• A Carrollian Perspective on Celestial Holography •

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Abstract

We show that a $3d$ sourced conformal Carrollian field theory has the right kinematic properties to holographically describe gravity in $4d$ asymptotically flat spacetime. The external sources encode the leaks of gravitational radiation at null infinity. The Ward identities of this theory are shown to reproduce those of the $2d$ celestial CFT after relating Carrollian to celestial operators.

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I. Flat space holography program

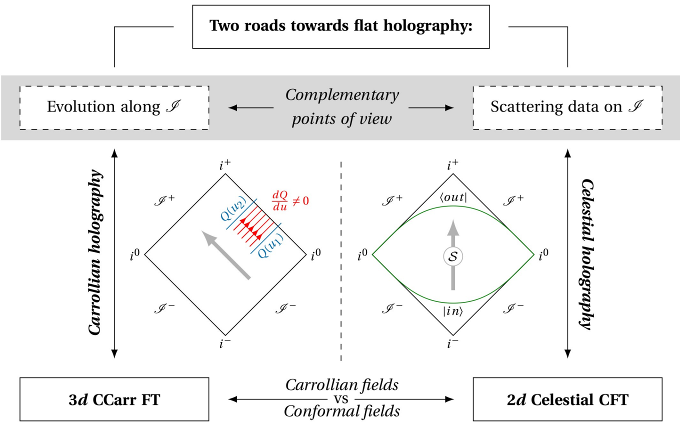
Holographic principle: *Gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region.* Explicit realization: *AdS/CFT correspondence.*
Does it extend to asymptotically flat spacetimes? → Obstructions: conformal boundary $\mathscr{I}$ is *null* + *radiation* leaking through.

The Carrollian picture

Inspired by asymptotic symmetry analysis.
[BMS group ≃ conformal Carrollian group]

Advantages:
- ✔ Similar pattern than AdS/CFT ($4d/3d$) + flat limit procedure $\Lambda \to 0$.
- ✔ $3d$ gravity (entropy matching, effective action, …) + fluid holography.

Drawbacks:
- ✘ Few is known about Carrollian QFTs.
- ✘ Deal with non-conserved charges.



The celestial picture

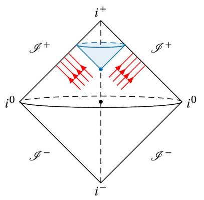
Inspired by quantum theory of scattering.
[$\mathcal{S}$-matrix elements ≃ $2d$ CFT correlators]

Advantages:
- ✔ Use the powerful techniques of CFT.
- ✔ Ward identities in the CCFT ⇔ soft theorems in the bulk.

Drawbacks:
- ✘ No clear path to relate with AdS/CFT.
- ✘ Information on the dynamics not manifest.

II. Gravity in $4d$ asymptotically flat spacetime

- Solution space of $4d$ asymptotically flat metrics at $\mathscr{I}^+$ in Bondi coordinates (u, r, x^A), $x^A = (z, \bar{z})$



$$ds^2 = \left(\frac{2M}{r} + \mathcal{O}(r^{-2})\right) du^2 - 2\left(1 + \mathcal{O}(r^{-2})\right) du dr$$
$$+ \left(r^2 \mathring{q}_{AB} + r\, C_{AB} + \mathcal{O}(r^{-1})\right) dx^A dx^B$$
$$+ \left(\frac{1}{2}\partial_B C^B_A + \frac{2}{3r}\left(N_A + \frac{1}{4} C^B_A \partial_C C^C_B\right) + \mathcal{O}(r^{-2})\right) du dx^A ,$$

with boundary metric $\mathring{q}_{AB} dx^A dx^B = 2 dz d\bar{z}$.

