Nuclear Matter and Neutron Stars: a window into Strong Interactions

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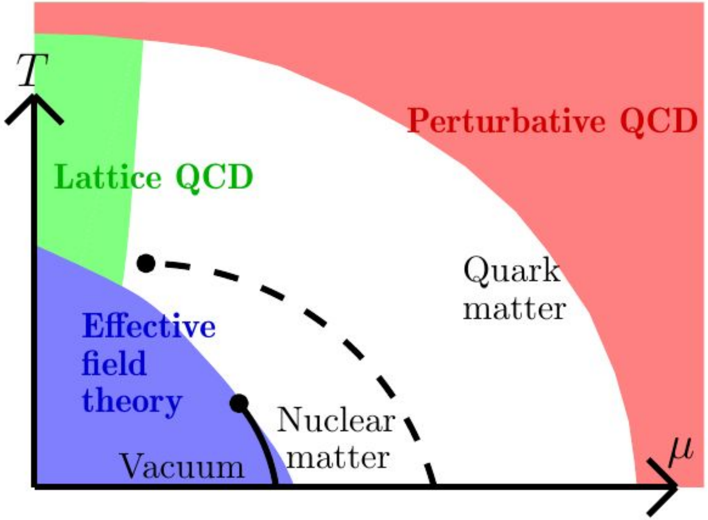
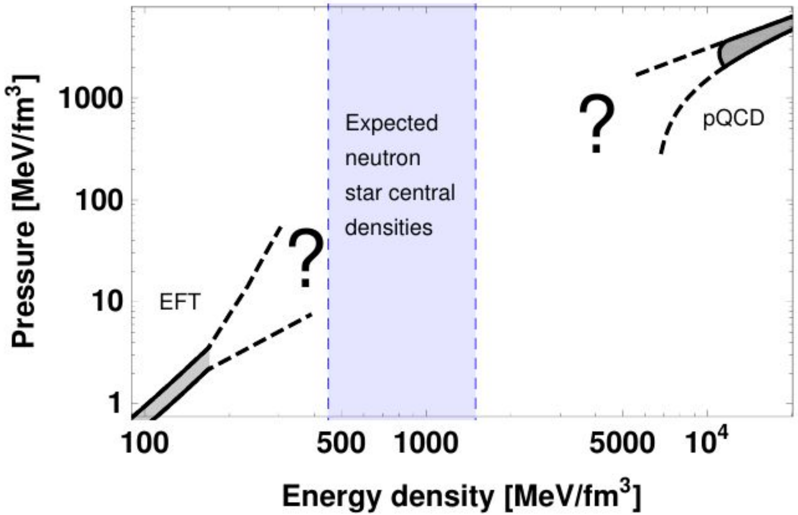
Overview

- The complicated puzzle of Strong Interactions
- Neutron Stars as precious source of information
- Modelling nuclear dynamics
- Understanding nuclear dynamics from astrophysical observations
- Summary and conclusions

QCD Phase Diagram

Quantum Chromodynamics (QCD) is well established as the fundamental theory of strong interactions...



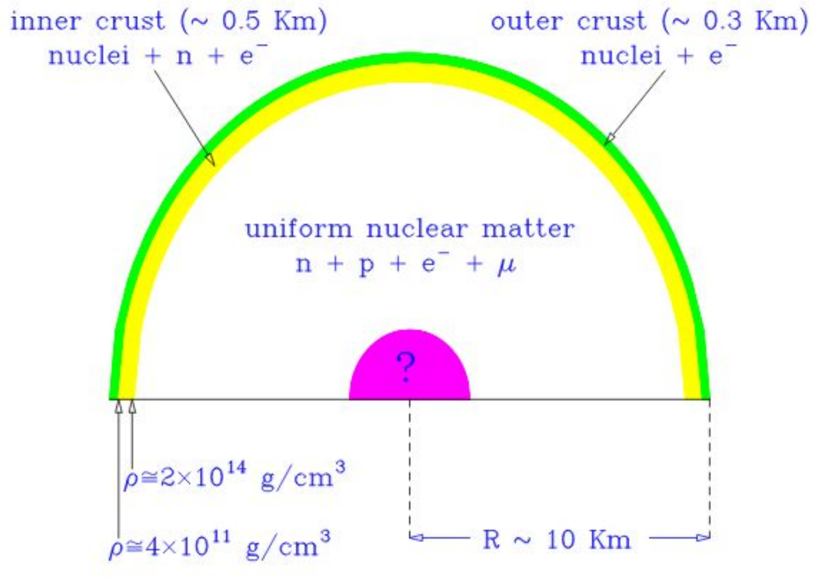


...however it turns out that it can't be used to describe the behavior of **dense and cold nuclear matter**!

