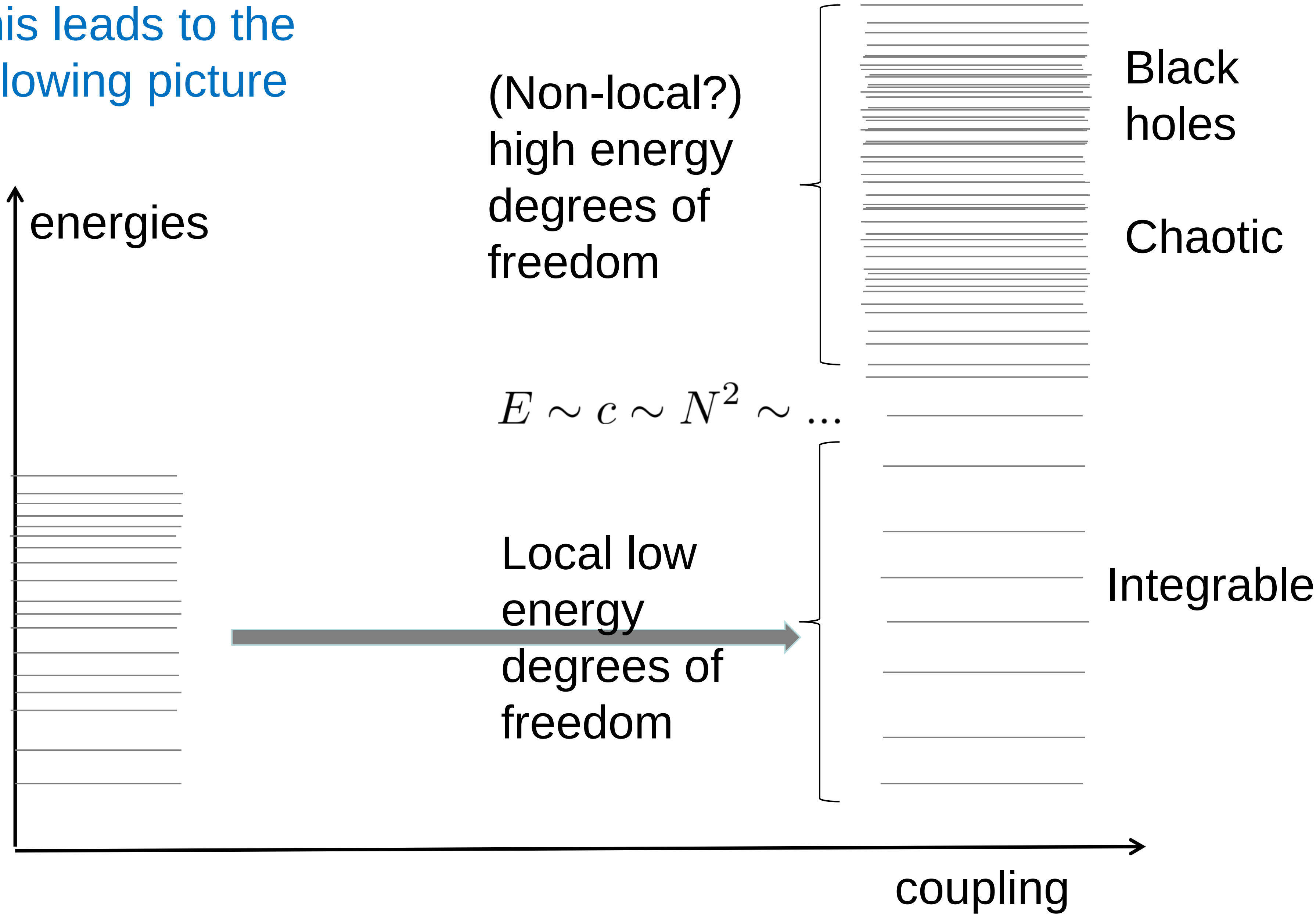
The entropy of black holes is given by the Bekenstein-Hawking equation

$$S = \frac{A}{4G}$$

This can be reproduced exactly for BPS black holes (many references starting with Strominger-Vafa '95) and approximately for some other black holes. The latter requires the computation of the finite temperature partition function of a strongly coupled CFT.

Importantly, this number is way too large to be explained by the low-energy degrees of freedom in AdS or the low dimension operators in the CFT.

This leads to the following picture



Take home message:

- Black holes are extreme objects in terms of their chaotic behavior and information scambling.
- The CFT description is in terms of a hot plasma with equally exreme properties
- While we can compute the number of states at high energy, we can not (yet?) see the individual states.
- The low and high energy part of the spectrum are very different in nature.

3. The role of quantum information

The Role of Quantum Information

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell (or EPR) pair, entanglement: measurements are correlated.

$$|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

Entanglement entropy S_A ~ number of Bell pairs that entangle A and B.

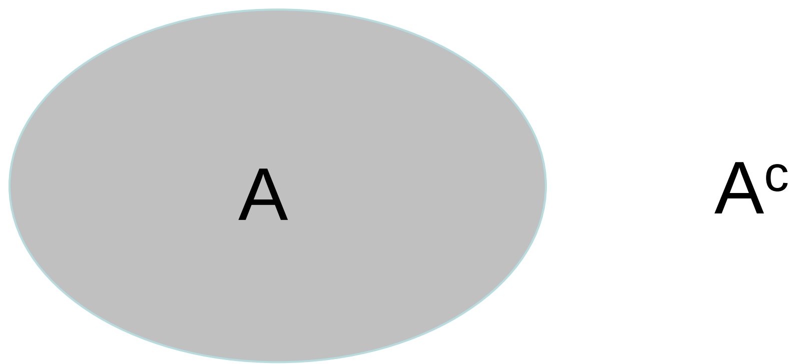
More precisely

$$\forall \mathcal{O}_A : \quad \mathrm{Tr}_{\mathcal{H}_A \otimes \mathcal{H}_B}(\rho\, \mathcal{O}_A \otimes \mathbb{I}_B) = \mathrm{Tr}_{\mathcal{H}_A}(\rho_A\, \mathcal{O}_A)$$

$$S_A(\rho) = -\mathrm{Tr}_{\mathcal{H}_A}(\rho_A \log \rho_A) = \lim_{n \to 1} \frac{1}{1-n} \log \mathrm{Tr}_{\mathcal{H}_A}(\rho_A^n)$$