- Evolution equations for mass (M) and angular momentum (N_A) aspects:

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4}\partial_A \partial_B N^{AB} \quad \text{(Bondi mass loss)},$$
$$\partial_u N_A = \partial_A M + \frac{1}{2}\partial^B \partial_{[A}\partial^C C_{B]C} + \text{quadratic terms in } N_{AB}, C_{AB} .$$

- Diffeomorphisms $\xi = \xi^\mu \partial_\mu$ preserving the solution space ⇒ extended $\mathfrak{bms}_4$ algebra.
  $\xi^u = T + \frac{u}{2}\partial_A Y^A$, $\quad \xi^z = Y^z + \mathcal{O}(r^{-1})$, $\quad \xi^{\bar{z}} = Y^{\bar{z}} + \mathcal{O}(r^{-1})$, $\quad \xi^r = -\frac{r}{2}\partial_A Y^A + \mathcal{O}(r^0)$.
  $T(z, \bar{z})$ = supertranslation; $(Y^z(z), Y^{\bar{z}}(\bar{z}))$ = superrotation (local CKV on the sphere).
  → Conformal symmetries of Carrollian structure at $\mathscr{I}^+$, (q_{ab}, n^c) with $q_{ab} n^b = 0$.
- BMS charges: not conserved if $N_{AB} \neq 0$ (Bondi news).

$$Q_\xi = \frac{1}{16\pi G}\int_S d^2 z \left(4TM + 2Y^A \bar{N}_A\right),$$
$$\bar{N}_A = N_A - u\partial_A M + \frac{1}{4} C^B_A \partial_C C^C_B + \frac{3}{32}\partial_A (C^B_C C^C_B) - \frac{u}{2}\partial^B \partial_{[A}\partial^C C_{B]C} . \tag{1}$$

- Flux-balance laws:

$$\frac{dQ_\xi}{du} = \int_S d^2 z \left(F^H_\xi + F^S_\xi\right) \quad \text{where} \quad \begin{cases} F^S_\xi(u, z, \bar{z}) \text{ is linear in } N_{AB}, \\ F^H_\xi(u, z, \bar{z}) \text{ is quadratic in } N_{AB} . \end{cases} \tag{2}$$

III. Sourced quantum field theories

Consider a QFT on a manifold $\mathscr{M}$ with boundary $\partial\mathscr{M}$ and local coordinates x^a.
- Fields: $\Phi^i(x)$, Noether symmetries $\delta_K \Phi^i = K^i[\Phi] \Rightarrow$ conserved currents $\mathrm{d}j_K(x) = 0$.
- Couple the theory with *external sources* $\sigma(x)$:
  → Noether currents no longer conserved: $\mathrm{d}j_K(x) = \boldsymbol{F}_K(x)$, $\boldsymbol{F}_K(x)\big|_{\sigma=0} = 0$.
- At quantum level ⇒ *sourced Ward identities*

$$\boxed{\partial_a \langle j^a_K(x)\, X\rangle + \frac{\hbar}{i}\sum_{i=1}^N \delta^{(n)}(x - x_i)\delta_{K^i}\langle X\rangle = \langle F_K(x)\, X\rangle} \tag{3}$$

where $\langle X\rangle = \langle \Phi^{i_1}(x_1) \dots \Phi^{i_N}(x_N)\rangle$. Integrating on $\mathscr{M}$ gives

$$\sum_{i=1}^N \delta_{K^i}\langle X\rangle = \frac{i}{\hbar}\left\langle \left(\int_{\mathscr{M}} \boldsymbol{F}_K - \int_{\partial\mathscr{M}} j_K\right) X\right\rangle . \tag{4}$$

IV. Holographic correspondence

Dual theory of 4d AF gravity: CCarrFT with external sources. Primary fields transform as

$$\delta_\xi \Phi_{(k,\bar{k})} \equiv \left[\left(T + \frac{u}{2}(\partial_z Y^z + \partial_{\bar{z}} Y^{\bar{z}})\right)\partial_u + Y^z \partial_z + Y^{\bar{z}}\partial_{\bar{z}} + k\partial_z Y^z + \bar{k}\partial_{\bar{z}} Y^{\bar{z}}\right]\Phi_{(k,\bar{k})},$$

under conformal Carrollian symmetries.
- Noether currents associated with the CCarr symmetries: $j^a_\xi = \mathcal{C}^a{}_b \xi^b$ where $\mathcal{C}^a{}_b$ plays the role of stress-energy tensor for a CCarrFT ("Carrollian momenta")
  Correspondence with gravitational charges (1) if

$$\langle \mathcal{C}^u{}_u\rangle = \frac{M}{4\pi G}, \quad \langle \mathcal{C}^u{}_A\rangle = \frac{1}{8\pi G}(\bar{N}_A + u\partial_A M), \quad \langle \mathcal{C}^A{}_B\rangle + \frac{1}{2}\delta^A{}_B \langle \mathcal{C}^u{}_u\rangle = 0 .$$

