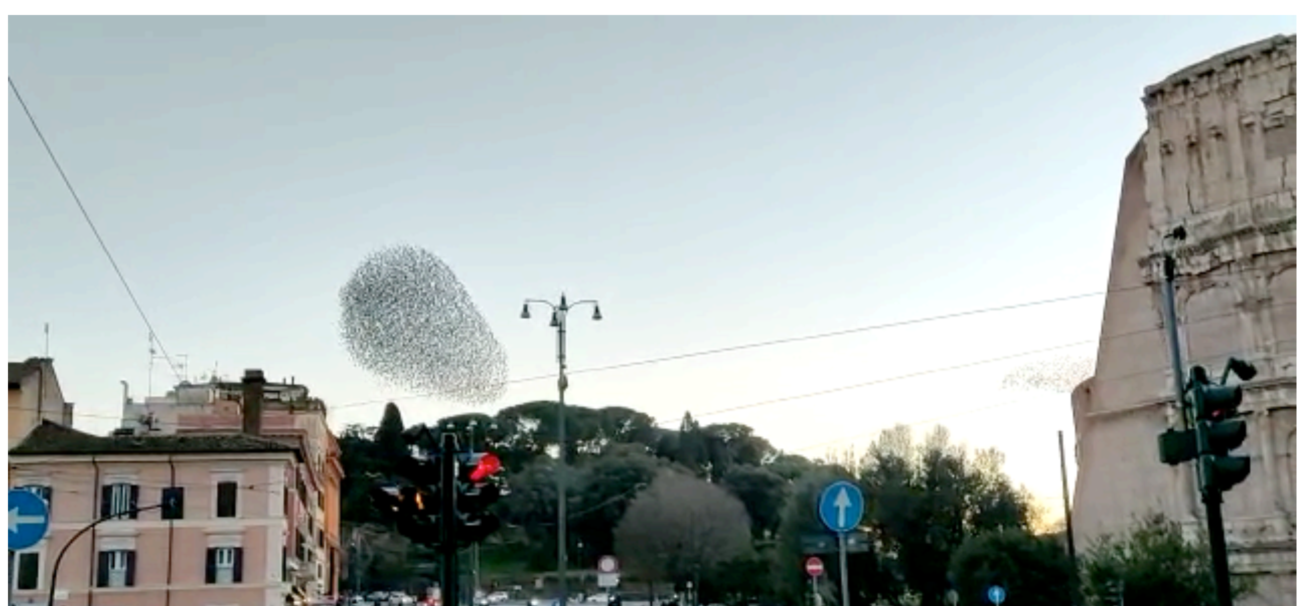
SAPIENZA Università di Roma

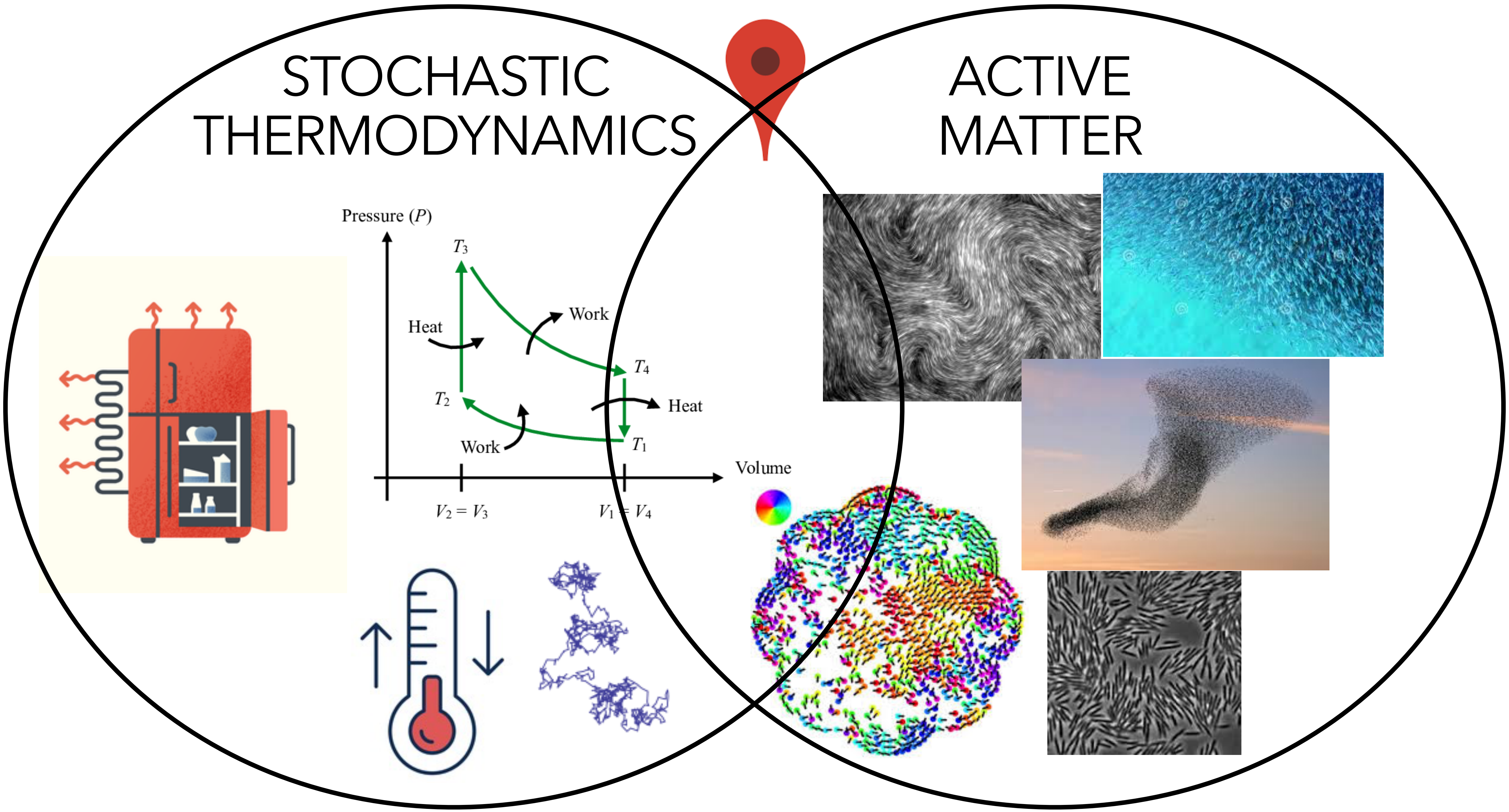
Istituto dei Sistemi Complessi

02/02/2022, PhD Seminar

Signatures of irreversibility in microscopic models of flocking

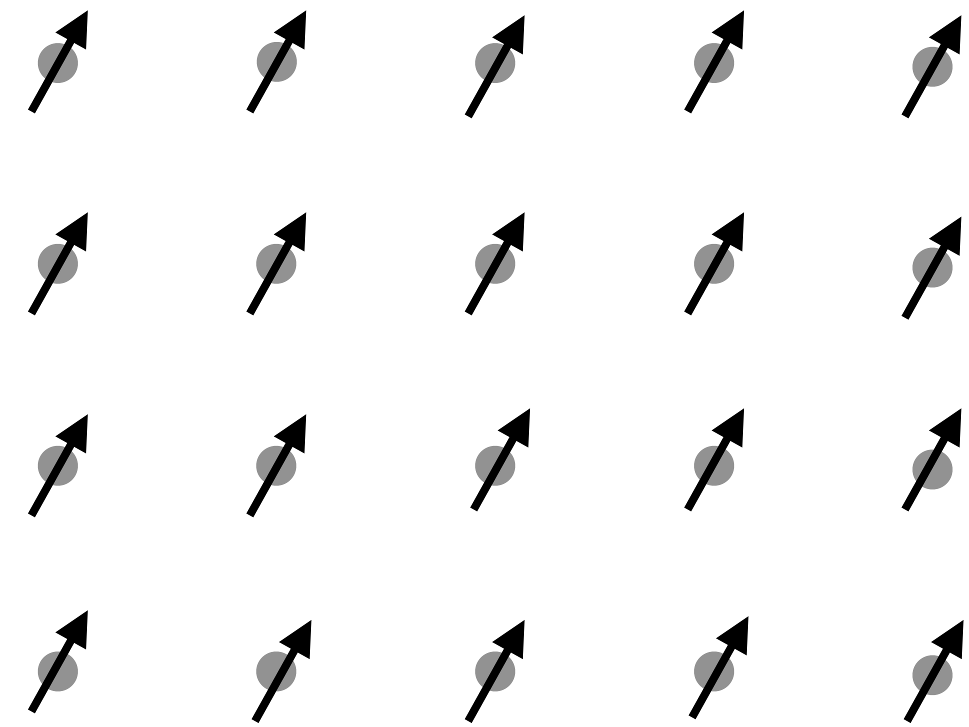


We are here


STOCHASTIC
THERMODYNAMICS
ACTIVE
MATTER
Pressure (P)
T_3
Heat
Work
T_4
T_2
Heat
Work
T_1
Volume
$V_2 = V_3$
$V_1 = V_4$