Neutron Stars

Neutron Stars (NSs) are extremely compact objects, with masses as large as one or two solar masses and with radii of about ten kilometers.

Matter inside a NS can reach very extreme conditions, impossible for Earth-based experiments.



In the innermost region

$$\rho > \rho_0$$

$$\rho_0 = 2.67 \times 10^{14}\, g/cm^3 \; (0.16\, fm^{-3})$$

$$T \sim 10^9\, K << T_F$$

$$T_F \sim 10^{12}\, K$$

NSs provide a unique opportunity to investigate the properties of **nuclear matter** at **high density** and **low temperature.**

Non relativistic stellar structure equations

$$\frac{d\Phi(r)}{dr} = -\frac{1}{\rho(r)}\frac{dP(r)}{dr}$$

Hydrostatic equilibrium

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi(r)}{dr}\right) = 4\pi G\rho(r)$$

Gravitational potential

$$\frac{dP}{dr} = -G\frac{M(r)\rho(r)}{r^2};$$

$$M(r) = 4\pi\int_0^r r'^2\rho(r')\,dr'.$$

Neutron Stars

High compactness → *General Relativity*.

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Solving the Einstein equations leads to the Tolman Oppenheimer and Volkov (TOV) equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$T^{\mu\nu} = u^\mu u^\nu(\epsilon + P) + g_{\mu\nu}P$$

TOV →

$$\begin{cases} \dfrac{dM(r)}{dr} = 4\pi r^2 \epsilon(r) \\ \dfrac{dP}{dr} = -\dfrac{[\epsilon(r) + P(r)]\,[M(r) + 4\pi r^3 P(r)]}{r\,[r - 2M(r)]} \end{cases}$$

In order to solve TOV equations we have to specify the equation of state (EOS) of neutron star matter

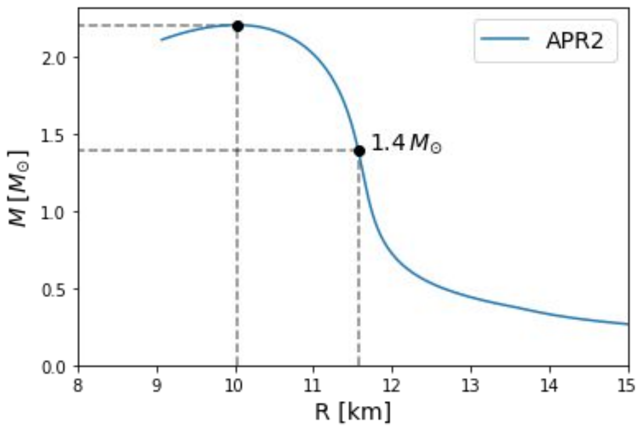
$$P = P(\epsilon)$$

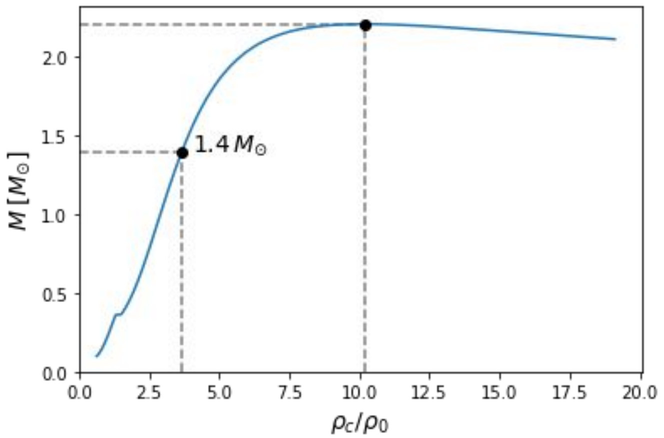
Neutron Stars

Non-interacting nucleons → $M_{\max} \sim 0.8\, M_{\odot}$

BUT we observe → $M \sim 1.4\, M_{\odot}$

Therefore we have to keep into account **interactions between nucleons!**





Nuclear Dynamics

Nuclear Dynamics: Nuclear Many Body Theory

In **Non-relativistic nuclear many body theory (NMBT)** we have point-like nucleons, interacting through nucleon-nucleon (NN) and three-nucleon (NNN) potentials.

