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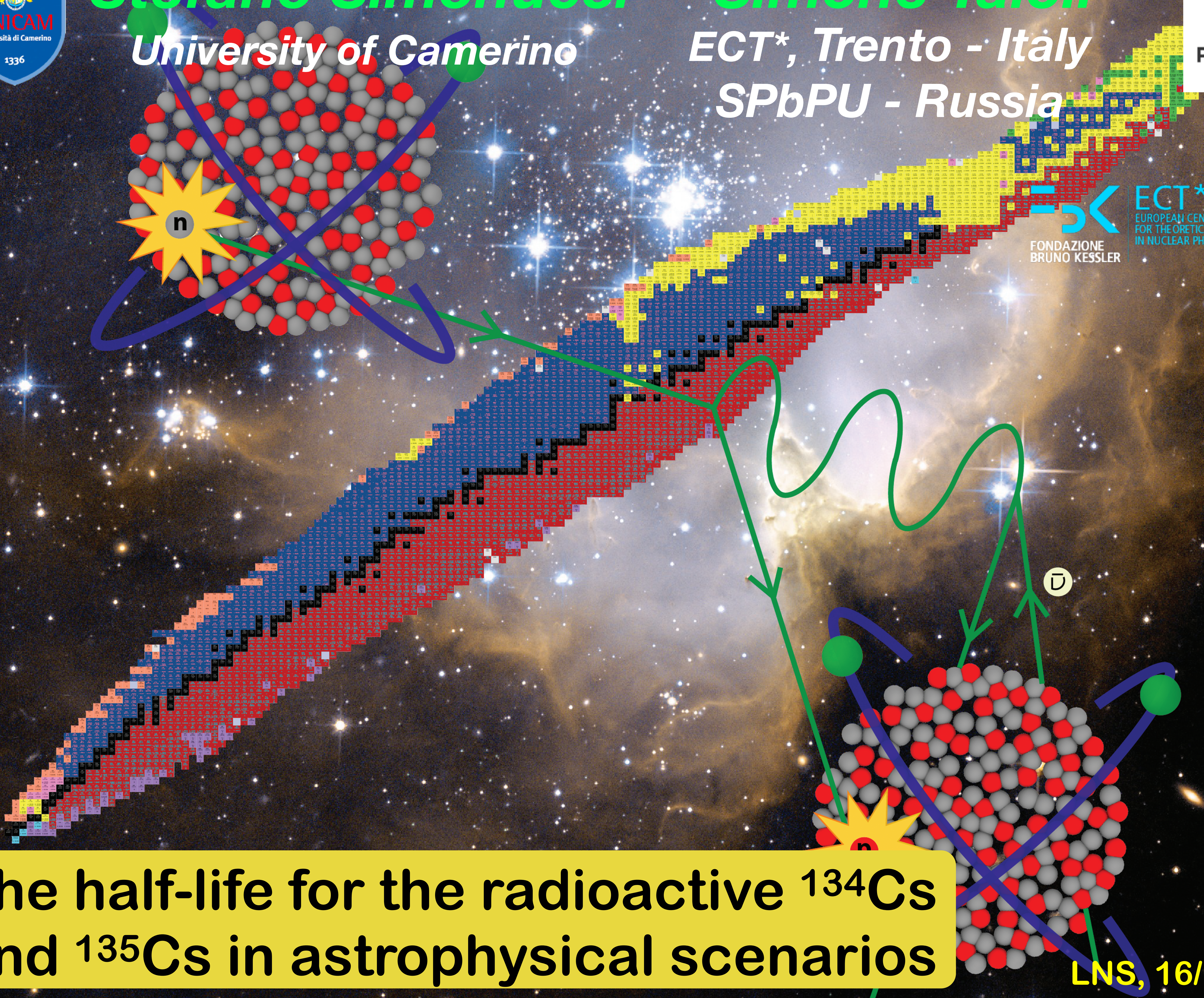
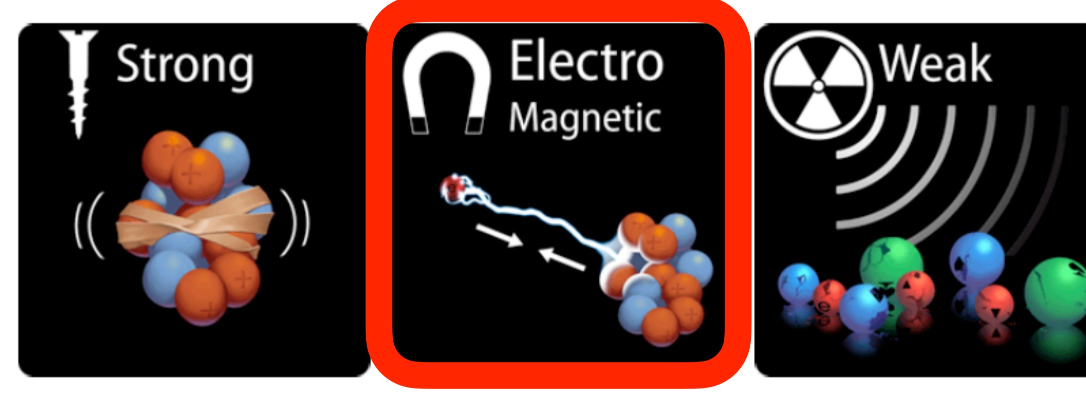


The half-life for the radioactive ^{134}Cs and ^{135}Cs in astrophysical scenarios

LNS, 16/12/2021

Outline



- Introduction to our relativistic approach to β-decay (β-decay and EC are not only purely nuclear but also atomic processes)

- Application of our approach to a number of β-decay processes in heavy atoms
  $^{36}Cl, ^{63}Ni, ^{129}I, ^{210}Bi, ^{241}Pu,$

- Application to β-decay of ^{134}Cs and ^{135}Cs in astrophysical environment

- Perspectives, future developments

D. Mascali
(INFN-LNS)

We are deep in debt to ....

T. Morresi
(Sorbonne)

S. Palmerini
(UniPG)

M. Busso
(UniPG)

Motivation of this work: provide the missing weak-interaction input data for Li nucleosynthesis calculations

$$ {}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{L}i + \nu_e $$

At ambient conditions ^{7}Be decays in 53 days into the ground state of ^{7}Li (3/2-) for 89.7% of cases, 10.3% it decays into the first excited state (1/2-)

Decay may occur by capture of an orbital e^{-} through the following process:

$$ p + e^{-} \rightarrow n + \nu_e $$

change Z to Z-1 of an atom, keeping fixed A

$$ W \propto \delta(\vec{r}) $$

insensitive to extra-nuclear factors, such as chemical environment, ionization degree, pressure and temperature.

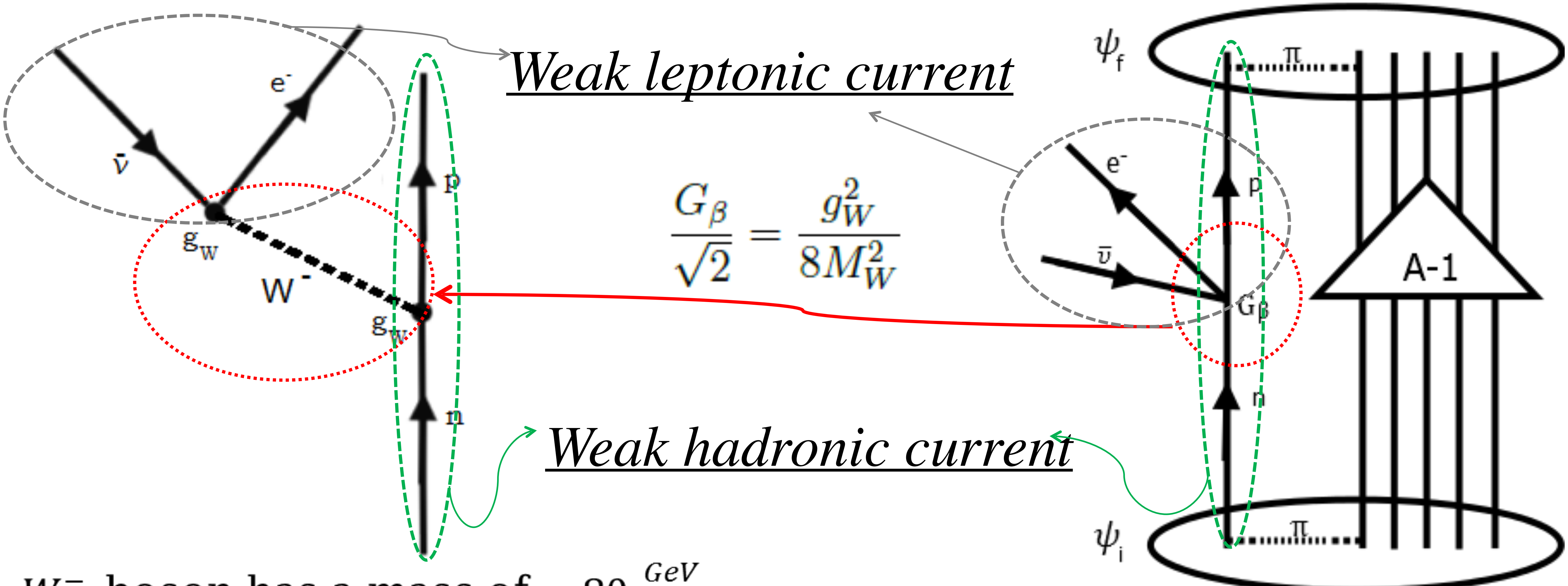
The driving force responsible for this decay is the weak interaction

One of the three β-decays mediated by the weak force.

Contrary to this simple view, there is evidence of changes in nuclear decay rates with these parameters. Why and how?

β-decay: tool basket

- **Standard Model of Particle physics: weak interaction is caused by emission or absorption of very massive bosons**



W^- boson has a mass of $\sim 80\ \frac{GeV}{c^2}$

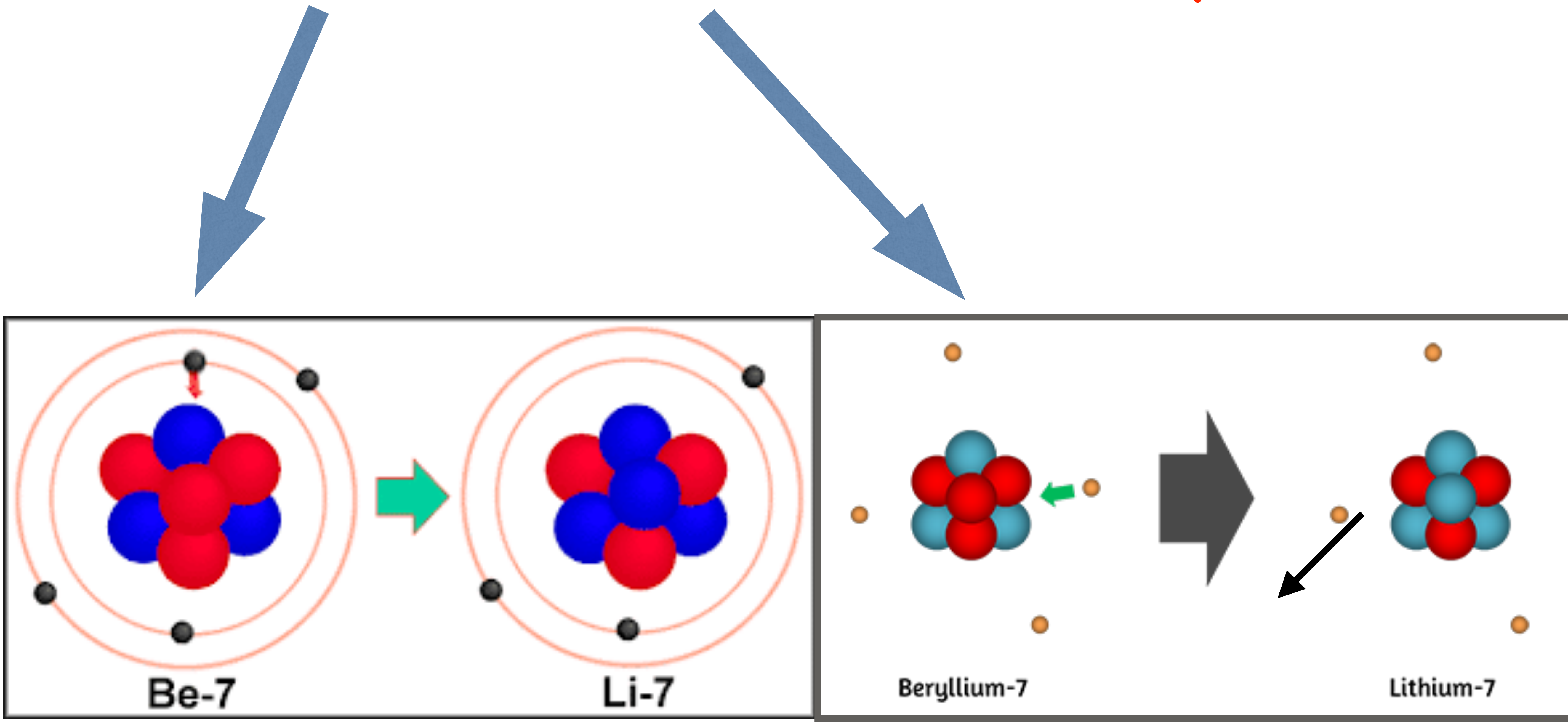
→ *Uncertainty principle*: W^- can only live $\Delta t \leq \frac{\hbar}{M_W \cdot c^2}$

→ During this time it can travel at most $c\Delta t \leq \frac{\hbar}{M_W \cdot c} \sim 10^{-3}\ fm \ll 1\ fm$

Short range=Fermi contact interaction

(range of strong interaction)

Bound or continuum electron capture



How do we actually calculate e-capture rates?

In particular we generalize the theory of scattering under two potentials in the center of mass, reducing the problem to a two-body scattering:

V = screened, short-range Coulomb potential
W = weak interaction coupling the Coulomb distorted initial state and the final decay channels

The cross section of the electron capture process can be written as:

$$\sigma_{i\to f} = \int \frac{d^3k}{(2\pi)^3}\frac{2\pi}{v}\left|\langle\psi^-_{f,\boldsymbol{k}}|W|\phi^+_{i,\boldsymbol{p}}\rangle+\langle\phi^-_{f,\boldsymbol{k}}|V|\phi_{i,\boldsymbol{p}}\rangle\right|^2\delta\left(\frac{p^2}{2m_e}+E_i-E_f-ck\right)$$

$$=\int \frac{d^3k}{(2\pi)^3}\frac{2\pi}{v}\left|\langle\phi^-_{f,\boldsymbol{k}}|T_w|\phi^+_{i,\boldsymbol{p}}\rangle\right|^2\delta\left(\frac{p^2}{2m_e}+E_i-E_f-ck\right)$$

Coulomb operator does not couple ini and fin channels

=0

$\phi_{i,\boldsymbol{p}}$ **= free-plane wave**

$\phi^-_{f,\boldsymbol{k}}$ **= Coulomb perturbed out-state (ν emitted and target in final state f)**

$\psi^-_{f,\boldsymbol{k}}$ **= Coulomb and weak perturbed out-state (ν emitted and target in final state f)**

$\phi^+_{i,\boldsymbol{p}}$ **= Coulomb perturbed in-state (Coulomb distort + outgoing spherical)**

E_i, E_f = internal energies of the target ^{7}Be and of the final decay product
$p = m_e v$ and k are relative e^- and neutrino momenta in the initial and final channels
v = electron velocity in the initial channel relative to ^{7}Be.

How do we actually calculate e-capture rates?