Entanglement entropy in quantum field theory

Typical situation: consider degrees of freedom associated to a spatial domain and its complement

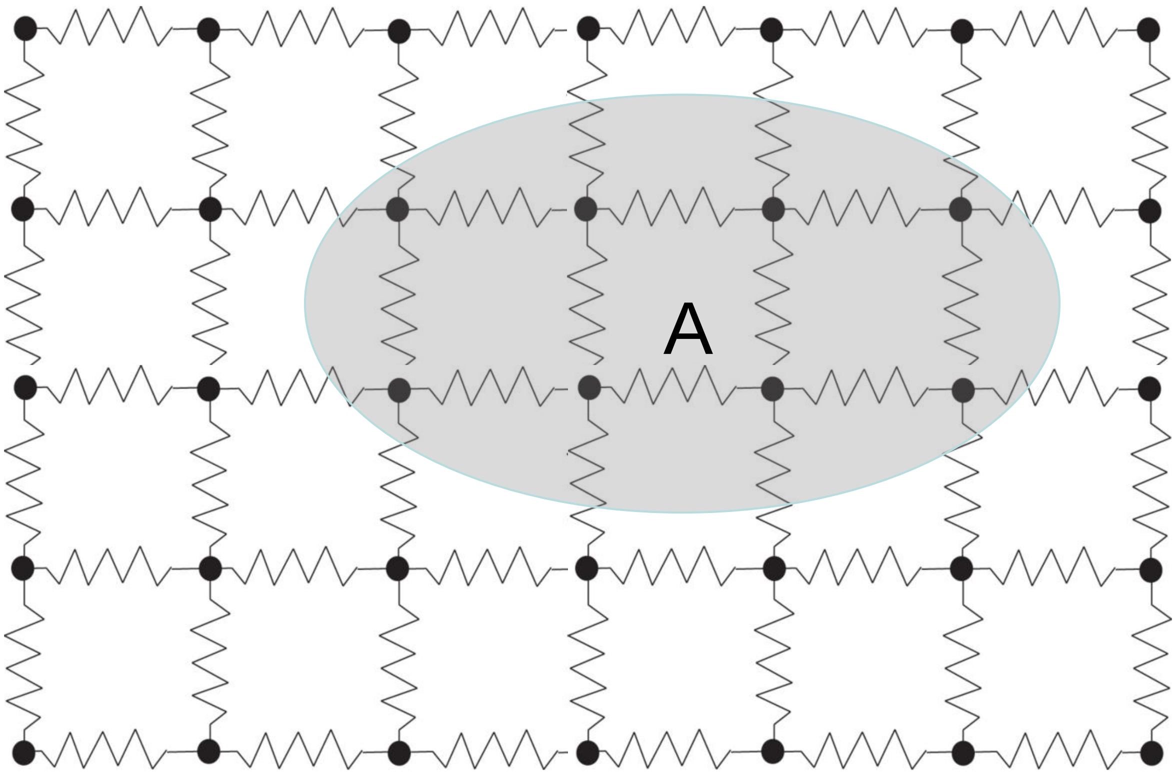


$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$$

Bombelli, Koul, Lee, Sorkin `86
Srednicki `93

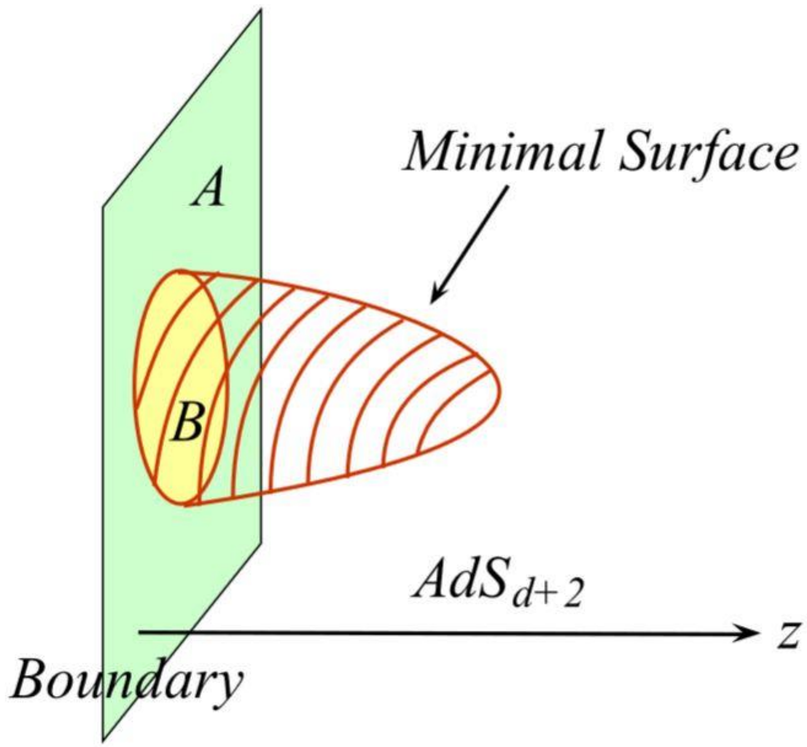
(various caveats)

Entanglement entropy = infinite in continuum field theory (and universal and fine-tuned)
Needs to be regulated: short distance regulator a.



$$S(A) = \frac{\text{area}(A)}{a^{D-2}} + \ldots$$

Ryu-Takayanagi ('06): entanglement entropy in CFT = area of minimal surface in gravity (S_A=area/4G).

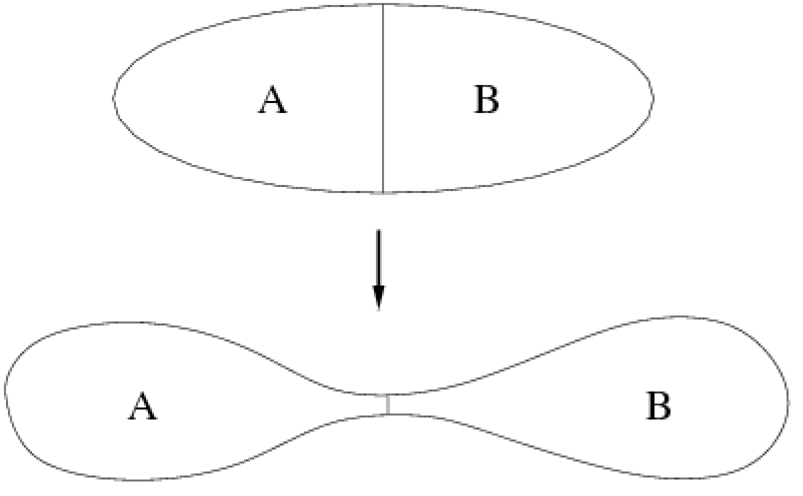


Postulating that entanglement entropy is computed by minimal area surfaces implies the linearized Einstein equations. Faulkner, Guica, Hartman, Myers, van Raamsdonk `13

Entanglement is needed to build op a connected spacetime (van Raamsdonk ‘10).