$$\mathcal{H} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

- Once the Hamiltonian is defined the equation of state of cold nuclear matter is carried out by computing the ground state energy by means of variational approaches.

$$E_0 = \langle \psi_0 | \mathcal{H} | \psi_0 \rangle$$

A variational approach is necessary because of the strong repulsive core of NN interaction which cannot be treated in perturbation theory.

Nuclear Dynamics: Relativistic Mean Field

- Define a Lagrangian density ⟹ $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_B + \mathcal{L}_{\text{int}}$

$$\mathcal{L}_{\text{int}} = g_\sigma \phi(x)\bar{\psi}(x)\psi(x) - g_\omega V_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$$

- Solve the field equations in the mean field approximation

$$\phi(x) \to \langle \phi(x) \rangle \,, \quad V_\mu(x) \to \langle V_\mu(x) \rangle$$

- Identify the thermodynamic quantities by means of the energy-momentum tensor

$$\langle T_{\mu\nu} \rangle = u_\mu u_\nu \left(\epsilon + P \right) - g_{\mu\nu} P \quad \Longleftrightarrow \quad T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$$

Summary

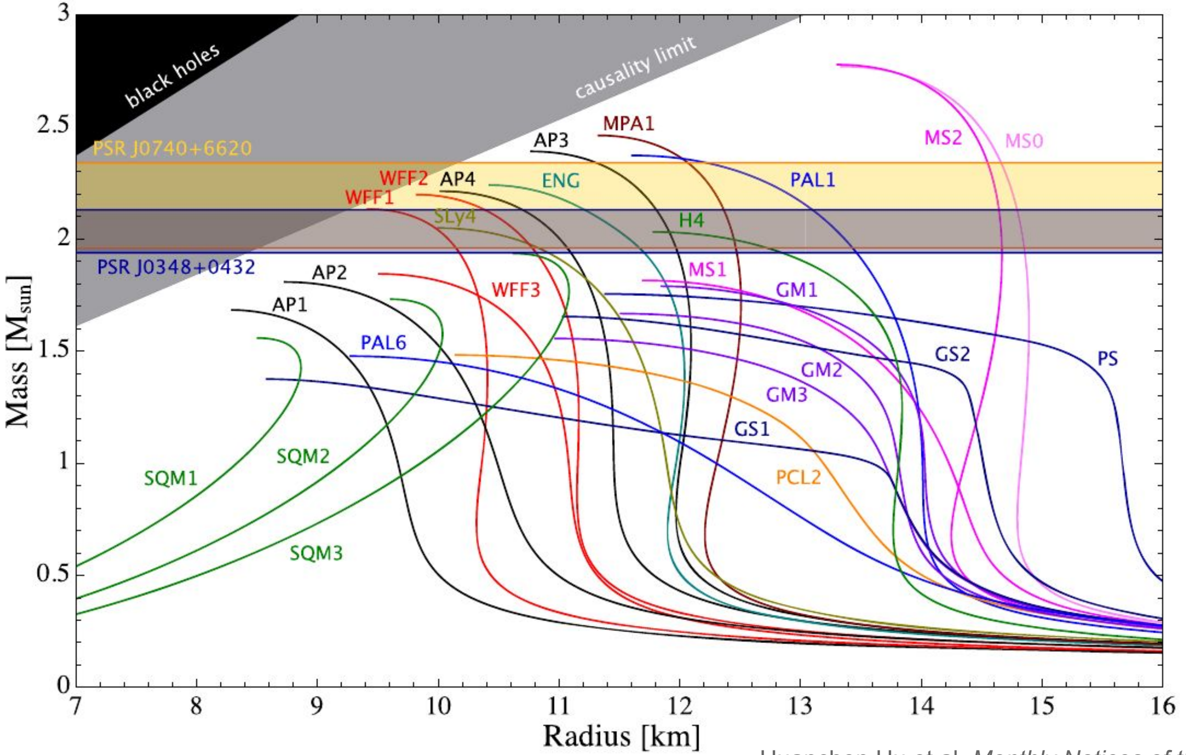
Non-relativistic many body theory

- Advantages
  - Well defined dynamics
  - Good description of nuclear systems
- Drawbacks
  - Causality violation at high density
  - Involves very complicated calculations related to the computation of many-body expectation values