We can define the T-matrix of the weak interaction as:

$$\left\langle \psi^-_{f,\boldsymbol{k}} | W | \phi^+_{i,\boldsymbol{p}} \right\rangle = \left\langle \phi^-_{f,\boldsymbol{k}} | \mathrm{T}_W | \phi^+_{i,\boldsymbol{p}} \right\rangle$$

By multiplying the c.s. by the e$^-$ current one obtains the e-capture rate:

$$\Gamma_{i\to f} = \int 2\pi \frac{d^3k}{(2\pi)^3} \left| \left\langle \phi^-_{f,\boldsymbol{k}} | T_w | \phi^+_{i,\boldsymbol{p}} \right\rangle \right|^2 \delta\left(\frac{p^2}{2m_e} + E_i - E_f - ck \right) = \frac{\bar{k}^2}{\pi c} \left| t_{f,i} \langle i, 0 | \phi^+_{i,\boldsymbol{p}}(0) \rangle \right|^2$$

$$= \frac{1}{\pi c^3} \left| t_{f,i} \right|^2 \langle i, 0 | \phi^+_{i,\boldsymbol{p}} \rangle \left(\frac{p^2}{2m_e} + E_i - E_f \right)^2 \langle \phi^+_{i,\boldsymbol{p}} | i, 0 \rangle$$

$H_0 + V$

$T_W \propto \delta(\boldsymbol{r})$

where $\bar{k} = \frac{1}{c}\left(\frac{p^2}{2m_e} + E_i - E_f \right)$ **and** $\langle \phi^+_{i,\boldsymbol{p}} | i, 0 \rangle$ **is the electronic w.f. at the Be nucleus.**

Approximations made

1. 1st Be e.s. is found at 429.4 keV=5X10^9 K above the ground state
2. $T_W \propto \delta(\boldsymbol{r})$ = very short range contact interaction
3. $t_{f,i}$ are chosen equal to those measured on the Earth, neglect dependence on T and $p^2/2m_e$

IMPORTANT OUTCOME!

^{7}Be-e$^-$ can be modelled as a two-body scattering process at a given relative electron momentum p.

The rate is proportional to $\rho_e(0)$.

Be e-capture

$${}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{L}i + \nu_e$$

At ambient conditions ^{7}Be decays in 53 days into the ground state of ^{7}Li (3/2-) for 89.7% of cases, 10.3% it decays into the first excited state (1/2-)

The energy of the Li excited state is 477.6 keV (~6X10^{9} K) higher than GS

Be Q_0 and Q_1 the kinetic energies of the neutrinos escaping from ^{7}Li in its ground and first excited state

Q_0 = 861.815 keV ⟷ $Q_1 = Q_0$ - 477. 6 = 384. 2 keV

Since the kinetic energy is higher in the first case, the available phase space will be larger. We can roughly estimate that for T = 10^{7} K:

$$BR = 89.7/10.3 \times (Q_0 + kT)^2/(Q_1 + kT)^2/(Q_0^2/Q_1^2) = 8.684$$

The percentage variation of BR due to an increase of the temperature by five orders of magnitude is thus only 0.3% → **neglect e.s. decay!!!**

1^{st} Be e.s. is found at 429.08 keV=5X10^{9} K above the ground state

Major problem:

find a good theory to model for different T and ϱ the hot plasma composed by ^{7}Be atoms surrounded by N_p protons (hydrogen nuclei) and N_e electrons, as a degenerate (quantum) Fermi gas, taking into account accurately the electron-electron interaction!

How do we actually calculate e-capture rates?

The e-capture rate for ^{7}Be is proportional to the electronic density at the nucleus!!!

Factors affecting this density, such as T (charge state distribution), ϱ, the level of ionization and the presence of other charged particles, screening the interaction, can appreciably modify the decay rate

How to calculate $\rho_e(0)$?

State-of-the-art techniques are based on the the Debye-Hückel (DH) models of screening, valid only for solar conditions and when electrons are not degenerate (but in RBG they could).

Does DH approximation really stand???

Our model system of stellar plasma is a Fermi gas in the presence of neutralising particles, such as proton, helium, etc…

Condition of the stellar material at high T

DEGENERACY CONDITIONS: CLASSICAL vs. QUANTUM

- The separation between identical particles is << λ_{DB}
- The density is >> N_q where N_q is the number of available quantum states

Solar core: T=15.6 X 10^6 K → ^{7}Be atoms are all ionized

(12000 K = 1 eV)!!!

De Broglie wavelength in the core of the Sun

$$l << \lambda_{DB} = h/p \simeq h/(3m_e kT)^{1/2} = 2.731 \times 10^{-11}\ m$$

Electronic density

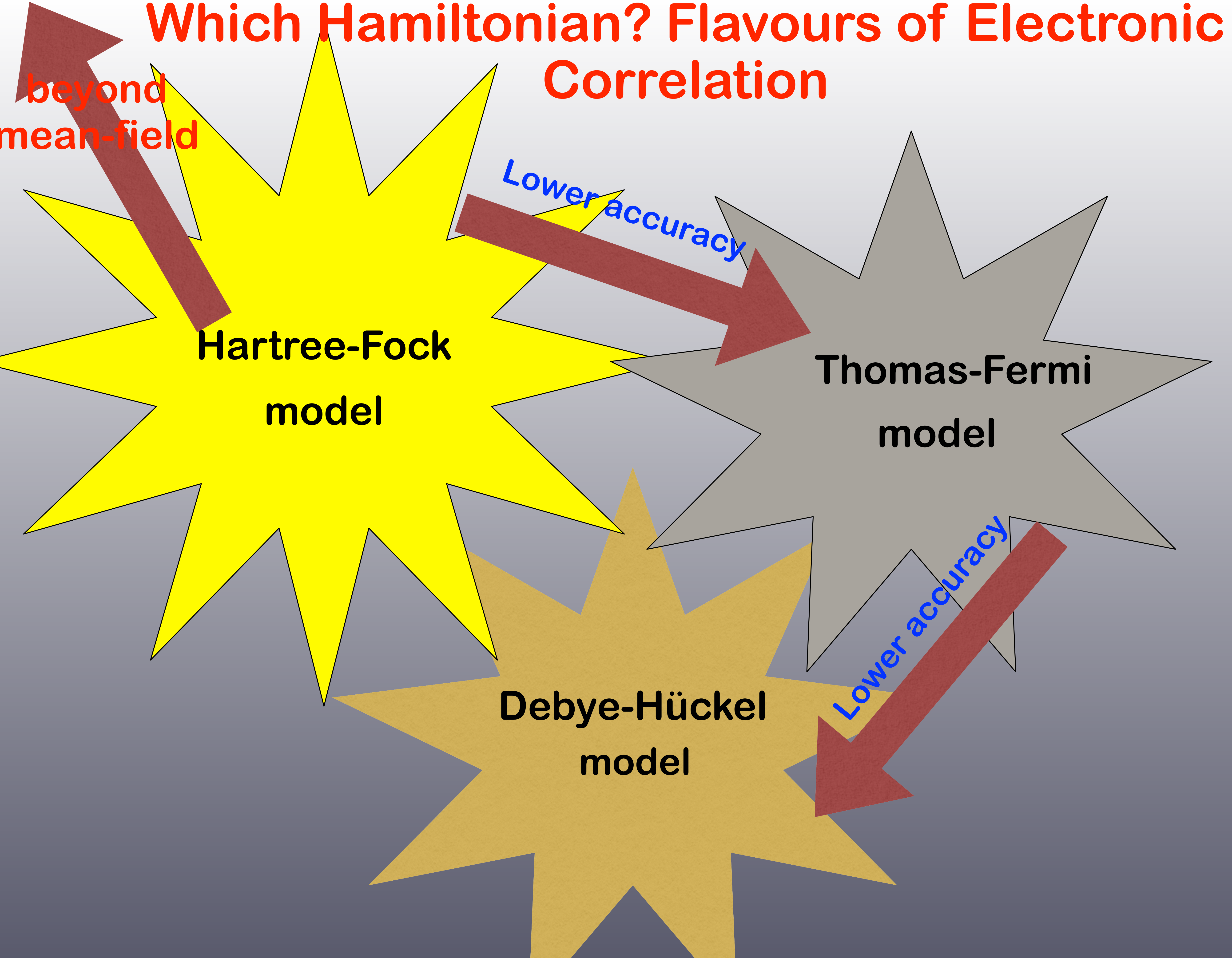
$$\rho_e >> n_{QNR} = (2\pi m_e kT/h^2)^{3/2} = 6.65 \times 10^{31}\ m^{-3}$$

To have degeneracy $T << h^2 \rho^{2/3}/(2\pi mk) = 9.12 \times 10^6$ K

In the solar core the temperature is marginally too high for degeneracy of electrons, but decreasing R can set it in...

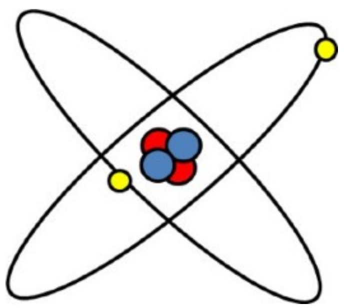
$T \propto 1/R$ and thus $n_{QNR} \propto T^{(3/2)} \propto R^{-3/2}$, which cannot keep the pace with $\rho_e \propto R^{-3}$ Cold? Fermi gas can be degenerate even at millions of K.

Which Hamiltonian? Flavours of Electronic Correlation



Hartree-Fock, TF, DH within BO approximation

There are 2 mechanisms to avoid each other: exchange and correlation, both lower the total energy and dress the e⁻-e⁻ bare interaction.



Many-body problem is replaced by many 1-body problem in which e⁻ are independent and feel an average potential $V_m(r)$

Correlation keeping the electrons apart is just among unsociable same spin electrons: Pauli exclusion principle

$$\Psi(r) = \psi_a(r_1)\psi_b(r_2) - \psi_a(r_2)\psi_b(r_1)$$

Thomas and Fermi (1920s) were the first to give an approximate expression of E as a function of the electronic density.

The kinetic, electronic exchange and correlations terms are taken from the theory of the uniform electron gas:

$$E[\rho] = T_{\mathrm{TF}}[\rho] + V_{\mathrm{ne}}[\rho] + J[\rho] + E_{\mathrm{x}}[\rho] + V_{\mathrm{nn}}$$

$$T_{\mathrm{TF}}[\rho] = C_{\mathrm{F}} \int_{\mathbb{R}^3} \rho^{5/3}(\mathbf{r})\, d^3\mathbf{r} \qquad E_{\mathrm{x}}[\rho] = -C_{\mathrm{x}} \int_{\mathbb{R}^3} \rho^{4/3}(\mathbf{r})\, d^3\mathbf{r}$$

Electronic density is far from uniform in a plasma

DH: Fermi-Dirac statistics to Boltzmann distribution linear in T

Some data...

Energy of the Isolated Beryllium Atom in Atomic Units and Spin-up Density at the Nucleus Obtained Through the HF and CI Calculations

	Energy	$\rho_{e\uparrow}(0)$
Hartree–Fock	−14.573	17.68521
Full-CI	−14.660	17.68060