The correlations in entangled states are reproduced by making spacetime connected.

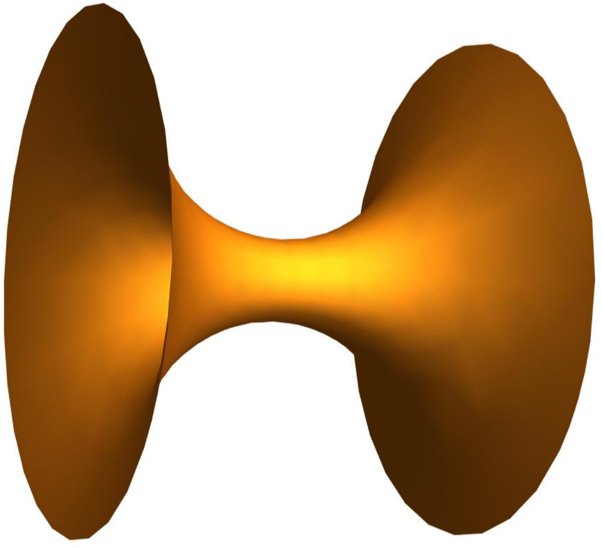
Exactly which types of entanglement have smooth geometric representations is not entirely clear.



Maximally extended space time for a black hole in AdS has two CFT boundaries.

$$|\Psi\rangle = \frac{1}{Z}\sum_i e^{-\beta E_i/2}|i\rangle_1|i\rangle_2$$

These are connected by a "wormhole", the Einstein-Rosen bridge (ER).



This is a state with a lot of entanglement – EPR (Einstein-Podolsky-Rosen) pairs.

ER=EPR

Maldacena, Susskind '13

Amazingly, many quantum information theoretic concepts have a gravitational interpretation:

- quantum error correction
- entanglement of purification
- various protocols
- differential entropy
- quantum teleportation
- relative entropy
- Renyi entropy
- mutual information
- entropy inequalities like strong subadditivity
- Algebraic QFT

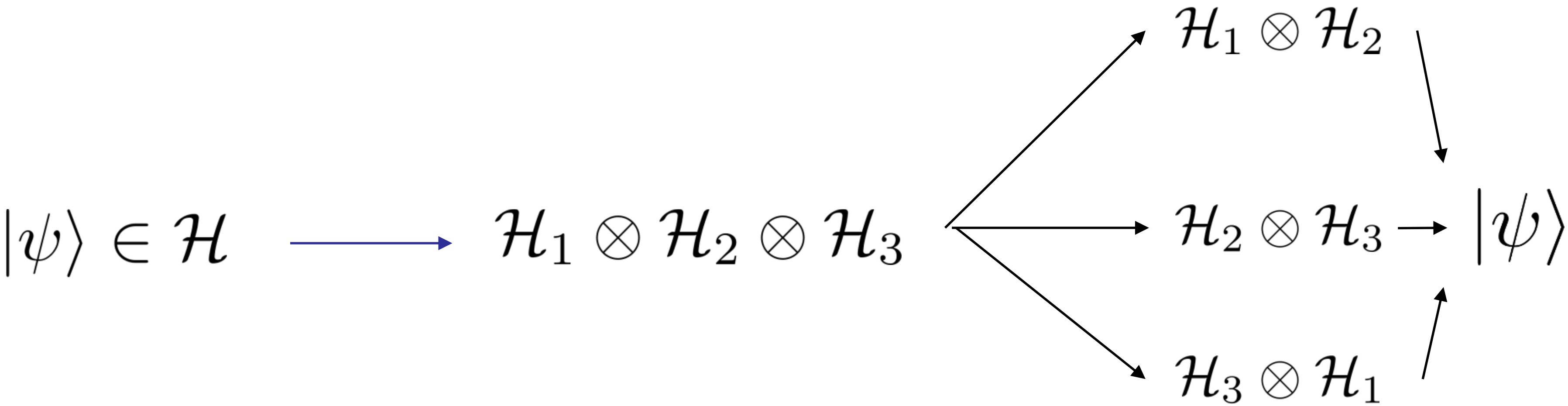
which led to the idea that perhaps quantum gravity can be formulated purely in information theoretic terms (but this has not been achieved yet).

Take home message

- Gravity geometrizes quantum information
- Smooth connected geometries correspond to particular entanglement patterns but the precise map is unknown ("ER=EPR")
- The maximally extended black hole spacetime corresponds to a particular pure entangled state in the copies of the same CFT, the thermofield double state.

4. Quantum error correction

Quantum error correction:



Is quantum gravity a local theory?