Relativistic Mean Field

- Advantages
  - Satisfies causality by construction
  - Does not involve too many technical difficulties
- Drawbacks
  - Oversimplified dynamics
  - Apparently not fully justified at neutron star densities

How can we choose between different EOS models?

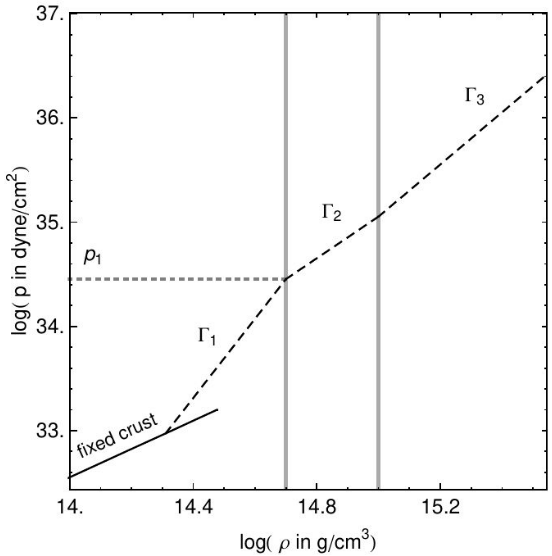


Huanchen Hu et al, *Monthly Notices of the Royal Astronomical Society*, Volume 497, Issue 32020, Pages 3118–3130

Bayesian Inference on Astrophysical Data

A large set of different EOSs can be described by a unified parametric model.
If we have a parametric EOS we can infer the probability distribution of its parameters from astrophysical data by means of Bayes theorem

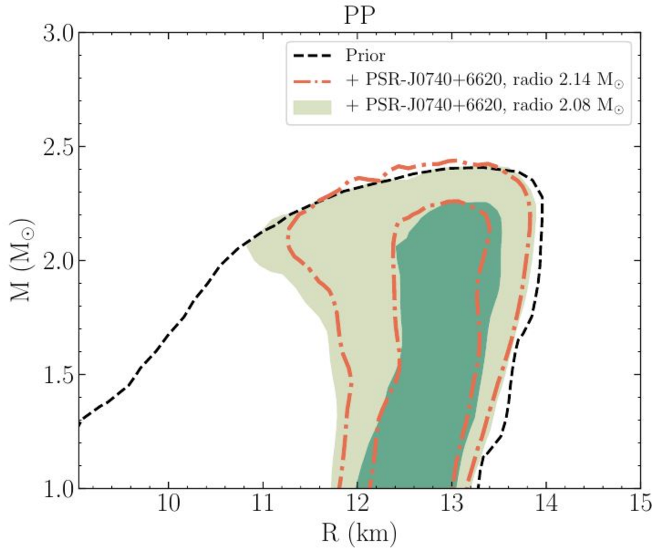
$$\mathcal{P}(\theta|O) \propto \mathcal{P}_0(\theta)\prod_{i=1}^{n}\mathcal{L}(O^{(i)}|D(\theta))$$