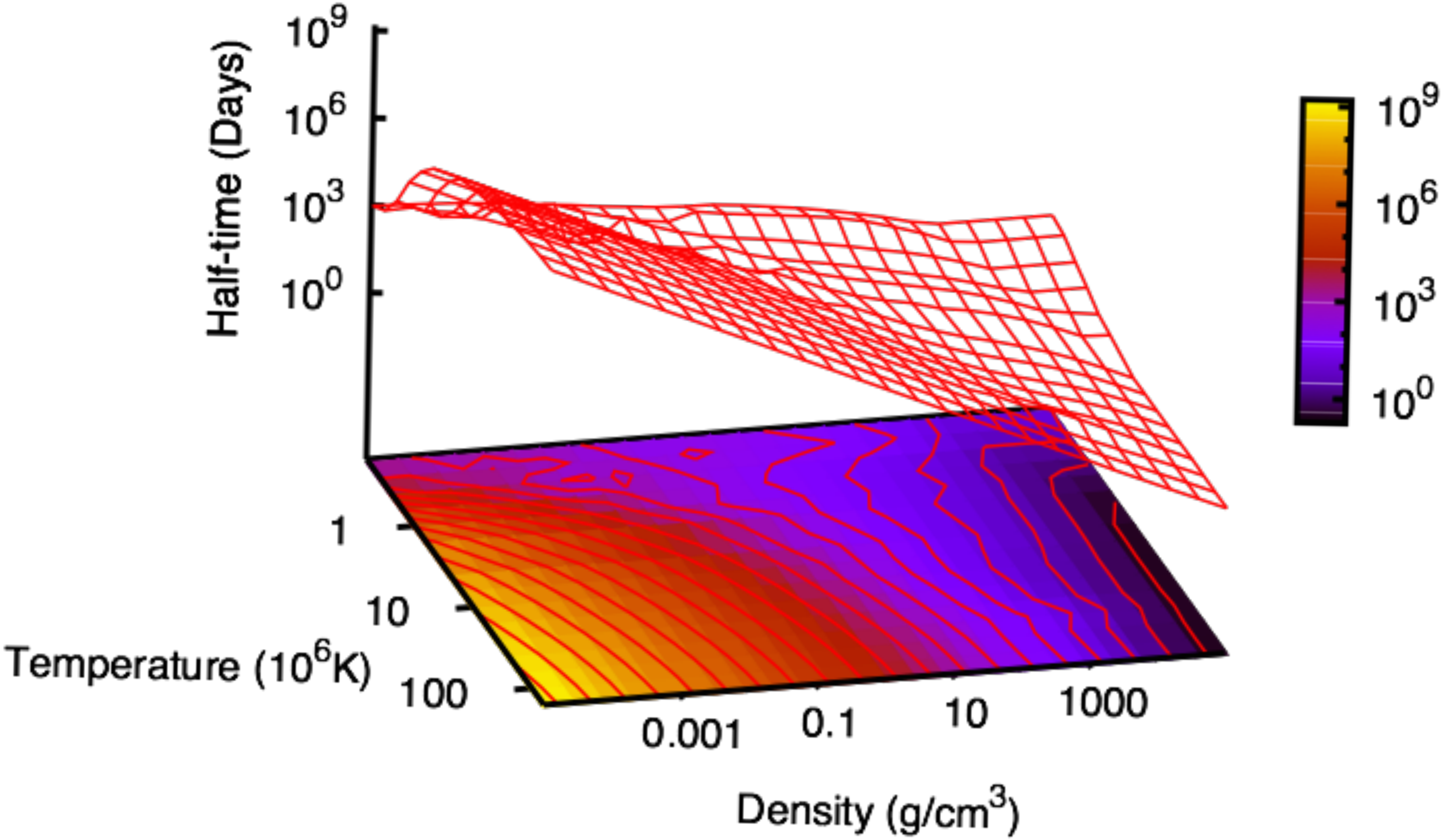
Degenerate condition

Solar condition

ρ (g cm^{-3})	T (10^6 K)	λ_{Debye} $a.u.$	$\lambda_{\text{De Broglie}}$ $(e-p)$	$\rho_{\text{HF}}(0)$ $a.u.$	$\rho_{\text{TF}}(0)$	$\rho_B(0)$	$\rho_{\text{DH}}(0)$
1000.	1.	0.038	1.409–0.0329	71.87 ÷ 71.97	68.99 ÷ 69.11	42.61 ÷ 42.74	47.46 ÷ 47.55
100.		0.119		33.52 ÷ 33.53	29.53 ÷ 29.55	4.027 ÷ 4.031	19.13 ÷ 19.14
10.		0.377		17.37 ÷ 17.37	13.83 ÷ 13.83	0.945 ÷ 0.945	13.33 ÷ 13.33
1.		1.193		7.839 ÷ 7.837	5.708 ÷ 5.707	0.184 ÷ 0.184	8.151 ÷ 8.149
0.1		3.771		1.940 ÷ 1.940	1.415 ÷ 1.415	0.044 ÷ 0.044	2.059 ÷ 2.058
0.01		11.93		0.278 ÷ 0.278	0.220 ÷ 0.220	0.0075 ÷ 0.0075	0.279 ÷ 0.279
0.001		37.71		0.0308 ÷ 0.0308	0.0264 ÷ 0.0264	0.0012 ÷ 0.0012	0.0303 ÷ 0.0303
1000.	10.	0.119	0.445–0.0103	122.43 ÷ 122.89	116.21 ÷ 116.68	51.77 ÷ 52.05	108.56 ÷ 109.01
100.		0.377		20.23 ÷ 20.27	19.53 ÷ 19.57	10.36 ÷ 10.39	19.54 ÷ 19.58
10.		1.193		2.578 ÷ 2.581	2.554 ÷ 2.558	2.515 ÷ 2.519	2.570 ÷ 2.573
1.		3.771		0.274 ÷ 0.275	0.274 ÷ 0.275	0.274 ÷ 0.274	0.274 ÷ 0.275
0.1		11.93		0.0281 ÷ 0.0282	0.0281 ÷ 0.0282	0.0281 ÷ 0.0282	0.0281 ÷ 0.0281
0.01		37.71		$(2.84 \div 2.84) \times 10^{-3}$	$(2.84 \div 2.84) \times 10^{-3}$	$(2.84 \div 2.84) \times 10^{-3}$	$(2.83 \div 2.83) \times 10^{-3}$
0.001		119.3		$(2.84 \div 2.84) \times 10^{-4}$	$(2.84 \div 2.84) \times 10^{-4}$	$(2.84 \div 2.84) \times 10^{-4}$	$(2.84 \div 2.84) \times 10^{-4}$
1000.	100.	0.377	0.141–0.0033	78.31 ÷ 80.39	78.24 ÷ 80.32	76.57 ÷ 78.64	78.22 ÷ 80.30
100.		1.193		9.051 ÷ 9.289	9.051 ÷ 9.288	9.031 ÷ 9.268	9.051 ÷ 9.288
10.		3.771		0.773 ÷ 0.787	0.773 ÷ 0.787	0.773 ÷ 0.787	0.773 ÷ 0.787
1.		11.93		0.0775 ÷ 0.0789	0.0775 ÷ 0.0789	0.0775 ÷ 0.0789	0.0775 ÷ 0.0789
0.1		37.71		$(7.75 \div 7.90) \times 10^{-3}$	$(7.75 \div 7.90) \times 10^{-3}$	$(7.75 \div 7.90) \times 10^{-3}$	$(7.75 \div 7.90) \times 10^{-3}$
0.01		119.3		$(7.75 \div 7.90) \times 10^{-4}$	$(7.75 \div 7.90) \times 10^{-4}$	$(7.75 \div 7.90) \times 10^{-4}$	$(7.75 \div 7.90) \times 10^{-4}$
0.001		377.1		$(7.75 \div 7.90) \times 10^{-5}$	$(7.75 \div 7.90) \times 10^{-5}$	$(7.75 \div 7.90) \times 10^{-5}$	$(7.75 \div 7.90) \times 10^{-5}$

A pictorial view of ^{7}Be half-life…

half-life (days)= 941.86881/$\varrho(0)$



β-decay: standard approach

A typical nuclear β-decay process reads:

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}X'_{N-1} + e^{-} + \bar{\nu}_{e} $$

Q-value: total energy released by the reaction ($m_\nu = 0$)

$$ Q_{\beta-} = m_N({}^{A}X) - m_N({}^{A}X') \qquad m({}^{A}X) = m_N({}^{A}X) + Zm_e - \sum_{i=1}^{Z}(B_i) $$

Extra-nuclear factor

$$ Q_{\beta-} = \{[m({}^{A}X) - Z\cdot m_e] - [m({}^{A}X') - (Z+1)\cdot m_e] - m_e\}\cdot c^2 + \{\sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i\} $$

In the traditional theory of β-decay processes, spectra are typically calculated as product of three factors:

$$ \frac{dN}{dW} \propto pWq^2 F(Z,W) C(W) $$

$$ C(W) = (2L'-1)! \sum_{k=1}^{L'} \lambda_k \frac{p^{2(k-1)} q^{2(L'-k)}}{(2k-1)![2(L'-k)+1]!} $$

$$ F(Z,W) = \frac{2\pi\nu}{1-\exp(-2\pi\nu)} \qquad \nu = \pm Ze^2/\hbar v $$

- **a phase-space factor to deal with the momentum sharing between the β-electron (p) and neutrino (q);**
- **a Fermi function F(Z,W) to take into account the static corrections due to the Coulomb field of the nucleus;**
- **a shape factor C(W) to include the coupling between nuclear and lepton dynamics.**

β-decay: standard approach

$$\frac{dN}{dW} \propto pWq^2F(Z,W)C(W)$$

It works well to predict the lineshape allowed and forbidden unique transitions, at variance, nuclear structure effects cannot be neglected when dealing with forbidden non-unique transitions, and there is no such a simple relation for C(W)

One can treat first forbidden non-unique transitions as allowed if

$$2\xi = \frac{\alpha Z}{R_{nuc}} >> E_{max}$$

where E_{max} is the maximum escaping energy of the β-electron and α is the fine structure constant

Still a rigours treatment of these transitions including electronic and nuclear DOF is missing!!!

Our approach to beta-decay helps to solve these issues, at least in the leptonic current term

Standard Model β-decay theory

β-decay rate is calculated by using Fermi's Golden Rule:

$$P_{i\to f} = 2\pi \int |\langle f|\hat{H}_\beta|i\rangle|^2 \rho(W_f)\delta(W_f - W_i)dW_f$$

Probability P per unit time that a system undergoes a transition from an initial state, *i*, to a number of final states, *f*, under the influence of a perturbation described by the Hamiltonian H_β

Weak Interaction Hamiltonian

$$\mathcal{H}_\beta = \frac{G_\beta}{\sqrt{2}}(\bar{\psi}_{f,p}(\mathbf{r})\gamma^\mu(1 - x\gamma^5)\hat{\psi}_{i,n}(\mathbf{r})) \cdot (\bar{\psi}_{f,e}(\mathbf{r})\gamma_\mu(1 - \gamma^5)\hat{\psi}_{i,\nu}(\mathbf{r})) + h.c.$$

Creates a proton

Destroys a neutron

Creates an electron

Destroys a neutrino (creates an antineutrino)

$\gamma^\mu \to$ Dirac gamma matrices

$G_\beta = G \cdot c_V \sim 1.13 \cdot 10^{-5} GeV^{-2}$; $x = \frac{c_A}{c_V} \sim 1.27$;

All the wavefunctions will be written as Dirac spinors

β-decay theory

Initial nuclear Fock-space state: $|\xi_n, j_n, \mu_n\rangle_A \equiv \hat{a}_n^\dagger |0\rangle_A$

Final nuclear Fock-space: $|\xi_p, j_p, \mu_p\rangle_A \equiv \hat{a}_p^\dagger |0\rangle_A$

Initial lepton Fock-space: $|0;\ 0\rangle_L$

Final lepton Fock-space: $\cancel{|(n_B, \kappa_B, \mu_B} + W_C^f, \kappa_C^f, \mu_C^f); W_\nu, \kappa_\nu, \mu_\nu\rangle_L \equiv (\cancel{\hat{a}^+_{B,e}} + \hat{a}^+_{C,e}) b_\nu^\dagger |0;0\rangle_L$

$j_{p,n,e}$ **nuclear spin**

$\mu_{p,n,e}$ **projection along the quantization axis**

$\xi_{p,n,e}$ **quantum number characterizing the nuclear state**

initial state: $|i\rangle \equiv |\xi_n, j_n, \mu_n\rangle_A \otimes |0;\ 0\rangle_L$

final state: $|f\rangle \equiv |\xi_p, j_p, \mu_p\rangle_A \otimes |W_C^f, \kappa_C^f, \mu_C^f;\ W_\nu, \kappa_\nu, \mu_\nu\rangle_L$

Field operators entering the Weak Interaction Hamiltonian

$$\hat{\psi}_n(\mathbf{r}) = \sum_{\xi_n, j_n, \mu_n} \langle \mathbf{r} | \xi_n, j_n, \mu_n\rangle\ \hat{a}_n + \text{antineutron creation term}$$

$$\hat{\psi}_e^+(\mathbf{r}) = \cancel{\sum_{n'_B, \kappa'_B, \mu'_B} \langle n'_B, \kappa'_B, \mu'_B | \mathbf{r}\rangle\ \hat{a}^+_{B,e}} + \int dW'_C \sum_{\kappa'_C, \mu'_C} \langle W'_C, \kappa'_C, \mu'_C | \mathbf{r}\rangle\ \hat{a}^+_{C,e} + \text{positron destruction term}$$

Inclusion of the antisymmetrization

$$\hat{\psi}_\nu(\mathbf{r}) = \int dW_\nu \sum_{\kappa_\nu, \mu_\nu} \left(\langle \mathbf{r} | W_\nu, \kappa_\nu, \mu_\nu\rangle\ \hat{a}_\nu + \langle \mathbf{r} | W_\nu, \kappa_\nu, \mu_\nu\rangle_-\ \hat{b}_\nu^\dagger \right)$$

$$\hat{\psi}_p^+(\mathbf{r}) = \sum_{\xi_p, j_p, \mu_p} \langle \xi_p, j_p, \mu_p | \mathbf{r}\rangle\ \hat{a}_p^+ + \text{antiproton destruction term}$$

→ In the standard approximation, one considers the particles entering the decay as non-interacting single particles

β-decay theory: total decay rate

$$\lambda_t = \frac{\pi G_\beta^2}{(2j_n+1)\cancel{(2J_B+1)}} \sum_{\gamma} \sum_{\mu_n,\mu_p} \sum_{\mu'_B,\mu'_C,\mu_B} \sum_{\mu_\nu} \int |I|^2 \rho(W_f)\delta(Q - T'_C - W_\nu)\, dW_\nu\, dW'_C$$

$$\langle f|\mathcal{H}_\beta|i\rangle$$

with electron energy $W_C^f = \sqrt{p^2c^2 + c^4} = c^2 + T_C^f$ and antineutrino energy $W_\nu = c \cdot p_\nu$

μ and γ' runs over magnetic and principal quantum number and where

$$I = \iiint\!\!\int \langle \xi_p, j_p, \mu_p|\, \hat{\psi}_p^{\dagger}(\mathbf{r}_h)\gamma^0\gamma^\mu(1 - x\gamma^5)\hat{\psi}_n(\mathbf{r}_h)\, |\xi_n, j_n, \mu_n\rangle \cdot$$

$$\langle \cancel{\Lambda_{B,C}}, n'_C, \kappa'_C, \mu'_C; W'_C, \kappa_\nu, \mu_\nu|\, \hat{\psi}_e^{\dagger}(\mathbf{r}_l)\gamma^0\gamma_\mu(1 - \gamma^5)\hat{\psi}_\nu(\mathbf{r}_l)\, |0; \cancel{\Lambda_B}\rangle\, \delta(\mathbf{r}_h - \mathbf{r}_l)\, d\mathbf{r}_h\, d\mathbf{r}_l$$

expresses the point-like nature of the decay

$$\delta(\mathbf{r}_h - \mathbf{r}_l) = \sum_{L',q} \delta(r_h - r_l) \cdot r_l^{-2} \cdot Y_{L',q}(\theta_h,\phi_h) Y_{L',-q}(\theta_l,\phi_l) \cdot (-)^q$$

Spherical armonics

This notation is useful because it allows to split the matrix element into nuclear and lepton parts

β-decay theory

To find the eigensolutions of the SM Hamiltonian for the β-decay we make a first major "approximation": we assume that one can factorize this operator as the tensorial product of of two interacting currents:

- hadronic (nuclear);
- leptonic (electron + neutrino)

Explicitly:

$$\langle f|\mathcal{H}_\beta|i\rangle = \frac{G_\beta}{\sqrt{2}} J^{H,\mu}_{i\to f}(\mathbf{r}) J^{L}_{i\to f,\mu}(\mathbf{r})$$

where:

e- and ν can be considered uncoupled

$$J^{L}_{i\to f,\mu}(\mathbf{r}) = \psi^{+}_{f,e}(\mathbf{r})\gamma_0\gamma_\mu\left(1-\gamma^5\right)\psi_{i,\nu}(\mathbf{r})$$

n and p w.f. can be factorized provided that the nucleus is "hydrogenic", that is composed by a closed shell with only one single nucleon in one open shell embedded in the mean field generated by the closed shell

$$J^{H.\mu}_{i\to f}(\mathbf{r}) = \psi^{+}_{f,p}(\mathbf{r})\gamma_0\gamma^\mu\left(1-x\gamma^5\right)\psi_{i,n}(\mathbf{r})$$

β-decay theory in central symmetry

Nuclear matrix element on a real space grid

$$J^{H,\mu}(r_h) = \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle \xi_p, j_p, \mu_p|\ \hat{\psi}_p^{+}(\mathbf{r}_h)\gamma^0\gamma^\mu(1-x\gamma^5)\hat{\psi}_n(\mathbf{r}_h)\ |\xi_n, j_n, \mu_n\rangle \cdot r_h^2 =$$

$$= \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle 0|\hat{a}_p\ \hat{\psi}_p^{+}(\mathbf{r}_h)\gamma^0\gamma^\mu(1-x\gamma^5)\hat{\psi}_n(\mathbf{r}_h)\ \hat{a}_n^{+}|0\rangle \cdot r_h^2$$

inserting the expressions for the field operators

$$J^{H,\mu}(r_h) = \sum_{\xi'_p, j'_p, \mu'_p}\ \sum_{\xi'_n, j'_n, \mu'_n} \langle 0|\hat{a}_p\ \hat{a}_{p'}^{+}\hat{a}_{n'}\ \hat{a}_n^{+}|0\rangle$$

$$\cdot \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle \xi'_p, j'_p, \mu'_p|\mathbf{r}_h\rangle\ \gamma^0\gamma^\mu(1-x\gamma^5)\langle \mathbf{r}_h|\xi'_n, j'_n, \mu'_n\rangle \cdot$$

and applying anti-commutation rules for creation/destruction Fock-space operators

$$\{\hat{a}_p, \hat{a}_n^\dagger\} = \{\hat{a}_p, \hat{a}_n\} = 0$$

$$\{\hat{a}_p, \hat{a}_{p'}^\dagger\} = \delta_{\xi_p,\xi'_p}\ \delta_{j_p,j'_p}\ \delta_{\mu_p,\mu'_p}$$

$$\{\hat{a}_n, \hat{a}_{n'}^\dagger\} = \delta_{\xi_n,\xi'_n}\ \delta_{j_n,j'_n}\ \delta_{\mu_n,\mu'_n}$$

<u>Selection rules</u>

one gets

$$\boxed{J^{H,\mu}(r_h) = \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle \xi_p, j_p, \mu_p|\mathbf{r}_h\rangle\ \gamma^0\gamma^\mu(1-x\gamma^5)\langle \mathbf{r}_h|\xi_n, j_n, \mu_n\rangle \cdot r_h^2}$$

