The complex structure of nucleon form factors exploring the Riemann surfaces of their ratio

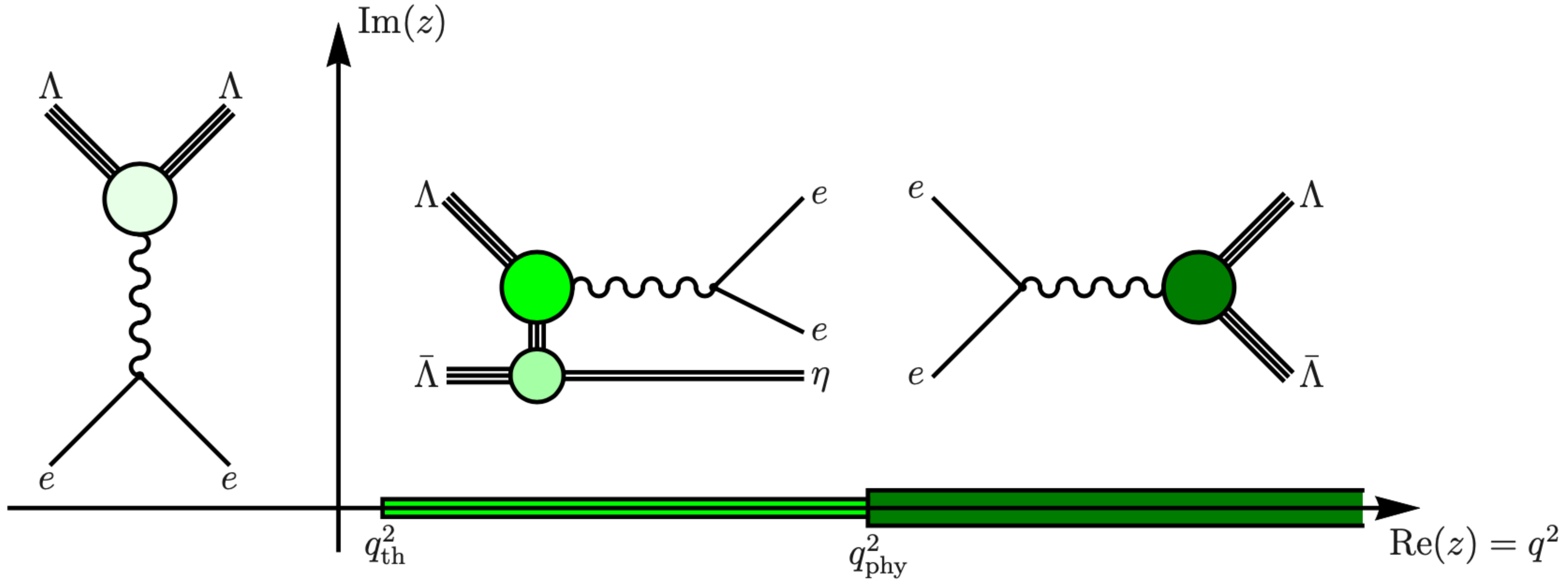
ALESSIO MANGONI

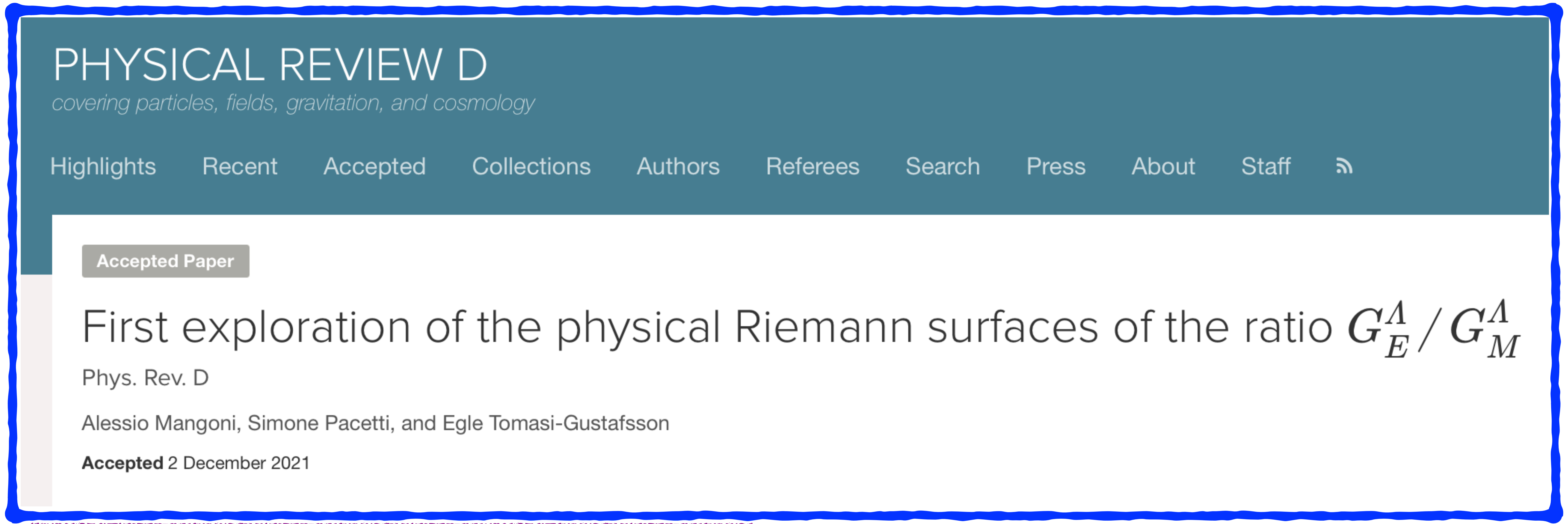
SIMONE PACETTI EGLE TOMASI-GUSTAFSSON

Istituto Nazionale di Fisica Nucleare - Sezione di Perugia

Agenda

- Electromagnetic form factors
- Λ form factors
- The Λ baryons
- Available data
- Theoretical aspects
- The G_E^Λ/G_M^Λ ratio
- The model
- Results
- Conclusions




PHYSICAL REVIEW D

covering particles, fields, gravitation, and cosmology

Highlights Recent Accepted Collections Authors Referees Search Press About Staff

Accepted Paper

First exploration of the physical Riemann surfaces of the ratio G_E^Λ/G_M^Λ

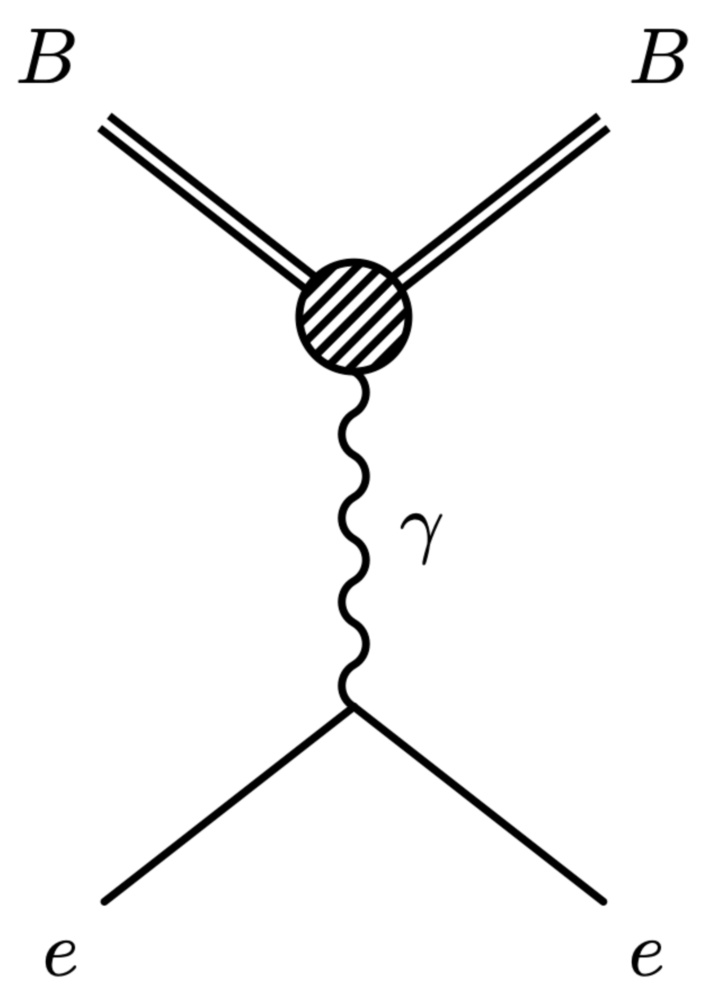
Phys. Rev. D

Alessio Mangoni, Simone Pacetti, and Egle Tomasi-Gustafsson

Accepted 2 December 2021

arXiv: 2109.03759 [hep-ph] - accepted by PRD

Electromagnetic form factors



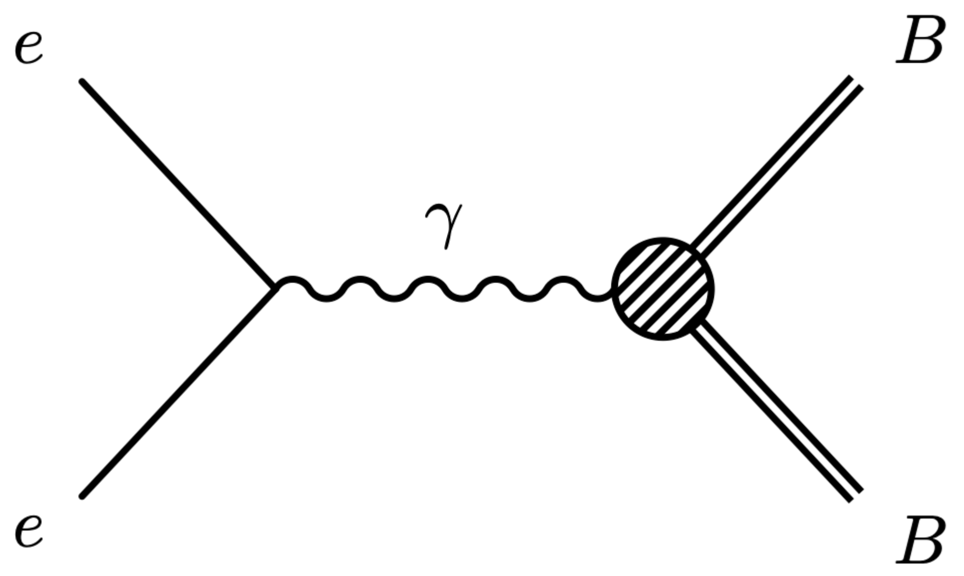
In quantum field theory the form factors (FFs) are crucial quantities used to describe the mechanisms that underlie the baryon dynamics

