Topological BF description of 2D accelerated chiral edge modes.

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Motivation

Recently observed *accelerated* edge modes on particular Quantum Hall Systems cannot be described by the usual approach of bulk-boundary correspondence in flat spacetime as for *constant* edge velocities. The common way out is to add a suitable *ad-hoc* potential in the boundary action, which reproduces the local modes. A formal description of this phenomenon is still lacking. The question is: how can a space-time dependence be "naturally" inherited by the parameters of the boundary theory? In a curved spacetime the bulk model, being topological, does not depend on the metric, but what happens when a boundary is introduced? Can the presence of a metric lead to nontrivial consequences on the boundary physics? Can this be a formal way of explaining *local* edge modes?

Abstract

What : Topological abelian BF theory with radial boundary on a generic 3D manifold. **Aim :** if, where and how the boundary keeps memory of the details of the background metric. **Why :** possible formal explanation for accelerated edge modes. **Outcome :** The 2D action holographically induced on the boundary can be decoupled in two Luttinger actions describing two chiral bosons moving on the edge of the 3D bulk. These edge excitations are *accelerated*, as a direct consequence of the non-flat nature of the bulk spacetime and acquire a local dependence through the determinant of the induced metric. Three possibilities arise for the motion of the edge quasiparticles: same directions (but $\mathcal{H} < 0$), opposite directions and a single-moving mode. Time Reversal symmetry uniquely identifies modes with equal and opposite velocities, which is the case of Topological Insulators. The novelty, with respect to the flat bulk background, is that the modes have local velocities, which corresponds to Topological Insulators with *accelerated* edge modes.

1) The Model

BF bulk term with radial boundary

$$S_{BF} = \kappa \int d^3x\, \theta(R-r)\, \epsilon^{\mu\nu\rho}\partial_\mu A_\nu B_\rho \tag{1}$$

Gauge fixing:

$$S_{gf} = \int d^3x\, \sqrt{-g}\, \theta(R-r)\, [(bA_\mu + d\, B_\mu)\, n^\mu] \tag{2}$$

External source:

$$S_J = \int d^3x\, \sqrt{-g}\, \theta(R-r)\, \left(J^\mu A_\mu + \hat{J}^\mu B_\mu\right) \tag{3}$$

Most general boundary term ($\leftrightarrow$ boundary conditions à la Symanzik):

$$S_{bd} = \int d^3x\, \sqrt{-\gamma}\, \delta(r-R) \left(\frac{\alpha^{ij}}{2} A_i A_j + \frac{\beta^{ij}}{2} B_i B_j + \zeta^{ij} A_i B_j\right) \tag{4}$$

Total action:

$$S = S_{BF} + S_{gf} + S_J + S_{bd}\ . \tag{5}$$

2) Boundary Conditions (BC)

BC from the equations of motion (EOM): $\lim_{\epsilon\to R} \int_\epsilon^R dr EOM \to$ BC

$$\alpha^{ij} A_j + \left(-\kappa \frac{\epsilon^{i1j}}{\sqrt{-\gamma}} + \zeta^{ij}\right) B_j\Big|_{r=R} = 0 \tag{6}$$

$$\zeta^{ji} A_j + \beta^{ij} B_j\big|_{r=R} = 0\ . \tag{7}$$

Solutions:

$$B_\theta(X) = -l_1(\gamma) B_t(X) \tag{8}$$

$$A_\theta(X) = -l_2(\gamma) B_t(X) \tag{9}$$

$$A_t(X) = -l_3(\gamma) B_t(X)\ , \tag{10}$$

with $l_i = l_i(\alpha, \beta, \zeta, \hat{\kappa}) = l_i(\gamma)$, γ induced metric.

3) Ward Identities

EOM $\Rightarrow$ broken Ward identities

$$\int_0^R dr\, \sqrt{-g}\nabla_\lambda J^\lambda = -\kappa\epsilon^{i1j}\partial_i B_j\big|_{r=R} \tag{11}$$

$$\int_0^R dr\, \sqrt{-g}\nabla_\lambda \hat{J}^\lambda = -\kappa\epsilon^{i1j}\partial_i A_j\big|_{r=R} \tag{12}$$

On-shell ($J = 0$) $\Rightarrow$ conserved currents $\Rightarrow$ Boundary dof:

$$\epsilon^{i1j}\partial_i B_j\big|_{r=R} = 0 \quad \Rightarrow \quad B_i(X) = \partial_i \psi(X) \tag{13}$$

$$\epsilon^{i1j}\partial_i A_j\big|_{r=R} = 0 \quad \Rightarrow \quad A_i(X) = \partial_i \varphi(X)\ . \tag{14}$$

4) Kaç-Moody Algebra

Broken Ward identities $\Rightarrow$ Kaç-Moody algebra:

$$\left[A_\theta(X), B_\theta(X')\right] = \frac{i}{\kappa}\partial_\theta \delta(\theta - \theta')\ , \tag{15}$$

all other commutators being zero.