β-decay theory: total decay rate

Lepton matrix element on a real space grid

$$J^L_\mu(r_h) = \int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle \bigwedge_{B,C} n'_B,\kappa'_B,\mu'_B,W'_C,\kappa'_C,\mu'_C;W_\nu,\kappa_\nu,\mu_\nu|$$

$$\hat{\psi}^+_e(\mathbf{r}_l)\gamma^0\gamma_\mu(1-\gamma^5)\hat{\psi}_\nu(\mathbf{r}_l)\ |\bigwedge_B n_B,\kappa_B,\mu_B;0\rangle\ \delta(r_h-r_l) =$$

$$= \int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle 0;0|\hat{a}'_{1,e}\dots\ \hat{a}'_{N,e}\hat{a}'_{C,e}\hat{b}_\nu\ \hat{\psi}^+_e(\mathbf{r}_l)\gamma^0\gamma_\mu(1-\gamma^5)\hat{\psi}_\nu(\mathbf{r}_l)\hat{a}^+_{1,e}\dots\ \hat{a}^+_{N,e}|0;0\rangle\delta(r_h-r_l)$$

inserting the expressions for the field operators

$$J^L_\mu(r_h) = \sum_{n'_B,\kappa'_B,\mu'_B}\int dW'_\nu \sum_{\kappa'_\nu,\mu'_\nu}\langle 0;0|\hat{a}'_{1,e}\dots\ \hat{a}'_{N,e}\hat{a}'_{C,e}\hat{b}_\nu\hat{a}'^+_{B',e}\hat{b}^+_{\nu'}\hat{a}^+_{1,e}\dots\ \hat{a}^+_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle n'_B,\kappa'_B,\mu'_B|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W'_\nu,\kappa'_\nu,\mu'_\nu\rangle\delta(r_h-r_l) +$$

$$+\int dW'_C \sum_{\kappa'_C,\mu'_C}\int dW'_\nu \sum_{\kappa'_\nu,\mu'_\nu}\langle 0;0|\hat{a}'_{1,e}\dots\ \hat{a}'_{N,e}\hat{a}'_{C,e}\hat{b}_\nu\hat{a}'^+_{C',e}\hat{b}^+_{\nu'}\hat{a}^+_{1,e}\dots\ \hat{a}^+_{N,e}|0;0\rangle$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle W'_C,\kappa'_C,\mu'_C|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W'_\nu,\kappa'_\nu,\mu'_\nu\rangle\delta(r_h-r_l)$$

β-decay theory: total decay rate

Lepton matrix element on a real space grid

and applying anti-commutation rules for creation/destruction Fock-space operators

$$\{\hat{a}'_{B,e}, \hat{a}'^{+}_{B',e}\} = \delta_{n_B,n'_B}\ \delta_{\kappa_B,\kappa'_B}\ \delta_{\mu_B,\mu'_B}$$

$$\{\hat{a}'_{C,e}, \hat{a}'^{+}_{C',e}\} = \delta(W_C - W'_C)\ \delta_{\kappa_C,\kappa'_C}\ \delta_{\mu_C,\mu'_C}$$

$$\{\hat{b}_{\nu}, \hat{b}^{+}_{\nu'}\} = \delta(W_\nu - W'_\nu)\ \delta_{\kappa_\nu,\kappa'_\nu}\ \delta_{\mu_\nu,\mu'_\nu}$$

$$\{\hat{a}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}^{+}_{B/C,e}, \hat{b}_{\nu}\} = \{\hat{a}_{C,e}, \hat{a}^{+}_{B,e}\} = 0$$

one gets

Inclusion of post-collisional effects: Fano's and Exchange interactions

$$J^L_\mu(r_h) = \sum_{j=1}^{N} \prod_{B\neq j} (-)^j \langle 0;0|\hat{a}'_{B,e}\hat{a}'_{C,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle \quad Q_{L',q,B;\mu}(r_h)$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle n'_B,\kappa'_B,\mu'_B|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h-r_l)\ +$$

Standard beta-decay

$$+\langle 0;0|\hat{a}'_{1,e}\ldots\ \hat{a}'_{N,e}\hat{a}^{+}_{1,e}\ldots\hat{a}^{+}_{N,e}|0;0\rangle \quad Q_{L',q,C;\mu}(r_h)$$

$$\int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l)\langle W'_C,\kappa'_C,\mu'_C|\mathbf{r}_l\rangle\gamma^0\gamma_\mu(1-\gamma^5)\langle\mathbf{r}_l|W_\nu,\kappa_\nu,\mu_\nu\rangle\delta(r_h-r_l)$$

$$J^L_\mu(r_h) = \begin{vmatrix} \langle\psi'_1|\phi_1\rangle & \langle\psi'_1|\phi_2\rangle & \cdots & \langle\psi'_1|\phi_N\rangle & Q_{L',q,1;\mu}(r_h) \\ \langle\psi'_2|\phi_1\rangle & \langle\psi'_2|\phi_2\rangle & \cdots & \langle\psi'_2|\phi_N\rangle & Q_{L',q,2;\mu}(r_h) \\ \vdots & & \ddots & & \vdots \\ \langle\psi'_N|\phi_1\rangle & \langle\psi'_N|\phi_2\rangle & \cdots & \langle\psi'_N|\phi_N\rangle & Q_{L',q,N;\mu}(r_h) \\ \langle\psi'_C|\phi_1\rangle & \langle\psi'_C|\phi_2\rangle & \cdots & \langle\psi'_C|\phi_N\rangle & Q_{L',q,C;\mu}(r_h) \end{vmatrix}$$

by combining the leptonic and the hadronic currents

$$J^{H,\mu}(r_h) = \int d\Omega_h Y_{L',q}(\theta_h,\phi_h)\langle\xi_p, j_p, \mu_p|\mathbf{r}_h\rangle\ \gamma^0\gamma^\mu(1-x\gamma^5)\langle\mathbf{r}_h|\xi_n, j_n, \mu_n\rangle \cdot r_h^2$$

Differential decay rate (electron energy spectrum)

$$\frac{d\lambda}{dW_e^t} = \frac{\pi G_\beta^2}{(2j_n+1)(2J_B+1)} \sum_{\gamma'} \sum_{\mu_n,\mu_p} \sum_{\mu'_B,\mu_B} \sum_{\kappa'_C,\mu'_C} \sum_{\kappa_\nu,\mu_\nu}$$

$$\left| \sum_{L',q}(-)^q \begin{vmatrix} \langle\psi'_1|\phi_1\rangle & \langle\psi'_1|\phi_2\rangle & \cdots & \langle\psi'_1|\phi_N\rangle & M_{L',q,1}(W_\nu = Q - W_e^t) \\ \langle\psi'_2|\phi_1\rangle & \langle\psi'_2|\phi_2\rangle & \cdots & \langle\psi'_2|\phi_N\rangle & M_{L',q,2}(W_\nu = Q - W_e^t) \\ \vdots & & \ddots & & \vdots \\ \langle\psi'_N|\phi_1\rangle & \langle\psi'_N|\phi_2\rangle & \cdots & \langle\psi'_N|\phi_N\rangle & M_{L',q,N}(W_\nu = Q - W_e^t) \\ \langle\psi'_C|\phi_1\rangle & \langle\psi'_C|\phi_2\rangle & \cdots & \langle\psi'_C|\phi_N\rangle & M_{L',q,C}(W_\nu = Q - W_e^t) \end{vmatrix} \right|^2$$

It gives the number of electrons per unit energy and per unit time

Differential decay rate (electron energy spectrum)

$$\frac{d\lambda}{dW_e^t} = \frac{\pi G_\beta^2}{(2j_n+1)(2J_B+1)} \sum_{\gamma'} \sum_{\mu_n,\mu_p} \sum_{\mu'_B,\mu_B} \sum_{\kappa'_C,\mu'_C} \sum_{\kappa_\nu,\mu_\nu}$$

$$\left| \sum_{L',q} (-)^q \begin{vmatrix} \langle\psi'_1|\phi_1\rangle & \langle\psi'_1|\phi_2\rangle & \cdots & \langle\psi'_1|\phi_N\rangle & M_{L',q,1}(W_\nu = Q - W_e^t) \\ \langle\psi'_2|\phi_1\rangle & \langle\psi'_2|\phi_2\rangle & \cdots & \langle\psi'_2|\phi_N\rangle & M_{L',q,2}(W_\nu = Q - W_e^t) \\ \vdots & & \ddots & & \vdots \\ \langle\psi'_N|\phi_1\rangle & \langle\psi'_N|\phi_2\rangle & \cdots & \langle\psi'_N|\phi_N\rangle & M_{L',q,N}(W_\nu = Q - W_e^t) \\ \langle\psi'_C|\phi_1\rangle & \langle\psi'_C|\phi_2\rangle & \cdots & \langle\psi'_C|\phi_N\rangle & M_{L',q,C}(W_\nu = Q - W_e^t) \end{vmatrix} \right|^2$$

The final orbital ψ'_i depend on γ' that identifies the possible final (shake-up, shake-off, excited) states

$$M_{L',q,B} = \int \left[\int d\Omega_h Y_{L',q}(\theta_h,\phi_h) \langle \xi_p, j_p, \mu_p | \mathbf{r}_h \rangle \; \gamma^0 \gamma^\mu (1 - x\gamma^5) \langle \mathbf{r}_h | \xi_n, j_n, \mu_n \rangle \cdot r_h^2 \cdot \right.$$
$$\left. \int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l) \langle n'_B, \kappa'_B, \mu'_B | \mathbf{r}_l \rangle \gamma^0 \gamma_\mu (1-\gamma^5) \langle \mathbf{r}_l | W_\nu, \kappa_\nu, \mu_\nu \rangle \delta(r_h - r_l) \right] dr_h;$$

$$M_{L',q,C} = \int \left[\int d\Omega_h Y_{L',q}(\theta_h,\phi_h) \langle \xi_p, j_p, \mu_p | \mathbf{r}_h \rangle \; \gamma^0 \gamma^\mu (1 - x\gamma^5) \langle \mathbf{r}_h | \xi_n, j_n, \mu_n \rangle \cdot r_h^2 \cdot \right.$$
$$\left. \int dr_l \int d\Omega_l Y_{L',-q}(\theta_l,\phi_l) \langle W'_C, \kappa'_C, \mu'_C | \mathbf{r}_l \rangle \gamma^0 \gamma_\mu (1-\gamma^5) \langle \mathbf{r}_l | W_\nu, \kappa_\nu, \mu_\nu \rangle \delta(r_h - r_l) \right] dr_h;$$

Using L' = 0, $\langle\psi'_i|\phi_j\rangle = \delta_{ij}$,e- wfs at nuclear radius, and γ_0 one recovers standard beta-decay

Calculation of the leptonic and hadronic wfs: DHF

The time independent Dirac Hamiltonian of a many particles system In the case of two different types of interactions, e.g. represented by scalar (g_S) and vector (g_V) potentials, the Dirac equation reads

$$\left\{\sum_i \left(c\boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i mc^2 + V_i\right) + \sum_{i<j} \left[\beta_i\beta_j g_{S,ij} + \left(1 - \boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j\right) g_{V,ij}\right]\right\} \psi\left(\mathbf{r}_1, \cdots \mathbf{r}_N\right) = E\psi\left(\mathbf{r}_1, \cdots \mathbf{r}_N\right)$$

which in second quantization can be written as follows:

$$H = \sum_{s_1 s_2} \int d\mathbf{r}\ \hat{\psi}^+_{s_1}(\mathbf{r}) \left[-ic\boldsymbol{\alpha}_{s_1 s_2} \cdot \boldsymbol{\nabla} + \beta_{s_1 s_2} mc^2 + \delta_{s_1 s_2} V(\mathbf{r})\right] \hat{\psi}_{s_2}(\mathbf{r}) +$$
$$+ \frac{1}{2} \sum_{s_1 s_2 s_1' s_2'} \int d\mathbf{r} d\mathbf{r}'\ \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}^+_{s_1'}(\mathbf{r}') \left[\beta_{s_1 s_2}\beta_{s_1' s_2'} g_S\left(\mathbf{r}, \mathbf{r}'\right) + \left(\delta_{s_1 s_2}\delta_{s_1' s_2'} - \boldsymbol{\alpha}_{s_1 s_2} \cdot \boldsymbol{\alpha}'_{s_1' s_2'}\right) g_V\left(\mathbf{r}, \mathbf{r}'\right)\right] \hat{\psi}_{s_2'}(\mathbf{r}') \hat{\psi}_{s_2}(\mathbf{r})$$

where s_1, s_2, s_1',s_2' index the bispinor two-components

To compute the electronic and hadronic current we use the HF approximation

$$\left\langle \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}^+_{s_1'}(\mathbf{r}') \hat{\psi}_{s_2'}(\mathbf{r}') \hat{\psi}_{s_2}(\mathbf{r}) \right\rangle = \left\langle \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}_{s_2}(\mathbf{r}) \right\rangle \left\langle \hat{\psi}^+_{s_1'}(\mathbf{r}') \hat{\psi}_{s_2'}(\mathbf{r}') \right\rangle - \left\langle \hat{\psi}^+_{s_1}(\mathbf{r}) \hat{\psi}_{s_2'}(\mathbf{r}') \right\rangle \left\langle \hat{\psi}^+_{s_1'}(\mathbf{r}') \hat{\psi}_{s_2}(\mathbf{r}) \right\rangle$$

Calculation of the leptonic and hadronic wfs: DHF

$$\begin{pmatrix} mc^2 + W_V + W_S + \mathbf{A}_P \cdot \boldsymbol{\sigma} - E & -c\boldsymbol{\sigma} \cdot i\boldsymbol{\nabla} - \boldsymbol{\sigma} \cdot \mathbf{A} + W_{PS} \\ -c\boldsymbol{\sigma} \cdot i\boldsymbol{\nabla} - \boldsymbol{\sigma} \cdot \mathbf{A} + W_{PS} & -mc^2 + W_V + \mathbf{A}_P \cdot \boldsymbol{\sigma} - W_S - E \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_S \end{pmatrix} = 0$$

where

$$\begin{array}{rcll} W_S & - & \text{scalar} & \text{potential} \\ W_V & - & \text{vectorial} & \text{potential} \\ W_{PS} & - & \text{pseudoscalar} & \text{potential} \\ \mathbf{A}_P & - & \text{pseudo-vectorial} & \text{potential} \end{array}$$

For leptons:

$$W_S = 0$$

$$W_V = \text{Coulomb interaction}$$

$$A_P = 0$$

For hadrons:

$$\begin{array}{lcll} W_V + W_S & - & \text{Wood-Saxon} & \text{potential} \\ W_V - W_S & - & \text{spin-orbit} & \text{potential} \end{array}$$

$$\mathbf{A}_P \quad - \quad \text{magnetic field} \; \mathbf{= 0}$$

Calculation of the leptonic wfs

Dirac equation in a spherical potential

$$H\psi(\mathbf{r}) = (c\boldsymbol{\alpha}\cdot\mathbf{p} + \beta mc^2 + V(r))\psi(\mathbf{r}) = W\psi(\mathbf{r})$$

solutions are of the form: $\psi(\mathbf{r}) = \psi_{\kappa,\mu}(\mathbf{r}) = \begin{pmatrix} f_\kappa(r)\,\chi_{\kappa,\mu}(\Omega) \\ ig_\kappa(r)\,\chi_{-\kappa,\mu}(\Omega) \end{pmatrix}$

where $\chi_{\kappa,\mu}(\Omega) = \sum_{m_s=-\frac{1}{2}}^{\frac{1}{2}} \langle l\mu - m_s; \frac{1}{2}m_s | j\mu\rangle Y_{l,\mu-m_s}(\Omega)\phi_{m_s}$ are the spherical harmonics tensor

and calling $f_\kappa = \frac{u_\kappa}{r}$; $g_\kappa = \frac{v_\kappa}{r}$ u_κ and v_κ are solutions of

$$\begin{cases} \dfrac{\partial u_\kappa}{\partial r} = -\dfrac{\kappa}{r}u_\kappa + \dfrac{1}{c}(W - V(r) + mc^2)v_\kappa(r) = 0 \\ \dfrac{\partial v_\kappa}{\partial r} = \dfrac{\kappa}{r}v_\kappa - \dfrac{1}{c}(W - V(r) - mc^2)u_\kappa(r) = 0 \end{cases}$$

where $V(r) = -\dfrac{Z_f}{r} + \displaystyle\int \dfrac{\rho(r')}{r_>} d^3r' - V_{ex}(r)$

and we assume $V_{ex} = \dfrac{3}{2}\alpha_X \left[\dfrac{3}{\pi}\rho(r)\right]^{1/3}$ **which is local (TF or LDA)**

To numerically solve the DHF equations we use the collocation methods, which is a Runge-Kutta type integration method

Calculation of the hadronic wfs: DHF

By changing the interaction potential, the calculation of the hadronic wavefunctions within the nuclear matrix elements can be performed