J. S. Read et al Phys. Rev. D 79, 124032 (2009)

$$\mathcal{L}_{EM}(M(\theta), R(\theta))$$

$$\vec{\theta} = (\Gamma_1, \Gamma_2, \Gamma_3, p_1)$$



G. Raaijmakers *et al* *ApJL* 918 L29 (2021)

A Quick Overview of Our Work

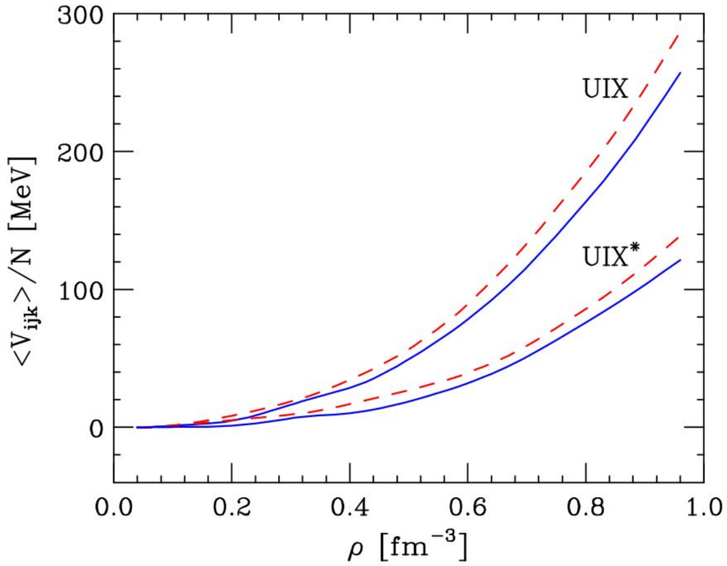
Relativistic Corrections

Boost-corrections to the NN potential are an attempt to estimate relativistic effects in the framework of Quantum Mechanics.

The Hamiltonian becomes:

$$\mathcal{H}^* = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} (v_{ij} + \delta v(\vec{P}_{ij})) + \sum_{i<j<k} V^*_{ijk}$$

The presence of the boost interaction accounts for the 37% of the the repulsive contribution of the NNN interaction.

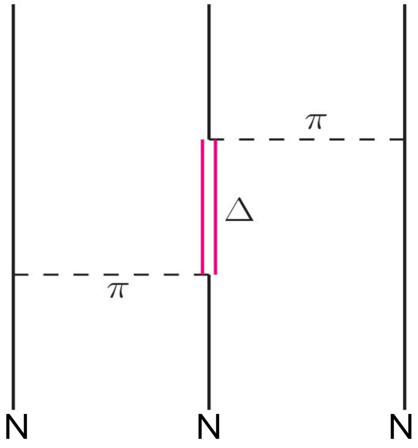


Three-Nucleon Potential

Three-nucleon interactions must be introduced in order to account for processes involving the internal structure of nucleons.

The **Urbana IX** (UIX) model of three-nucleon potential, used in the derivation of the APR EOS comprises two terms.

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$



Two free parameters that are adjusted in order to reproduce the **binding energy of tritium** and the correct value of the **nuclear saturation density.**

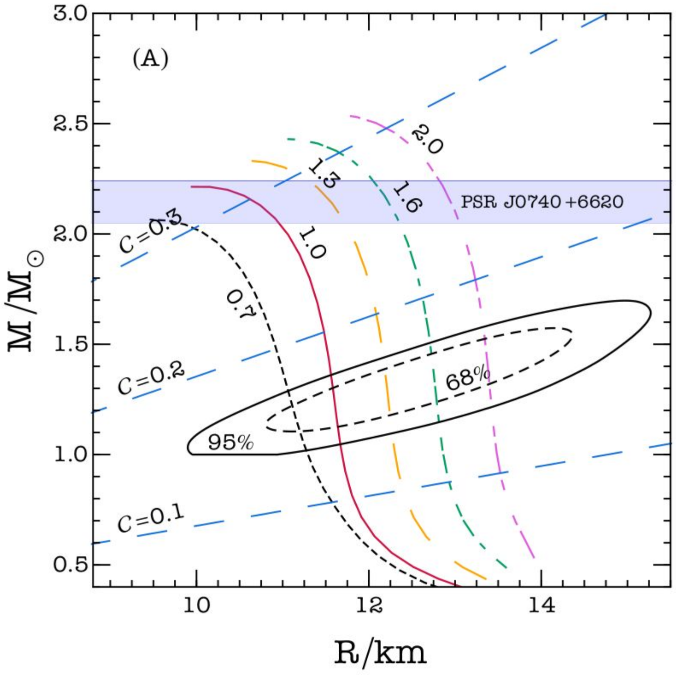
The value of these free parameters strongly depend on the strength of the NN potential!