Central charge:

$$c = \frac{1}{\kappa} \quad \Rightarrow \quad \kappa > 0\ . \tag{16}$$

No dependence on the bulk metric.

5) Boundary Theory

Most general 2D action :

$$S_{2D}[\varphi, \psi] = \int d^2X\, \sqrt{-\gamma}\, \left(a^{ij}\partial_i\varphi\partial_j\varphi + b^{ij}\partial_i\psi\partial_j\psi + c^{ij}\partial_i\varphi\partial_j\psi + d^i\partial_i\varphi + f^i\partial_i\psi + h\right)\ , \tag{17}$$

with a^{ij}, b^{ij}, c^{ij}, d^i, f^i, h depending on γ_{ij}, φ, ψ. Constraints

1. Shift symmetry :

$$\delta_s\varphi = \eta \quad ; \quad \delta'_s\psi = \eta' \qquad \Rightarrow \qquad \delta_s S_{2D} = \delta'_s S_{2D} = 0 \tag{18}$$

2. KM algebra $\leftrightarrow$ canonical commutators $[q(x), p(x')] = i\delta(x - x')$:

$$\left[\varphi(X), \partial_{\theta'}\psi(X')\right] = \frac{i}{\kappa}\delta(\theta - \theta') \quad \Rightarrow \quad q_1 \equiv \varphi \quad ; \quad p_1 \equiv \kappa\partial_\theta\psi \tag{19}$$

3. Holographic Contact (bulk-boundary correspondence) :

Bulk(BC)	$\leftrightarrow$	Boundary(EOM)
$\{\hat{\kappa}, \alpha, \beta, \zeta\}$	$\leftrightarrow$	$\{a, b, c, d, f, h\}$

4. Field redefinition and decoupling:

$$\Phi^\pm \equiv \varphi \pm \psi \quad ; \quad S_{2D}[\Phi^+, \Phi^-] = S_{2D}[\Phi^+] + S_{2D}[\Phi^-] \tag{20}$$

6) Boundary Physics

$$\mathbf{1.} + \mathbf{2.} + \mathbf{3.} + \mathbf{4.} \quad \Rightarrow \quad S_{2D}[\Phi^\pm] = \frac{\kappa}{2}\int d^2X\ \partial_\theta\Phi^\pm(\pm\partial_t\Phi^\pm + v_\pm\partial_\theta\Phi^\pm)\ , \tag{21}$$

$\Rightarrow$ Luttinger model for two edge chiral bosons with

$$v_\pm(\gamma) = \frac{1 \mp l_3}{l_2 \pm l_1} \qquad \gamma \equiv \det\gamma_{ij}\ , \tag{22}$$

If

$$n_\pm \equiv \frac{1}{2\pi}\partial_\theta\Phi_\pm \tag{23}$$

2D EOM $\equiv$ continuity equations for the density operator $n_\pm$

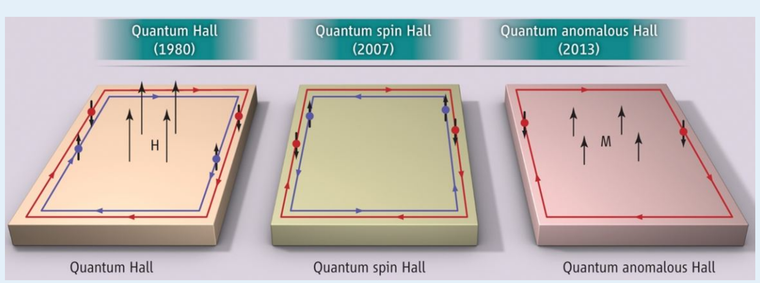
$$\partial_t n_+ + \partial_\theta(v_+ n_+) = 0 \qquad \text{and} \qquad \partial_t n_- - \partial_\theta(v_- n_-) = 0\ . \tag{24}$$

$$v_\pm = v_\pm(\gamma(t, \theta)) \quad \Rightarrow \quad \text{non-constant velocities !} \tag{25}$$

Summary of Results

- Memory of the bulk metric in the chiral velocities of the edge states : $v = v(\gamma) \ \Rightarrow\ v = v(t)$;
- Theoretical framework for the recently observed accelerated chiral bosons, which cannot be explained by a Topological Field Theory with boundary on a flat bulk background ;
- Three possible physical interpretations: chiral Luttinger liquids ($v_+ v_- > 0$), Topological Insulators ($v_+ = v_-$), Quantum Anomalous Hall ($v_+ = 0$ or $v_- = 0$).

7) Pheno

Tuning the l_i parameters

- $\boldsymbol{v_+ v_- > 0}$: opposite velocities
  chiral Luttinger liquids
- $\boldsymbol{v_+ = v_-}$: equal opposite velocities
  Topological Insulators
  (**Time Reversal**)
- $\boldsymbol{v_+ v_- = 0}$: L or R mover not moving
  Quantum Anomalous Hall Insulators
- $\boldsymbol{v_+ v_- < 0}$: LR movers in the same direction
  violation of $\mathcal{H} > 0$.