Polar active matter

- Directional d.o.f.
- Alignment (short-range, ferromagnetic)
- Self-propulsion



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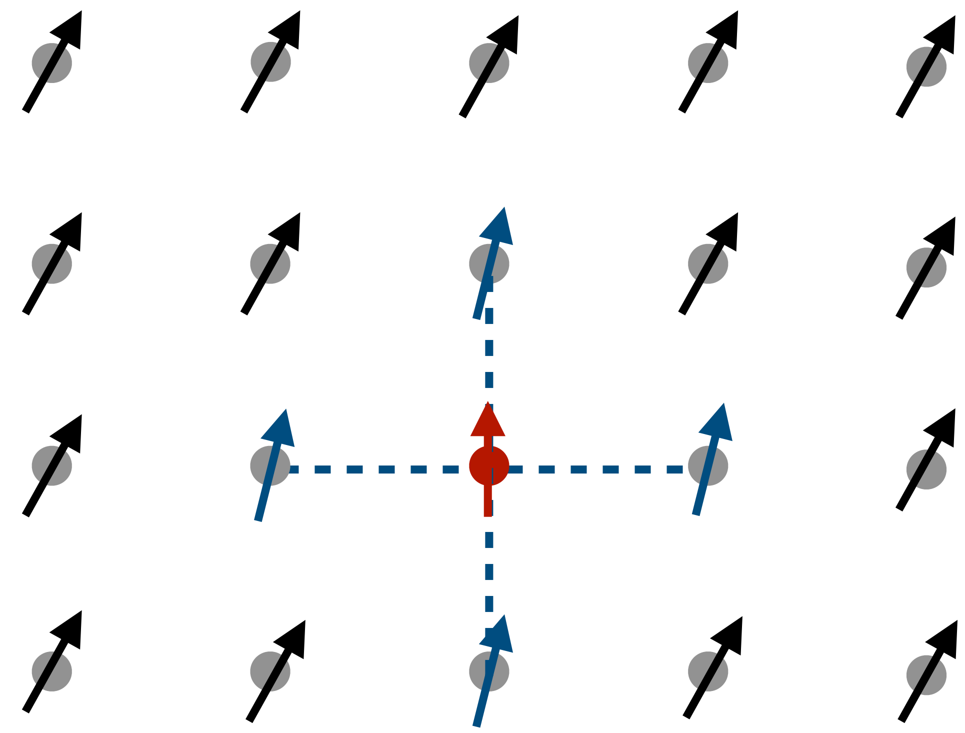

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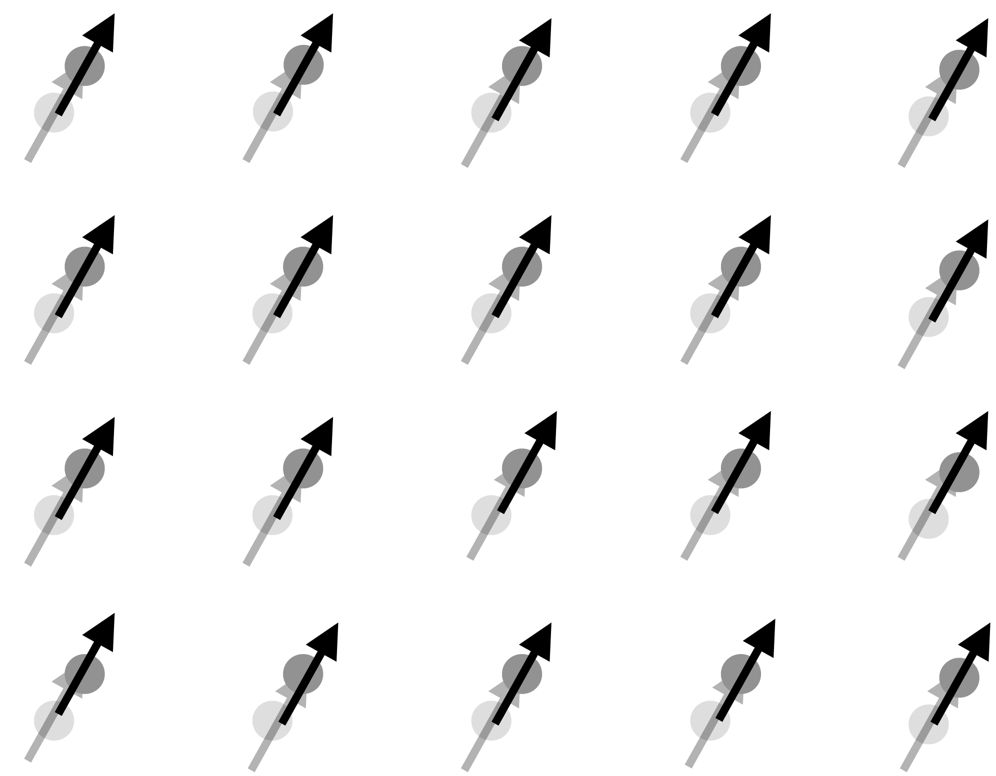

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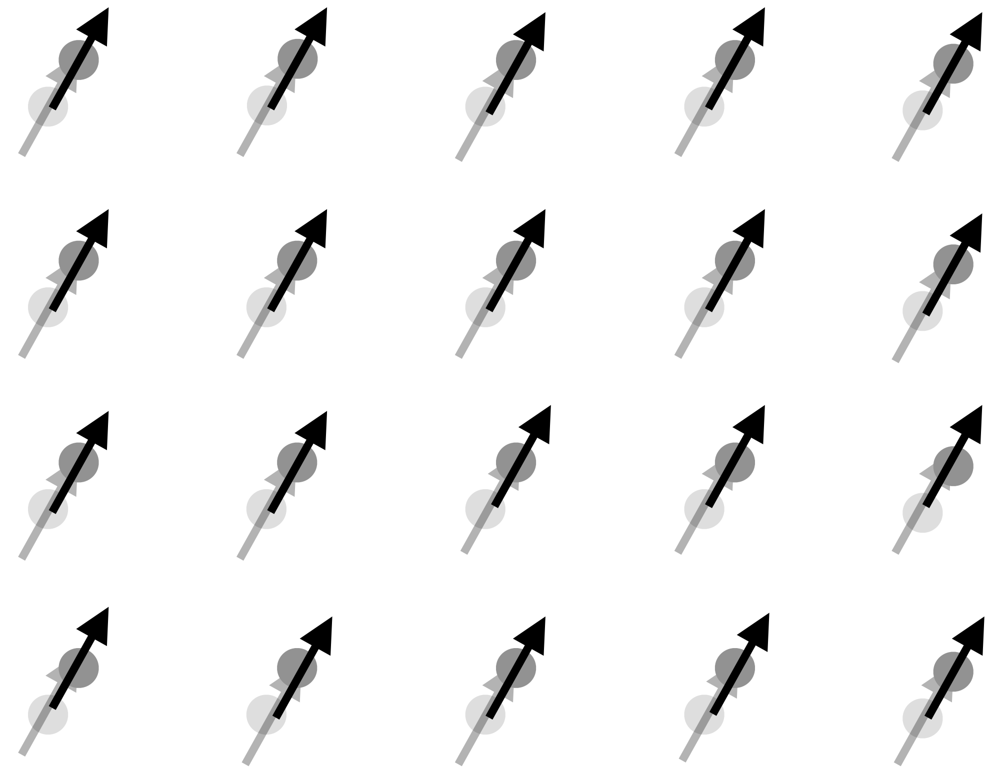

➡ Spontaneous symmetry breaking in $d = 2$. True Long Range Order.

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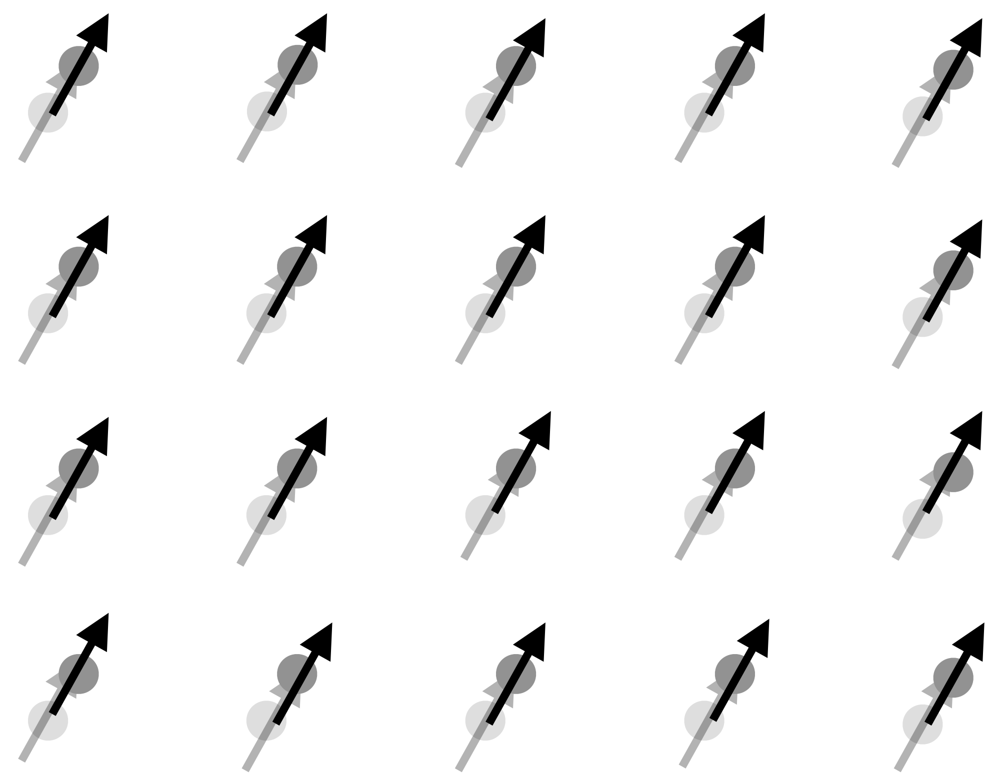


