Goldstino Condensates and Anti-brane Uplifting Instability

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Motivation

Anti-branes are a common ingredient in string theory constructions, which can be used to induce **spontaneous supersymmetry breaking**. This results into a **goldstino sector** on the world-volume of the membranes, whose dynamics can be captured by **constrained superfields**.

The minimal supersymmetric theory describing the low energy dynamics of a goldstino is the **Volkov–Akulov model**.
Goldstino self-interactions allow for the possible appearance of **composite states**.

Then: do these composite states of the goldstino form**?** If so, what are the consequences of their formation**?**

The Volkov–Akulov model

It is a theory of a single goldstino fermion, which we will indicate by G_α, whose action is

$$S_{\text{VA}} = \int \mathrm{d}^4x \left(-f^2 + iG\sigma^m\partial_m\overline{G} - \frac{1}{4f^2}G^2\partial^2\overline{G}^2 - \frac{1}{16f^6}G^2\overline{G}^2\partial^2G^2\partial^2\overline{G}^2 \right), \tag{1}$$

$\sqrt{f}$ being related to the supersymmetry breaking energy scale.
An equivalent formulation of the model is given in terms of two chiral superfields by

$$\begin{aligned} S_{\text{VA}} &= \int \mathrm{d}^4x \left[\int \mathrm{d}^4\theta K + \left(\int \mathrm{d}^2\theta W + \text{c.c.} \right) \right] = \\ &= \int \mathrm{d}^4x \left\{ \int \mathrm{d}^4\theta \mathcal{X}\overline{\mathcal{X}} + \left[\int \mathrm{d}^2\theta \left(f\mathcal{X} + \frac{1}{2}\mathcal{X}^2\mathcal{T} \right) + \text{c.c.} \right] \right\}, \end{aligned} \tag{2}$$

where

$$\mathcal{X} = X + \sqrt{2}\theta G + \theta^2 F^{\mathcal{X}} \quad \text{and} \quad \mathcal{T} = T + \sqrt{2}\theta\lambda + \theta^2 F^{\mathcal{T}}. \tag{3}$$

The superfield $\mathcal{T}$ plays the role of a Lagrange multiplier that, once integrated out, imposes the nilpotency condition on $\mathcal{X}$, $\mathcal{X}^2 = 0$.

Composite states detection via the renormalization group

The complex scalar fields of $\mathcal{X}$ and $\mathcal{T}$, namely X and T, are related to goldstino bilinears via equations of motion:

$$\langle X \rangle \sim \left\langle \frac{G^2}{f} \right\rangle + \dots \quad \text{and} \quad \langle T \rangle \sim \left\langle \frac{\partial^2\overline{G}^2}{f^2} \right\rangle + \dots. \tag{4}$$

The fields X and T are initially non-dynamical.
The renormalization group (RG) method applied to the Volkov–Akulov model produces a kinetic term for T. As a consequence, X and T become **dynamical** and represent **composite states**.

The exact renormalization group approach

A small kinetic term for T, which corresponds to a strong coupling regime, requires a non-perturbative approach to the RG flow. Then, the adoption of the **exact renormalization group** (ERG) technique is needed.
A Wilsonian effective action $S[\Phi;\mu]$ with cut-off μ can be arranged into a propagator contribution and an interaction piece as

$$S_{\text{prop.}} = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\Phi^A(-k)C^{-1}_{AB}(k,\mu)\Phi^B(k) \tag{5}$$

and

$$S_{\text{int.}} = \sum_\lambda \int \prod_A \left(\frac{\mathrm{d}^4k_A}{(2\pi)^4} \right) g_\lambda(\mu) \prod_{\{A\}_\lambda} \left(k_A^{n_{A,\lambda}}\Phi^A(k_A) \right) \delta\left(\sum_A k_A \right), \tag{6}$$

where $C^{AB}(k,\mu)$ is a regularized propagator for the field Φ, which suppresses high-momentum modes.
The invariance of the corresponding partition function under changes of μ results in the crucial equation

$$\begin{aligned} \dot{S}_{\text{int.}} &\equiv -\mu\partial_\mu S_{\text{int.}} = \\ &= \int \frac{\mathrm{d}^4k}{(2\pi)^4}\tilde{C}^{AB}(k,\mu) \left(\frac{\delta S_{\text{int.}}}{\delta\Phi^A(-k)\delta\Phi^B(k)} + \frac{\delta S_{\text{int.}}}{\delta\phi^A(-k)}\frac{\delta S_{\text{int.}}}{\delta\Phi^B(k)} \right), \end{aligned} \tag{7}$$

with $\tilde{C}^{AB}(k,\mu)$ related to $C^{AB}(k,\mu)$ in a prescribed way.
This equation gives a system of coupled ODEs, one for each of the couplings in $S_{\text{int.}}$.

