Spin field for N=1 particle in the worldline

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Based on: Boffo, E. and Sachs, I. *"Spin fields for the spinning particle"*, arXiv:2206.03243

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Introduction

The supersymmetric *worldline* of a relativistic massless particle plays an important role as a toy model for string theory. Furthermore its BRST quantization induces the Batalin-Vilkovisky completion of ordinary QFT in target space. Moreover, BRST operators for the RNS worldline are manifestly background independent.
Backgrounds like *Ramond-Ramond backgrounds* are notoriously hard to incorporate on the worldsheet and even more on the worldline, where "bosonization" of the Clifford algebra generators is not possible, and the typical branch cuts of the multi-valued OPE of spin fields are missing on the line.

Objectives

The aims are:

- **to recover the *Ramond-Ramond fields* and their equations in BRST-cohomology (not necessarily of the RNS worldline);**
- **to explore *different backgrounds* by deforming the BRST differential;**
- **to provide a *sigma model*,**

from the supersymmetric RNS worldline in 4 dimensions. Firstly, a definition for the spin field, analogue to the spin field of the superstring, is proposed.

Setting

The odd part of the graded Lie algebra of the $N = 1$ worldline in 4 dimensions, $[x^\mu, p_\nu] = \mathrm{i}\delta^\mu_\nu, \{\psi^\mu, \psi^\nu\} = -2\eta^{\mu\nu}$, can be recovered from an associative operator algebra, that leaves also the representation space defined. As generators $\vartheta^\alpha, \varepsilon^\alpha, \lambda_\beta$ can be taken, with the relations

$$[\lambda_\beta, \vartheta^\alpha] = \delta^\alpha_\beta = [\lambda_\beta, \varepsilon^\alpha], \quad [\lambda_\alpha, \lambda_\beta] = 0.$$

as well as the anti-chiral partners $\tilde{\vartheta}_{\dot\alpha}, \tilde{\varepsilon}_{\dot\alpha}, \tilde{\lambda}^{\dot\beta}$ with identical relations, and no commutators between the two sets. Jacobi identity does not hold.
The Clifford gamma matrices are defined from the Pauli matrices $\sigma^\mu = (\mathrm{id}, \sigma^i)$ and a Grassmann degree shifting operator $\uparrow$, satisfying $\lambda_\alpha \uparrow = \uparrow \lambda_\alpha$ and $\lambda^{\dot\alpha} \uparrow = \uparrow \lambda^{\dot\alpha}$:

$$\psi^\mu := \left(\vartheta^\alpha \sigma^\mu_{\alpha\dot\alpha} \tilde{\lambda}^{\dot\alpha} + \tilde{\vartheta}_{\dot\alpha} \tilde{\sigma}^{\mu\,\dot\alpha\alpha} \lambda_\alpha\right) \uparrow \tag{1}$$

The representation space is a complex vector space with states generated from the vacuum by the *spin fields* ϑ^α and $\tilde{\vartheta}_{\dot\alpha}$. Iteration of this action as well as that of the other creation operators or spin fields ε^α and $\tilde{\varepsilon}_{\dot\alpha}$, makes the representation space infinite dimensional. The subspace of "2-particles" states consists of:

$$\begin{aligned} &F_{\alpha\beta}(x)\,|\mathrm{e}^{\alpha\beta}\rangle\,,\ A_\alpha{}^{\dot\beta}(x)\,|e_\alpha{}^{\dot\beta}\rangle\,,\\ &\tilde{F}^{\dot\alpha\dot\beta}(x)\,|\mathrm{e}_{\dot\alpha\dot\beta}\rangle\,,\ \tilde{A}_{\dot\alpha}{}^{\beta}(x)\,|e^{\dot\alpha}{}_{\beta}\rangle\,, \end{aligned} \tag{2}$$

There is no definite symmetry between the indices.

Coh(Q)

The BRST-differential of the $N = 1$ massless particle is:

$$Q = cp^2 + \gamma\psi^\mu p_\mu + \gamma^2 b, \qquad \underbrace{\{c, b\} = 1}_{\text{worldline diffeos ghosts}}, \qquad \underbrace{[\gamma, \beta] = 1}_{\text{supersymmetry ghosts}}, \qquad \psi^\mu \text{ in (1)}$$

The submodule (2) must be extended with a representation for the algebra of ghosts in order to study coh(Q). The physical states are in the cohomology at ghost degree 0. There are:

- $F_{\alpha\beta} = F^{[0]}_{[\alpha\beta]} + F^{[2]}_{(\alpha\beta)}$ such that $\delta F^{[2]} = 0, \quad 2\mathrm{d}F^{[0]} - \star\mathrm{d}F^{[2]} = 0$.
- $A_\alpha{}^{\dot\beta}$ such that $\delta A = 0 = \mathrm{d}A$.

(And the same holds for the chiral counterparts.)

Result:

These are ***Ramond-Ramond fields equations*** for a **1-form and a 2-form field strength**, when $F^{[0]} \overset{!}{=} 0$.

Q-twistings

Any twisted BRST operator is *always nilpotent* on the representation space as a consequence of associativity.