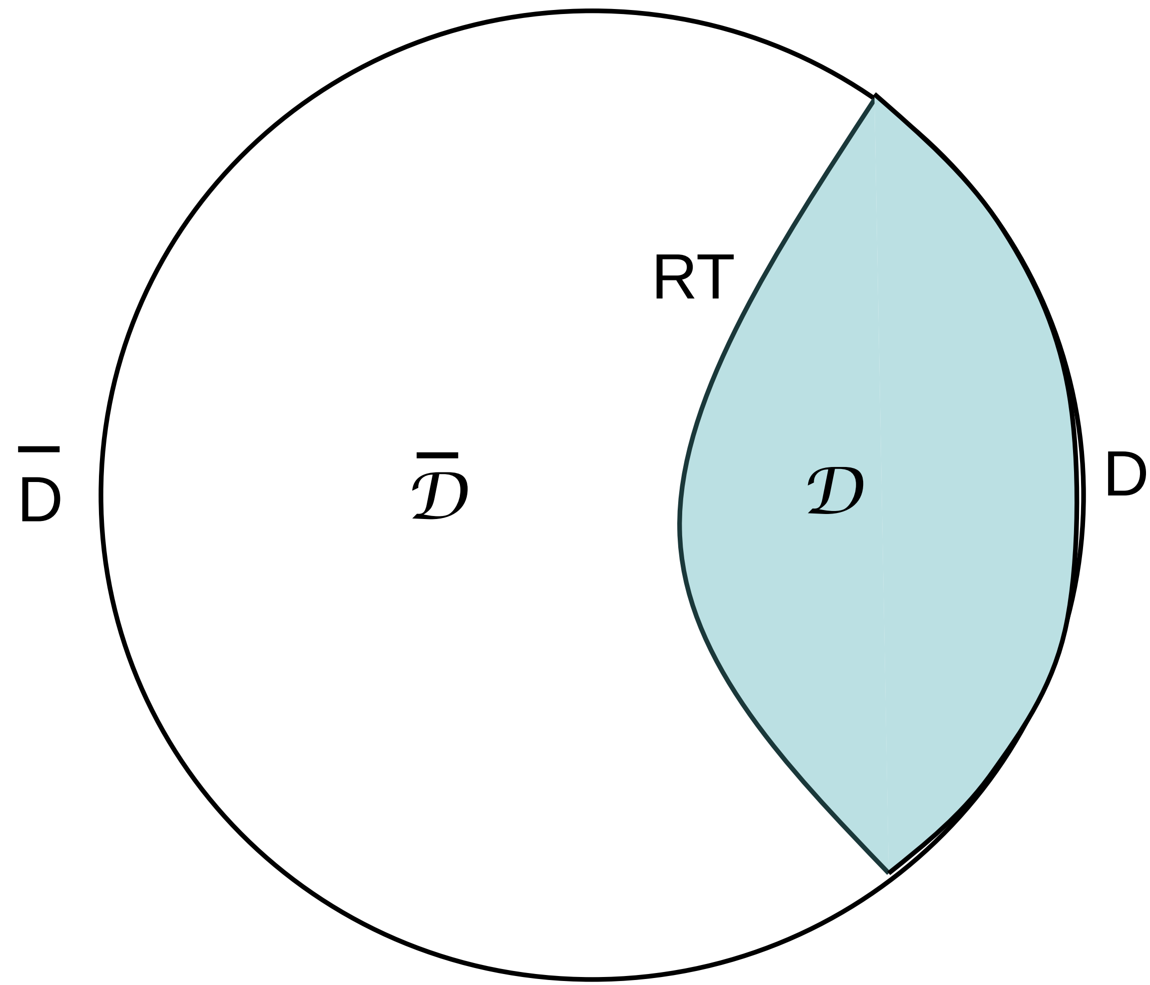
If the degrees of freedom of quantum gravity were approximately local, one should be able to compute their entanglement between some spatial domain and its complement.

This requires a factorization

$$\mathcal{H} = \mathcal{H}_{\rm outside} \otimes \mathcal{H}_{\rm inside}$$

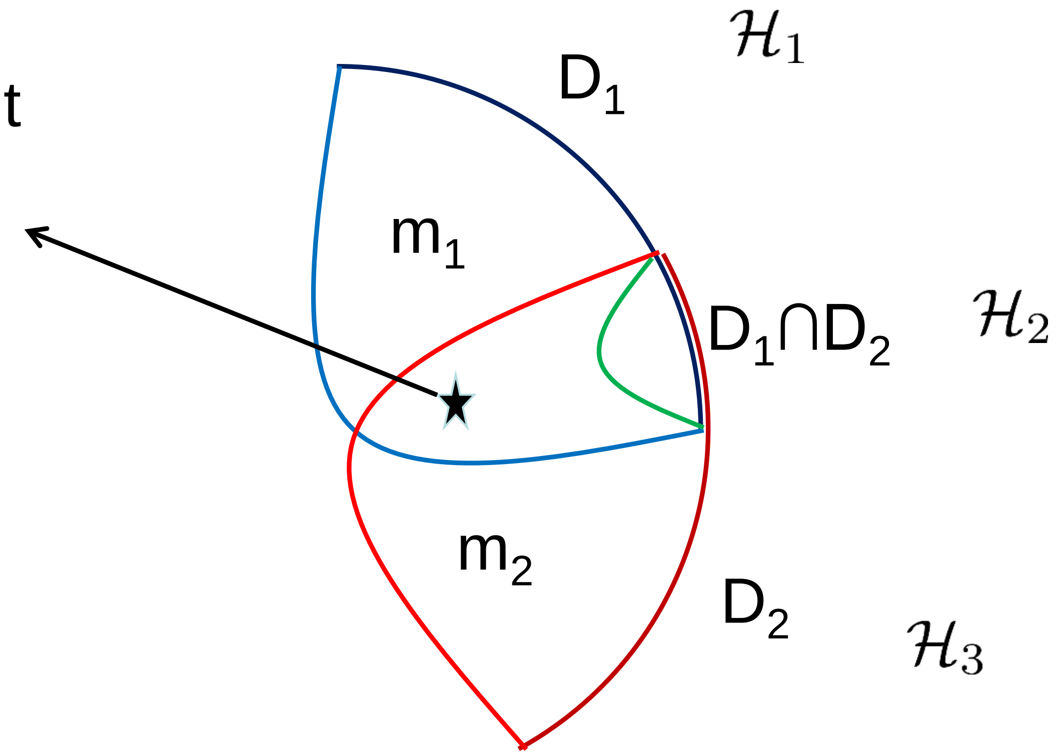
Such a factorization is often used when computing Hawking radiation, when discussing the information loss paradox, and in many arguments pertaining to the (non)existence of firewalls.

$$\mathcal{H} = \mathcal{H}_D \otimes \mathcal{H}_{\bar{D}} \simeq \mathcal{H}_{\mathcal{D}} \otimes \mathcal{H}_{\bar{\mathcal{D}}}$$



A local operator here would act entirely in D_1 but also entirely in D_2; but it does not act in $D_1 \cap D_2$. This is a contradiction. Local operators do not exist?

Connection to quantum error correction!



Local operators only act properly on a subset of the degrees of freedom: the so-called "code subspace".