FFs

encode all the information concerning baryon dynamics

complex energy-dependent coupling constants

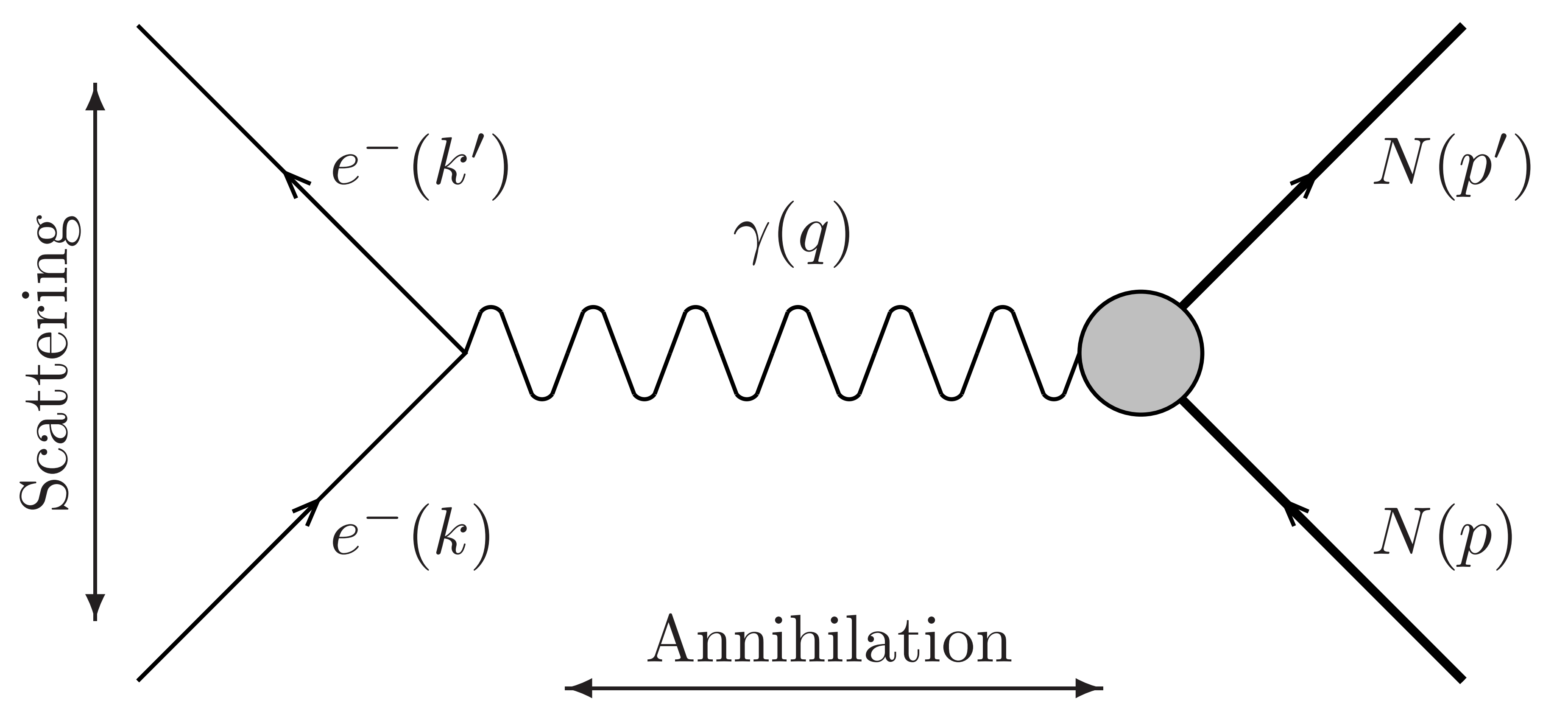
parametrize the baryon four-current



Electromagnetic form factors

The nucleon electromagnetic (EM) FFs are Lorentz scalar functions of q^2
(squared four-momentum transfer of the photon)

Space-like FFs are related to the elastic scattering process
$e^- N \to e^- N$



EM FFs give informations about the internal structure of nucleons

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_N} F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Time-like FFs are related to the annihilation processes
$e^+ e^- \leftrightarrow N\overline{N}$

$$\Gamma^\mu = F_1(q^2)\gamma^\mu - \frac{\sigma^{\mu\nu} q_\nu}{2M_N} F_2(q^2)$$

F_1 and F_2 are the Dirac and Pauli FFs

Istituto Nazionale di Fisica Nucleare

Λ form factors

G_E^Λ G_M^Λ

Λ FFs

Lorentz scalar functions of the four-momentum transferred squared q^2

Analytic functions with a branch cut along the positive real axis, for $q^2 \geq q^2_{\rm th}$ with $q^2_{\rm th} = (2M_\pi + M_{\pi^0})^2$

In the space-like region ($q^2 \leq 0$):
$G_E^\Lambda, G_M^\Lambda \in \mathbb{R}$
data can be extracted studying the scattering process $e^-\Lambda \to e^-\Lambda$

In the time-like region where $q^2 \geq q^2_{\rm phys} = (2M_\Lambda)^2$:
$G_E^\Lambda, G_M^\Lambda \in \mathbb{C}$
data can be extracted studying the annihilation processes $e^+e^- \leftrightarrow \Lambda\overline{\Lambda}$

FFs are real for real values of $q^2 \leq q^2_{\rm th}$
(thanks to the Schwarz reflection principle)

The Λ baryons

Scattering and $\Lambda\overline{\Lambda}$ annihilation experiments are hindered due to the impossibility to obtain stable beams or targets of Λ baryons

The weak decay $\Lambda \to p\pi^-$ can be used to obtain information about the polarization of the Λ baryon

The component of the polarization vector orthogonal to the scattering plane xz of the spin-1/2 baryon B in a generic annihilation process $e^+e^- \to B\overline{B}$ can be written as

$$\mathcal{P}_y = -\frac{\sqrt{\frac{q^2}{4M_B^2}}\frac{|G_E^\Lambda|}{|G_M^\Lambda|}\sin(2\theta)\sin\left(\arg\left(\frac{G_E^\Lambda}{G_M^\Lambda}\right)\right)}{\frac{q^2}{4M_B^2}\left(1+\cos^2(\theta)\right)+\frac{|G_E^\Lambda|^2}{|G_M^\Lambda|^2}\sin^2(\theta)}$$

the knowledge of the sinus does not provide information on the determination of the relative phase $\arg\left(\frac{G_E^\Lambda}{G_M^\Lambda}\right)$

q is the momentum transferred, M_B is the baryon mass and θ is the scattering angle in the e^+e^- CM reference frame

INFN Istituto Nazionale di Fisica Nucleare

STRONG 2020

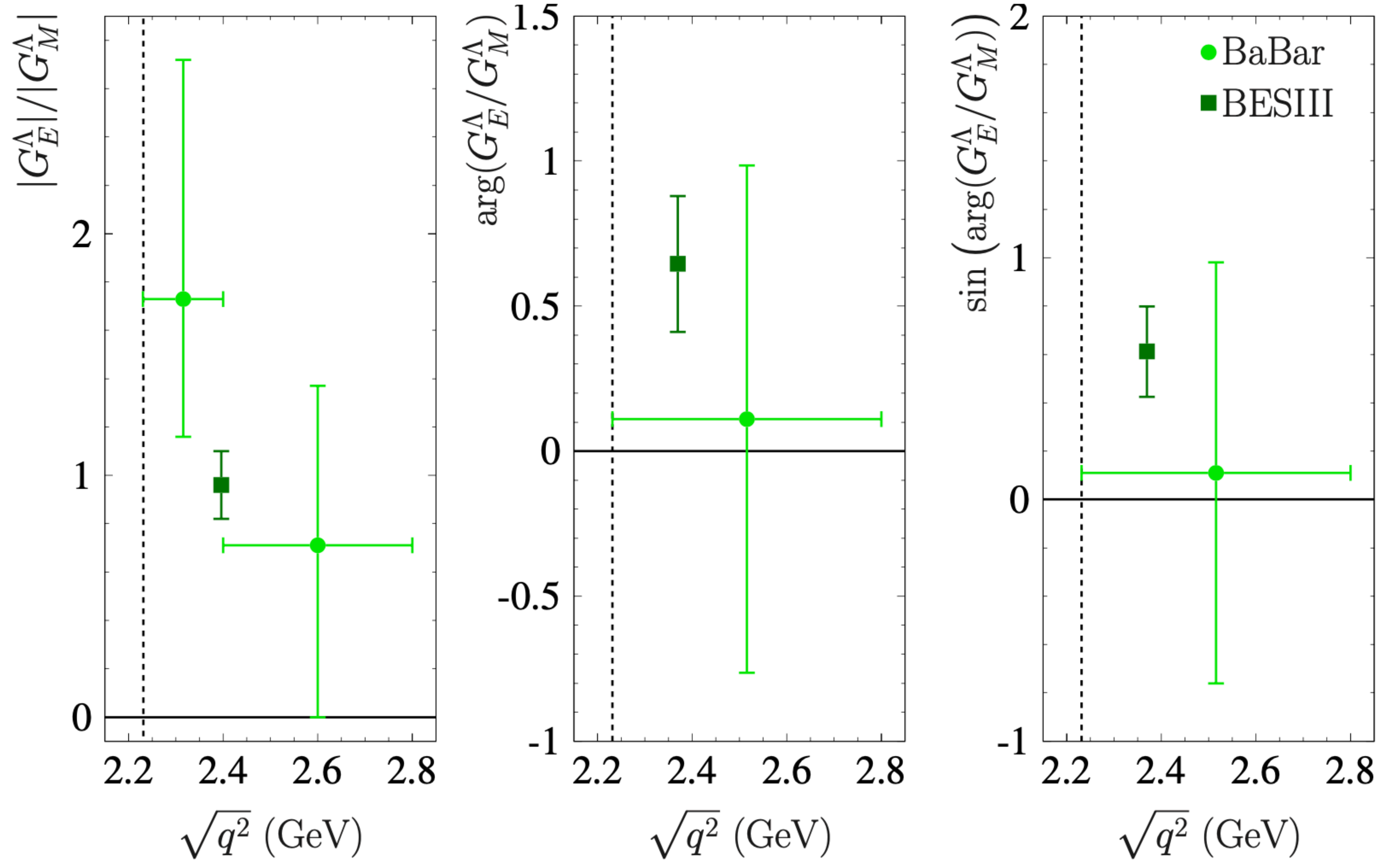
Available data

Total available data at present:
5 data points

There are only two sets of data for the Λ baryon from BESIII (2019) and BaBar (2006) experiments

- 1 datum on modulus of G_E^Λ/G_M^Λ
- 1 datum on phase of G_E^Λ/G_M^Λ

- 2 data on modulus of G_E^Λ/G_M^Λ
- 1 datum on phase of G_E^Λ/G_M^Λ



Data for the ratio:

$$\{q_j^2, |R_j|, \delta|R_j|\}_{j=1}^M$$

$$M = 3$$

Data for the phase:

$$\{q_k^2, \sin(\phi_k), \delta\sin(\phi_k)\}_{k=1}^P$$

$$P = 2$$

Theoretical aspects

$R(z)$: analytic multivalued function, defined in the domain D and with real brunch cut (x_0, ∞)