Nuclear wfs simulations out of scope (WS model potential)

$$V_C(r) = -V_C\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}$$

Protons

$$V_C = V_0\left(1 \pm \chi\frac{N-Z}{A}\right)$$

Neutrons

$$\tilde{V}_{SO}(r) = \tilde{V}_{SO}\left[1+\exp\left(\frac{r-R_{SO}}{a_{SO}}\right)\right]^{-1}$$

$$\tilde{V}_{SO} = \lambda V_C$$

$R = R_0 A^{1/3}$ **and** $R_{SO} = R_{0,SO}A^{1/3}$ **= nuclear radius**

a **and** a_{SO} **= diffuseness**

$V_0, \chi, \lambda, a = a_{SO}, R_0, R_{0,SO}$ are parameters to be optimised according to experiments or ab-initio nuclear structure simulations

$$V_0 = 52.06\ MeV, \chi = 0.639, R_0 = 1.260\ fm, R_{0,SO} = 1.160\ fm, \lambda = 24.1, a = a_{SO} = 0.662\ fm$$

The beta-decay spectrum of $^{63}Ni, ^{129}I, ^{241}Pu$

Allowed Gamow-Teller

$^{63}_{28}Ni_{35}$ **= even-odd**

$^{63}_{29}Cu_{34}$ **= odd-even**

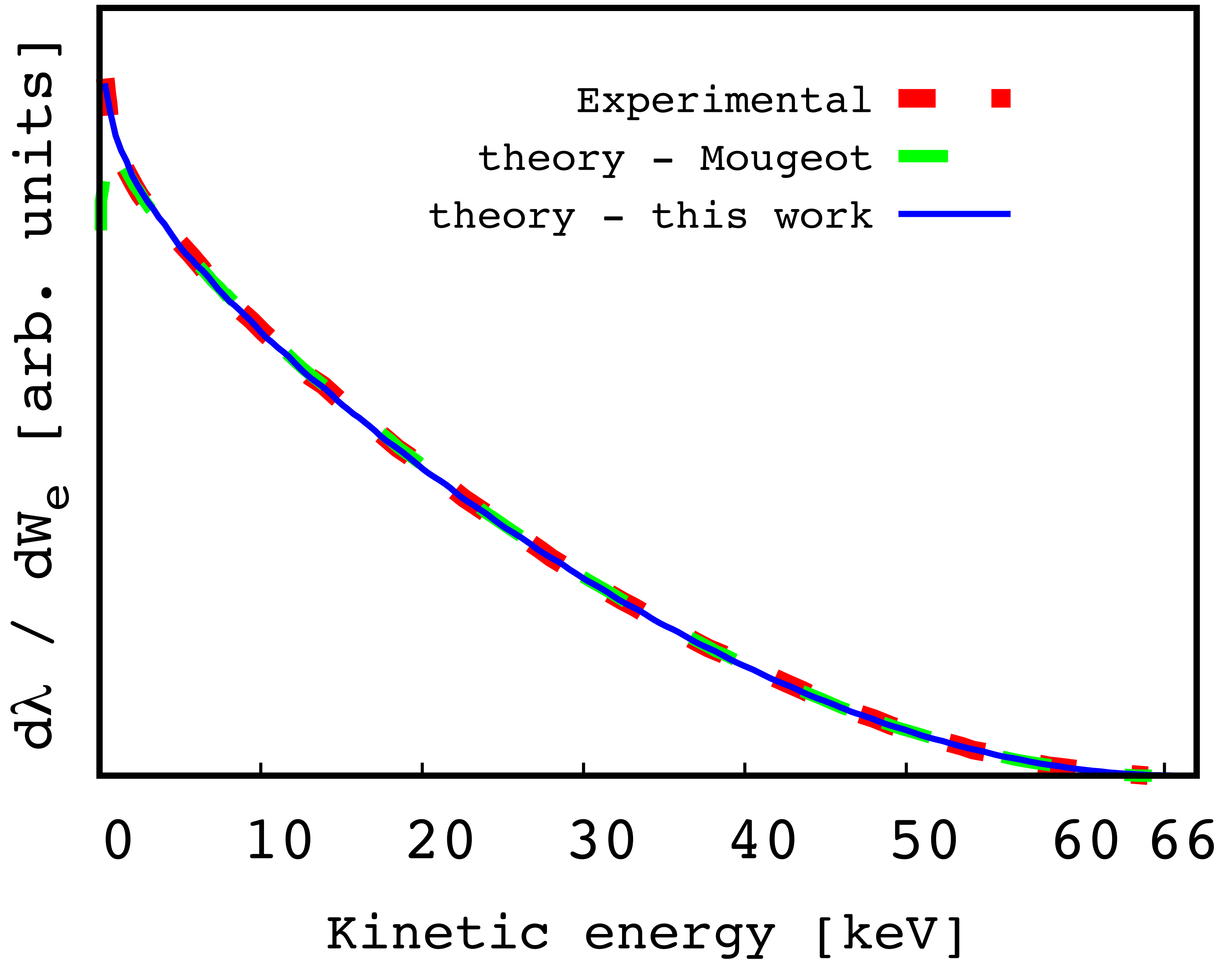
100% via β^- half-life = 101.2 y

Q-value = 66.945 keV
(ground state to ground state)

Mean-field DHF + screening + exchange works just fine as other approaches!!!!

$$^{63}_{28}\mathrm{Ni}_{35} \rightarrow ^{63}_{29}\mathrm{Cu}_{34} + e^- + \bar{\nu}_e$$

$$J = 1/2^- \rightarrow J = 3/2^-$$



The beta-decay spectrum of $^{63}Ni, ^{129}I, ^{241}Pu$

$^{129}I \rightarrow \;\; 17374.6321 \;\; > \;\; 5486.6741$

$^{241}Pu \rightarrow \;\; 30566.4823 \;\; >> \;\; 763.6509$

$$^{129}_{53}\mathrm{I}_{76} \rightarrow \, ^{129}_{54}\mathrm{Xe}^{*}_{75} + e^{-} + \bar{\nu}_e$$

$$J = 7/2^{+} \rightarrow J = 3/2^{+}$$

Second forbidden

$^{129}_{53}I_{76}$ **= odd-even**

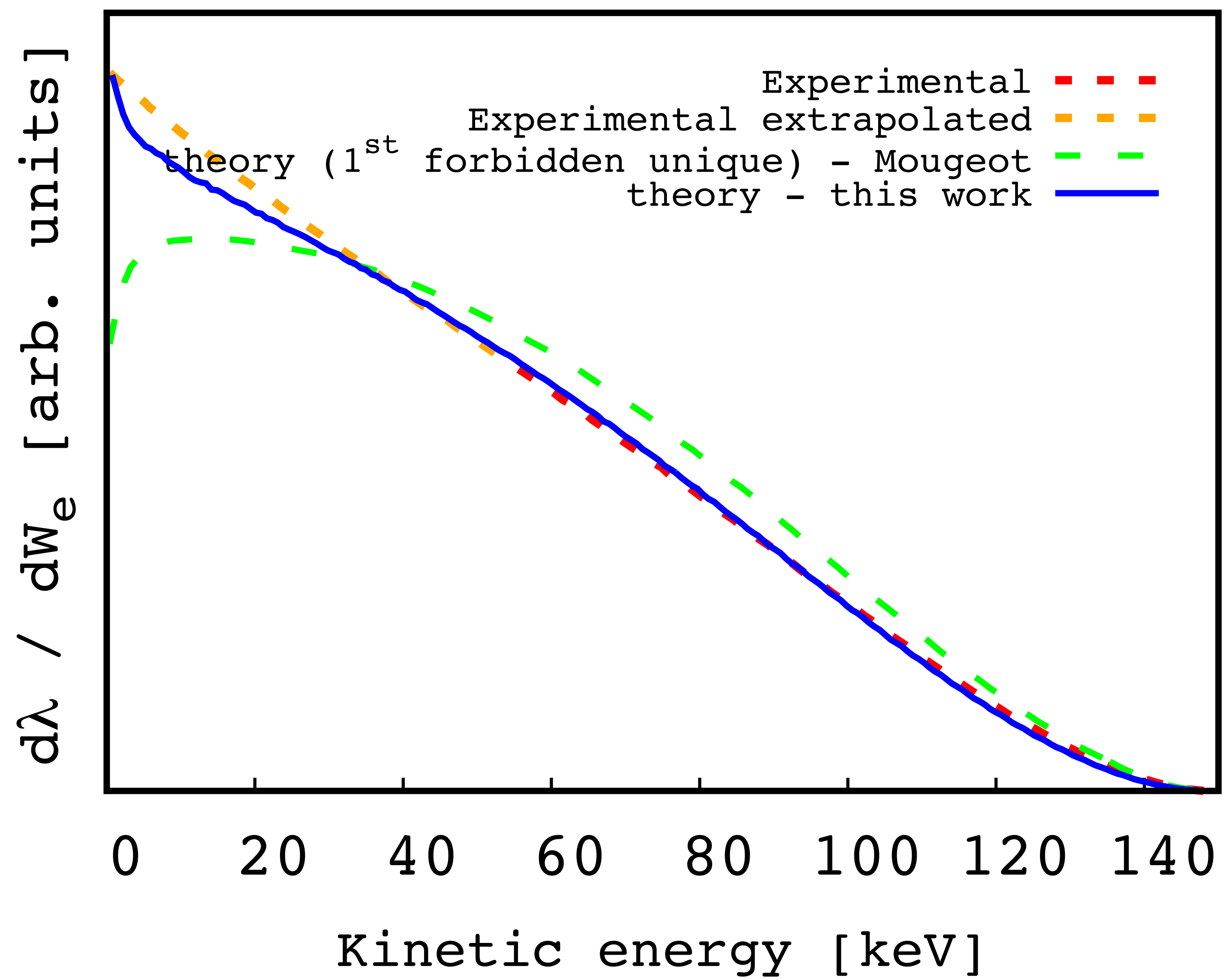
$^{129}_{54}\mathrm{Xe}_{75}$ **= even-odd**

β^{-} **half-life = 1.57X10^{7} y (10.05)**

Q-value = 149.4 keV
(ground state to excited state)

γ **= 39.578 keV**

Mean-field DHF + screening + exchange works just better than other approaches!!!!



The beta-decay spectrum of $^{63}Ni, ^{129}I, ^{241}Pu$

$$^{129}I \rightarrow \quad 17374.6321 \quad > \quad 5486.6741$$
$$^{241}Pu \rightarrow \quad 30566.4823 \quad >> \quad 763.6509$$

$$^{241}_{94}\mathrm{Pu}_{147} \rightarrow \, ^{241}_{95}\mathrm{Am}_{146} + e^- + \bar{\nu}_e$$
$$J = 5/2^+ \rightarrow J = 5/2^-$$

First forbidden

$^{241}_{94}\mathrm{Pu}_{147}$ **= even-odd**

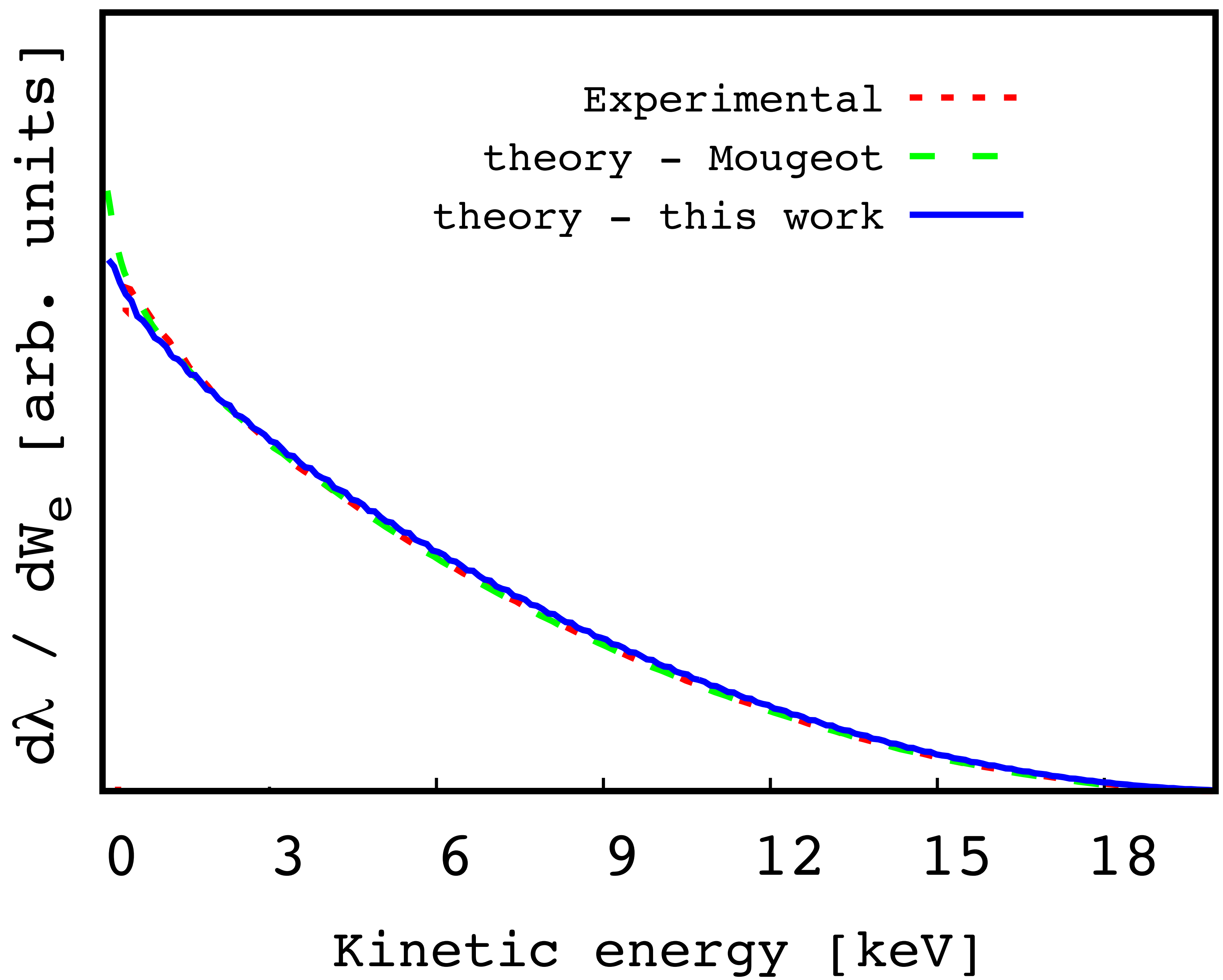
$^{241}_{95}\mathrm{Am}_{146}$ **= odd-even**

β^- **half-life = 14.329 y (6.85)**

Q-value = 20.78 keV
(ground state to excited state)

γ **= 39.578 keV**

Mean-field DHF + screening + exchange works just fine as other approaches!!!!



