Near-BPS Skyrmions and nuclear matter

Marco Barsanti⋆, Stefano Bolognesi⋆, Sven Bjarke Gudnason†

⋆ University of Pisa

† Henan University

UNIVERSITÀ DI PISA

1. What is a topological soliton?

- A *soliton*, or solitary-wave, is a stable and well-localized collective excitation of a medium. Mathematically, it represents the solution of a non-linear system of equations. A *topological soliton* is a specific kind of soliton, characterized by a conserved *topological charge* Q, arising from the geometrical properties of the system.

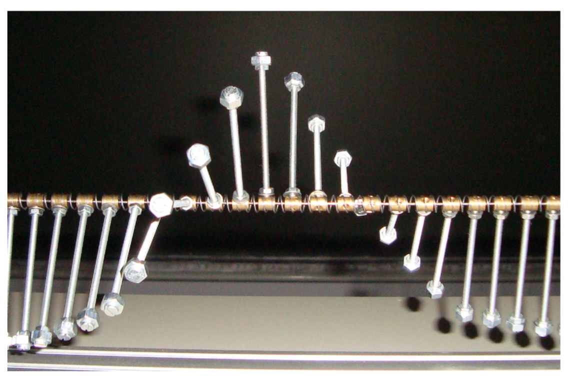


Figure: Example of topological soliton trapped in a system of pendulums coupled by torsional springs. The topological charge represents in this case the number of twists made by the pendulums' series.

Solitons can be found in a large variety of contexts, such as optics, fluidodynamics, superconductors, superfluids and Quantum Field Theory.

2. Baryons as topological solitons: the Skyrmions

- An important example of topological solitons in QFT is given by the *Skyrmions*. These solitons, defined in $3+1$ Minkowski space-time, are classical solutions of the low-energy QCD Lagrangian for the pion field, the so-called *Skyrme model* [1]:

$$\mathcal{L}_{Skyrme} = c_2 \,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + c_4 \,\mathrm{Tr}\left([U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2\right),$$

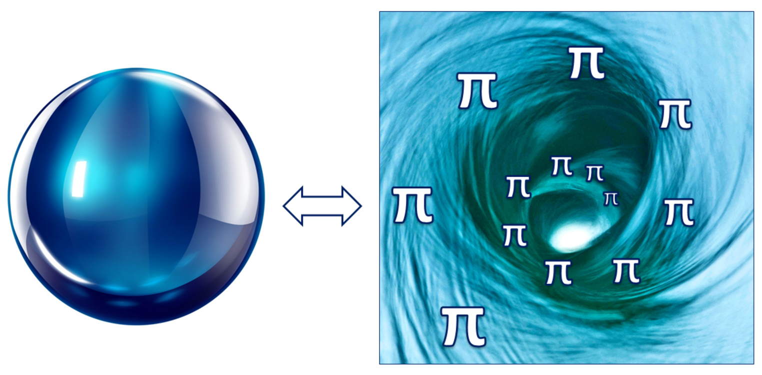
where c_2 and c_4 are constants and $U \in SU(2)$.

- In the static case, the field $U(x)$ is a map between two iper-spheres:

$$U(x) : \mathbb{R}^3 \sim S^3 \Rightarrow SU(2) \sim S^3 .$$

The topological charge Q counts here the number of times the second iper-sphere is wrapped by the first one.

- Remarkably, in [2] the conserved topological charge Q was identified with the conserved *baryon number* B, and thus the Baryons can be described by Skyrmions in a pion field-theory. In this context, therefore, a Baryon appears as a "vortex" in the pion field $U(x)$:

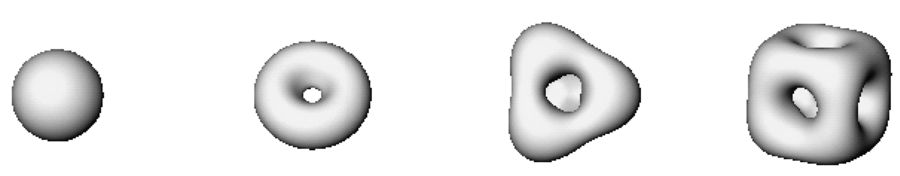


Baryon Pion-field vortex

Obviously, the skyrmionic solution needs a semi-classical quantization procedure to really describe a quantum state, such as the proton or the neutron. This procedure is meaningful only in the Large-N limit of QCD, in which the Baryons become semi-classical objects.

3. The binding-energy problem of the Skyrme model

- The solitonic description of the Baryons represents an important example of analytic non-perturbative approach to QCD. The Skyrme model is able to predict several physical quantities, such as the nucleon and Δ-baryon masses, the proton and neutron radii and magnetic moments (adding QED), within an accuracy of the 30% with respect to the experimental values.
- In the case of topological charge $Q > 1$, and thus baryon number $B = Q > 1$, the Skyrmions are conjectured to describe nuclei.



1: $O(3)$ 2: $O(2)$ 3: T_d 4: O_h

Figure: Baryon density for Skyrmions with $B = 1, 2, 3, 4$ and the corresponding symmetry of the solution.