Dispersion relation (DR) for imaginary part

$$R(z) \in \mathbb{R}, \forall z \in D \cap \mathbb{R}$$

$$R(z) = o(1/\ln(|z|)) \qquad z \to \infty$$

$$R(z) \propto_{z \to x_0} (z - x_0)^{\alpha}, \quad \mathrm{Re}(\alpha) > -1$$

$$f(z) = \frac{1}{\pi} \int_{x_0}^{\infty} \frac{\mathrm{Im}(f(x))}{x - z} dx$$

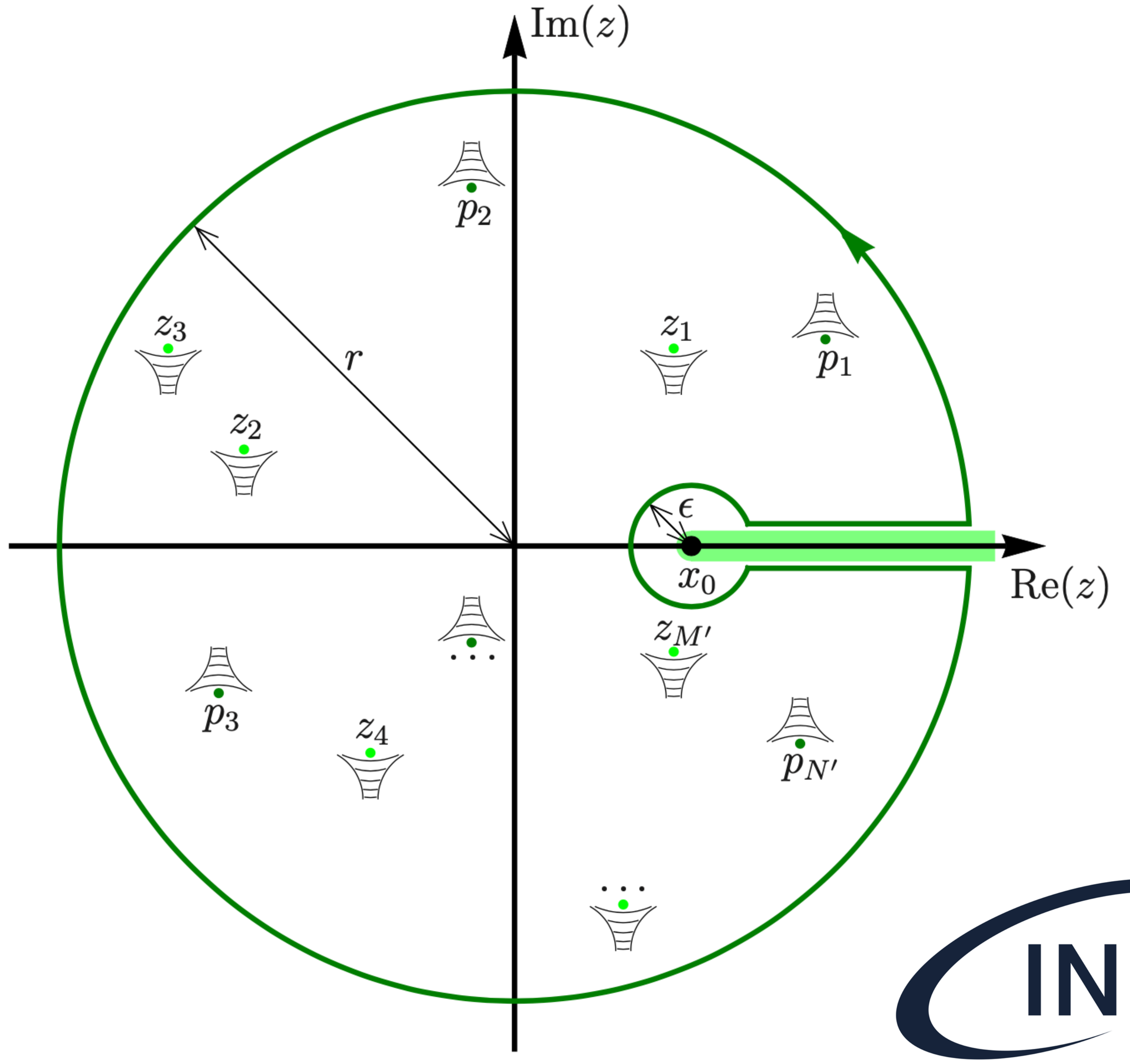
One-subtracted DR at x_1, when $R(z) = \mathcal{O}(1) \qquad z \to \infty$

$$f(z) = f(x_1) + \frac{z - x_1}{\pi} \int_{x_0}^{\infty} \frac{\mathrm{Im}(f(x))}{(x - x_1)(x - z)} dx$$

$$x_0 > x_1 \in \mathbb{R}$$

Levinson's theorem

$$\arg(R(\infty)) - \arg(R(x_0)) = \pi(M - N)$$



M: # of zeros
N: # of poles

The $G_E^\Lambda / G_M^\Lambda$ ratio

G_E^Λ and G_M^Λ form factors → Analyticity domain $D = \{z \in \mathbb{C} : z \notin (q_{\text{th}}^2, \infty)\}$

$R = \dfrac{G_E^\Lambda}{G_M^\Lambda}$ ratio

Multivalued meromorphic function with:

- Brunch cut $(q_{\text{th}}^2, \infty)$
- A set of isolated poles
- Schwarz reflection principle: $R^*(q^2) = R(q^{2*})$
- Same asymptotic behavior in space-like and time-like regions

$R(q^2) = \mathcal{O}(1) \quad |q^2| \to \infty$

Under the hypothesis that G_M^Λ has no zeros

- **Domain** $D = \{z \in \mathbb{C} : z \notin (q_{\text{th}}^2, \infty)\}$
- **Same zeros as G_E^Λ**
- **R has at least 1 zero in the origin, being $G_E^\Lambda(0) = Q_\Lambda = 0$**
- **R fulfills the requirement of the dispersion relation**

The model

Parametrization for the imaginary part of the ratio

$$R(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$$ in terms of Chebyshev polynomials T_j

$$\mathrm{Im}\left[R(q^2)\right] \equiv Y\left(q^2; \vec{C}, q^2_{\rm asy}\right)$$

$$Y\left(q^2; \vec{C}, q^2_{\rm asy}\right) = \begin{cases} \sum_{j=0}^{N} C_j T_j\left[x(q^2)\right] & q^2_{\rm th} < q^2 < q^2_{\rm asy} \\ 0 & q^2 \geq q^2_{\rm asy} \end{cases}$$

$$x(q^2) = 2\frac{q^2 - q^2_{\rm th}}{q^2_{\rm asy} - q^2_{\rm th}} - 1$$

$$\mathrm{Re}[R(q^2)] = \frac{q^2}{\pi}\mathrm{Pr}\int_{q^2_{\rm th}}^{q^2_{\rm asy}} \frac{\mathrm{Im}[R(s)]}{s(s-q^2)}ds$$

$$\begin{cases} R(q^2_{\rm th}) \in \mathbb{R} & \Rightarrow Y\left(q^2_{\rm th}; \vec{C}, q^2_{\rm asy}\right) = 0 \\ R(q^2_{\rm phy}) \in \mathbb{R} & \Rightarrow Y\left(q^2_{\rm phy}; \vec{C}, q^2_{\rm asy}\right) = 0 \\ R(q^2 \geq q^2_{\rm asy}) \in \mathbb{R} & \Rightarrow Y\left(q^2 \geq q^2_{\rm asy}; \vec{C}, q^2_{\rm asy}\right) = 0 \end{cases}$$

Theoretical constraints

$N+2$ **free parameters:**
$\vec{C} = \{C_0, C_1, \ldots, C_N\}$ and the asymptotic threshold $q^2_{\rm asy} \in (q^2_{\rm phy}, \infty)$

$N-1$ **independent parameters:** $\{C_3, C_4, \ldots, C_N, q^2_{\rm asy}\}$
(degrees of freedom for the parametrization)

Istituto Nazionale di Fisica Nucleare

The model

$$R(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$$

$$X(q^2) = \mathrm{Re}[R(q^2)] = \frac{q^2}{\pi}\mathrm{Pr}\int_{q_{th}^2}^{q_{asy}^2} \frac{\mathrm{Im}[R(s)]}{s(s-q^2)} ds$$

(Pr = Principal Value)

Constraints for $\mathrm{Re}[R(q^2)]$

$$\chi^2_{\mathrm{phy}} = \left(1 - X(q^2_{\mathrm{phy}})\right)^2$$

$$\chi^2_{\mathrm{asy}} = \left(1 - X(q^2_{\mathrm{asy}})^2\right)^2$$

with weights $\tau_{\mathrm{phy}}, \tau_{\mathrm{asy}}$

χ^2 **definition**

$$\chi^2(\vec{C}, q^2_{\mathrm{asy}}) = \chi^2_{|R|} + \chi^2_\phi + \tau_{\mathrm{phy}}\chi^2_{\mathrm{phy}} + \tau_{\mathrm{asy}}\chi^2_{\mathrm{asy}} + \tau_{\mathrm{curv}}\chi^2_{\mathrm{curv}}$$