The beta-decay spectrum of ${}^{210}_{83}\mathrm{Bi}_{127}$

$${}^{210}_{83}\mathrm{Bi}_{127} \rightarrow {}^{210}_{84}\mathrm{Po}_{126} + e^- + \bar{\nu}_e$$

$$J = 1^- \rightarrow J = 0^+$$

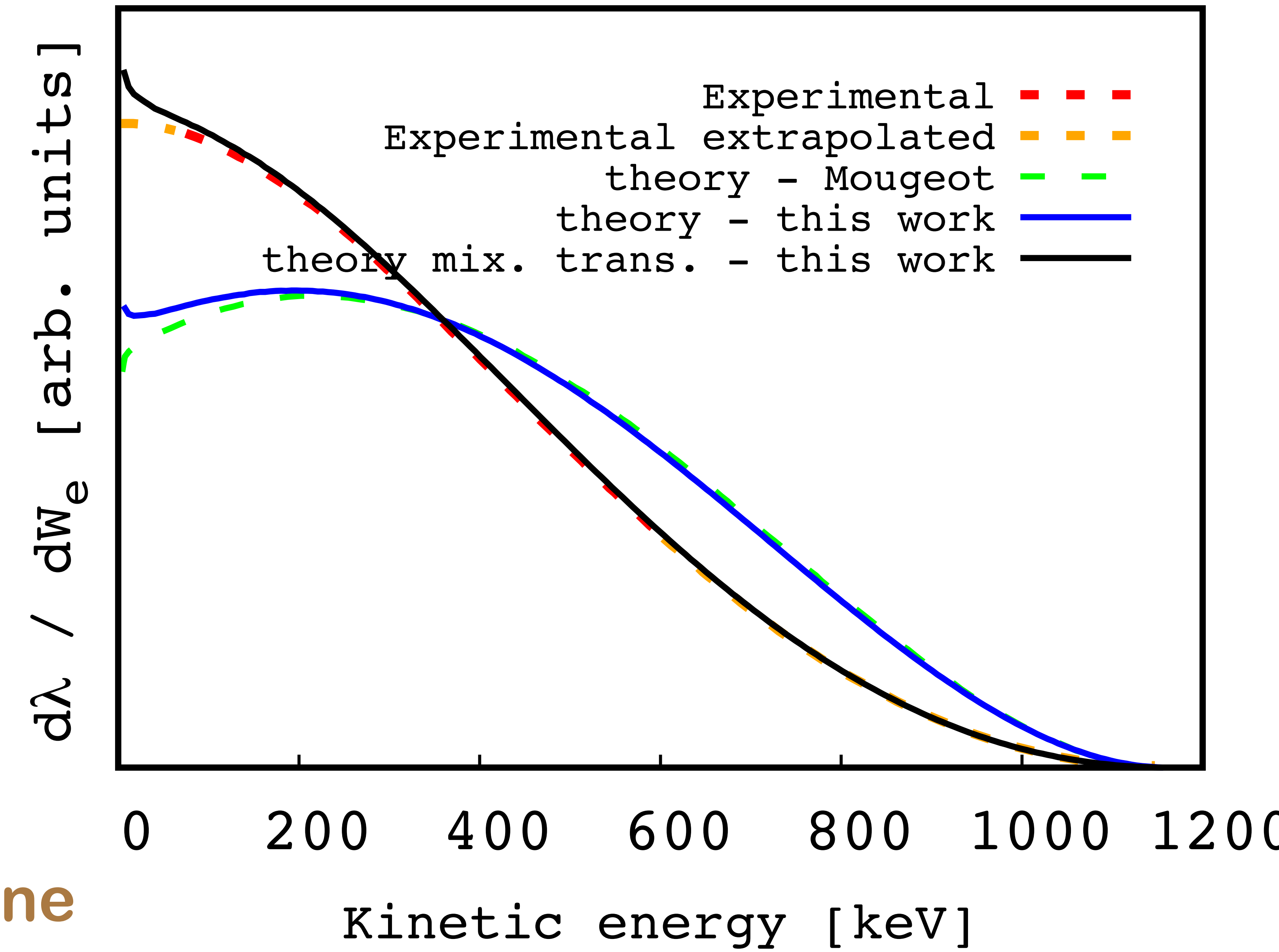
First forbidden non -unique

${}^{210}_{83}\mathrm{Bi}_{127}$ **= odd-odd**

${}^{210}_{84}\mathrm{Po}_{126}$ **= even-even**

β^- **half-life = 5.012 d**

Q-value = 1162.2 keV (ground state to excited state)



Mean-field DHF + screening + exchange does not work fine as standard approaches !!!!

Shake-up and shake-off modifies the decay by only 5%

The beta-decay spectrum of $^{36}_{17}\mathrm{Cl}_{19}$

$$^{36}_{17}\mathrm{Cl}_{19} \rightarrow {}^{36}_{18}\mathrm{Ar}_{18} + e^{-} + \bar{\nu}_e$$

$$J = 2^{+} \rightarrow J = 0^{+}$$

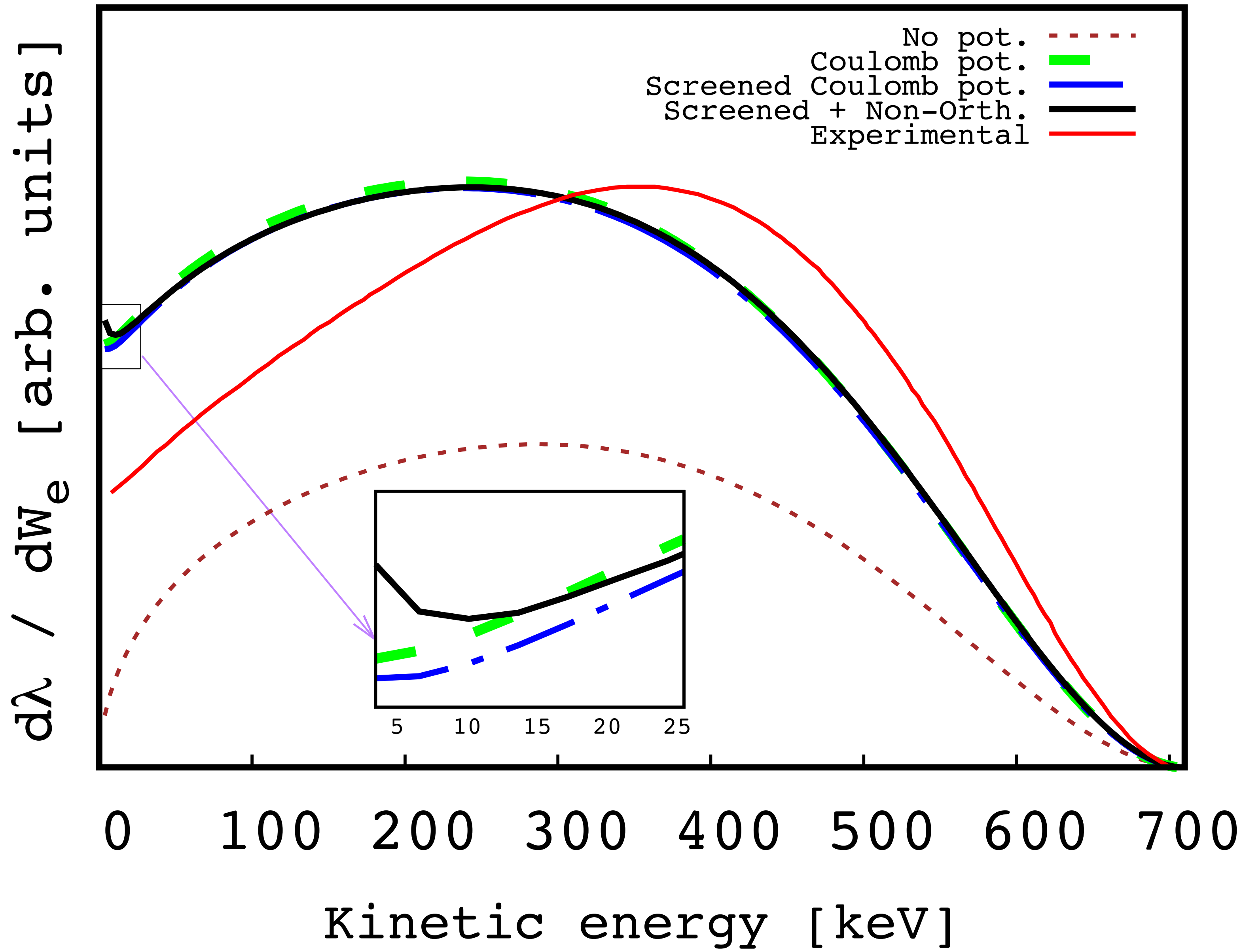
Second forbidden non-unique

$^{36}_{17}\mathrm{Cl}_{19}$ **= odd-odd**

$^{36}_{18}\mathrm{Ar}_{18}$ **= even-even**

β^{-} **half-life = 3.01X10^{5} y**

Q-value = 709.547 keV
(ground state to excited state)



Mean-field DHF + screening (self-consistent DHF) + exchange (discrete-continuum interaction) does not work fine as standard approaches !!!!

Shake-up and shake-off modifies the decay by only 5%

Final-state nuclear many-body affects on beta-decay spectra of odd-odd nuclei?

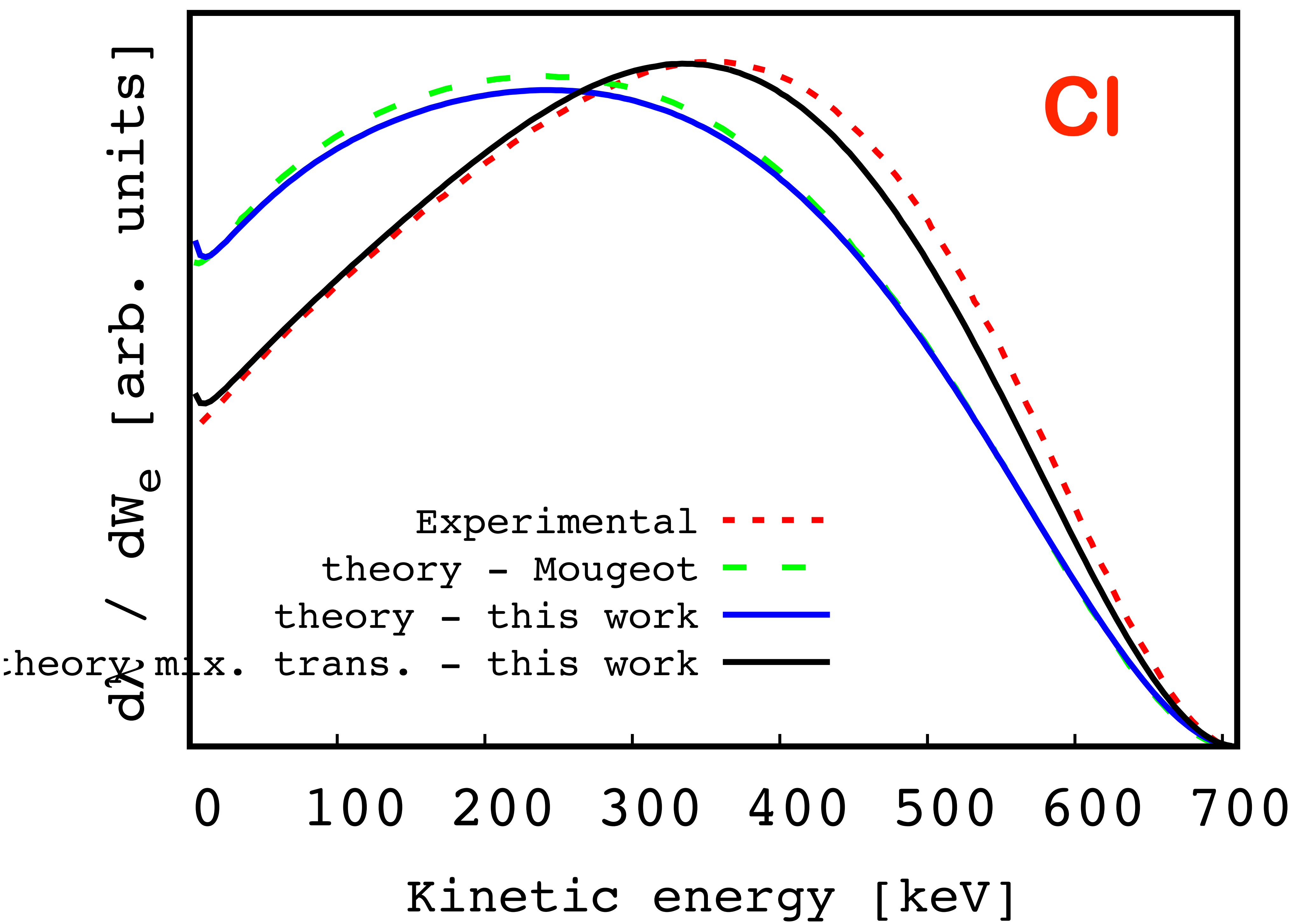
Spectroscopic term	n	l	J	Number of states 2J+1	Number of nucleons in a shell	Magic Numbers
$1s_{1/2}$	1	0	1/2	2	2	2
$1p_{3/2}$	1	1	3/2	4	6	8
$1p_{1/2}$	1	1	1/2	2		
$1d_{5/2}$	1	2	5/2	6	12	20
$2s_{1/2}$	2	0	1/2	2		
$1d_{3/2}$	1	2	3/2	4		
$1f_{7/2}$	1	3	7/2	8	8	28
$2p_{3/2}$	2	1	3/2	4	22	50
$1f_{5/2}$	1	3	5/2	6		
$2p_{1/2}$	2	1	1/2	2		
$1g_{9/2}$	1	4	9/2	10		
$1g_{7/2}$	1	4	7/2	8	32	82
$2d_{5/2}$	2	2	5/2	6		
$2d_{3/2}$	2	2	3/2	4		
$3s_{1/2}$	3	0	1/2	2		
$1h_{11/2}$	1	5	11/2	12		
$1h_{9/2}$	1	5	9/2	10	44	126
$2f_{7/2}$	2	3	7/2	8		
$2f_{5/2}$	2	3	5/2	6		
$3p_{3/2}$	3	1	3/2	4		
$3p_{1/2}$	3	1	1/2	2		
$1i_{13/2}$	1	6	13/2	14		
$2g_{9/2}$	2	4	9/2	10	58	184
$3d_{5/2}$	3	2	5/2	6		
$1i_{11/2}$	1	6	11/2	12		
$2g_{7/2}$	2	4	7/2	8		
$4s_{1/2}$	4	0	1/2	2		
$2d_{3/2}$	2	2	3/2	4		
$1j_{15/2}$	1	7	15/2	16		

The experimentally determined final state of the ${}^{36}_{18}\mathrm{Ar}_{18}$ daughter nucleus is 0+. Within the nuclear shell model two protons and two neutrons all occupy the $1d_{3/2}$ single-particle state. By coupling the $1d_{3/2}$ n to p and to a $1d_{3/2}$ "core" to construct a 0+ final symmetry state, and by calculating the hadronic matrix element for this transition only, we obtain the lineshape reported as a blue curve in the previous figure. We could not yet find a good agreement between simulations and experimental data.