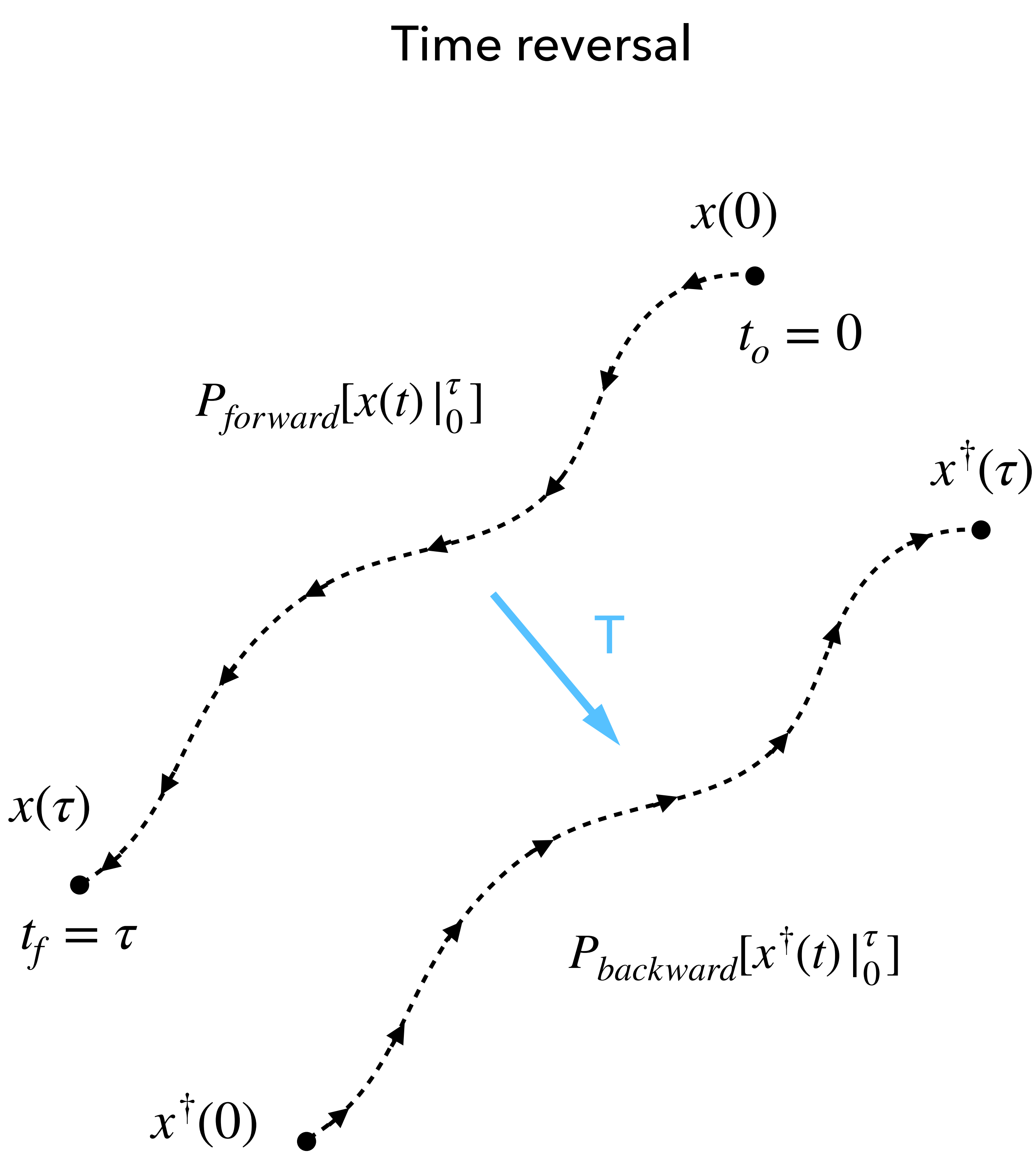
➡ Spontaneous symmetry breaking in $d = 2$. True Long Range Order.

➡ No SSB in $d = 2$ if motion is detailed balanced (i.e. reversible)

Our goals

- Quantify irreversibility in models of flocks
- Connect non-equilibrium-ness with the rotational symmetry breaking
- Understand how dissipation occurs at a microscopic level
- Identify signatures of irreversibility that can be exploited to learn something new from real systems

Quantifying irreversibility



Fluctuating Entropy Production

$$\sigma[x(t)\,|_0^\tau] = \log \frac{P_{forward}[x(t)\,|_0^\tau]}{P_{backward}[x^\dagger(t)\,|_0^\tau]}$$

Average Entropy Production

$$S(\tau) = \langle \sigma[x(t)\,|_0^\tau] \rangle = S^{hk}(\tau) + \Delta S_0$$

Entropy Production Rate (EPR)

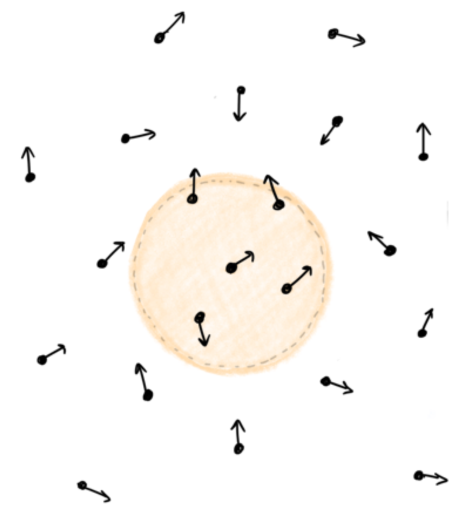
$$\dot{S} = \lim_{\tau\to\infty} \frac{S(\tau)}{\tau}$$

The irreversible flocking process

- The model $(d = 2)$:

$$\begin{cases} \dot{\mathbf{x}}_i = v_0 \hat{\mathbf{e}}(\theta_i) \\ \dot{\theta}_i = -\dfrac{\partial H(\Theta; \mathbf{n}(X))}{\partial \theta_i} + \sqrt{2D}\xi_i, \qquad \langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\delta(t - t') \end{cases}$$

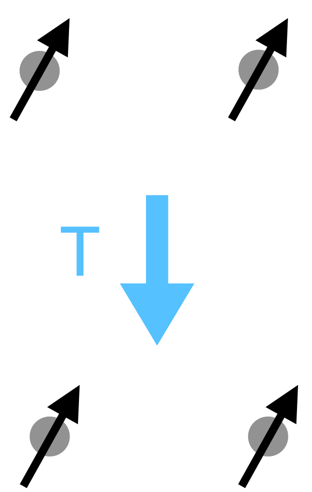
$$H_{XY}(\Theta; \mathbf{n}) = -\frac{J}{2} \sum_{ij} n_{ij} \cos(\theta_i - \theta_j)$$



$$n_{ij} = \Theta(R - |\mathbf{x}_i - \mathbf{x}_j|)$$

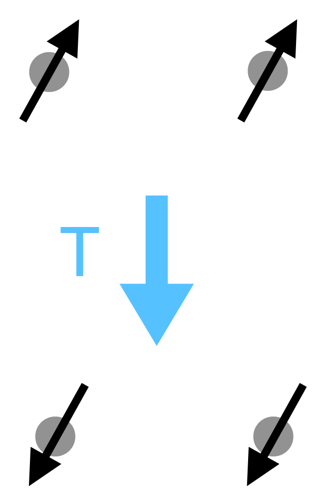
- Time reversal:

EVEN



$$x_i^\alpha(t) \to x_i^\alpha(\tau - t); \quad \theta_i(t) \to \theta_i(\tau - t)$$

ODD



$$x_i^\alpha(t) \to x_i^\alpha(\tau - t); \quad \theta_i(t) \to \theta_i(\tau - t) + \pi$$

The irreversible flocking process

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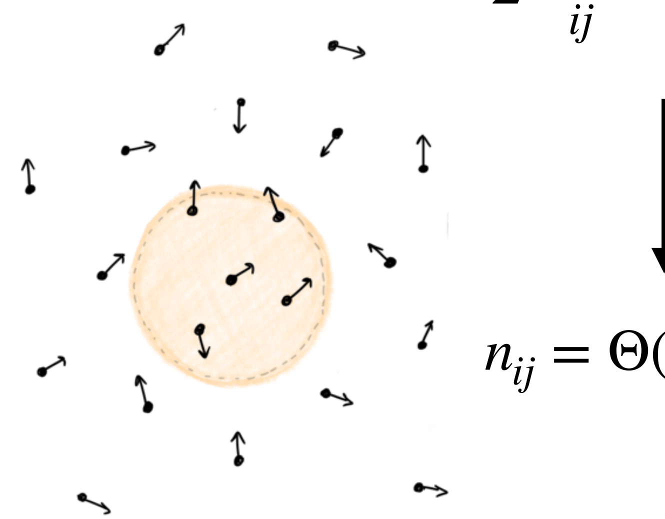

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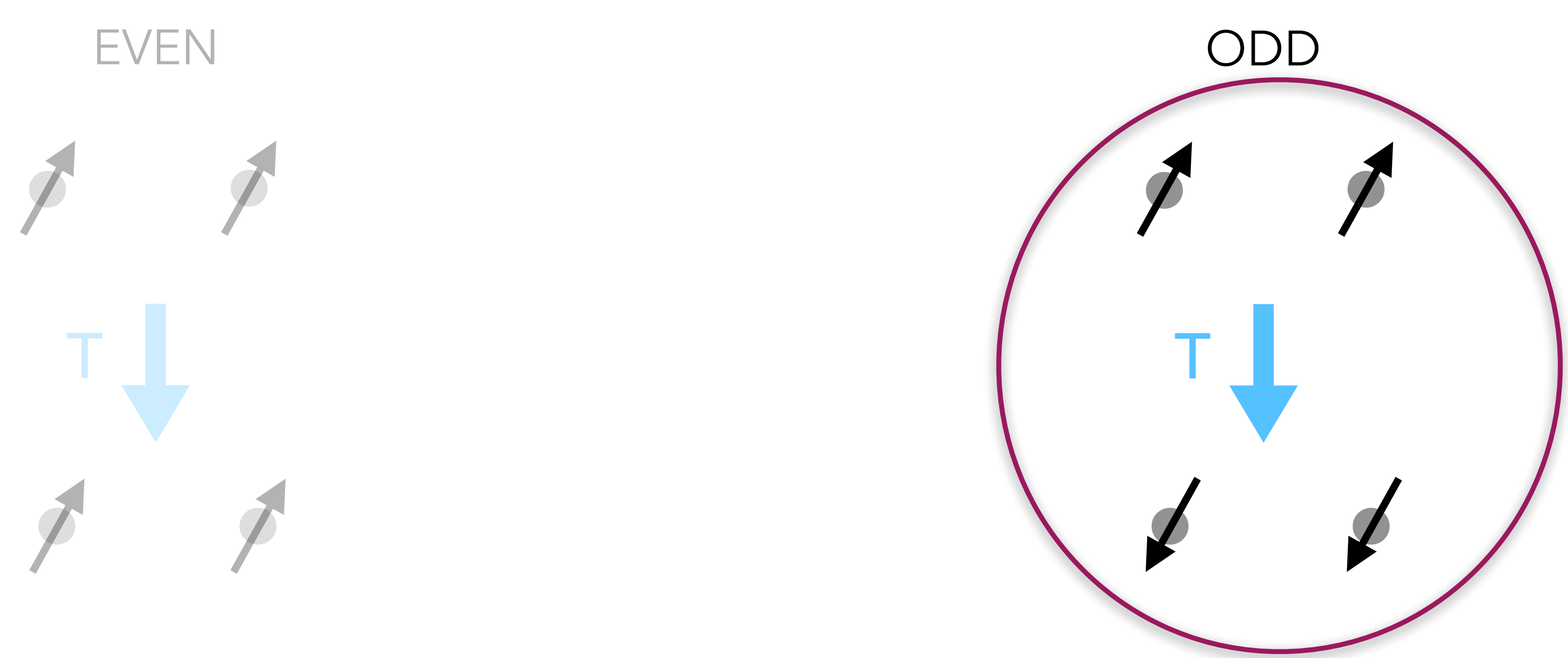