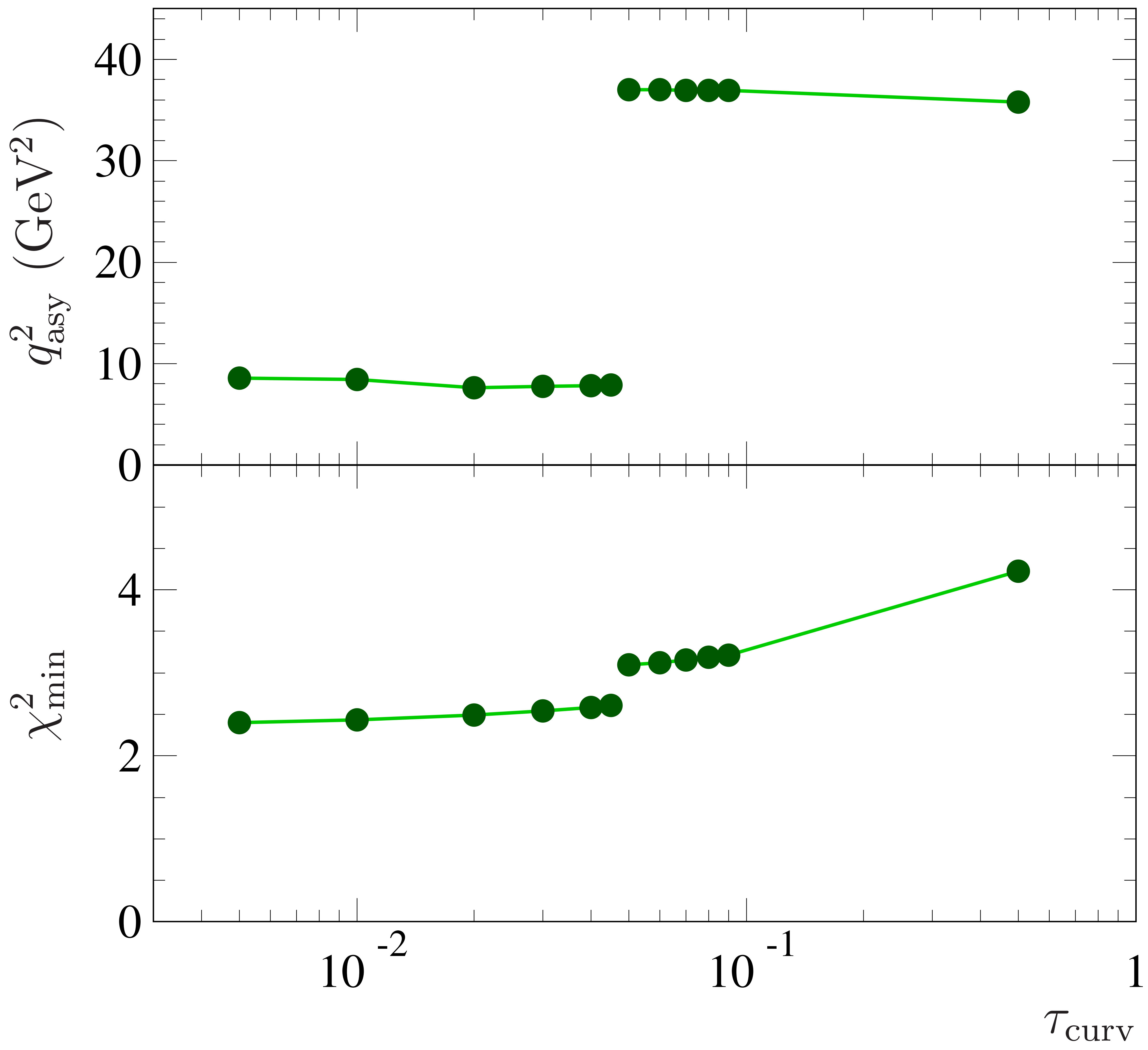
$$\chi^2_{|R|} = \sum_{j=1}^{M}\left(\frac{\sqrt{X(q_j^2)^2 + Y(q_j^2)^2} - |R_j|}{\delta|R_j|}\right)^2 \qquad \chi^2_\phi = \sum_{k=1}^{P}\left(\frac{\sin\left(\arctan\left(Y(q_k^2)/X(q_k^2)\right)\right) - \sin(\phi_k)}{\delta\sin(\phi_k)}\right)^2$$

$\tau_{\mathrm{curv}}\chi^2_{\mathrm{curv}}$ is a contribution to stabilize the solution, with $\chi^2_{\mathrm{curv}} = \int_{q_{th}^2}^{q_{asy}^2}\left|\frac{d^2Y(s)}{ds^2}\right|^2 ds$

The model

We have to choose N and $\tau_{\rm curv}$ balancing the increase of the total curvature as N increases and the suppression of the oscillations as $\tau_{\rm curv}$ increases

Best values of $q^2_{\rm asy}$ and the corresponding χ^2 minima for $N = 5$ as a function of $\tau_{\rm curv}$



Final values chosen:
$N = 5$ and $\tau_{\rm curv} = 0.05$

Results

The ratio $R(q^2) = \dfrac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$ **is real** for $q^2 = q^2_{\rm th}$ and $q^2 = q^2_{\rm asy}$

$$N_{\rm th} = \frac{1}{\pi}\arg[R(q^2_{\rm th})]$$

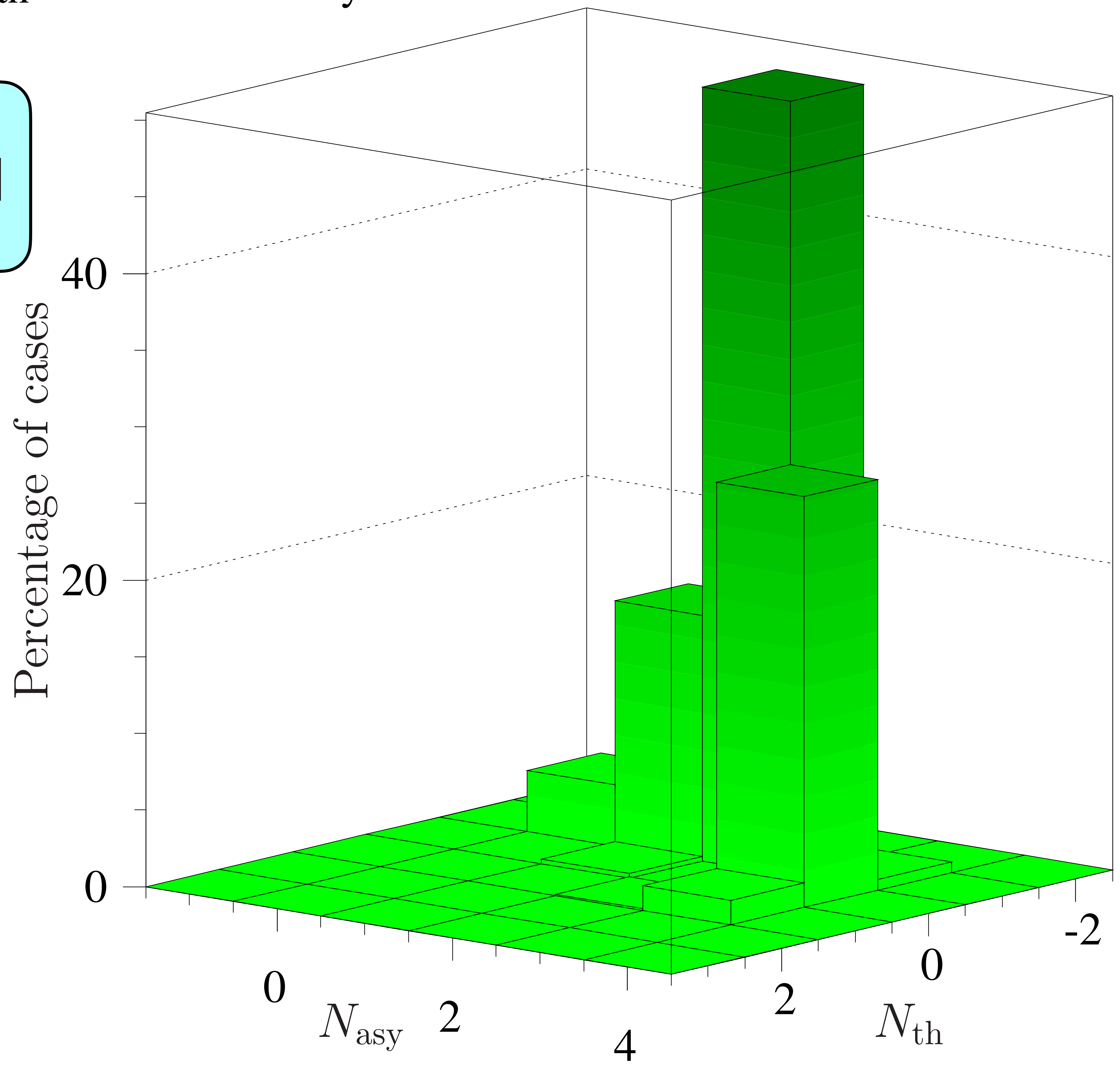
$N_{\rm th} \in \mathbb{N}$ $\quad$ $N_{\rm asy} \in \mathbb{N}$

$$N_{\rm asy} = \frac{1}{\pi}\arg[R(q^2_{\rm asy})]$$

Using the available data we obtain the results shown in table, where the probability is given by a Monte Carlo procedure

$N_{\rm th}$	$N_{\rm asy}$	%	Visual percentage
-1	0	4.0	
-1	1	16.0	
-1	2	50.5	
-1	3	0.7	
0	1	0.3	
0	3	26.8	
1	2	0.1	
1	3	1.6	