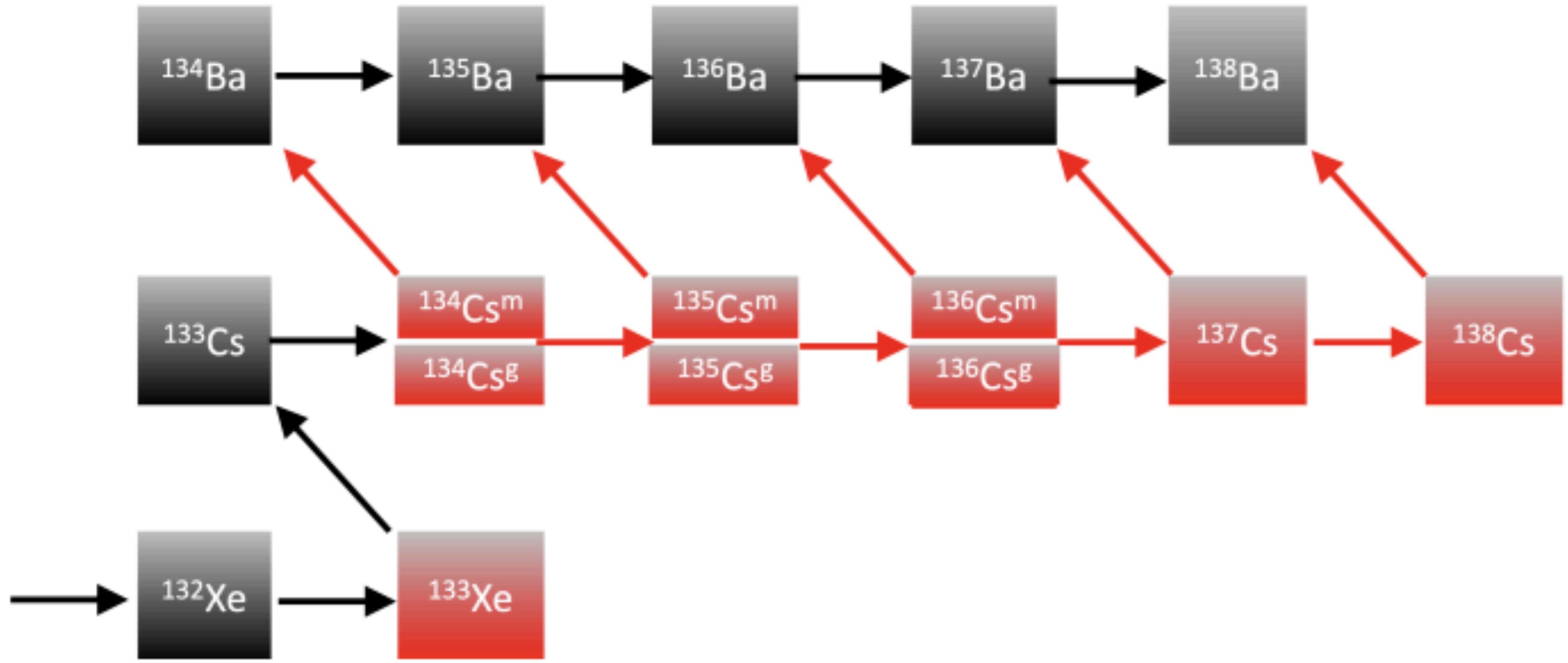
Adding "nuclear many-body effects" by mixing transitions to the $1d_{3/2}$ orbital with the $2s_{1/2}$ level, which is energetically close, we find good agreement with experiments

Final-state nuclear many-body affects on beta-decay spectra of odd-odd nuclei?

$$1 \times J^{H,\mu}_{1d_{3/2}\to 1d_{3/2}} - 2.55 \times J^{H,\mu}_{1d_{3/2}\to 2s_{1/2}}$$

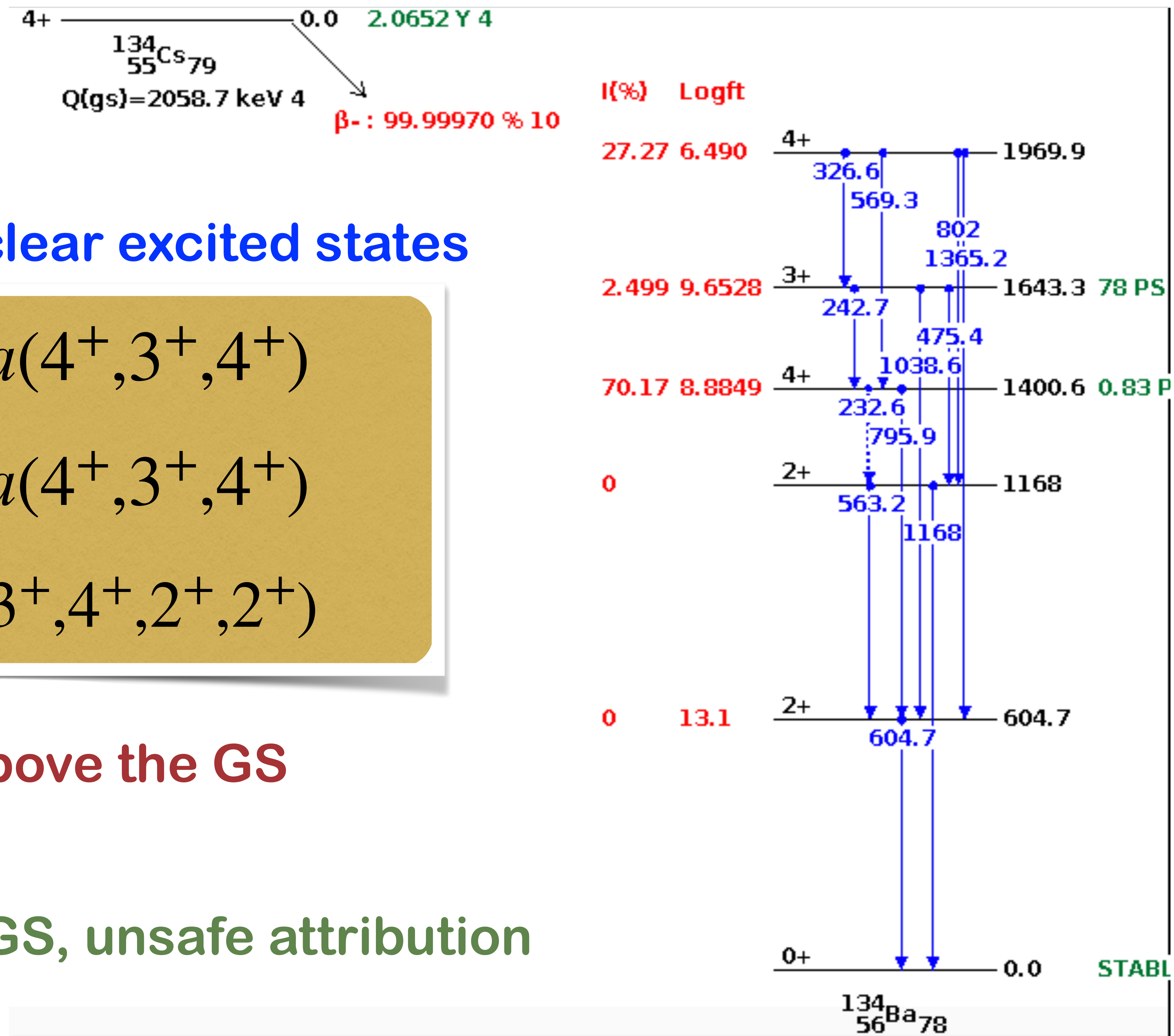


The half-life for the radioactive ^{134}Cs and ^{135}Cs in astrophysical scenarios



- The abundance of Ba in AGB stars depends solely on slow (s) n-captures
- The s-process contribution to the element Ba starts from neutron captures on the stable isotope ^{133}Cs
- The flux proceeds through a branching point at the radioactive ^{134}Cs, where n-captures compete mainly with β–decay (laboratory half-life = 2 yr) to excited states of ^{134}Ba and, much less effectively, with electron captures to 134Xe (half-life = 6.8·105 yr)
- From ^{134}Cs, n-captures feed the longer-lived ^{135}Cs, and then ^{136}Cs (half-life = 13.16 d) and ^{137}Cs (half-life = 30.07 y), which are sites of branching points for the s-process path, but whose decay rates remain essentially unchanged for varying temperatures

$$^{134}_{55}Cs \rightarrow ^{134}_{56}Ba + e^- + \bar{\nu}$$



^{134}Cs short-lived nuclear excited states

$$Cs(4^+) \rightarrow Ba(4^+,3^+,4^+)$$

$$Cs(5^+) \rightarrow Ba(4^+,3^+,4^+)$$

$$Cs(3^+) \rightarrow Ba(4^+,3^+,4^+,2^+,2^+)$$

11 keV above the GS

60 keV above the GS, unsafe attribution

1 keV≈11.6 MK

How rates are typically assessed

Experimental tests of Fermi's theory

Start from the allowed beta transition Fermi Formula

$$\frac{\mathrm{d}\lambda^0_\beta}{\mathrm{d}p} = g^2 \frac{|M^0_{if}|^2}{2\pi^3\hbar^7 c} p^2 q^2$$

$$\begin{aligned}\lambda_\beta &= g^2 \frac{|M^L_{if}|^2}{2\pi^3\hbar^7 c}\int_0^{p_{\max}} \mathrm{d}p\, S^L(p,q) F^{\pm}(Z',p) p^2 q^2 \\ &= g^2 \frac{m_e^5 c^4 |M^L_{if}|^2}{2\pi^3\hbar^7}\left[\frac{1}{(m_e c)^5}\int_0^{p_{\max}} \mathrm{d}p\, S^L(p,q) F^{\pm}(Z',p) p^2 q^2\right] \\ &\equiv g^2 \frac{m_e^5 c^4 |M^L_{if}|^2}{2\pi^3\hbar^7} f_L(Z',Q)\ ,\end{aligned}$$

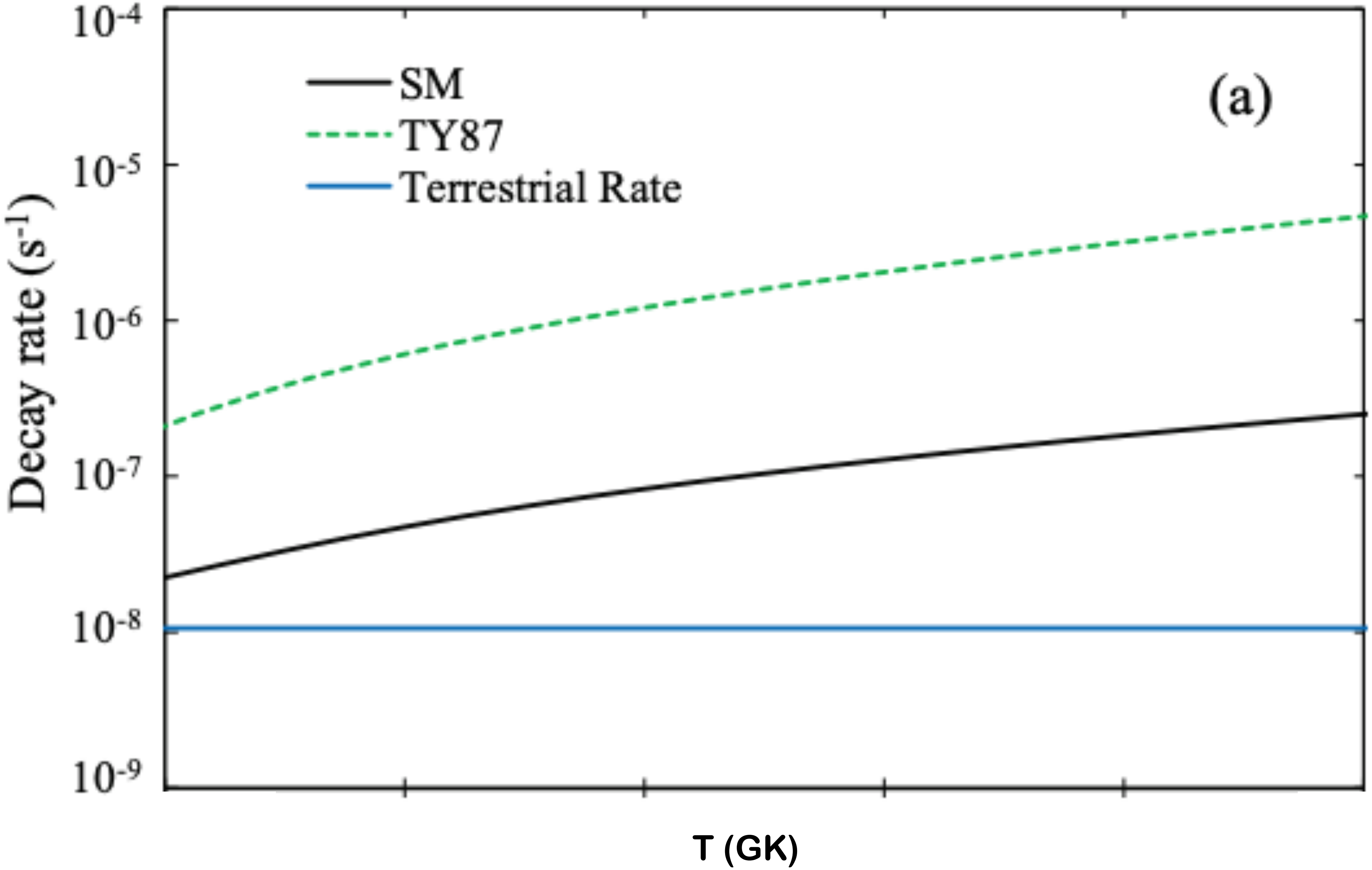
Include shape factor (accounting of "forbiddeness") and Fermi function (accounting of Coulomb distortion of the e wf)

$$t_{1/2} = \log(2)/\lambda_\beta$$

Invert the eq.

$$f_L(Z',Q) t_{1/2} \equiv f t_{1/2} = \frac{\log_e(2) 2\pi^3\hbar^7}{g^2 m_e^5 c^4 |M^L_{if}|^2}\ . \quad \rightarrow \quad |M^L_{if}|$$

The ft's can be quite large, and sometimes the "log ft" value is quoted. log(ft) can be measured, this is called systematics



^{134}Cs stellar β–decay rate of TY87 and of Li. et al. obtained with the shell model (Kuo-Ang Li et al. 2021 ApJL 919 L19)

The half-life for the radioactive ^{134}Cs and ^{135}Cs in astrophysical scenarios: our model

The calculations of β decay have been carried out by solving the Dirac-Hartree-Fock (DHF) equations for both the electron liquid and the nucleus, using the following approximations:

- The hadronic and leptonic currents have been factorised in two non-interacting parts.
- The nucleon-nucleon interaction is modelled by a relativistic one-body Wood-Saxon potential.
- Nuclear dynamic correlation is neglected.

Assumptions in the nuclear simulations

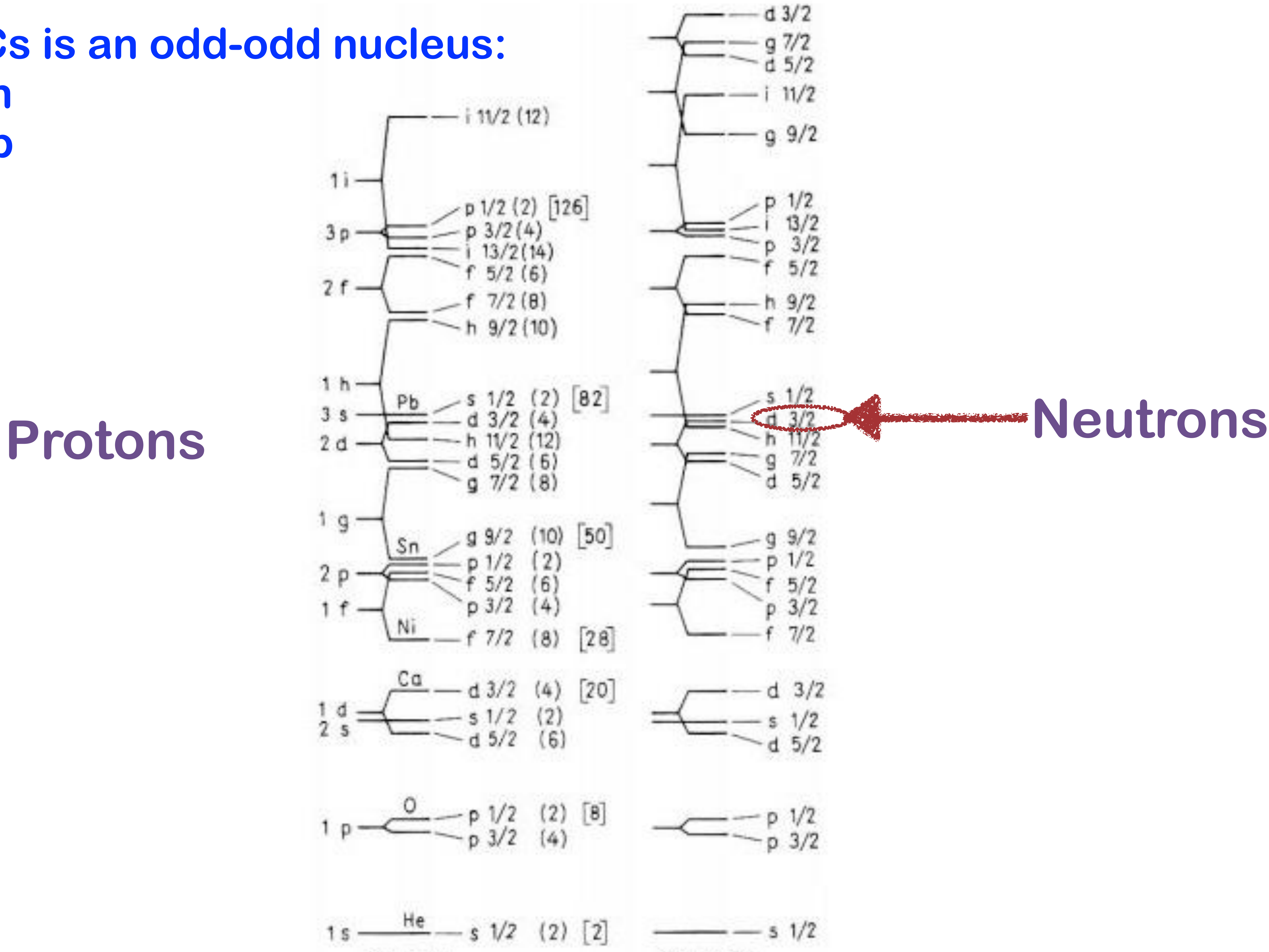
- The decaying neutron in the Cs nucleus is found in the $2d_{3/2}$ shell and weak decays into a proton in the $1g_{7/2}$ shell of Ba. This was deduced according to the nuclear shell model and can be a crude approximation particularly for the excited decays, where several states may participate in the decay. This state is geometrically coupled to the "core" of the other nucleons to recover the total J.