A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998)

Constraining three-body force with multimessenger data

$$\langle V_{ijk}^R \rangle \rightarrow \alpha \langle V_{ijk}^R \rangle$$

NICER PSR J0030+0451



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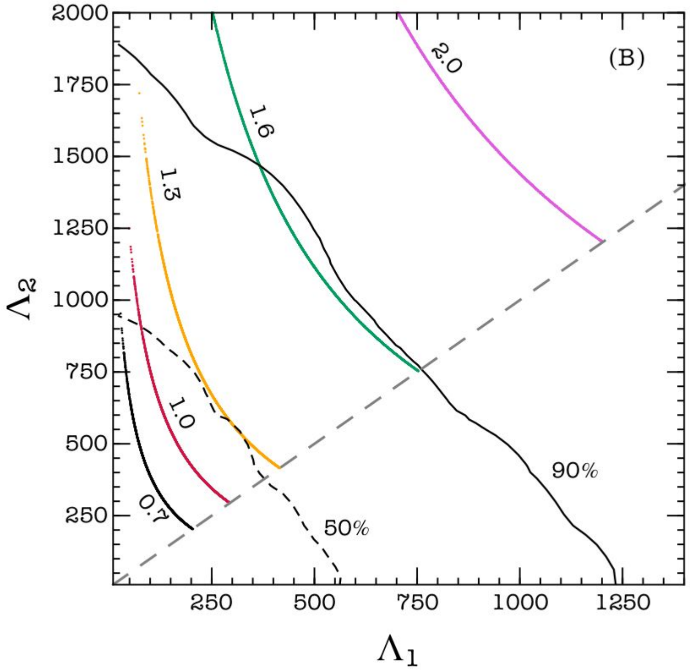
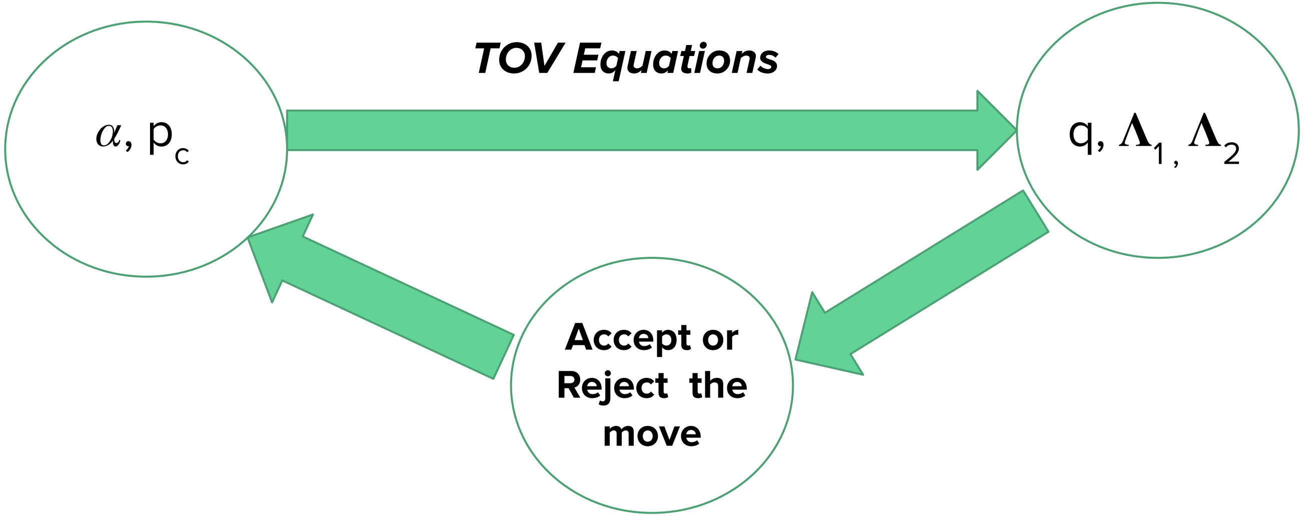
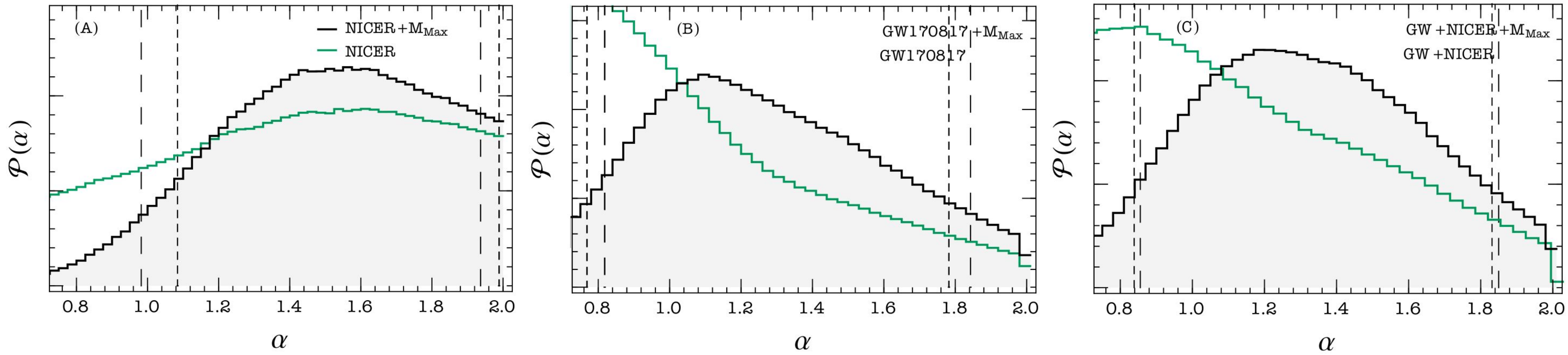