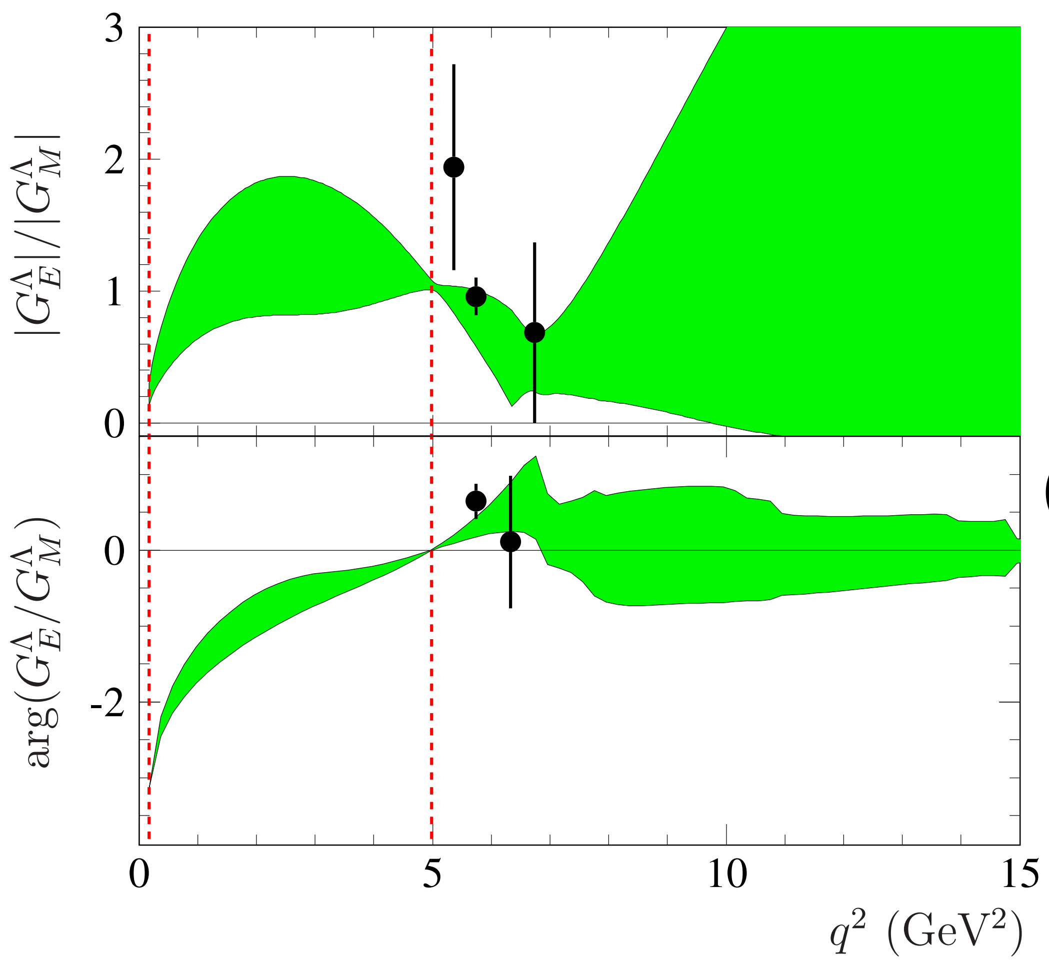
6 relevant cases



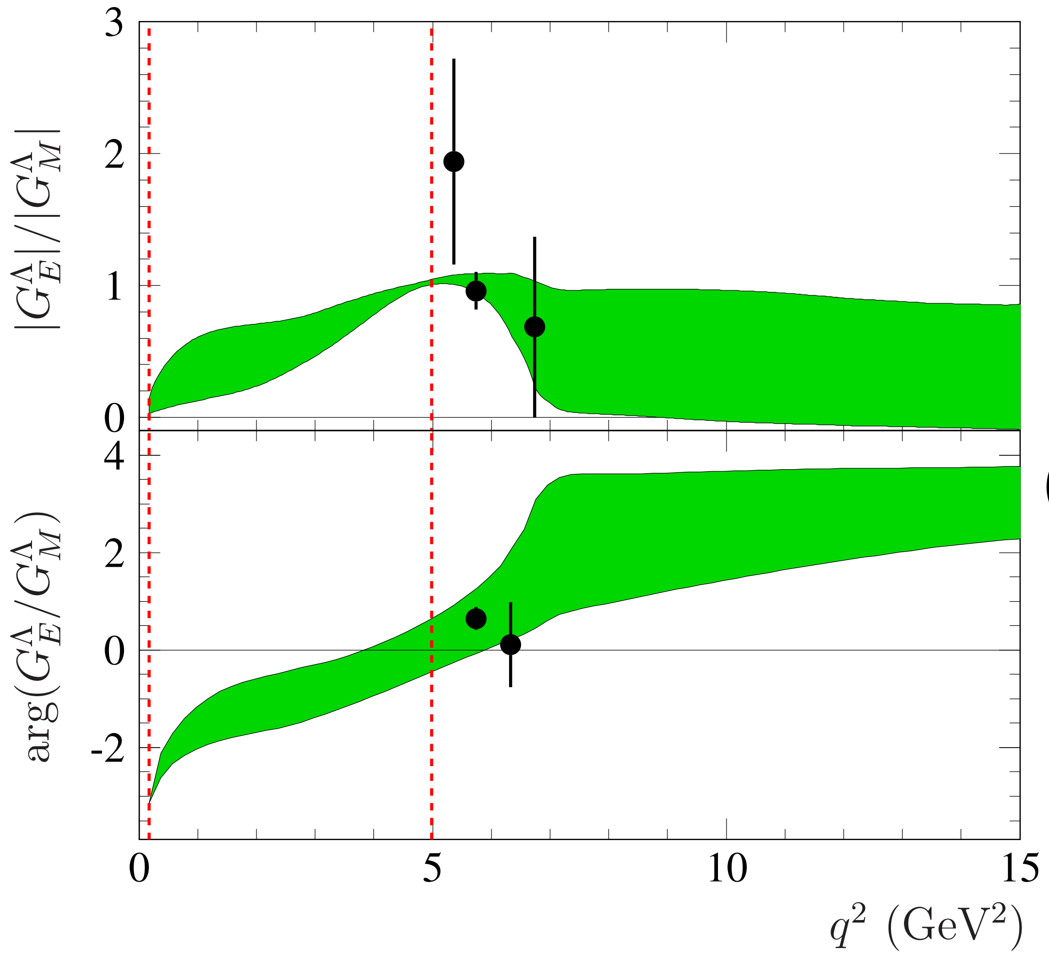
Results

Modulus and phase of R for the six relevant cases

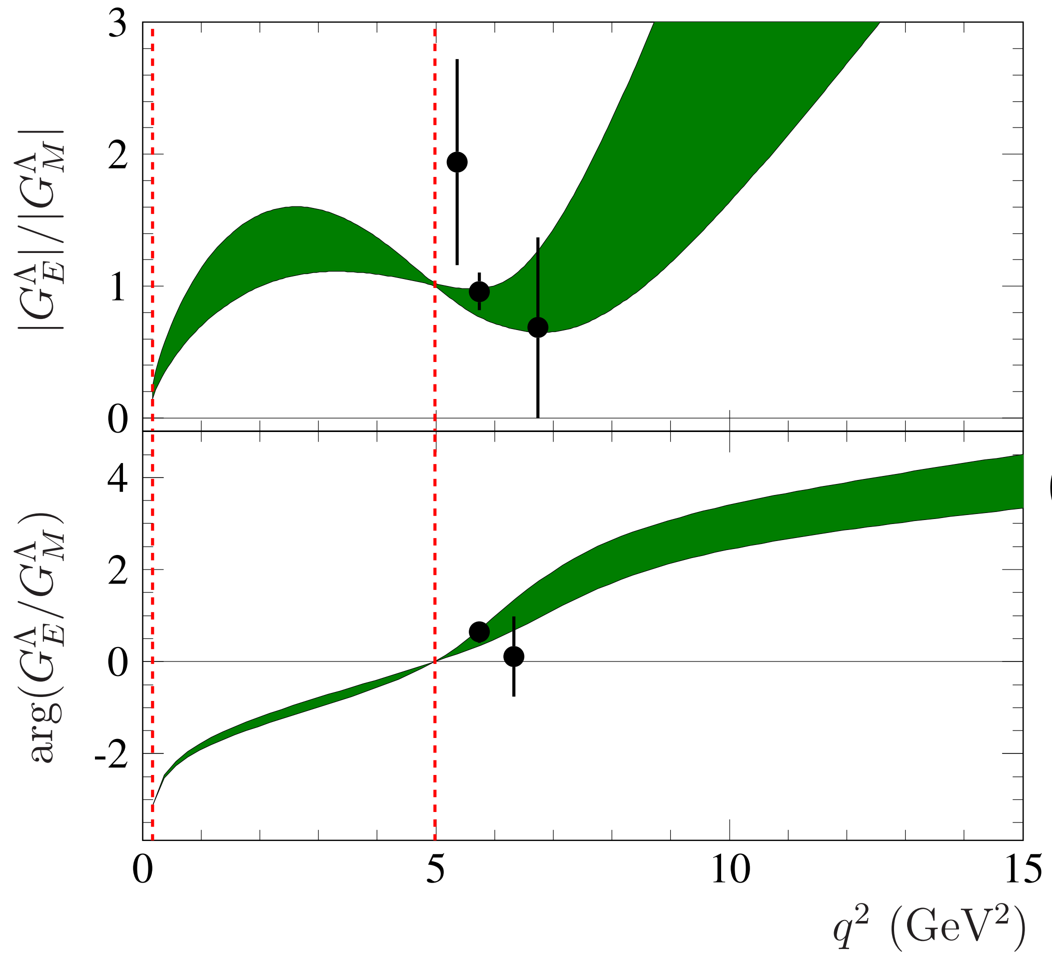
$$(N_{\text{th}}, N_{\text{asy}}) = (-1,0), (-1,1), (-1,2), (-1,3), (0,3), (1,3)$$



