On the Geometric Approach to the Boundary Problem in Supergravity

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This poster is about...

(SU)GRA Lagrangians in the presence of a non-trivial spacetime bdy: York-Gibbons-Hawking, Horava-Witten, AdS/CFT ⟶ General lesson:

For $\partial\mathcal{M} \neq 0 \longrightarrow$ Bulk thy + Bdy terms

> Here: Geometric construction of SUGRA Lagrangians for $\partial\mathcal{M} \neq 0$ and applications relevant in holography

Geometric approach in the presence of a boundary

Gravity ➜ Cartan approach (Aros, Contreras, Olea, Troncoso, Zanelli): Diffeomorphism invariance of the bulk Einstein Lagrangian + Λ broken for $\partial\mathcal{M}_4 \neq 0 \longrightarrow$ Restored by adding the topological Euler-Gauss-Bonnet term

$$\mathcal{L}_{\mathrm{EGB}} = \mathcal{R}^{ab} \wedge \mathcal{R}^{cd}\epsilon_{abcd} = \mathrm{d}\left(\omega^{ab} \wedge \mathcal{R}^{cd} + \omega^{a}{}_{d} \wedge \omega^{db} \wedge \omega^{cd}\right)\epsilon_{abcd}$$

- Expansion of $\mathcal{L}_{\mathrm{EGB}}$ in the radial coordinate orthogonal to the bdy ⟶ Contribution needed to regularize the action and the related (background-independent) conserved charges

SUGRA ➜ Systematic way to face the bdy problem:

Geometric approach to SUGRA in superspace

[hep-th:2005.13593, hep-th:1405.2010]

Action: $\mathcal{S} = \int_{\mathcal{M}_4 \subset \mathcal{M}_{4|4N}} \mathcal{L}[\mu^{\mathcal{A}}] \qquad (D=4)$

SUSY transf.: $\ell_\epsilon = \imath_\epsilon \mathrm{d} + \mathrm{d}\imath_\epsilon$

SUSY inv. of $\mathcal{L}$: $\delta_\epsilon \mathcal{L} = \ell_\epsilon \mathcal{L} = \imath_\epsilon(\mathrm{d}\mathcal{L}) + \mathrm{d}(\imath_\epsilon \mathcal{L}) = 0$

Necessary cond.: $\imath_\epsilon(\mathrm{d}\mathcal{L}) = 0 \quad \longrightarrow \quad \mathcal{L}_{\mathrm{bulk}}$ ✓

SUSY inv. of $\mathcal{S}$: $\delta_\epsilon \mathcal{S} = \int_{\mathcal{M}_4} \mathrm{d}(\imath_\epsilon \mathcal{L}_{\mathrm{bulk}}) = \int_{\partial\mathcal{M}_4} \imath_\epsilon \mathcal{L}_{\mathrm{bulk}} = 0$

$\Rightarrow \quad \imath_\epsilon \mathcal{L}_{\mathrm{bulk}}|_{\partial\mathcal{M}_4} = \mathrm{d}\varphi$ ✗ for $\partial\mathcal{M}_4 \neq 0$

SUSY invariance requires to add bdy terms ⟶ Consider:

$\mathcal{L}_{\mathrm{full}} = \mathcal{L}_{\mathrm{bulk}} + \mathcal{L}_{\mathrm{bdy}} \quad \Rightarrow \quad \imath_\epsilon(\mathrm{d}\mathcal{L}_{\mathrm{full}}) = 0$ and $\imath_\epsilon \mathcal{L}_{\mathrm{full}}|_{\partial\mathcal{M}_4} = 0$

Let's apply to $\mathcal{N}=1$ AdS$_4$ pure SUGRA with boundary

[hep-th:1405.2010]

$$\mathcal{L}^{\mathcal{N}=1}_{\mathrm{bulk}} = \frac{1}{4}\mathcal{R}^{ab}V^{cd}\epsilon_{abcd} - \bar{\psi}\gamma_5\gamma_a\rho V^a - \frac{\mathrm{i}}{2\ell}\bar{\psi}\gamma_5\gamma_{ab}\psi V^{ab} - \frac{1}{8\ell^2}V^{abcd}\epsilon_{abcd}$$