Sampling The Posterior

$$\mathcal{P}(\alpha, p_c^{(1)} | O_{\rm GW}) \propto \mathcal{P}_0(\alpha, p_c^{(1)}, p_c^{(2)}) \mathcal{L}_{\rm GW}(q, \Lambda_1, \Lambda_2)$$



Posterior Distributions



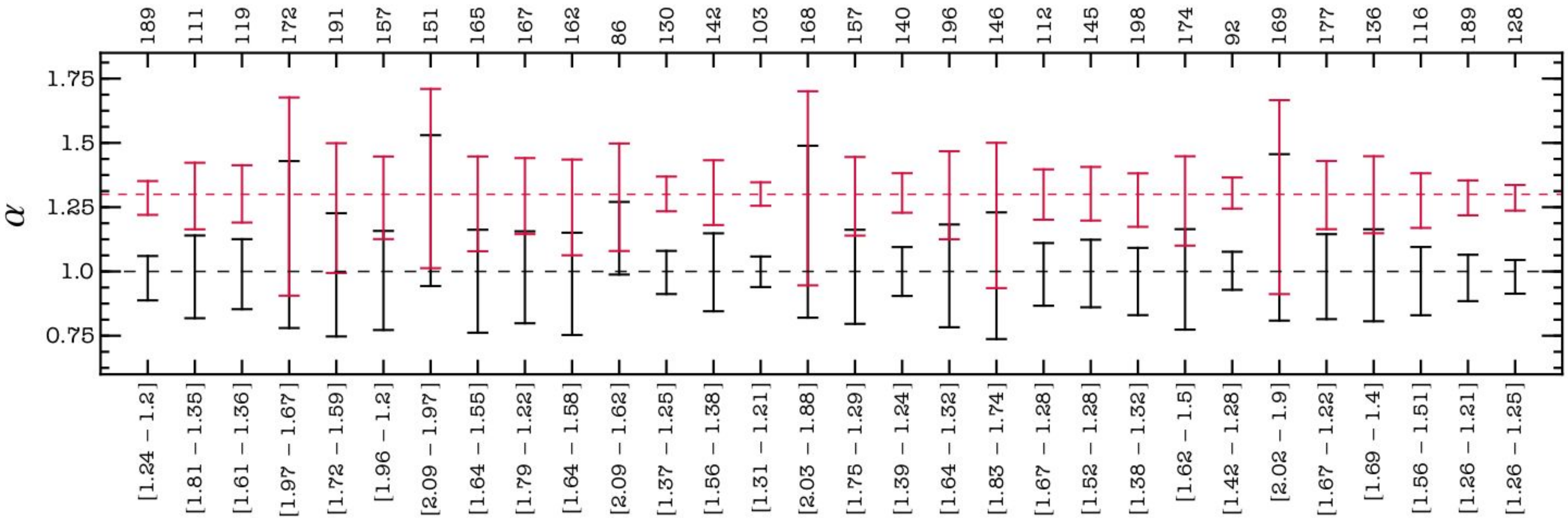
The GWs alone turn out to be not enough to extract relevant information about the strength of NNN repulsion and the shape of the PDF appears to be dominated by the maximum mass requirement.