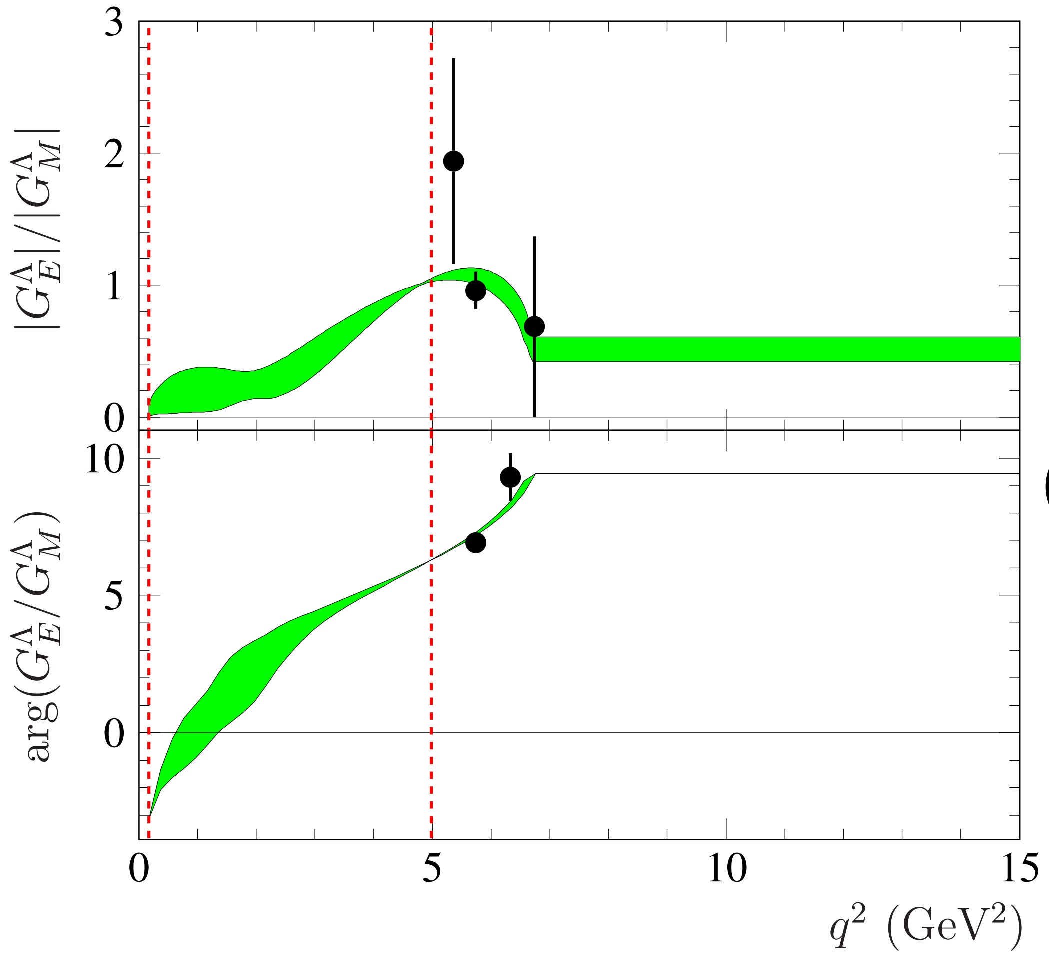
(−1,0)
4 %



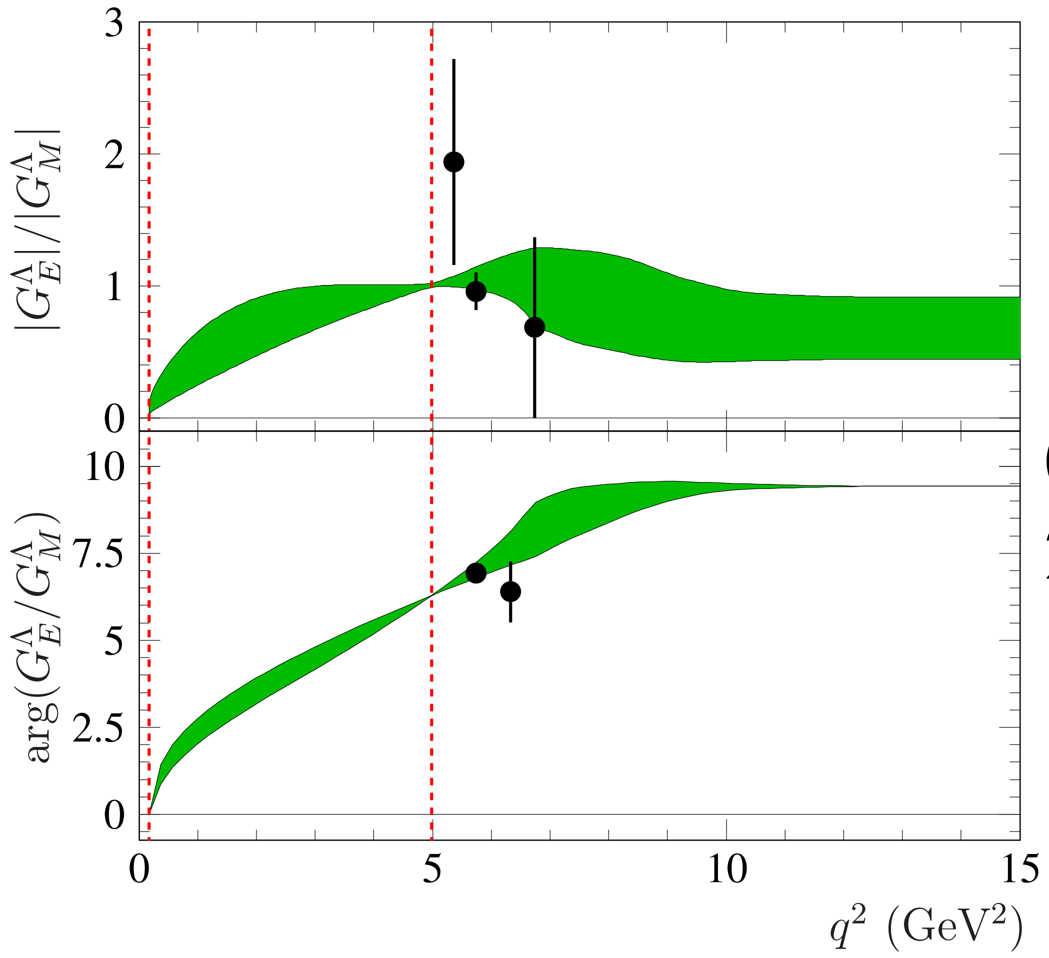
(−1,1)
16 %



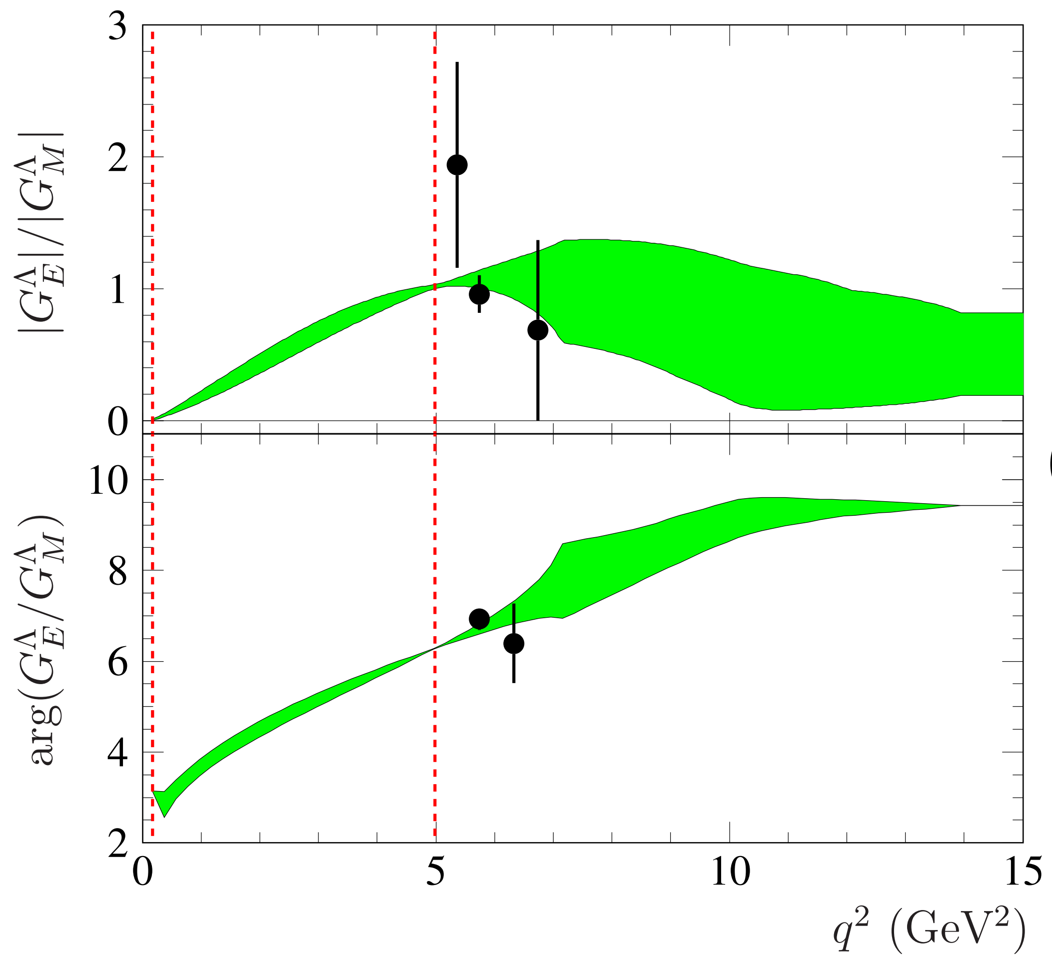
(−1,2)
50.5 %



(−1,3)
0.7 %



(0,3)
26.8 %



(1,3)
1.6 %

Istituto Nazionale di Fisica Nucleare

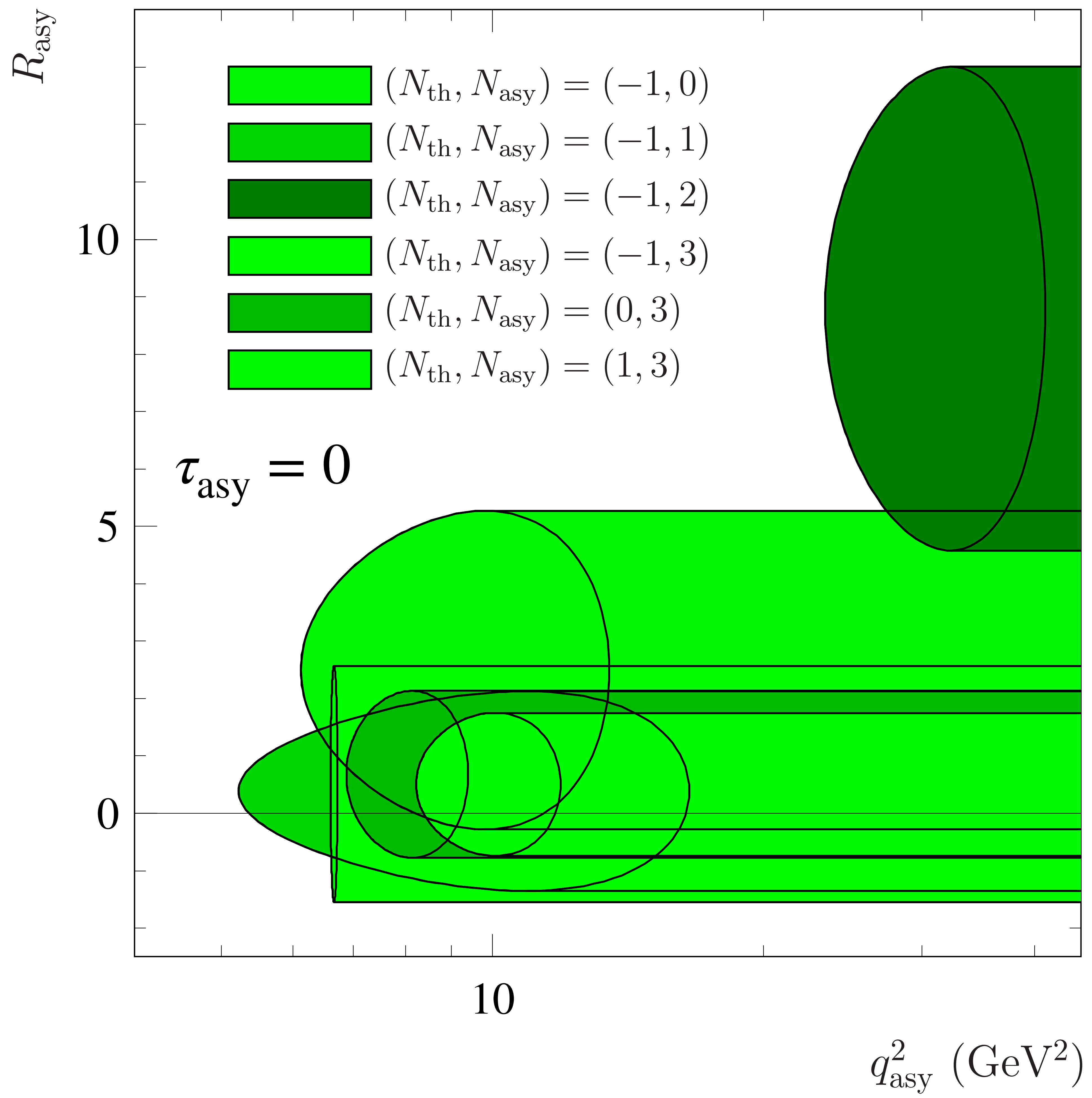
Results

Asymptotic behavior for the six relevant cases

$(N_{th}, N_{asy}) = (-1,0), (-1,1), (-1,2), (-1,3), (0,3), (1,3)$

We obtain the asymptotic threshold $(q^2_{asy} \pm \delta q^2_{asy})$ and the corresponding values of the modulus of the ratio $(R_{asy} \pm \delta R_{asy})$

$$\boxed{R_{asy} = |R(q^2_{asy})|}$$



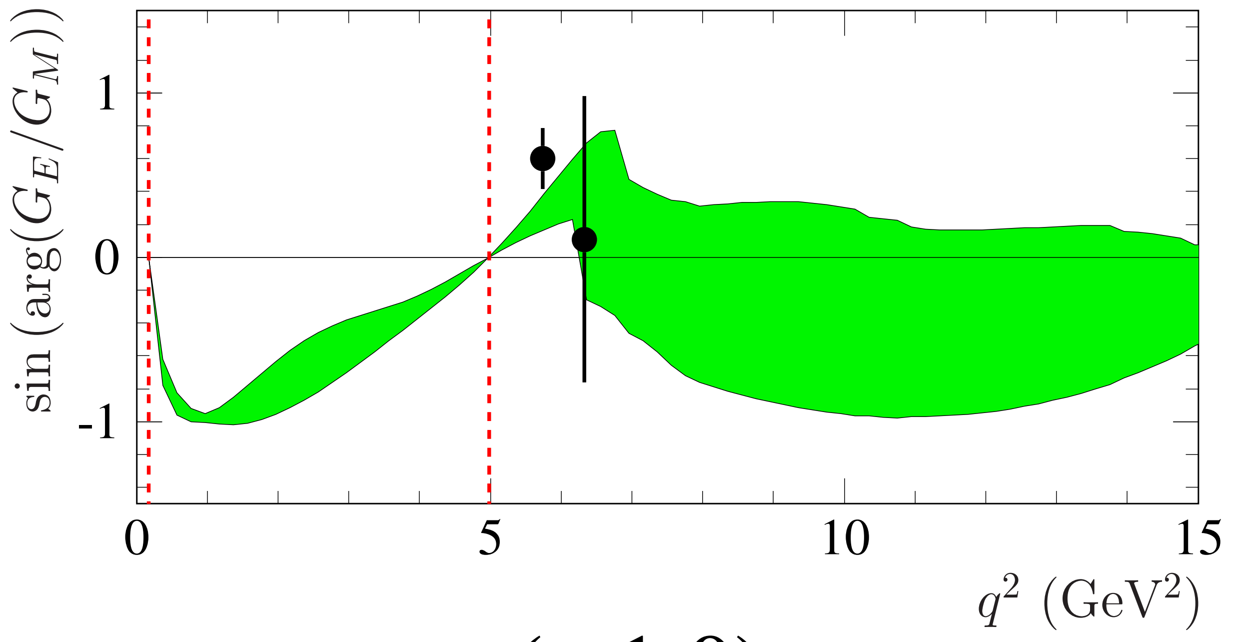
The knowledge of $|R(q^2)|$ at higher time-like q^2 could give hints to choose or neglect the case $(-1,2)$