For $\partial\mathcal{M}_4 \neq 0$: $\imath_\epsilon \mathcal{L}^{\mathcal{N}=1}_{\mathrm{bulk}}|_{\partial\mathcal{M}_4} \neq \mathrm{d}\varphi \quad \Rightarrow \quad \delta_\epsilon \mathcal{S}_{\mathrm{bulk}} \neq 0$ ✗

$$\mathcal{L}^{\mathcal{N}=1}_{\mathrm{bdy}} = d\mathcal{B}_{(3)} = \alpha \mathcal{R}^{ab}\mathcal{R}^{cd}\epsilon_{abcd} - \mathrm{i}\beta\left(\bar{\rho}\gamma_5\rho - \frac{1}{4}\mathcal{R}^{ab}\bar{\psi}\gamma_5\gamma_{ab}\psi\right)$$

$\Rightarrow$ Consider: $\mathcal{L}^{\mathcal{N}=1}_{\mathrm{full}} \equiv \mathcal{L}^{\mathcal{N}=1}_{\mathrm{bulk}} + \mathcal{L}^{\mathcal{N}=1}_{\mathrm{bdy}}$

- Bdy contributions to the e.o.m. ⇒ Supercurvatures dynamically fixed on $\partial\mathcal{M}_4$ to const. values in the anholonomic basis $\{V^a, \psi\}$ ⟶ Constraints (✯)
- SUSY invariance ⇒ Eq. relating α and β ⟶ Setting $\alpha = -\ell^2/8$ and $\beta = \ell$:

$\mathcal{L}^{\mathcal{N}=1}_{\mathrm{bdy}}$ ➜ $\mathcal{N}=1$ extension of $\mathcal{L}_{\mathrm{EGB}}$
&
$\mathcal{L}^{\mathcal{N}=1}_{\mathrm{full}}$ ➜ MacDowell-Mansouri Lagrangian ⟶ OSp(1|4) supercurvatures

$$\mathcal{L}^{\mathcal{N}=1}_{\mathrm{full}} = -\frac{\ell^2}{8}\mathbf{R}^{ab} \wedge \mathbf{R}^{cd}\epsilon_{abcd} - \mathrm{i}\ell\bar{\boldsymbol{\rho}}\gamma_5 \wedge \boldsymbol{\rho} \quad ✓$$

and (✯) become: $\mathbf{R}^{ab}|_{\partial\mathcal{M}_4} = 0\,, \quad \boldsymbol{\rho}|_{\partial\mathcal{M}_4} = 0\,, \quad \mathbf{R}^{a}|_{\partial\mathcal{M}_4} = 0$

⟶ OSp(1|4) supercurv. = 0 at $\partial\mathcal{M}_4$ ⇒ Bdy: global OSp(1|4) inv.

For $\mathcal{N}=2$ AdS$_4$ pure SUGRA with bdy: $\mathcal{L}^{\mathcal{N}=2}_{\mathrm{full}}$ à la MacDowell-Mansouri, OSp(2|4) supercurv. = 0 at $\partial\mathcal{M}_4$ ⇒ Bdy enjoys global OSp(2|4) invariance

What about "flat" $D=4$ SUGRA with boundary?

[hep-th:1809.07871]

1) Direct limit $\ell \to \infty$ of the MM $\mathcal{L}^{\mathcal{N}=1}_{\mathrm{full}}$ not well-defined ✗

2) Bdy terms using ω^{ab}, V^a, ψ scale as L^0, L, while EH, RS scale as L^2 ✗

➜ Alternative approach: Add new fields A^{ab} and χ (s.w. L^2, $\mathrm{L}^{3/2}$)

> - Act as auxiliary fields (off-shell matching of B and F d.o.f.) from the bulk perspective ⟶ Implement through their e.o.m. the Bianchi identities of Lorentz and SUSY
> - Only in bdy Lagrangian, restoring SUSY ⟶ Topological role

➜ Full Lagrangian is MM-like, $\ell \to \infty$ limit of a thy originating from AdS$_4$ SUGRA redefining ω^{ab} and ψ with A^{ab} and χ (with A^{ab} and χ also in the bulk)

Applications to asymptotic boundaries and outlooks

- AVZ $D=3$ model of unconventional SUSY [hep-th:1109.3944] from $\mathcal{N}=2$ AdS$_4$ pure SUGRA with a 3D bdy ➜ Local AdS$_3$ at spatial infinity [hep-th:1801.08081]
- $\mathcal{N}=2$ SUSY extension of EGB term gives counterterms for holographic renormalization? ➜ Holographic framework for $\mathcal{N}=2$ AdS$_4$ pure SUGRA, with all contributions from the fermionic fields [hep-th:2010.02119]
- Applications to flat SUGRA in holography? We know: Bdy dual to flat gravity identified in Carrollian fluids [hep-th:1802.06809] ➜ At SUSY level?

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