Backgrounds

$\varphi(\vartheta^\alpha\lambda_\alpha \pm \tilde{\vartheta}_{\dot\alpha}\tilde{\lambda}^{\dot\alpha}) \uparrow$	Yukawa coupling (mass term).
$(\vartheta^\alpha \mathbf{A}_{\alpha\beta}\lambda^\beta + \tilde{\vartheta}_{\dot\alpha}\tilde{\mathbf{A}}^{\dot\alpha\dot\beta}\tilde{\lambda}_{\dot\beta}) \uparrow$	spin connection in warped compactification from 5d.

Coh(q)

On the same complex vector space, we can equivalently consider

$$\mathbf{q} = \tilde{\vartheta}_{\dot\alpha}\tilde{\sigma}^{\mu\,\dot\alpha\alpha}\lambda_\alpha \uparrow p_\mu, \quad \mathbf{q}^2 = 0 \tag{3}$$

since there are no target space ghost fields in the BRST of the $N = 1$ worldline. Now $\mathbf{q}$-exact states can be decoupled.

Result:

On (2), $F_{\alpha\beta}(x)$ and $A_\alpha{}^{\dot\beta}(x)$ as in Q-cohomology, but now those $\tilde{F}^{\dot\alpha\dot\beta}$ and $\tilde{A}^{\dot\alpha}{}_\beta$ which are not in the kernel of $p_{\alpha\dot\alpha}$ are $\mathbf{q}$-exact.

q-twistings

Deformations of $\mathbf{q}$ by backgrounds are now dynamical and yield the correct amplitudes for interactions, when we perturb the differential $\mathbf{q} + \delta\mathbf{q}$ while at the same time perturbing the representation space.

Backgrounds

$\delta\mathbf{q}_1 := \tilde{\vartheta}_{\dot\beta}\tilde{F}^{\dot\beta\beta}\lambda_\beta \uparrow$	the first order perturbed $F_{\alpha\beta}$ and $A_\alpha{}^{\dot\beta}$ get a $U(1)$ charge.
$\delta\mathbf{q}_2 := \vartheta^\beta F_{\beta\dot\beta}\tilde{\lambda}^{\dot\beta} \uparrow$	$\tilde{F}^{\dot\alpha\dot\beta}$ and $\tilde{A}^{\dot\alpha}{}_\beta$ are set to zero, as they are not $\mathbf{q} + \delta\mathbf{q}_2$ exact.
$\delta\mathbf{q}_3 := \tilde{\vartheta}_{\dot\beta}\tilde{F}^{\dot\beta\dot\gamma}\tilde{\lambda}_{\dot\gamma} \uparrow$	$A_\alpha{}^{\dot\beta}$ is deformed by Chern-Simons interaction.
$\delta\mathbf{q}_4 := \vartheta^\alpha F_{\alpha\beta}\lambda^\beta \uparrow$	$F_{\alpha\beta}$ and $A_\alpha{}^{\dot\beta}$ must be null, $\tilde{F}^{\dot\alpha\dot\beta}$ and $\tilde{A}^{\dot\alpha}{}_\beta$ are off-shell, and not $\mathbf{q} + \delta\mathbf{q}_4$ exact.

Sigma model

The $\mathbf{q}$-cohomology admits a sigma-model:

$$S = \int \mathrm{d}\tau\, p_\mu \dot{x}^\mu + \vartheta^\alpha \dot{\lambda}_\alpha + \tilde{\vartheta}_{\dot\alpha}\dot{\tilde{\lambda}}^{\dot\alpha} + \tilde{w}_{\dot\alpha}\tilde{p}^{\dot\alpha\alpha}\lambda_\alpha + w^\alpha p_{\alpha\dot\alpha}\tilde{\lambda}^{\dot\alpha}\,. \tag{4}$$

There are constraints

$$D_\alpha := p_{\alpha\dot\alpha}\tilde{\lambda}^{\dot\alpha}, \quad \tilde{D}^{\dot\alpha} := \tilde{p}^{\dot\alpha\alpha}\lambda_\alpha, \quad [D_\alpha, \tilde{D}^{\dot\alpha}] = 0 = [D_\alpha, D_\beta] = [\tilde{D}^{\dot\alpha}, \tilde{D}^{\dot\beta}],$$

generating a supersymmetry. It can be gauge fixed and taken into account by ghost pairs $\{\tau^\alpha, \omega_\beta\} = \delta^\alpha_\beta$, $\{\tilde{\tau}_{\dot\alpha}, \tilde{\omega}^{\dot\beta}\} = \delta^{\dot\beta}_{\dot\alpha}$ and the BRST operator $\tau^\alpha D_\alpha + \tilde{\tau}_{\dot\alpha}\tilde{D}^{\dot\alpha}$.

Result:

When $\tilde{\tau}_{\dot\alpha}\tilde{D}^{\dot\alpha}$ is represented on fields, choosing the polarization $\psi(x, \vartheta^\alpha, \tilde{\tau}_{\dot\alpha})$, we get $\delta A = 0 = \mathrm{d}A$ for the R-R vector. However the $\tilde{D}^{\dot\alpha}$**-deformation by the YM vector** $A^{\alpha\dot\alpha}\lambda_\alpha$ is **not nilpotent on the R-R vector ghost**.

- **RNS particle**: recovered by considering BRST action as a standard action, and adding Lagrange multiplier, so that $[\lambda, \vartheta + \tau \uparrow] = 1$.

Acknowledgement

GAČR CZECH SCIENCE FOUNDATION EXPRO 19-28628X