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The irreversible flocking process

- The model ($d = 2$) :

$$
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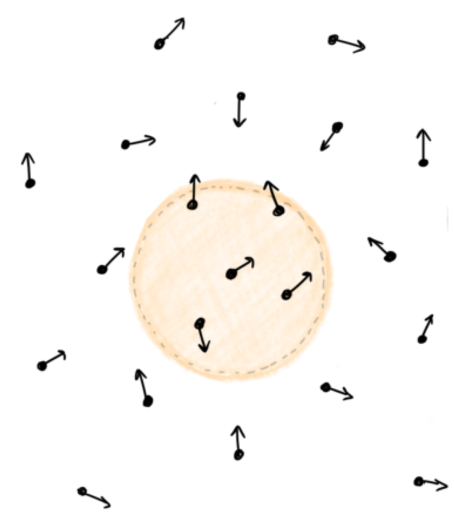


$$n_{ij} = \Theta(R - |\mathbf{x}_i - \mathbf{x}_j|)$$

- Time reversal: $\theta_i(t) \to \theta_i(\tau - t) + \pi$

Two equivalent formulas for EPR

$$\dot{S}_1 = -\langle \frac{J}{D}\sum_{ij}\frac{d\theta_i}{dt} \circ n_{ij}\sin(\theta_i - \theta_j) \rangle$$

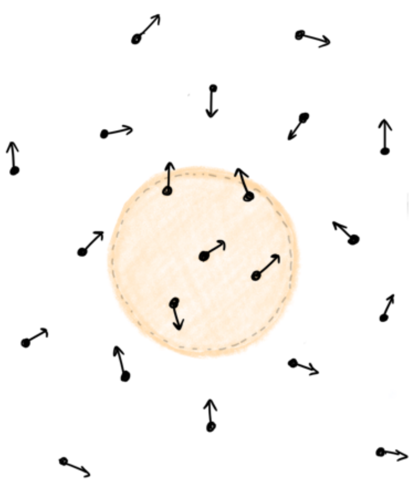
Heat dissipated into bath

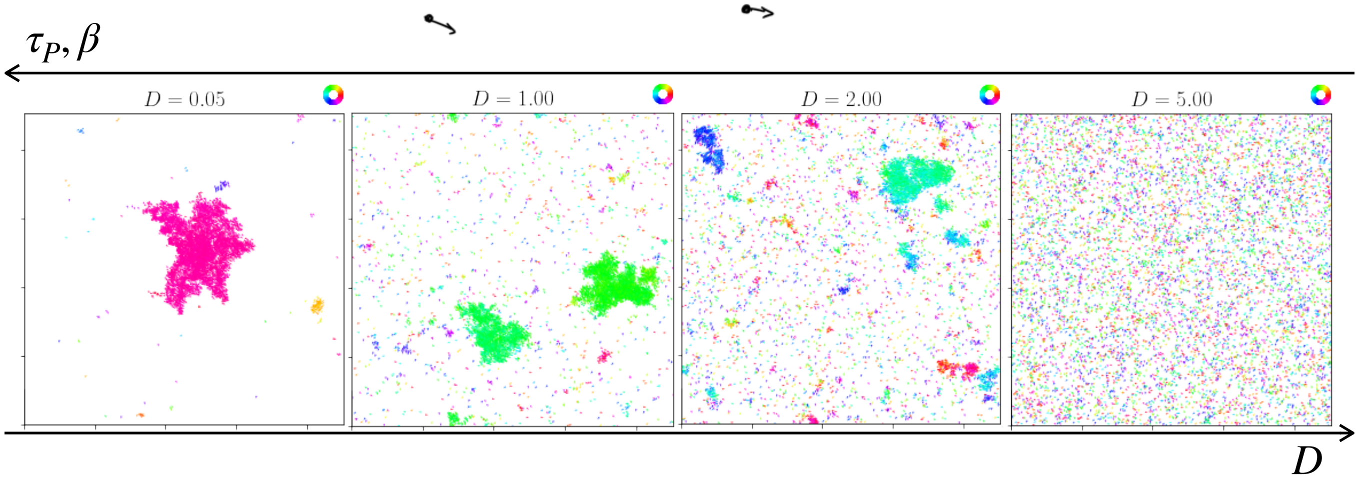
$$\dot{S}_2 = -\langle \frac{J}{2D}\sum_{ij}\frac{dn_{ij}}{dt} \circ \cos(\theta_i - \theta_j) \rangle$$

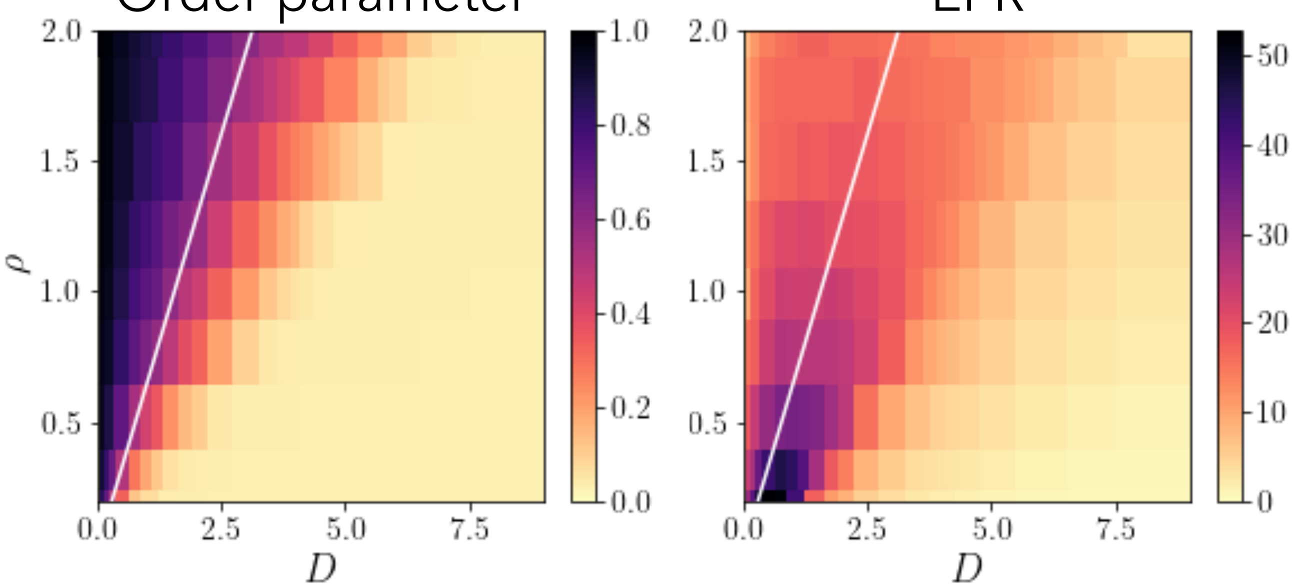
Work of fictitious external protocol that reshuffles the network

Irreversibility and phase diagram

$$H_{XY}(\Theta; \mathbf{n}) = -\frac{J}{2}\sum_{ij} n_{ij}\cos(\theta_i - \theta_j)$$