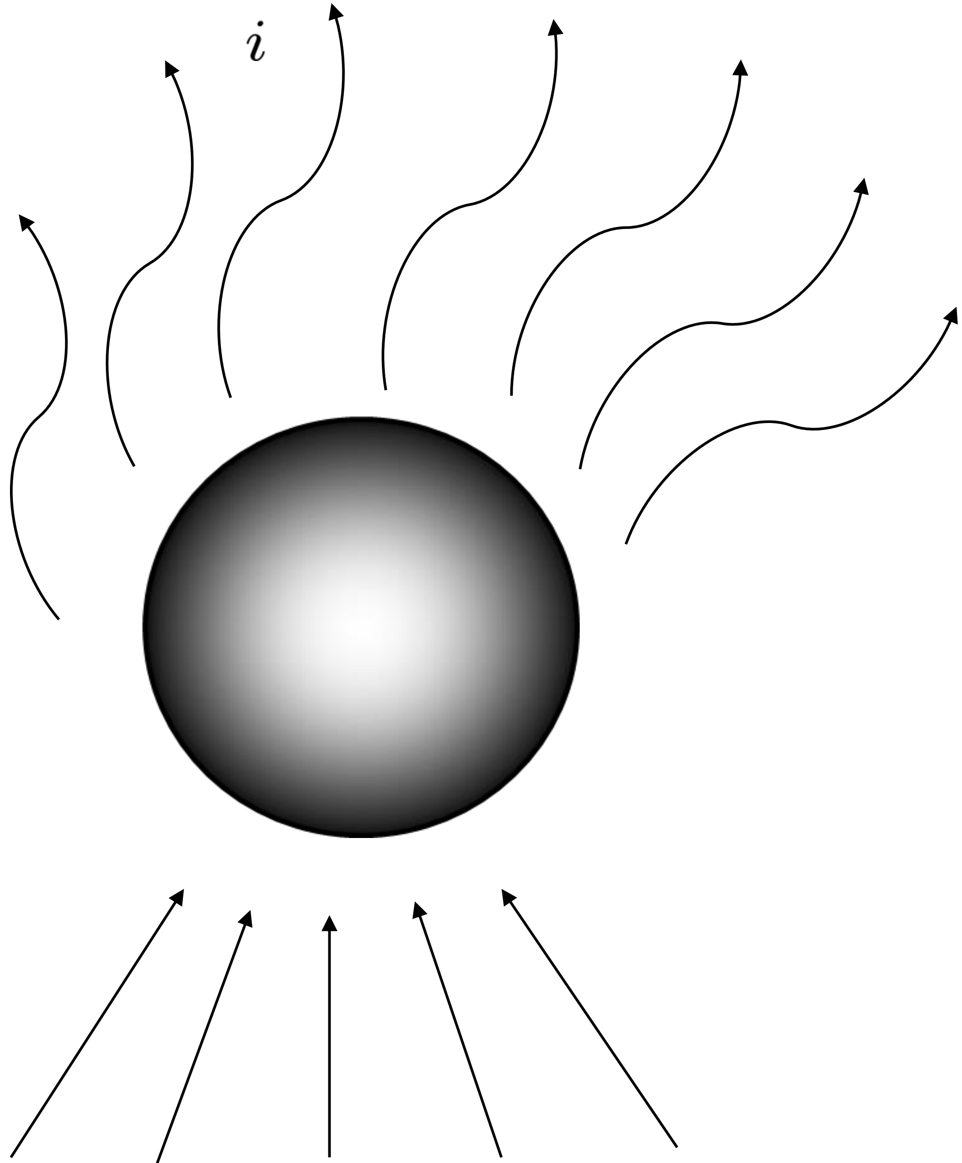
Almheiri, Dong, Harlow `14
Mintun, Polchinski, Rosenhaus `15

Take home message

- An approximate local operator in AdS only acts as expected on a (small) subset of the degrees of freedom, the code subspace.
- Operators have different representations in different entanglement wedges just as in quantum error correction.
- The Hilbert space of quantum gravity does not factorize across spatial regions.

5. Black hole information paradox

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$



$$\rho = |\psi\rangle\langle\psi|$$

According to a famous computation Hawking ’74’75 black holes emit purely thermal radiation.

As a result a pure state apparently becomes a mixed state under time evolution: a violation of unitarity.

This violation cannot be undone by higher order curvature corrections to general relativity. Mathur ‘09

So there seems to be fundamental conflict between

- Unitarity
- Locality
- The equivalence principle

It has been suggested that the equivalence principle should be abandoned and the horizon is really a “firewall”

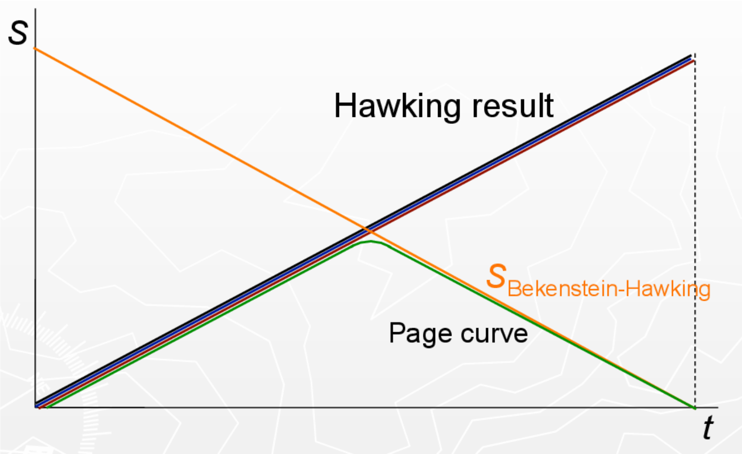
Almheiri, Marolf, Polchinski, Sully ‘13

The AdS/CFT correspondence seems to respect unitarity, locality, and the equivalence principle.

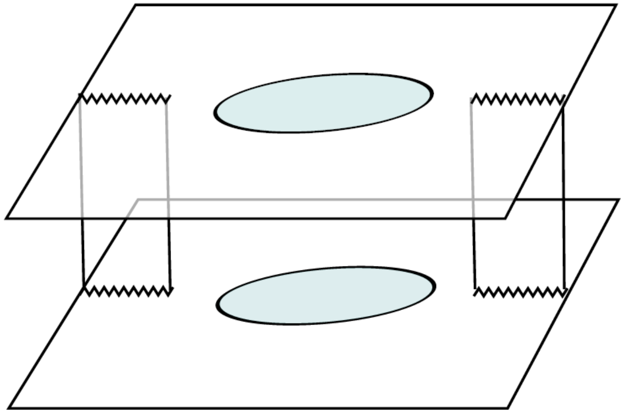
So how does it resolve the information loss paradox?