- An exhaustive physical description of nuclei represents an hard challenge for the Skyrme model. In particular, the main issue of the theory in this context is the highly incorrect prediction of the nuclear binding-energy, at least one order of magnitude larger then the experimental value.

  To fix this problem, we proposed a new type of Skyrme-like model, called *near-BPS model.*

4. The near-BPS Skyrme model

- In order to introduce the near-BPS theory, it is necessary to start form the pure BPS model. The so-called *BPS Skyrme model* has Lagrangian [3]

$$\mathcal{L}_{BPS} = c_6 \left[\mathrm{Tr}(\epsilon^{\mu\nu\rho\sigma} U^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U)\right]^2 - c_0 V(U, U^\dagger),$$

where c_6 and c_0 are constants and again $U \in SU(2)$.

The energy of the BPS Skyrmion is proportional to the topological charge (and thus to the baryon number). Physically, a nucleus built with this model possesses zero binding-energy.

- A small deformation of the pure BPS model should reproduce a nuclear system with small binding-energy. The so-called near-BPS model consists of

$$\mathcal{L}_{near-BPS} = \mathcal{L}_{BPS} + \epsilon \mathcal{L}_{pert} \qquad \epsilon \ll 1,$$

where ϵ is a small dimensionless parameter and $\mathcal{L}_{pert}$ is the perturbation.

- In $3+1$ Minkowski space-time, the full-numerical resolution of the system in the limit of very small ϵ becomes highly non-trivial. To avoid that obstacle, we developed a perturbative method that we firstly tested on a 2+1-dimensional toy model (see next) [4][5].

5. The near-BPS baby Skyrmions and perturbative method

- The near-BPS model can be more easily solved in the 2+1-dimensional version of the Skyrme theory, the so-called *baby Skyrme model*. The baby-Skyrme field consists of a 3-vector $\vec{\phi}$ with constraint $\vec{\phi} \cdot \vec{\phi} = 1$.
- In this case, the field $\vec{\phi}(x)$ is a map between two spheres

$$\vec{\phi}(x) : \mathbb{R}^2 \sim S^2 \Rightarrow S^2 .$$

The topological charge Q counts the number of windings between the spheres.

- The existence of a BPS sector even in the baby Skyrme theory, allows us to build a near-BPS model in the same way of the previous block [4][5]. To solve the system, we implemented a perturbative expansion of the field around a BPS background as

$$\vec{\phi}(x) = \vec{\phi}_{BPS}(x) + \delta\vec{\phi}(x, \epsilon)$$

and calculate the near-BPS energy at different orders in ϵ.

- The accuracy of the perturbative method was checked by comparing the results with the full-numerical analysis.

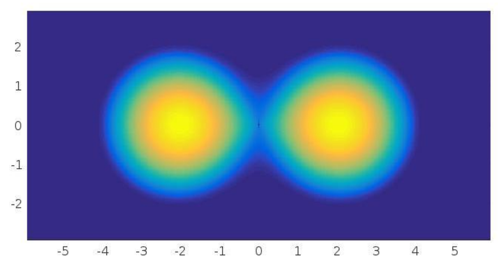


Figure: Example of baby-baryon density for the (toy-model) deuteron for $\epsilon = 0.02$.

- The analysis performed in 2+1 dimensions for the near-BPS model confirmed the presence of a small binding-energy tunable with ϵ and the validity of the perturbative method proposed.

6. The near-BPS Skyrmions and perturbative method

- Once tested the perturbative method on the baby Skyrme model, we applied it to the 3D near-BPS Skyrme theory [6]. The model was chosen to be

$$\mathcal{L}_{near-BPS} = \mathcal{L}_{BPS} + \epsilon \mathcal{L}_{Skyrme} \qquad \epsilon \ll 1,$$

with the original Skyrme model as perturbation.

- To implement the perturbative scheme, we expanded the field $U(x) \in SU(2)$ as

$$U(x) = U_{BPS}(x) + \delta U(x, \epsilon),$$

and we approximated the energy order by order in ϵ.

- For the specific model considered, the binding-energy turned out to be of order ϵ^2. Thus, after the proper calibration, that quantity finally results to be of order ~ 1 Mev (at least for the deuteron).
- A quantization of such classical solitonic solutions is needed in order to finally compare our results with nuclear physics.
- The paper with the 3D near-BPS Skyrme analysis is still on working [6].

References

[1] T.H.R. Skyrme, "A nonlinear Field Theory", Proc. Roy. Soc. Lond. A 260 (1961), 127-138.
[2] E. Witten, "Global aspect of current algebra", Nucl.Phys.B 223 (1983) 422-432.
[3] C. Adam *et al.*, "A Skyrme-type proposal for baryonic matter", Phys.Lett.B 691 (2010) 105-110
[4] S.B. Gudnason, **M. Barsanti**, S. Bolognesi, "Near-BPS baby Skyrmions", JHEP 11 (2020) 062.
[5] S.B. Gudnason, **M. Barsanti**, S. Bolognesi, "Near-BPS baby Skyrmions with Gaussian tails", JHEP 05 (2021) 134
[6] S.B. Gudnason, **M. Barsanti**, S. Bolognesi, "Near-BPS Skyrmions", on working