Istituto Nazionale di Fisica Nucleare

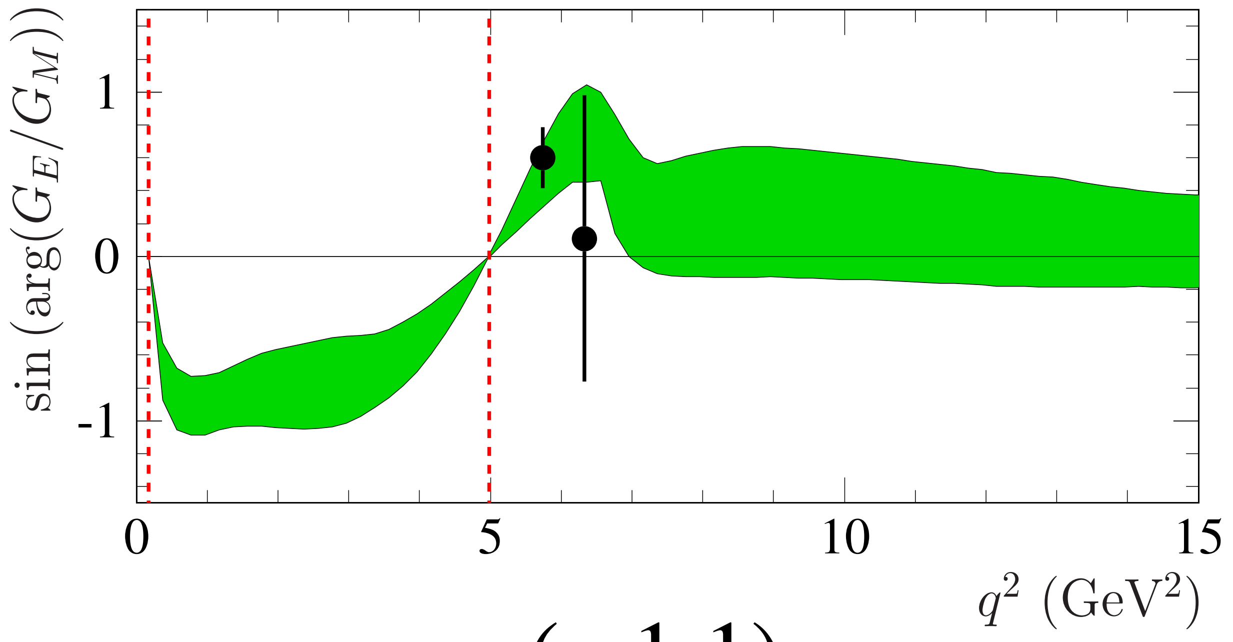
Results

Sinus of the phase of R for the six relevant cases

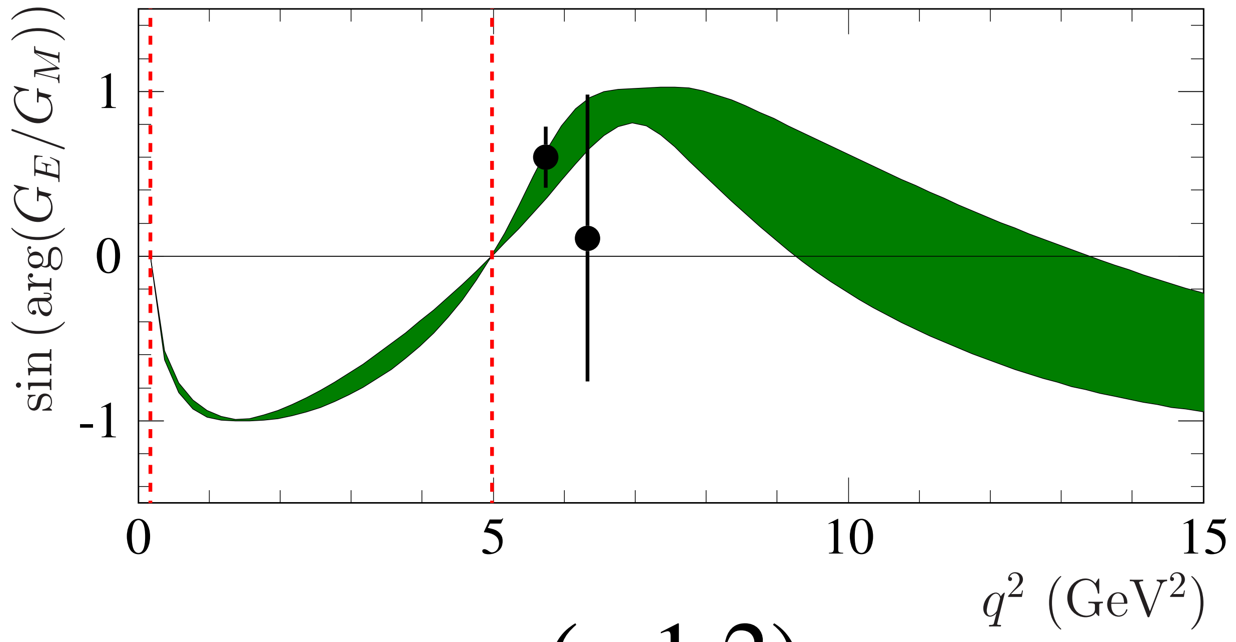
$$(N_{\text{th}}, N_{\text{asy}}) = (-1,0), (-1,1), (-1,2), (-1,3), (0,3), (1,3)$$