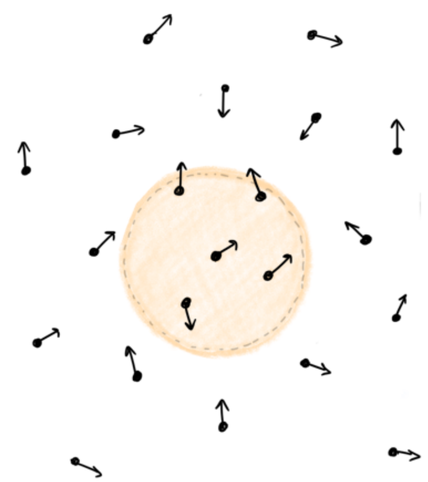






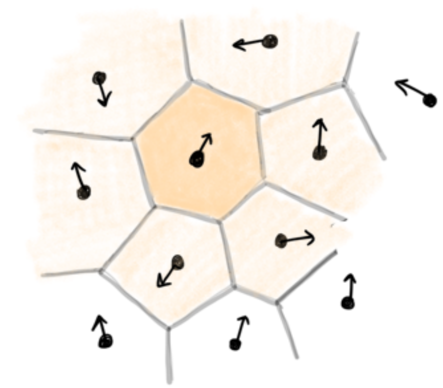
Irreversibility and phase diagram

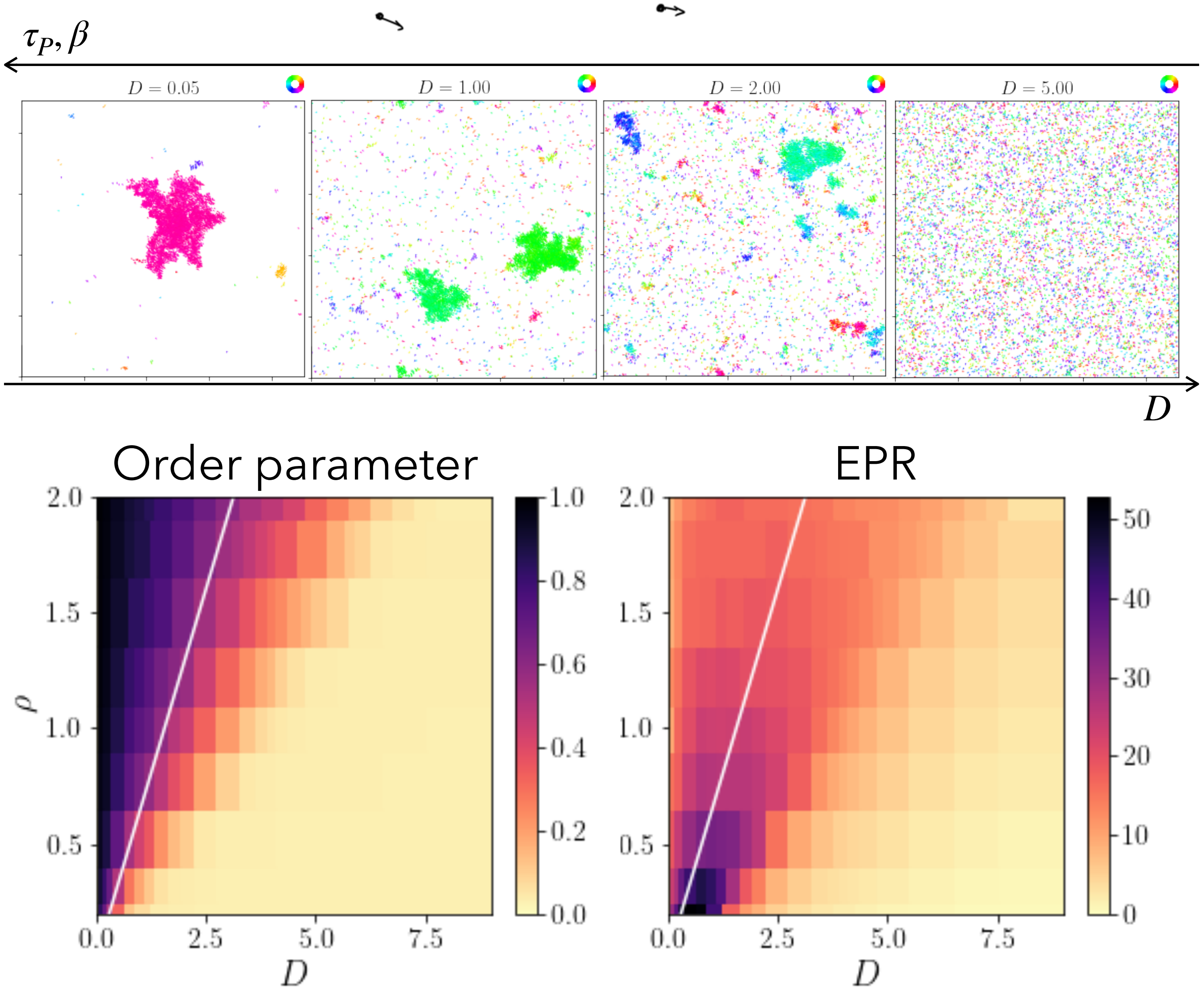
Model I (metric)

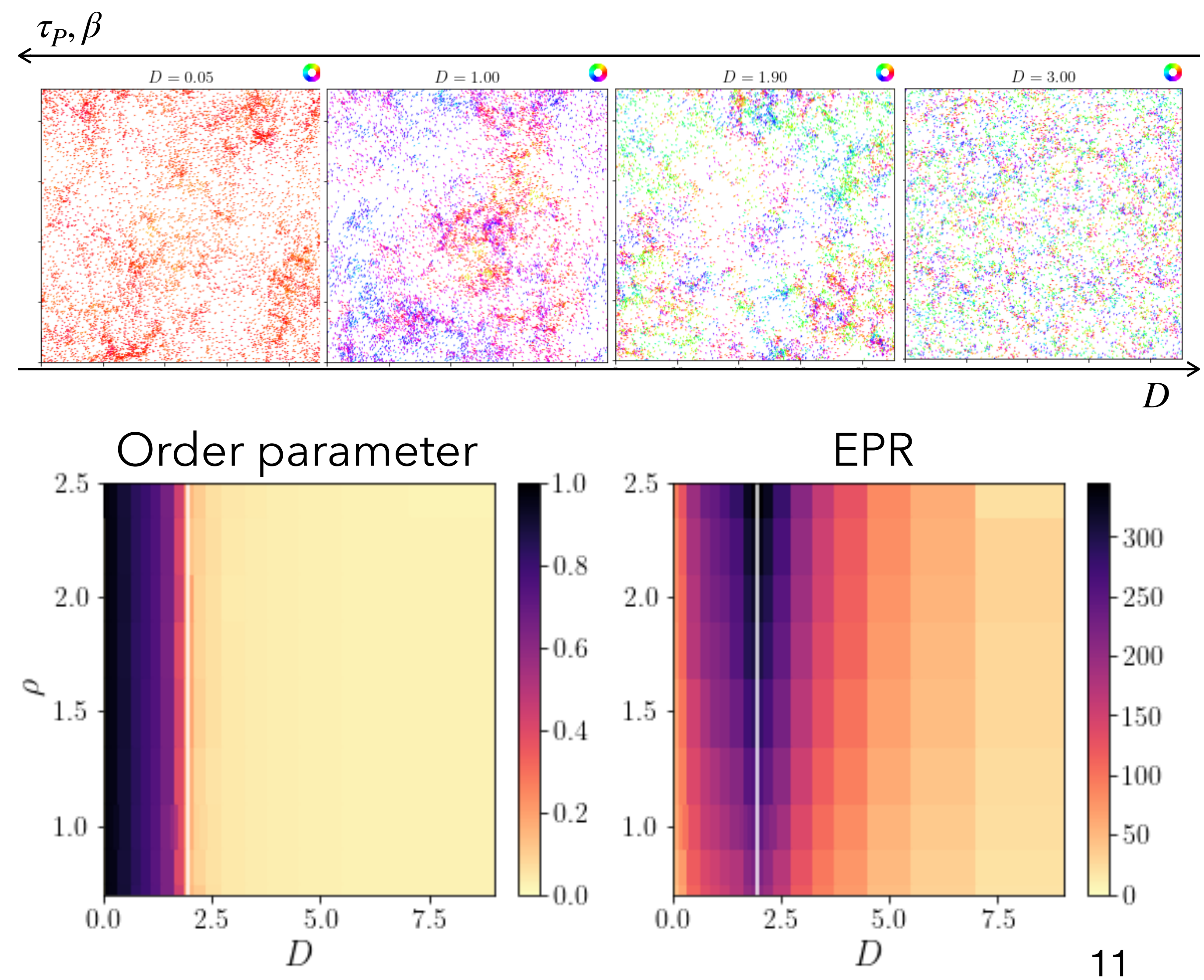


$$H_{XY}(\Theta;\mathbf{n}) = -\frac{J}{2}\sum_{ij} n_{ij}\cos(\theta_i-\theta_j)$$

Model II (topological)