However this analysis, yielding $\alpha_{\mathrm{GW+EM}} = 1.32^{+0.48}_{-0.51}$ has shown that there is sensitivity of NS observables with respect to the considered microscopic parameter.

A. Maselli, A. Sabatucci and O. Benhar, Phys. Rev. C 103, 065804, 2021

Einstein Telescope

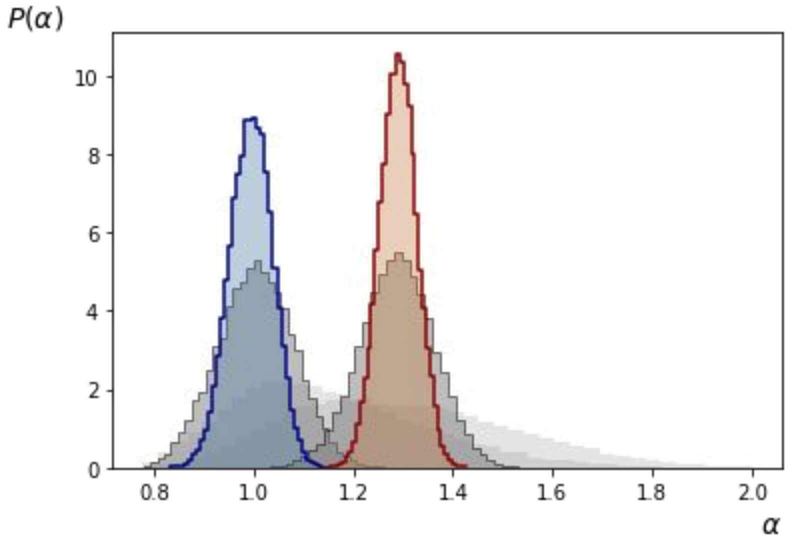
Here we can see the 90 % confidence level of α for a set of 30 simulated Einstein Telescope (ET) events.

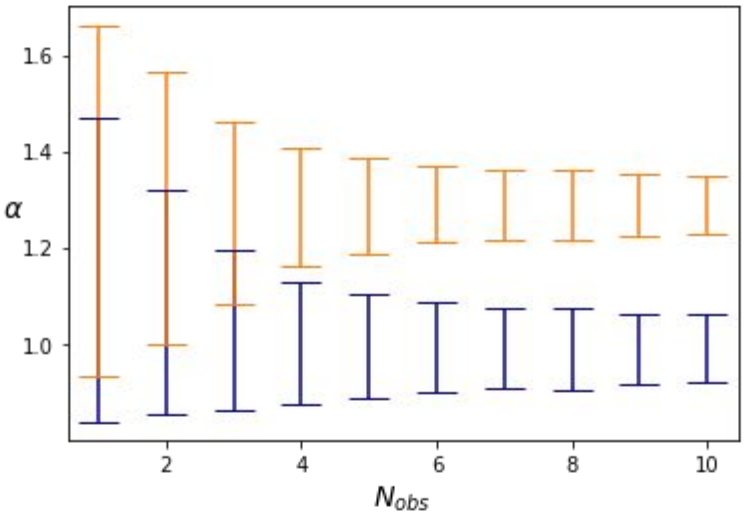


A. Sabatucci, A. Maselli, O. Benhar and C. Pacilio (coming soon)

Einstein Telescope

Combining up to 10 different ET observations.





Summary and Outlook

- It is currently impossible to understand the properties of dense and cold nuclear matter starting from QCD
- Neutron Stars represent a precious source of information
- We have to carry out realistic models as much as possible constrained by empirical data
- There exist different approaches to describe nuclear matter
- Astrophysical observations have the potential to drive us towards the right direction
- With the expected improvements and the upcoming new generation of Gravitational Wave detectors the future looks bright

Thank you for the attention!