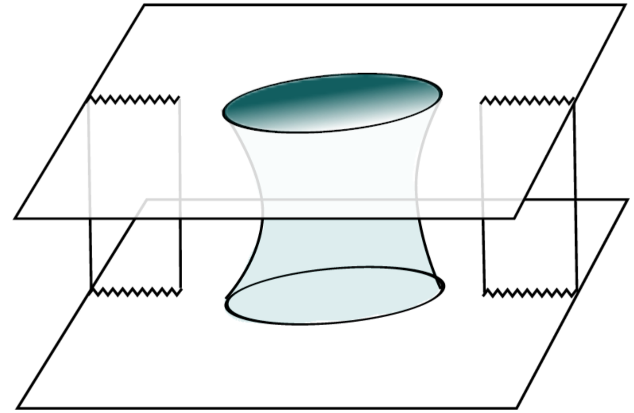
Penington ’19
Almheiri, Engelhardt, Marolf, Maxfield ‘19
Penington, Shenker, Stanford, Yang ‘19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ‘19

Couple the boundary of AdS to an external, large system, which captures all the radiation. Use AdS/CFT technology to compute the entropy contained in that radiation.



Page ‘93


(a)


(b)

In the computation different semiclassical configurations appear, depending on whether or not spacetime is connected by a wormhole. For late times the latter dominates the computation.

The page curve is reproduced.

Comments:

- The computation shows that the radiation carries information about part of the black hole interior (“islands”)
- To uncover the information, extremely complex measurements have to be made. The relevant apparatus will create a substantial backreaction on the black hole geometry.
- The computation only relies on semi-classical general relativity.
- The computation does not admit a direct translation in the language of effective field theory.
- The computation does not elucidate the nature of the individual microstates which make up the black hole.
- Very small but very non-local effects seem to be key.

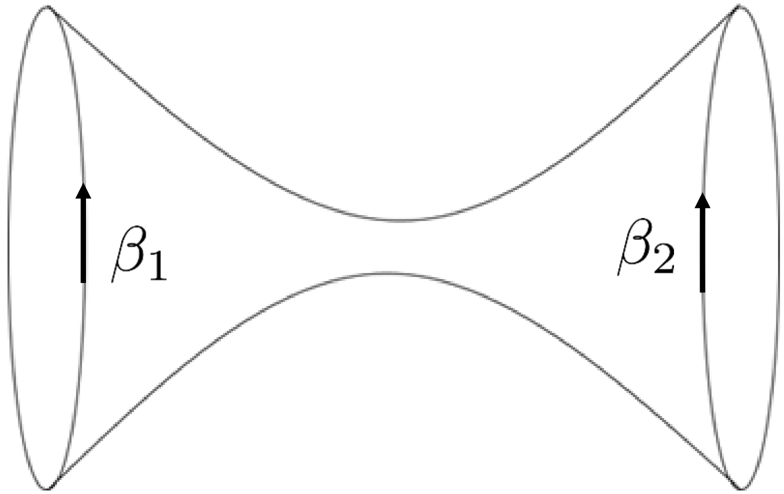
$$\begin{pmatrix} e^{-S} & 0 & 0 & 0 & 0 \\ 0 & e^{-S} & 0 & 0 & 0 \\ 0 & 0 & e^{-S} & 0 & 0 \\ 0 & 0 & 0 & e^{-S} & 0 \\ 0 & 0 & 0 & 0 & e^{-S} \end{pmatrix} \Longrightarrow \begin{pmatrix} e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \\ e^{-S} & e^{-S} & e^{-S} & e^{-S} & e^{-S} \end{pmatrix}$$

Take home message

- By coupling AdS/CFT to an external system we can precisely factorize the Hilbert space in “black hole” and “radiation”.
- The computation of the entropy of the radiation can be formulated as a suitable Euclidean path integral question.
- A wormhole solution dominates the computation at late times precisely reproducing the Page curve.
- One once more sees that one has to give up locality.

6. Wormholes and the factorization puzzle

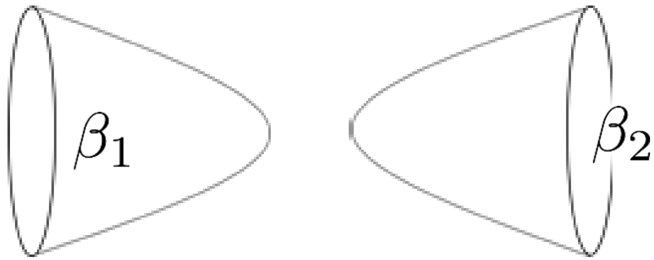
Semi-classical gravity seems to give rise to correlations between copies of the same theory due to the existence of Euclidean wormhole solutions



Such correlations (lack of factorization) could arise due to disorder averages but in standard AdS/CFT there was no need for (or a sign of) disorder.

“factorization puzzle”

$$\langle Z(\beta_1)Z(\beta_2)\rangle =$$



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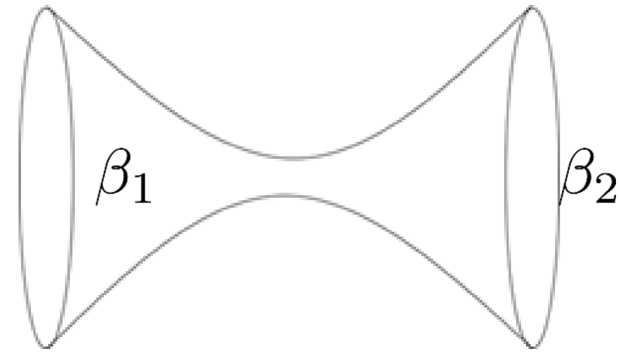
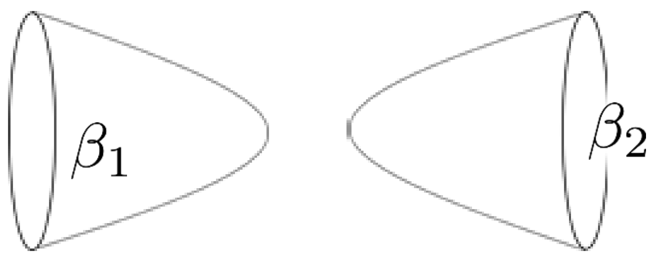


$$\langle Z(\beta_1)\rangle\langle Z(\beta_2)\rangle =$$



$$\Longrightarrow \langle Z(\beta_1)Z(\beta_2)\rangle \neq \langle Z(\beta_1)\rangle\langle Z(\beta_2)\rangle$$

This would be fine if the theory would carry additional parameters that we need to average over because then

$$\sum_{\alpha} \langle Z_\alpha(\beta_1) Z_\alpha(\beta_2) \rangle \neq \sum_{\alpha} \langle Z_\alpha(\beta_1) \rangle \sum_{\alpha'} \langle Z_{\alpha'}(\beta_2) \rangle$$

Does gravity at a fundamental level involve some sort of averaging over parameters?