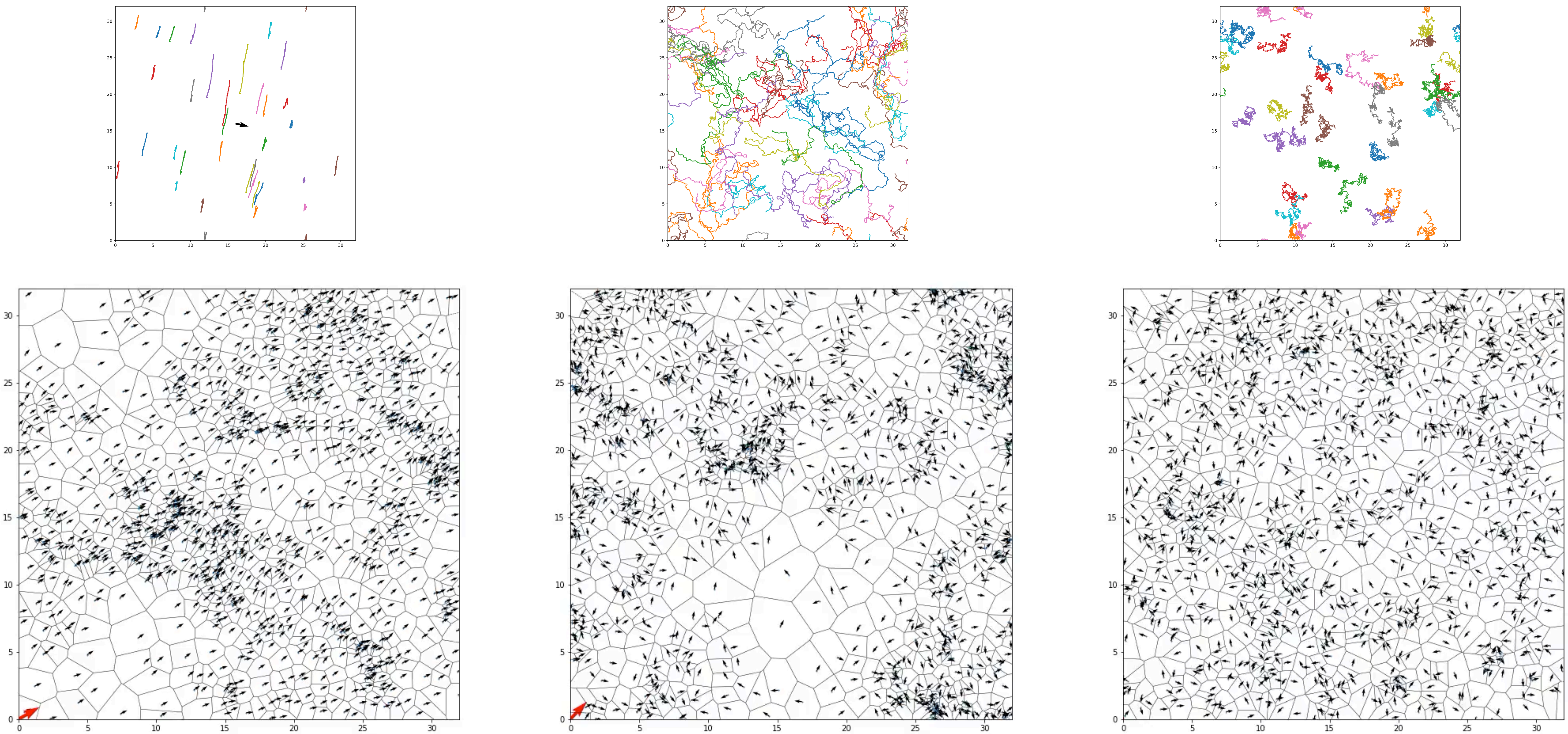






Irreversibility and phase diagram

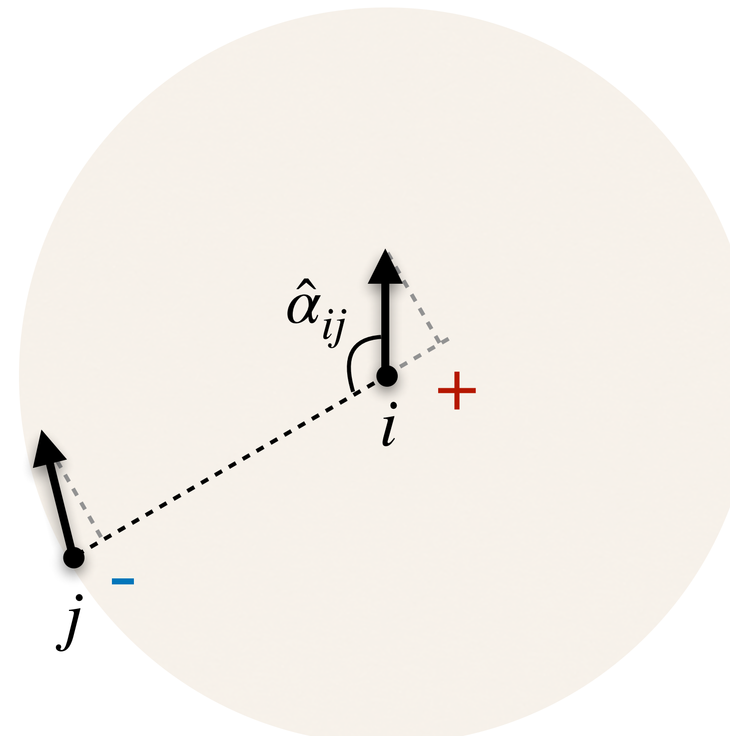
- $\dot{S} = -\langle \frac{J}{2D} \sum_{ij} \frac{dn_{ij}}{dt} \circ \cos(\theta_i - \theta_j) \rangle$
- Rewiring of the interaction network is mostly efficient close to the transition



Signatures of irreversibility

- Model I (metric, step-like): $n_{ij} = \Theta(R - |\mathbf{x}_i - \mathbf{x}_j|)$

$$\dot{S} = \frac{\langle \dot{w}_{\text{resh}} \rangle}{D} = - \langle \frac{J}{2D} \sum_{ij} \frac{dn_{ij}}{dt} \circ \cos(\theta_i - \theta_j) \rangle$$



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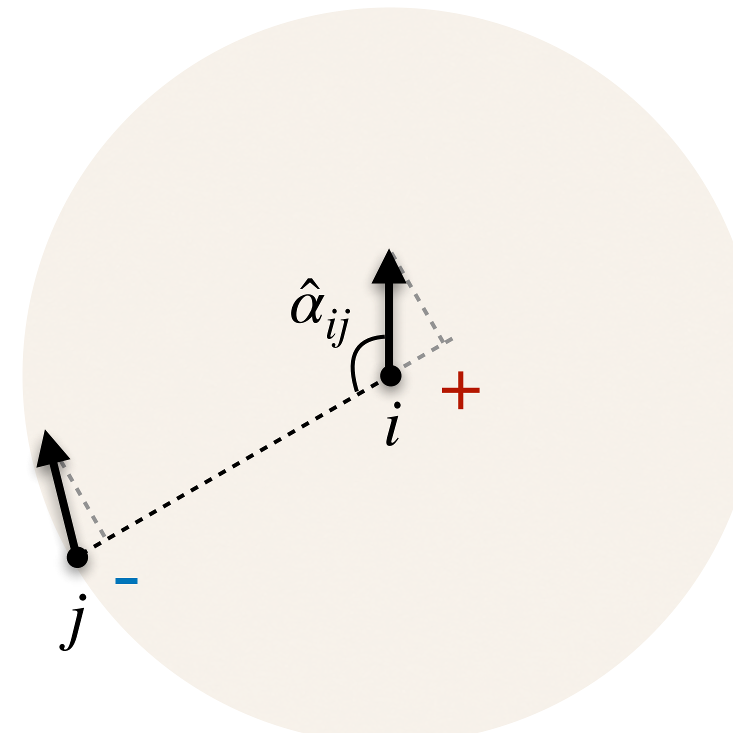
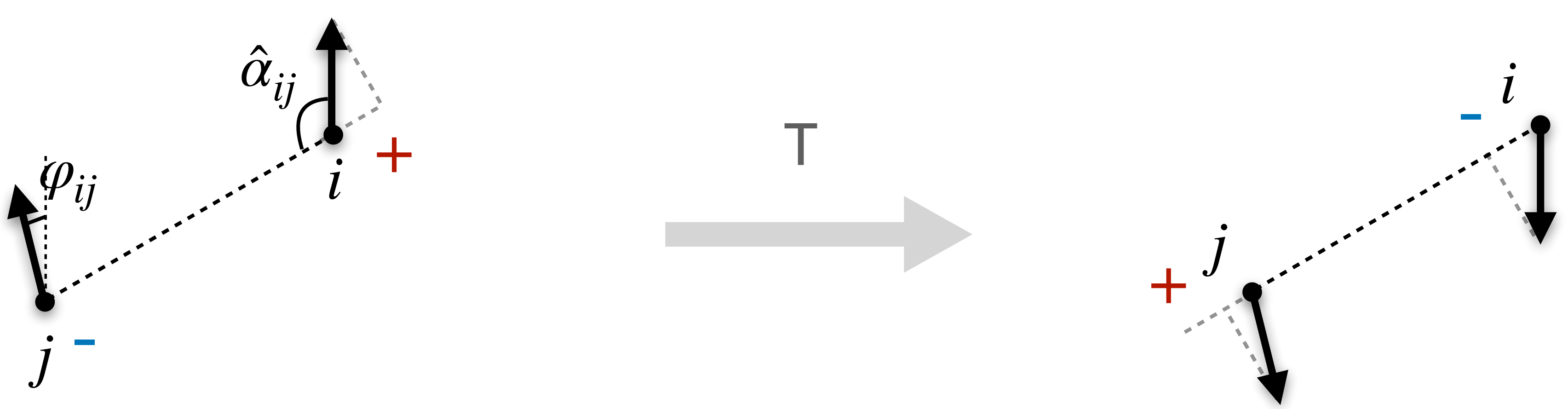


- $\dot{S} > 0 \implies$ asymmetries in the steady state distribution of pairs of particles