The specific equations and their solutions

To solve the ERG flow equations while preserving supersymmetry we exploit the **supersymmetric local potential approximation** (SLPA). We thus project the equations onto those interactions that can be described by a Kähler potential and a superpotential, ignoring all contributions due to higher superderivative terms.
The **results** that we get within the SLPA are trustable for small changes of the Wilson cut-off. We expect that the qualitative features of our findings do not change, when following the RG flow longer.
We consider the Kähler potential

$$K_{\text{prop.}} = c^{-1}\mathcal{X}\overline{\mathcal{X}} + c^{-1}\mathcal{T}\overline{\mathcal{T}} \tag{8}$$

and

$$K_{\text{int.}} = (\alpha-1)\mathcal{X}\overline{\mathcal{X}} + (\beta-1)\mathcal{T}\overline{\mathcal{T}} + \mu^{-2}\gamma\mathcal{X}\overline{\mathcal{X}}\mathcal{T}\overline{\mathcal{T}} + \frac{1}{4}\mu^{-2}\zeta\mathcal{X}^2\overline{\mathcal{X}}^2, \tag{9}$$

together with the superpotential

$$W = f\mathcal{X} + \frac{1}{2}\mathcal{T}\mathcal{X}^2. \tag{10}$$

The quantity $c(k/\mu)$ is a regulator function, which we can choose to be, for instance, $c(k/\mu) = 1 + \sum_{n=1} c_n\frac{k^{2n}}{\mu^{2n}}$.
Heavily relying on supersymmetry, we determine the ERG flow equations of the couplings by focusing on the terms that are quadratic in the auxiliary fields $F^{\mathcal{X}}$ and $F^{\mathcal{T}}$. We obtain

$$\dot{\gamma} = -2\gamma - 2c_1; \quad \dot{\zeta} = -2\zeta - 2c_1; \quad \dot{\alpha} = -2N(\gamma+\zeta) \quad \text{and} \quad \dot{\beta} = -2N\gamma, \tag{11}$$

which, once the boundary conditions at the UV matching point specified by the energy scale Λ

$$\gamma\Big|_{\mu=\Lambda} = 0; \quad \zeta\Big|_{\mu=\Lambda} = 0; \quad \alpha\Big|_{\mu=\Lambda} = 1 \quad \text{and} \quad \beta\Big|_{\mu=\Lambda} = 0 \tag{12}$$

are imposed, give

$$\begin{aligned} &\gamma = \zeta = -c_1(1-e^{-2t}) = -2c_1t + \mathcal{O}(t^2); \\ &\alpha = 1 - 2c_1N + 4c_1N\left(t + \frac{1}{2}e^{-2t}\right) = 1 + \mathcal{O}(t^2) \quad \text{and} \\ &\beta = -c_1N + 2c_1N\left(t + \frac{1}{2}e^{-2t}\right) = 2c_1Nt^2 + \mathcal{O}(t^3), \end{aligned} \tag{13}$$

where $t = \log\frac{\Lambda}{\mu}$, and $N < 0$ and $c_1 < 0$ for typical choices of the regulator.
The scalar field T acquires a positive kinetic term.

"Vacuum" structure (in)stability analysis

The central critical point corresponding to the original Volkov–Akulov model develops a **tachyonic instability**, signaling **goldstino condensation** (see Figure 1-left).

As an interesting comment, we observe that, when naively coupling the model to the (unwarped) KKLT construction, ignoring supergravity interactions, similar results are obtained (see Figure 1-right).

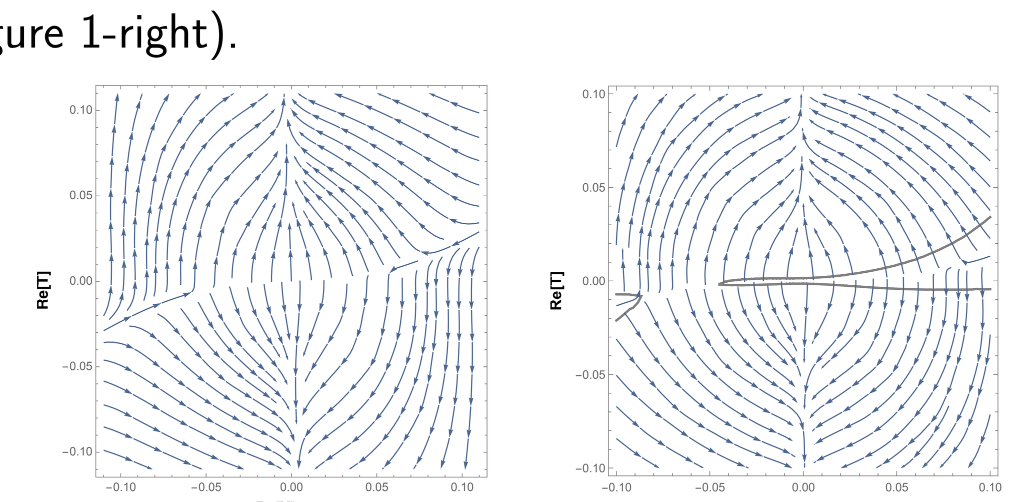

Figure 1. This figure presents two illustrative stream-plots of the (negative) gradient of the scalar potential in the (X,T) field space, for $t = 0.1$. On the right hand side the black contour shows the location where the gradient vanishes for the real component of the additional KKLT Kähler modulus.

Conclusions and Outlook

The pure Volkov–Akulov model studied through the RG flow technique has an instability towards goldstino condensation, which seems to persist even including higher order corrections.

Further investigations are in order! For instance:
What is the endpoint of this instability**?** We need to go beyond the SLPA.
What comes out from a well-posed supergravity analysis**?** We need a supergravity-consistent regulator.
As far as a stringy description of a $\overline{D3}/O3$ system is concerned, does the anti-brane survive or annhilate**?**

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