This has been suggested before in the context of “baby universes”.

Coleman ‘88
Giddings Strominger ‘88

In JT gravity, a two-dimensional theory, one can show that the theory is indeed an averaged theory; it is described by a random matrix model.

Saad, Stanford, Shenker '19

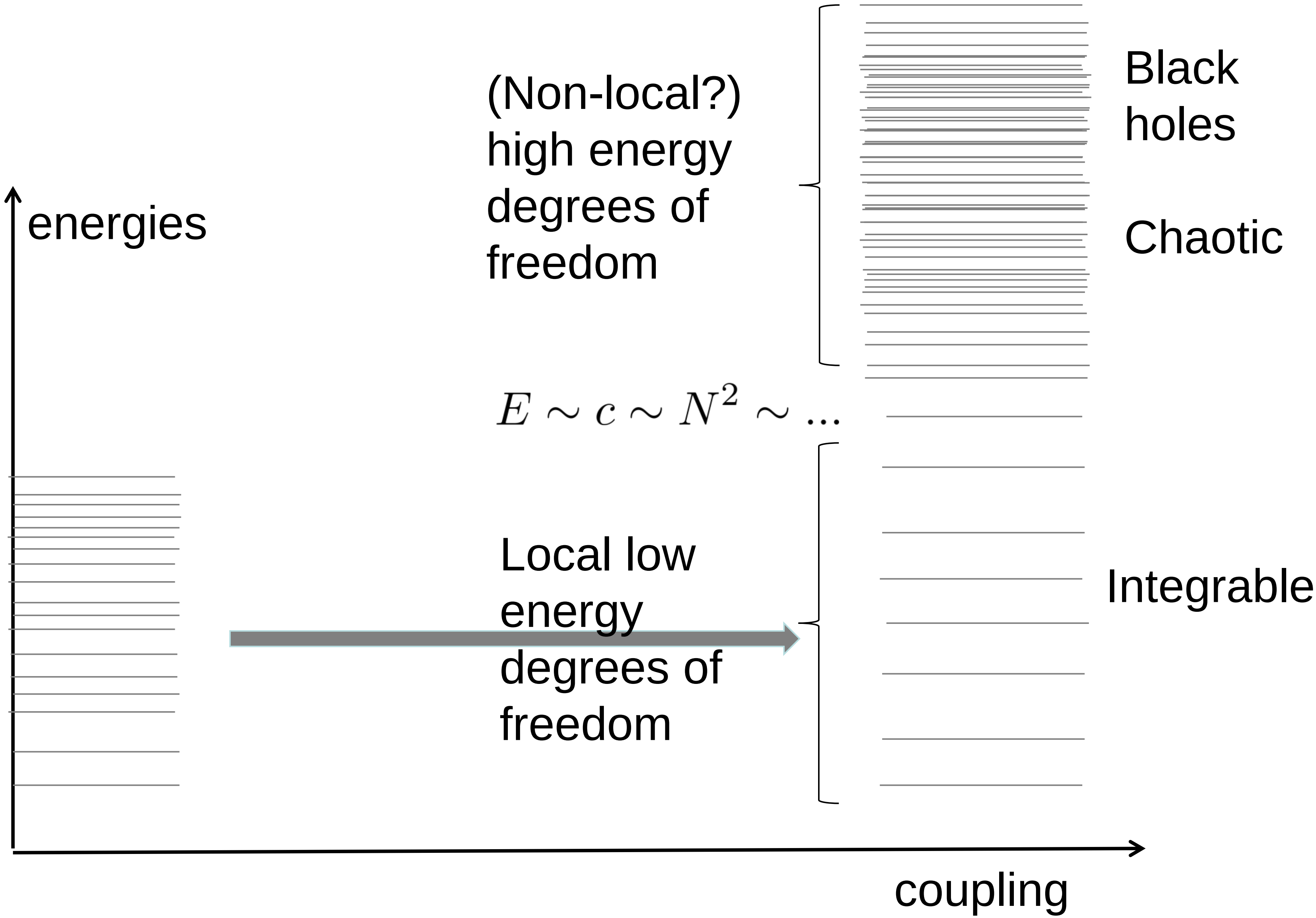
In this example the 2d theory is treated as an exact description rather than a semi-classical theory.

$$S_{JT} \sim \int d^2x \sqrt{-g}\, \Phi(R - \Lambda)$$

In AdS/CFT, no semi-classical gravitational computations resolve *exact* information about the UV physics of the theory.

Rather, they provide *coarse grained* information about the UV physics

$$\sum_i \delta(E - E_i) \Rightarrow \text{smooth } \rho(E)$$


(Non-local?) high energy degrees of freedom
Black holes
Chaotic
energies
$E \sim c \sim N^2 \sim \ldots$
Local low energy degrees of freedom
Integrable
coupling

Is it conceivable that coarse graining over states in the chaotic part of the spectrum imitates disorder averaging and that this explains the appearance of wormholes in semi-classical gravity?

For example:

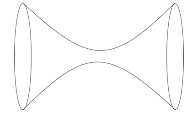
Consider a large set of N=e^S random phases $e^{i\phi_i}$

$$\left(\sum e^{i\phi_i}\right)\left(\sum e^{i\phi_i}\right)^* = \sum_{i=j} 1 + \sum_{i\neq j} e^{i(\phi_i - \phi_j)}$$

N=e^S
visible in
gravity

erratic
invisible in
gravity

So semi-classical gravity is sensitive to the average size of fluctuations (stable under coarse graining) but not to the individual fluctuations themselves which disappear under coarse graining.



$$\left(\sum e^{i\phi_i}\right)\left(\sum e^{i\phi_i}\right)^* = \sum_{i=j} 1 + \sum_{i\neq j} e^{i(\phi_i - \phi_j)}$$

What happens in the full UV theory? Possibilities:

- The relevant gravitational solution (eg wormhole) is unstable and factorization is restored (but solution remains as off-shell configuration)
- UV physics adds the fluctuating contributions $\sum_{i\neq j} e^{i(\phi_i - \phi_j)}$ and factorization is restored
- The UV theory is an average of theories, averaging makes the fluctuating term exactly zero, and factorization is not restored

MAIN CLAIM:

- Semi-classical gravity is the theory of the statistics of the chaotic sector of the theory.