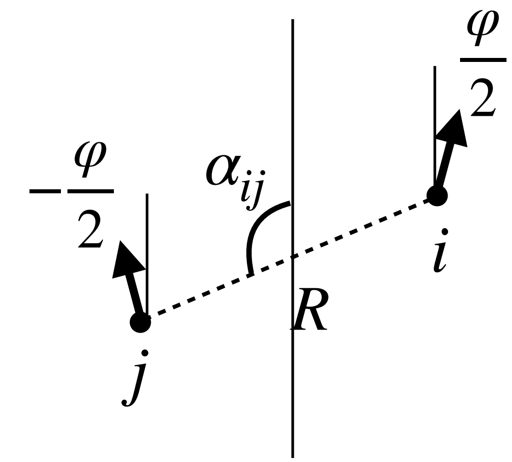


$$\mathrm{Prob(config.1)} \neq \mathrm{Prob(config.2)}$$

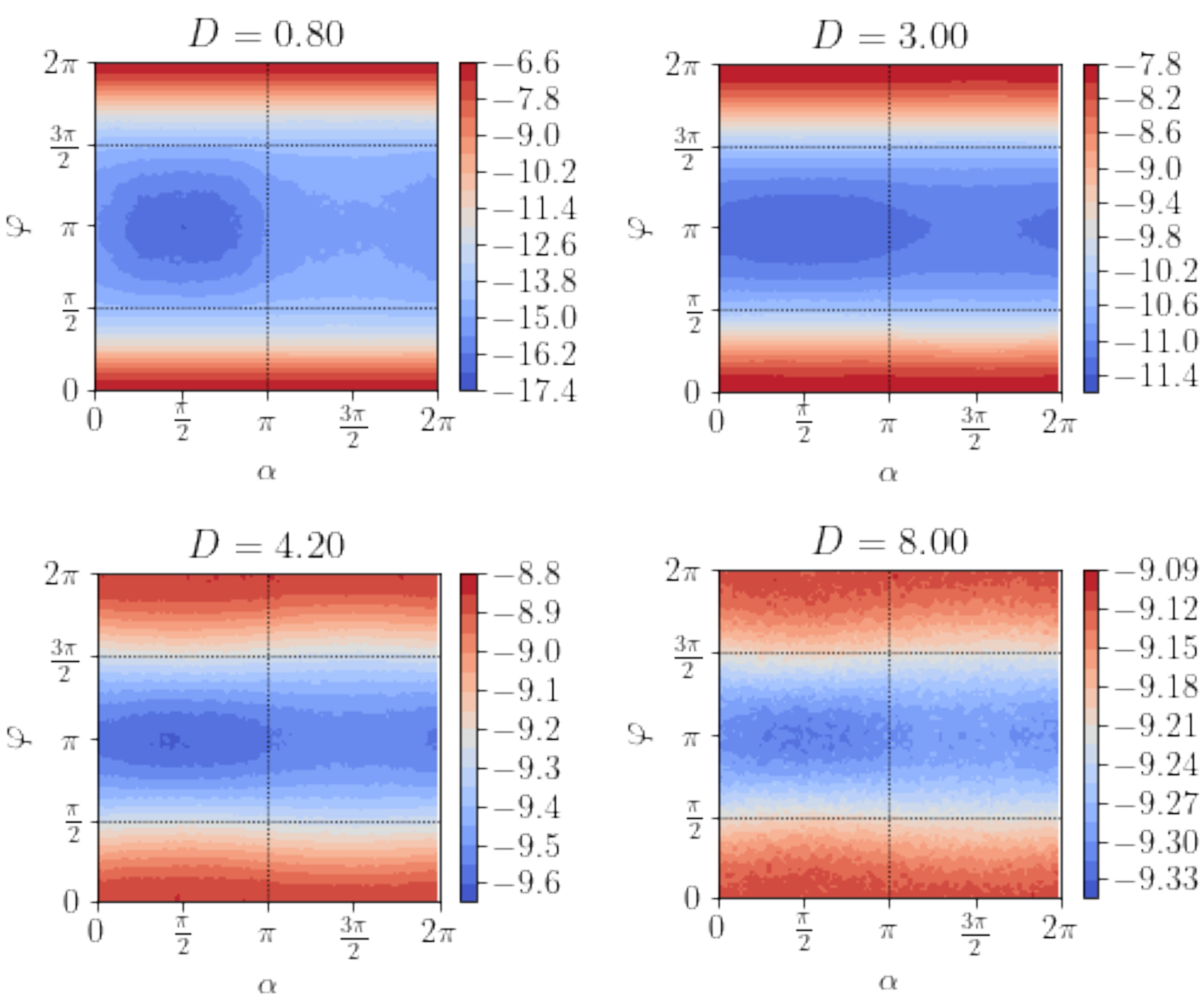
Asymmetries of the pair distribution

- Pair configuration parametrised by $\alpha, \varphi \; - \; q(\alpha, \varphi)$ number density

- Time reversal: $\alpha \mapsto \alpha + \pi$



$\log q(\alpha, \varphi)$



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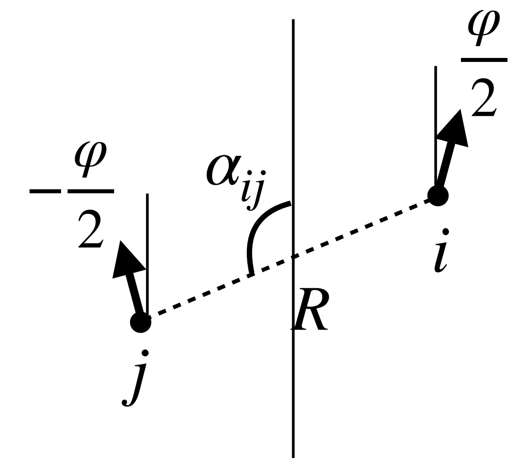

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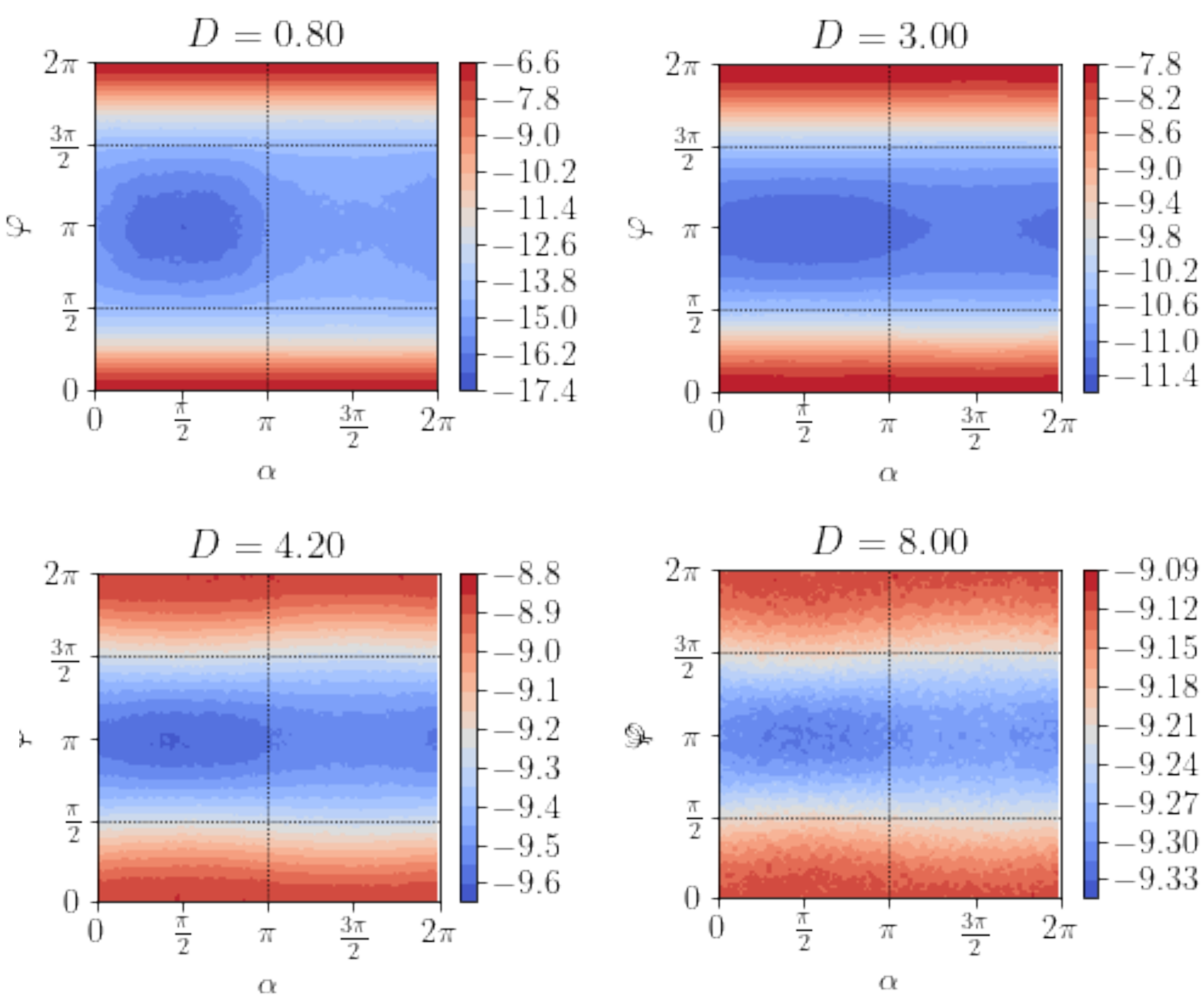
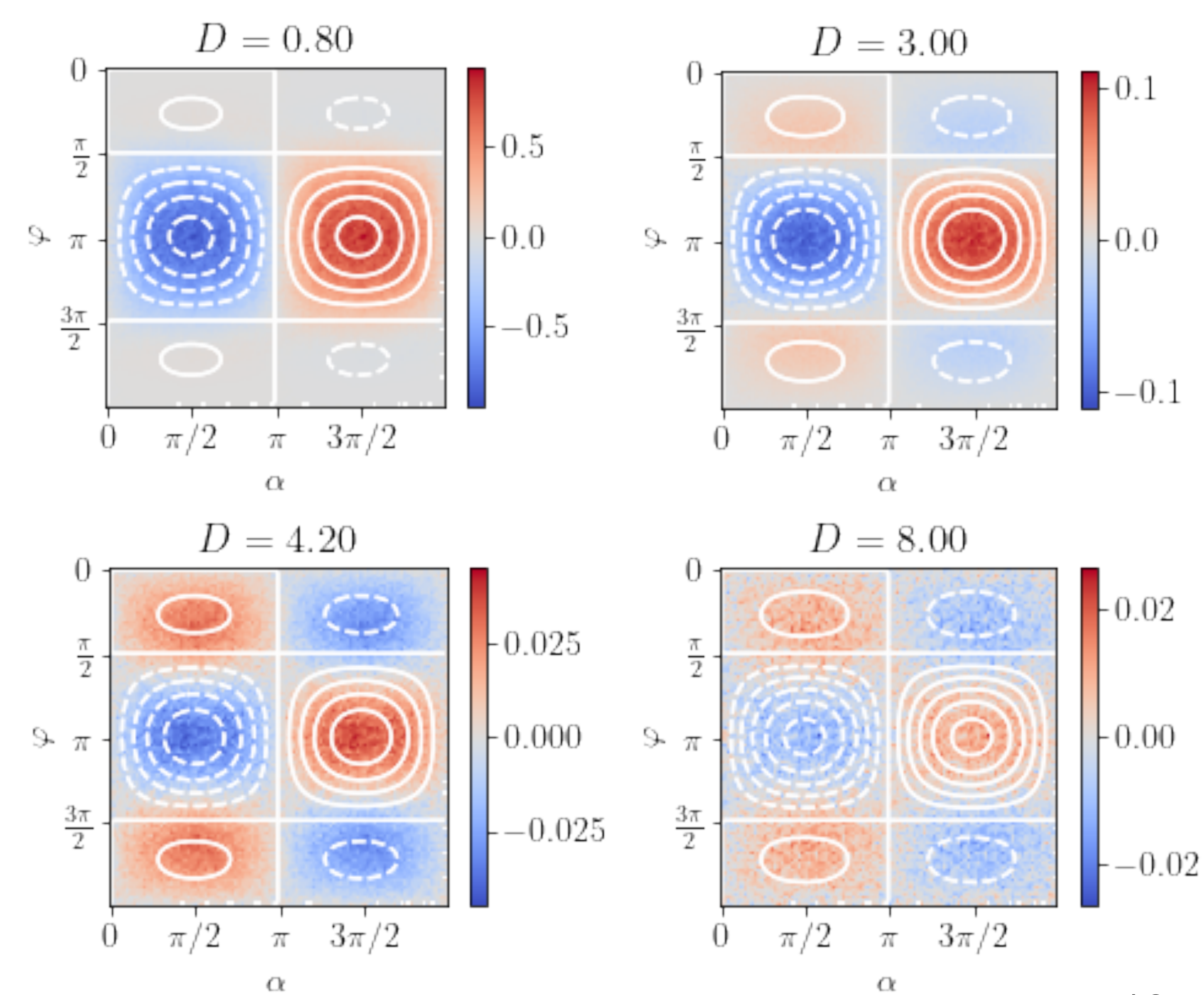
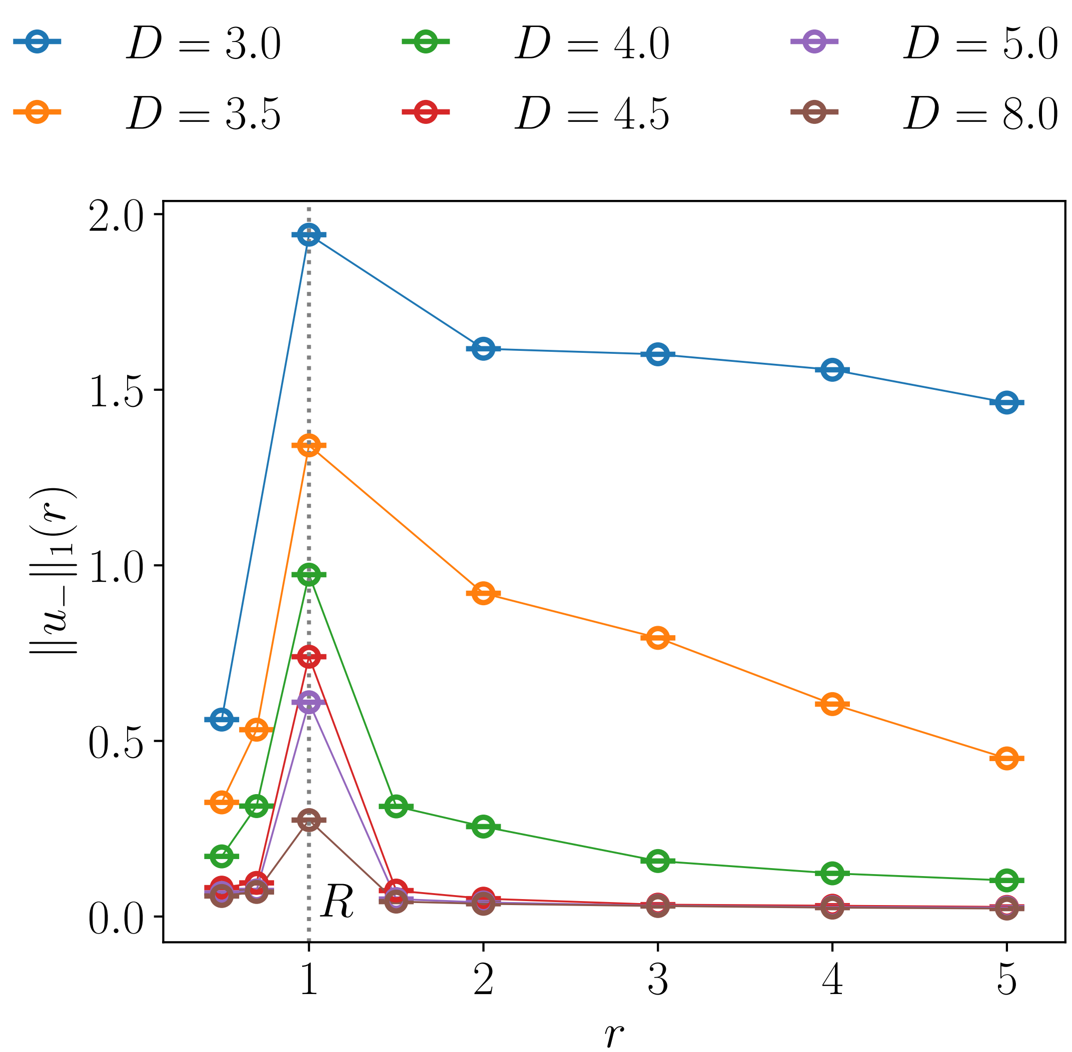