- Flux-balance law: $\partial_a j^a_\xi = F_\xi[\sigma]$ with external flux $F_\xi[\sigma] = F_a[\sigma]\xi^a$.
  → Sources σ_{AB} identified with Bondi news, $\sigma_{AB} = N_{AB}$;
  → Perturbation in the shear Φ_{AB}: quasi-primary CCarr field, $\langle \Phi_{AB}\rangle = C_{AB}$.
- Sourced CCarr Ward identities reproduce the BMS flux-balance laws (2).

V. Massless scattering and antipodal matching

Massless scattering in AF spacetime = relate initial data on $\mathscr{I}^-$ with final states on $\mathscr{I}^+$.
The dual theory lives on $\hat{\mathscr{I}} = \mathscr{I}^- \sqcup \mathscr{I}^+$ glued by antipodal identification of $\mathscr{I}^-_+$ and $\mathscr{I}^+_-$.
- Antipodal matching implied by continuity at the gluing;
- Gluing surface distinguished by a vanishing n^a (stable under supertranslations);
- Carrollian symmetries acting on $\hat{\mathscr{I}}$ = antipodally matched diagonal BMS symmetries.

Hypotheses:
- $\boldsymbol{j}_\xi = 0$ on $\mathscr{I}^+_+$ and $\mathscr{I}^-_-$ (no massive particles);
- $\int_{\mathscr{I}^+} \boldsymbol{F}_\xi = -\int_{\mathscr{I}^-} \boldsymbol{F}_\xi$ (constraint on the sources).

} (4) ⇒ $\delta_\xi \langle X\rangle = 0$ (BMS invariant correlators)

VI. Relation with Celestial CFT

Central claim: *the Ward identities of the sourced CCarr FT reproduce the BMS Ward identities of the CCFT after performing the right integral transforms. Ex.* for ξ = supertranslation.

1. Split the variations $\delta_\xi\Phi$ into homogeneous $\delta^H_\xi\Phi$ and non-homogeneous $\delta^S_\xi\Phi$ parts;
2. Rewrite $\delta^S_\xi\langle X\rangle \propto \langle F^S_\xi X\rangle$ in $\delta_\xi\langle X\rangle = 0$ using the canonical relation:
$$[\Pi_{zz}(u, z, \bar{z}), \Phi_{\bar{w}\bar{w}}(u', w, \bar{w})] = 16\pi G\, i\hbar\, \delta(u - u')\delta^{(2)}(z - w) ;$$
3. Particularize for $\xi = \frac{1}{z-w}\partial_u$ and perform the $\mathcal{B}$-transform $\qquad (J = k - \bar{k})$
$$\mathcal{O}^{out}_{\Delta,J}(z, \bar{z}) \propto \int_{-\infty}^{+\infty} \frac{du}{u^\Delta}\Phi^{out}_{(k,\bar{k})}(u, z, \bar{z}), \quad \mathcal{O}^{in}_{\Delta,J}(z, \bar{z}) \propto \int_{-\infty}^{+\infty} \frac{dv}{v^\Delta}\Phi^{in}_{(k,\bar{k})}(v, z, \bar{z}).$$
4. Define $P(z, \bar{z}) \equiv \frac{1}{4G}\left(\int du + \int dv\right)\partial_{\bar{z}}\Pi_{zz}$ to get the *soft graviton theorem*
$$\left\langle P(z, \bar{z})\prod_{i=1}^N \mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i)\right\rangle + \hbar\sum_{q=1}^N \frac{1}{z - z_q}\left\langle \dots \mathcal{O}_{\Delta_q+1, J_q}(z_q, \bar{z}_q)\dots\right\rangle = 0 .$$