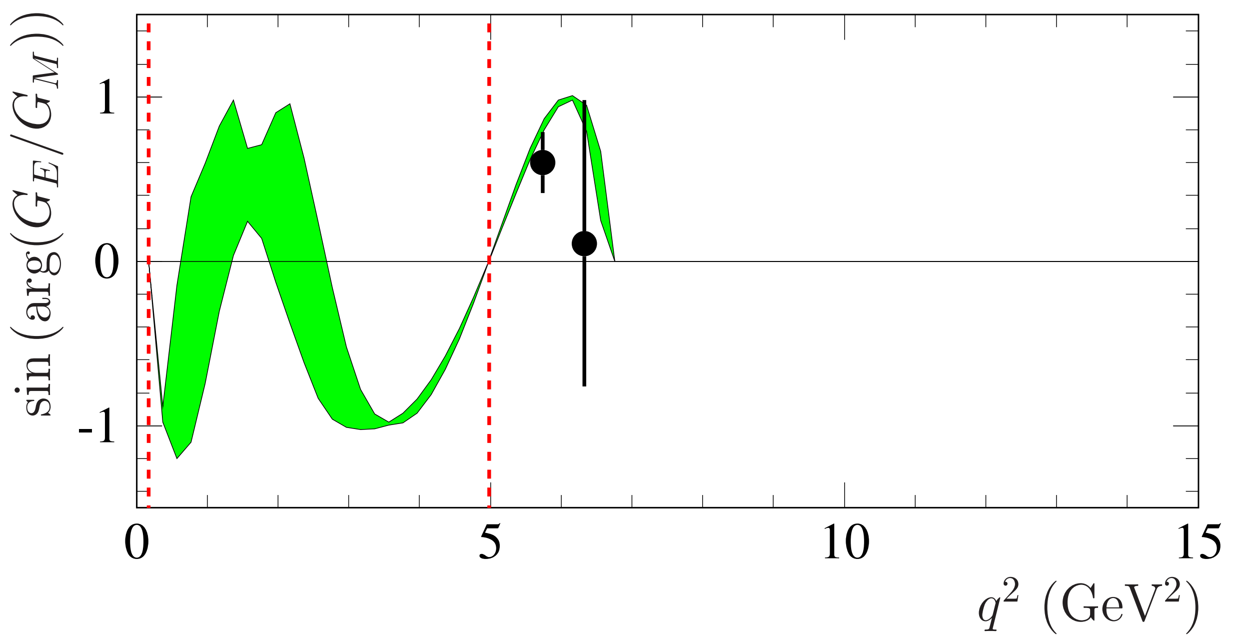
(−1,0)
4 %



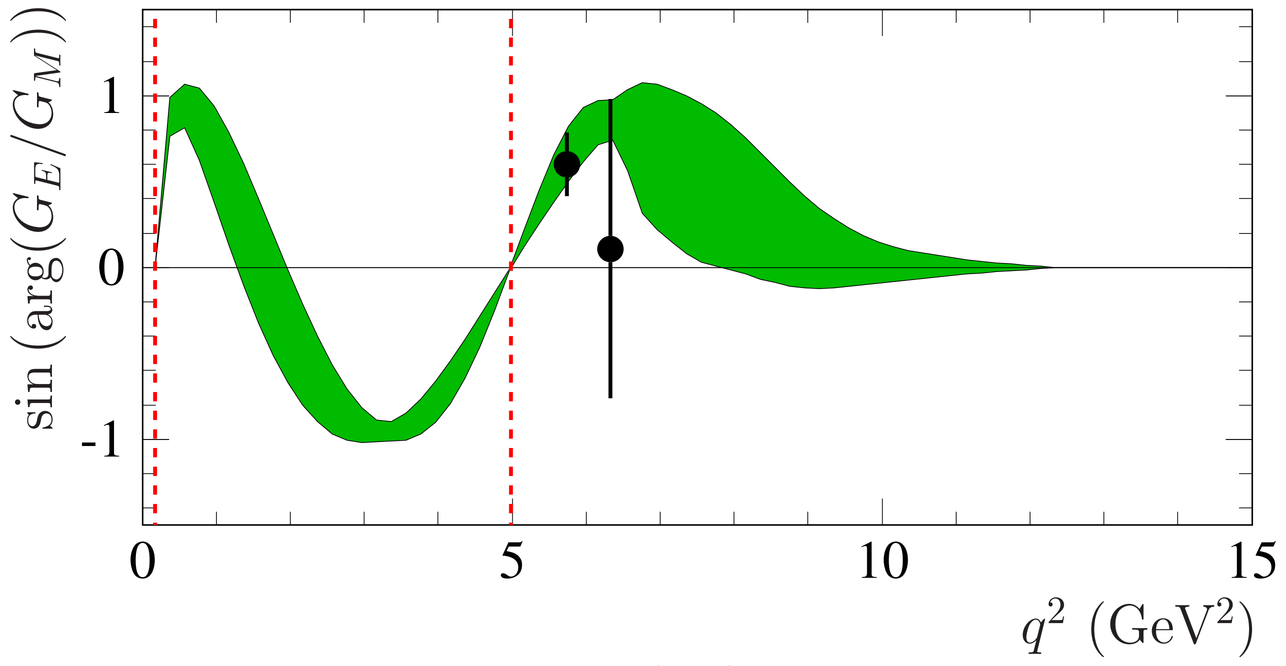
(−1,1)
16 %



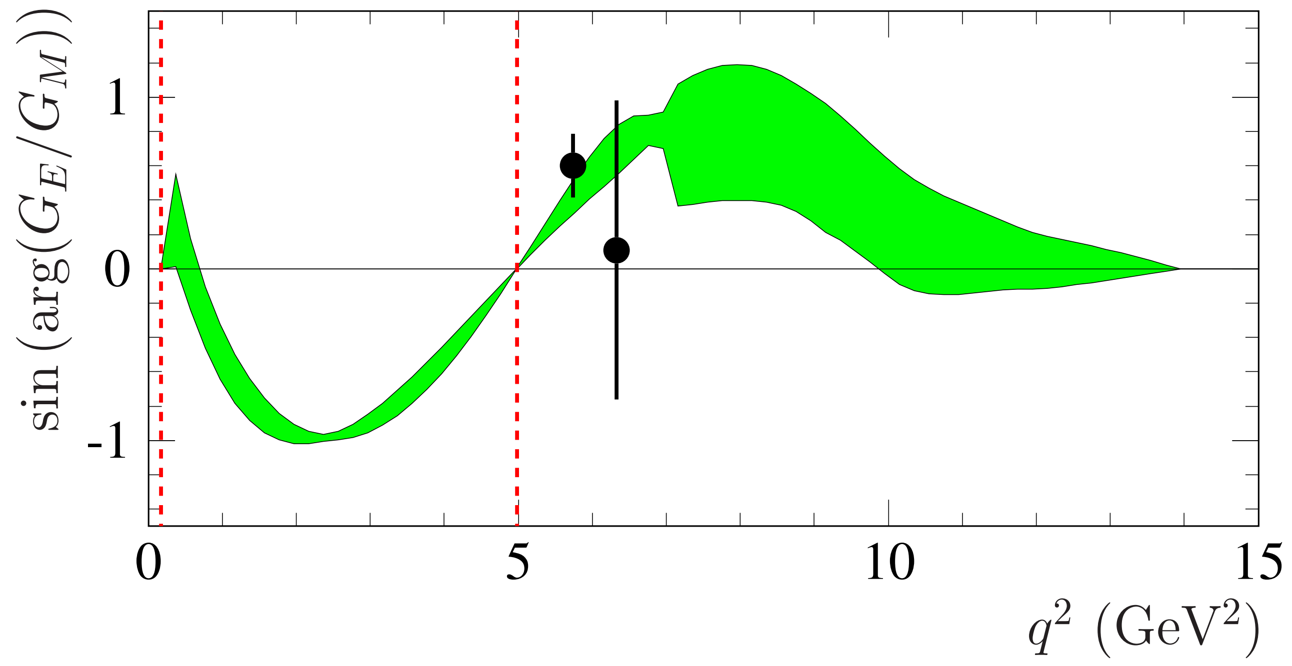
(−1,2)
50.5 %



(−1,3)
0.7 %



(0,3)
26.8 %



(1,3)
1.6 %

INFN Istituto Nazionale di Fisica Nucleare

STRONG 2020

Results

The complete knowledge of the ratio $R(q^2) = \dfrac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}$ can be used to calculate the the so-called charge root-mean square radius $\langle r_E \rangle$

$$\langle r_E \rangle^2 = 6 \frac{dG_E^\Lambda(q^2)}{dq^2}\bigg|_{q^2=0}$$

For the Λ baryon, being $G_E^\Lambda(0) = 0$ and $G_M^\Lambda(0) = \mu = (-0.613 \pm 0.004)\mu_N \neq 0$

$$\begin{aligned} \frac{dR(q^2)}{dq^2}\bigg|_{q^2=0} &= \frac{1}{G_M(q^2)}\left(\frac{dG_E(q^2)}{dq^2}\right. \\ &\quad \left.-\frac{G_E(q^2)}{G_M(q^2)}\frac{dG_M(q^2)}{dq^2}\right)\bigg|_{q^2=0}, \\ &= \left(\frac{1}{G_M(q^2)}\frac{dG_E(q^2)}{dq^2}\right)\bigg|_{q^2=0} = \frac{1}{\mu}\frac{\langle r_E \rangle^2}{6} \end{aligned}$$

$$\Longrightarrow \quad \boxed{\langle r_E \rangle^2 = 6\mu \frac{dR(q^2)}{dq^2}\bigg|_{q^2=0}}$$

Results

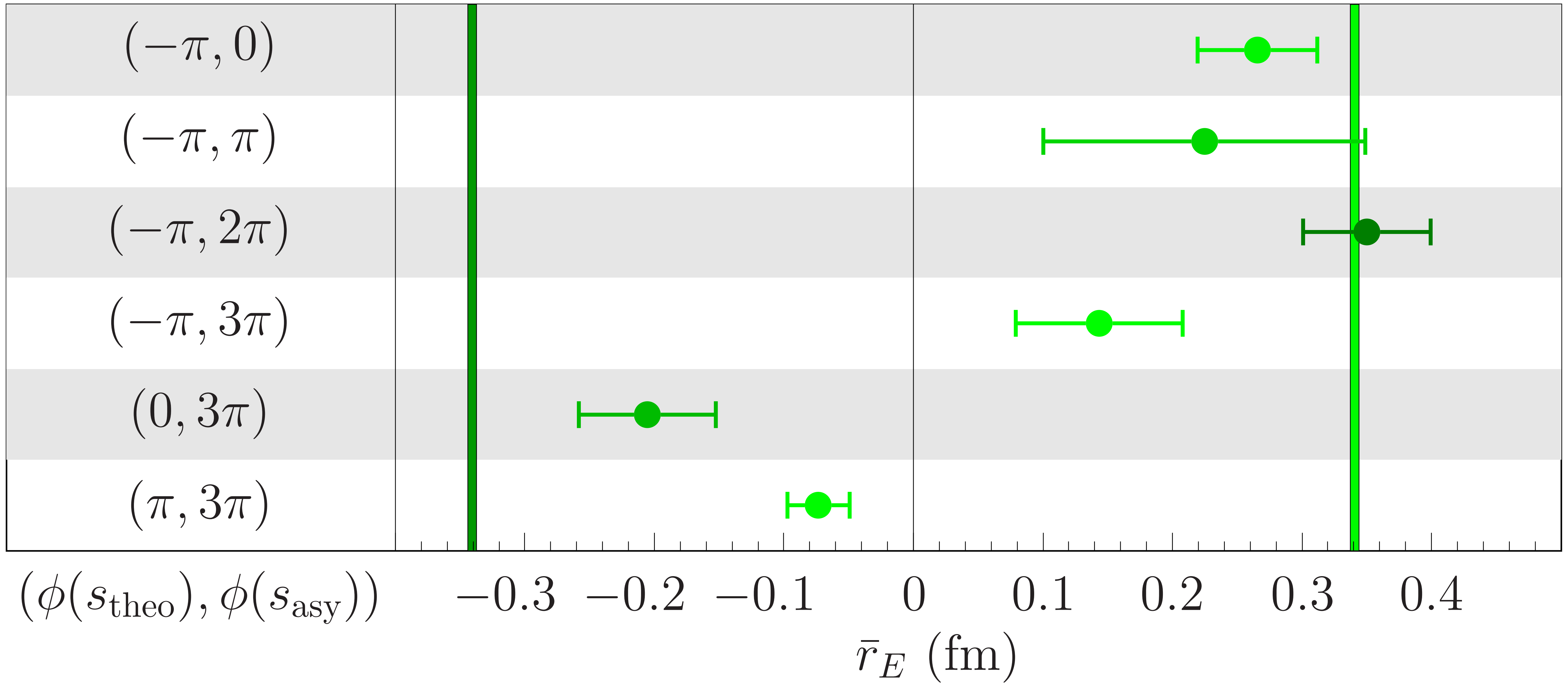
Using the following expression from our model

$$\left.\frac{dR(q^2)}{dq^2}\right|_{q^2=0} = \frac{1}{\pi}\int_{q^2_{\rm th}}^{\infty}\frac{\mathrm{Im}\,(R(s))}{s^2}ds = \frac{1}{\pi\Delta q^2}\sum_{j=0}^{N}C_j\int_{-1}^{1}\frac{T_j\,(x)}{\left(x+1+q^2_{\rm th}/\Delta q^2\right)^2}dx$$

$$\Delta q^2 = (q^2_{\rm asy} - q^2_{\rm th})/2$$

$$\boxed{\langle r_E^\Lambda\rangle^2 = \frac{6\mu_\Lambda}{\pi\Delta q^2}\sum_{j=0}^{N}C_j\int_{-1}^{1}\frac{T_j\,(x)}{\left(x+1+q^2_{\rm th}/\Delta q^2\right)^2}dx}$$

$$\bar{r}_E = \mathrm{Sign}\left[\langle r_E\rangle^2\right]\,\sqrt{|\langle r_E\rangle^2|}$$



the symmetric vertical green bars indicate the negative normalized neutron charge radius and its reflection

Conclusions

- We propose a phenomenological model based on first principles, as analyticity, to study the Λ baryon electromagnetic FFs, G_E^Λ and G_M^Λ and their ratio $R = G_E^\Lambda / G_M^\Lambda$

- For the Levinson's theorem, in our case, the difference $(N_{\rm asy} - N_{\rm th})$ gives the total number of zeros for R and, hence, for G_E^Λ, assuming $G_M^\Lambda \neq 0$

$$N_{\rm th,asy} = \frac{1}{\pi} \arg[R(q^2_{\rm th,asy})]$$

- We determine, **for the first time**, the complex structure of the ratio knowing the experimental values of its modulus and phase

- Despite the few available data, the model allows to select the following acceptable six cases (with a probability $> 0.5\,\%$): $(N_{\rm th}, N_{\rm asy}) = (-1,0), (-1,1), (-1,2), (-1,3), (0,3), (1,3)$

- The model gives information about the number of space-like zeros and the determination of the phase that is not directly accessible by experiments (the sinus of the phase is insensitive to its determination)

This work (arXiv: 2109.03759) has been accepted by PRD on 2 December 2021

Istituto Nazionale di Fisica Nucleare