$\log q(\alpha, \varphi) - \log q(\alpha + \pi, \varphi)$

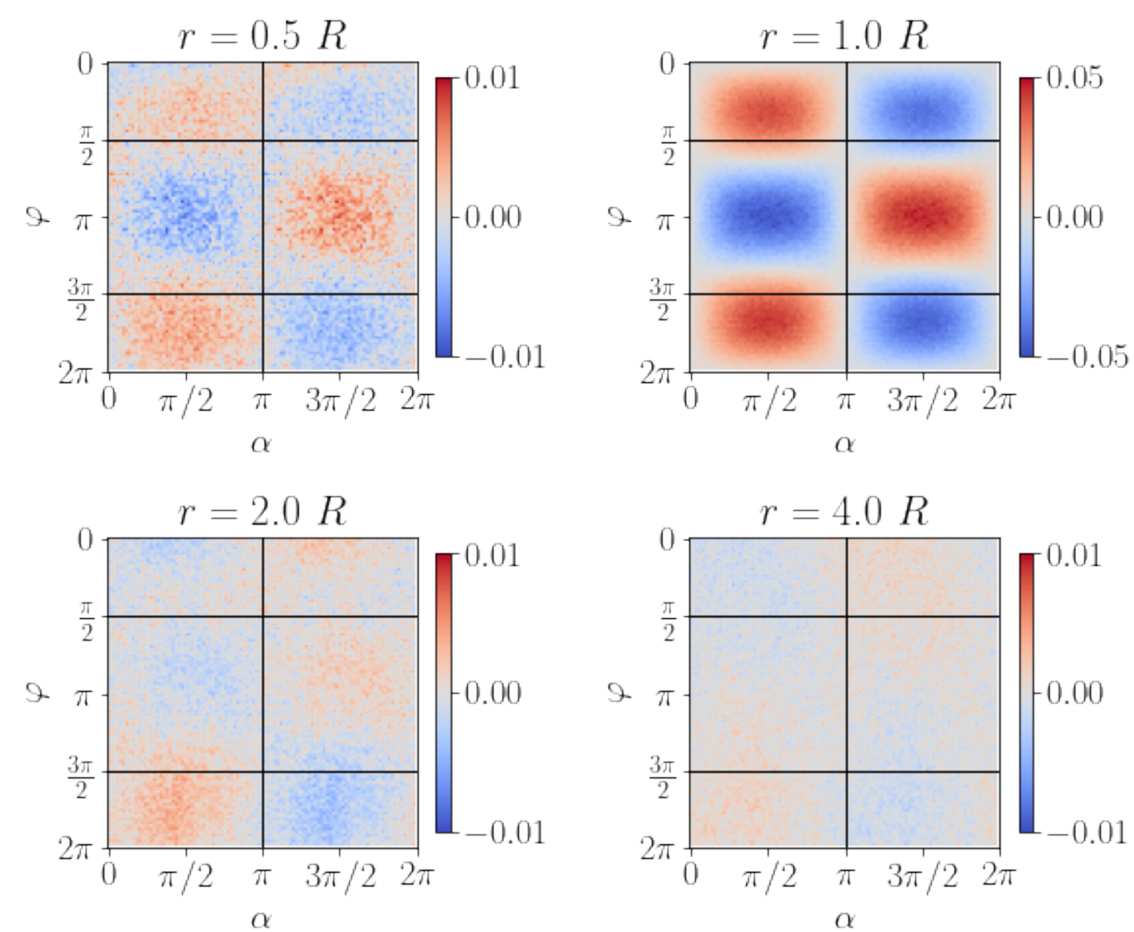


Asymmetries of the pair distribution

- Positiveness of the EPR constrains the way the asymmetry is realized.
- The asymmetry peaks at $r = R$ (inter-particle distance equal to interaction range)



$$u_- = \log q(\alpha, \varphi) - \log q(\alpha + \pi, \varphi)$$



A more general theoretical result

- In our metric model:

$$\dot{S} > 0 \implies q(\alpha, \varphi) \neq q(\alpha + \pi, \varphi)$$

Pair asymmetry can be seen as a low-dimensional projection of a more general asymmetry.

- In general for a flocking model of polar particles:

$$\dot{S} > 0 \implies \psi(X, \Theta) \neq \psi(X, \Theta + \pi)$$ (Steady-state pdf)

$X = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ positions

$\Theta = (\theta_1, \ldots, \theta_N)$ phases

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↓

$$\psi(X, V) \neq \psi(X, \mathscr{R}_{\theta_0}(V))$$

Explicit rotational symmetry breaking!

BUT…

$$\psi(X, V) = \psi\left(\mathscr{R}_{\theta_0}(X), \mathscr{R}_{\theta_0}(V)\right)$$

… The rotational symmetry in the coupled (X, V) space is only spontaneously broken!

Summary

- Collectively moving 2D flocks are genuinely out of equilibrium.
- Irreversibility is due to **rewiring** of the interaction network.
- Strongly polarized flocks (zero-temperature limit) or disordered groups of particles (infinite temperature) are affectively at equilibrium.
- General signatures of irreversibility in flocking models are asymmetries in the steady-state distribution.
- In the metric model, this asymmetry is best visible in the **low-dimensional** space of pair pdfs.

↓

Good for experiments!

Summary

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- Strongly polar … of particles (infinite tempe…

Thank you!

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Good for experiments!