- It can probe (coarse-grained) higher moments of the relevant statistical distributions but not individual values.

- It cannot distinguish averaged from non-averaged theories as long as the averages yield the same moments of the statistical distribution (up to the accuracy of semi-classical gravity).

Is this a fundamental limitation on how much information low-energy observers can obtain?

In a series of papers, we collected a lot of evidence for this picture:

Alex Belin, JdB, arXiv:2006.05499
Alex Belin, JdB, Pranyal Nayak, Julian Sonner, arXiv:2012.07875
Alex Belin, JdB, Diego Liska, arXiv:2110.14649
Alex Belin, JdB, Pranyal Nayak, Julian Sonner, arXiv:2111.06373
Tarek Anous, Alex Belin, JdB, Diego Liska, arXiv:2112.09143

A lot of related work has appeared, e.g:

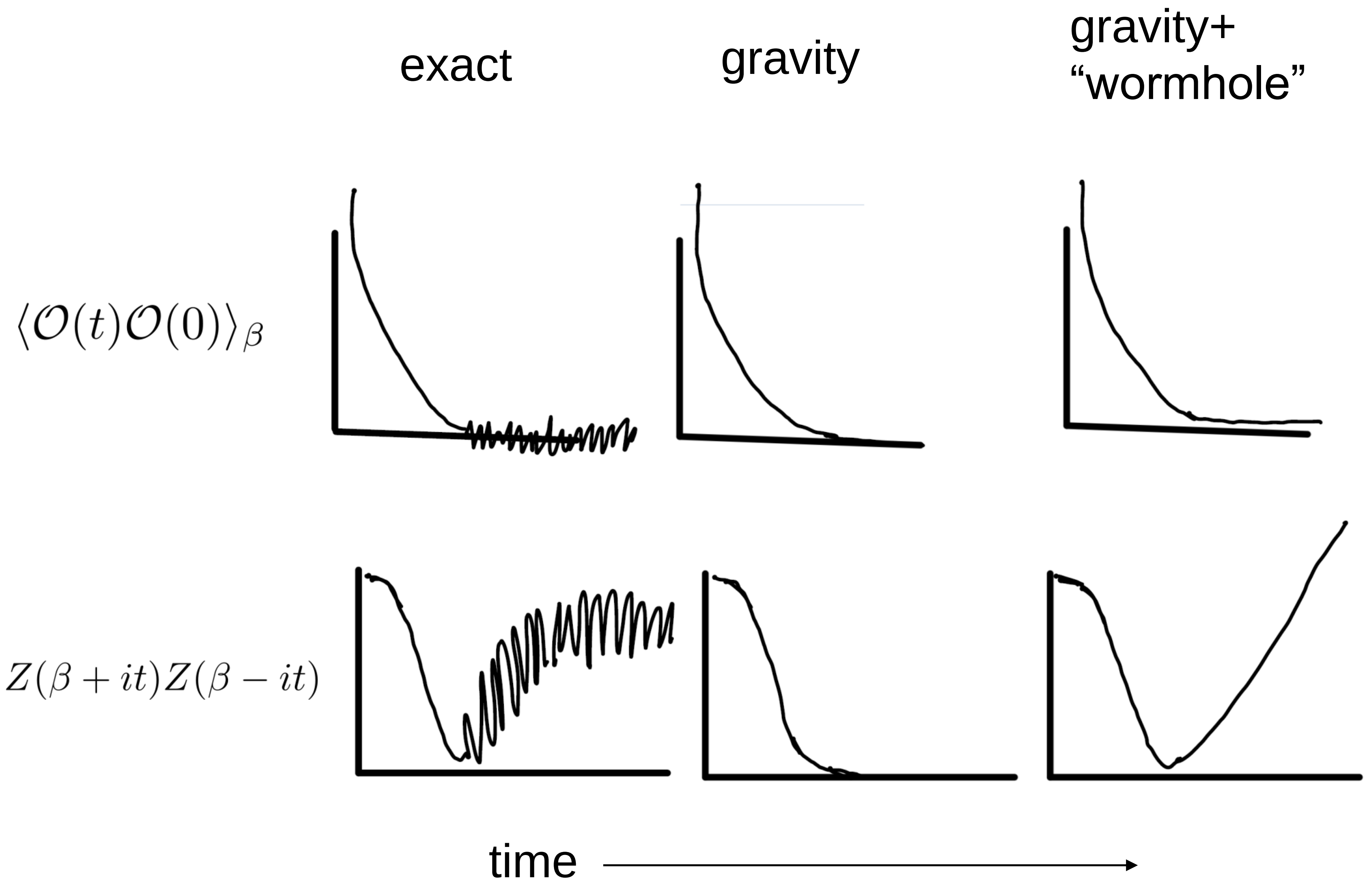
Pollack, Rozali, Sully, Wakeham ‘20
Liu, Vardhan ‘20
Altland, Sonner ‘20
Janssen, Mirbabayi, Zograf ’21
Sasieta ‘21
Altland, Bagrets, Nayak, Sonner, Vielma ‘21
Freivogel, Nikolakopoulou, Rotundo ’21
Schlenker, Witten ‘22
Chandra, Collier, Hartman, Maloney ‘22

Take home message

- Semi-classical gravity is the theory of the statistics of the chaotic sector of the theory.
- It can probe (coarse-grained) higher moments of the relevant statistical distributions but not individual values.
- There is no need to interpret semiclassical gravity fundamentally as an averaged theory
- Some special theories where gravity has no propagating degrees of freedom have a precise description as averaged theories.

7. What happened to the microstates? Have we seen them?

Not quite yet….



CONCLUSIONS

Great progress in understanding quantum gravity and black holes

Semi-classical gravity is the theory of the statistics of the high-energy, chaotic sector of the theory.

Picture is consistent with known wormhole solutions and predicts new wormhole solutions.

It seems very difficult to probe interesting aspects of quantum gravity with semi-classical observers alone.

The main remaining open questions (in my opinion) are (i) to accurately predict the fate of the infalling observer and (ii) to extend everything to other space-times.