- In a many-body approach, such as CI, the decaying neutron wave function is a superposition of several configuration of nearby energy. In ^{134}Cs those are the $1h_{11/2}$ and $3s_{1/2}$ single-particle orbitals, respectively. However, this level of forbiddance is higher than the d to f owing to a bigger jump in ΔJ.

- The population of nuclear states has been assumed to follow a Boltzmann probability distribution, i.e. $\exp(-E/K_B T)$, where E is the energy of the nuclear level, T the temperature, and K_B the Boltzmann constant. We also took into account the degeneration of the three nuclear levels, which is 9(4^+), 11(5^+), and 7(3^+).

The nuclear shell model: practical view

^{134}Cs is an odd-odd nucleus:
79 n
55 p



Assumptions in the electronic structure calculations

- The chemical potential of electrons and positrons is calculated under the assumption to deal with an ideal Fermi gas in a box using a relativistic energy-momentum dispersion $E^2 = c^2p^2 + m_e c^4$. Protons are non-relativistic particles.

- The density of protons n_p (protons/cm³) and is equal to the density of electrons minus the density of positrons at that given temperature (energy can be high enough to form e⁺-e⁻ couples):

$$n_p = n_{e^-} - n_{e^+}$$

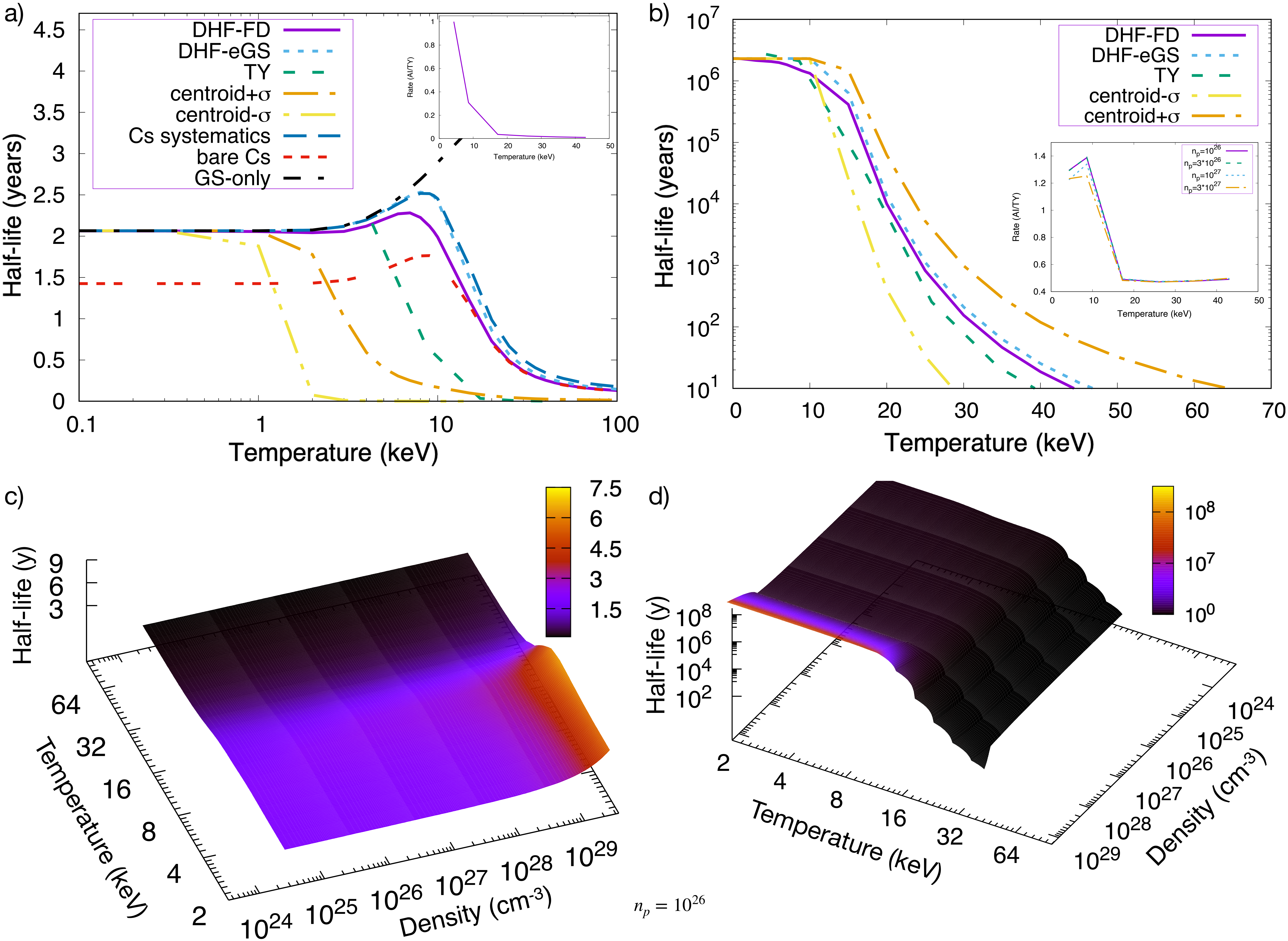
- The electronic levels of the Cs atom have not been re-optimized at each temperature. It is assumed that they are the same at any temperature, and we populate them according the Fermi-Dirac (FD) distribution $n^i_{e^-} = \dfrac{1}{1 + e^{(\epsilon_i - \mu_{e^-})/(KT)}} = F(T, \mu)$, where the energies ϵ_i of the i-th level is obtained via the self-consistent solution of the DHF equation and the chemical potential from the implicit relation valid for a Fermi gas:

$$n_{e^-} = \int_0^\infty dp\ p^2/\pi^2 \times (F((c \times \sqrt{(}p^2 + c^2) - \mu_e)/kT) - F((c \times \sqrt{(}p^2 + c^2) + \mu_e)/kT))$$

Assumptions in the rate calculations

- In the neutral atom, electrons move within the mean-field potential of the other electrons, while for the completely ionized atom (bare nucleus) the orbitals are optimized by considering only a bare Coulomb potential (Cs1s BE = 36.12 keV and 41 keV for neutral and completely ionized atom, respectively).

- We renormalize the rate at all temperatures by a constant factor obtained so as to recover the room temperature experimental log(ft) in simulations (~10). We expect this to be due mainly to the accuracy of nuclear wavefunction calculations more than that of the electronic part.



a)
DHF-FD
DHF-eGS
TY
centroid+σ
centroid-σ
Cs systematics
bare Cs
GS-only
Half-life (years)
Temperature (keV)
Rate (Al/TY)
Temperature (keV)
b)
DHF-FD
DHF-eGS
TY
centroid-σ
centroid+σ
Half-life (years)
Temperature (keV)
$n_p=10^{26}$
$n_p=3*10^{26}$
$n_p=10^{27}$
$n_p=3*10^{27}$
Rate (Al/TY)
Temperature (keV)
c)
Half-life (y)
Temperature (keV)
Density (cm-3)
d)
Half-life (y)
Temperature (keV)
Density (cm-3)
$n_p = 10^{26}$

Important messages from rate calculations

The β decay rate of Cs is affected concurrently by two major factors:

1. **the presence of 3 nuclear excited states of Cs;**
2. **the electronic excitation, also up to a complete ionization**

Nuclear DOF

The nuclear excited state dynamics is the most relevant of the two, as it can increase the rate by a factor of 15 at 100 KeV (1 GK) to 23 at 1000 KeV with reference to room temperature conditions and by a factor of 3 at $T>10^8$ K for ^{134}Cs as compared to previous works based on systematics.

This is basically due to populating fast-decaying nuclear excited states, in particular the 60 keV excited state of ^{134}Cs which delivers a rate ~ 80 times higher than the 4^+ GS decay. This number is obtained by comparing the decay rates from 4^+ and 3^+, as if they were the only occupied nuclear states from which the decay occurs.

Electronic DOF

At variance, in the range [0:15] keV the temperature of electrons has the most pronounced impact on the rate. Typically increase the rate as electron can be accommodated also in empty bound orbitals. Despite being a quark-level process, the contribution of the electronic degrees of freedom to the rate is crucial.

Increasing temperature means both populating electronic excited states and changing the charge state. This may decrease the half-life even by 20% at 10 keV

To summarize some data: owing to the temperature acting on both nuclei and electrons we find an increase of the rate of about 3 times at 20 KeV (~ 230 MK), of 6 times at 30 keV, of 8 times at 40 keV (~ 464 MK) with respect to the GS decay only.

Table I. Comparison between ^{134}Cs rates obtained using our model and by TY [2] (units in $s^{-1} \times 10^{-8}$).

T_8[a]	TY[b]	This work[b]
0.5 (4.31)	1.02	1.02
1 (8.62)	3.28	1.01
2 (17.23)	63.1	2.28
3 (25.85)	211.0	4.73
4 (34.47)	481.0	7.22
5 (43.09)	889.0	9.36

[a] $T_8 = 10^8$ K (corresponding values in keV in parentheses).
[b] $n_p = 10^{26}$ cm^{-3}

Table II. Comparison between ^{135}Cs rates obtained using our model and by TY [2] (units in s^{-1}).

T_8[a]	TY[b]	This work[b]	TY[c]	This work[c]	TY[d]	This work[d]	TY[e]	This work[e]
0.5 (4.31)	8.12e-15	1.05e-14	7.90e-15	1.02e-14	7.92e-15	9.70e-15	7.39e-15	9.11e-15
1 (8.62)	1.04e-14	1.44e-14	8.78e-15	1.22e-14	8.04e-15	1.08e-14	7.81e-15	9.79e-15
2 (17.23)	6.91e-13	3.39e-13	6.65e-13	3.27e-13	6.09e-13	3.01e-13	5.52e-13	2.66e-13
3 (25.85)	8.64e-11	4.08e-11	8.55e-11	4.04e-11	8.24e-11	3.91e-11	7.74e-11	3.64e-11
4 (34.47)	9.77e-10	4.66e-10	9.65e-10	4.64e-10	9.52e-10	4.57e-10	9.17e-10	4.38e-10
5 (43.09)	4.18e-09	2.05e-09	4.15e-09	2.05e-09	4.08e-09	2.03e-09	3.96e-09	1.97e-09

[a] $T_8 = 10^8$ K (corresponding values in keV in parentheses).
[b] $n_p = 10^{26}$ cm^{-3}
[c] $n_p = 3 \times 10^{26}$ cm^{-3}
[d] $n_p = 10^{27}$ cm^{-3}
[e] $n_p = 3 \times 10^{27}$ cm^{-3}

Major differences with state-of-the art methods

- We do not use semi-empirical approaches based on *log(ft)*

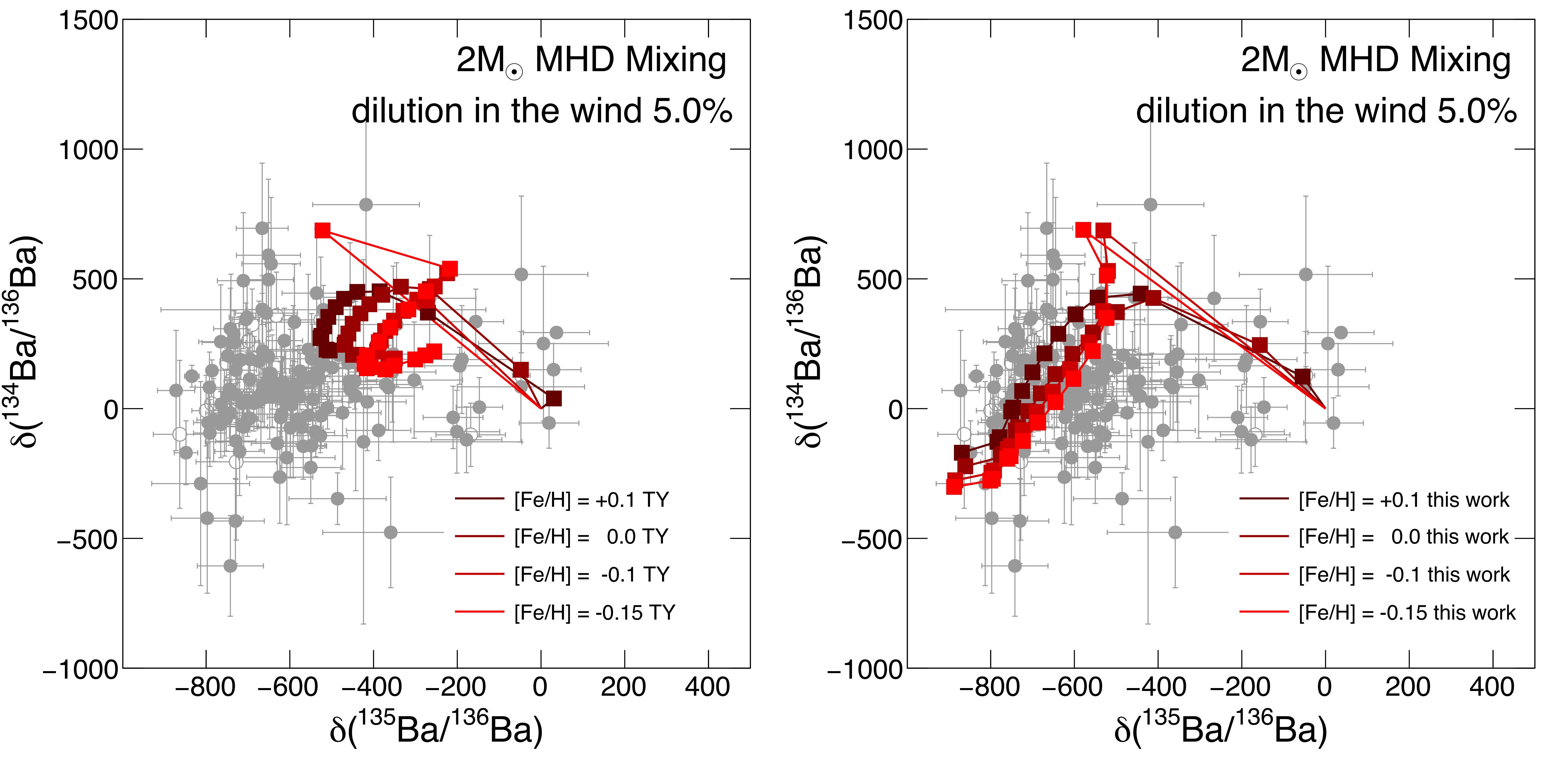
- We do not calculate *log(ft)* by e.g. using the nuclear shell model to obtain the stellar rate of ^{134}Cs within the standard approach to β-decay spectra.

- At variance, in our work we extend the theory and the computational methods by using a fully relativistic approach.

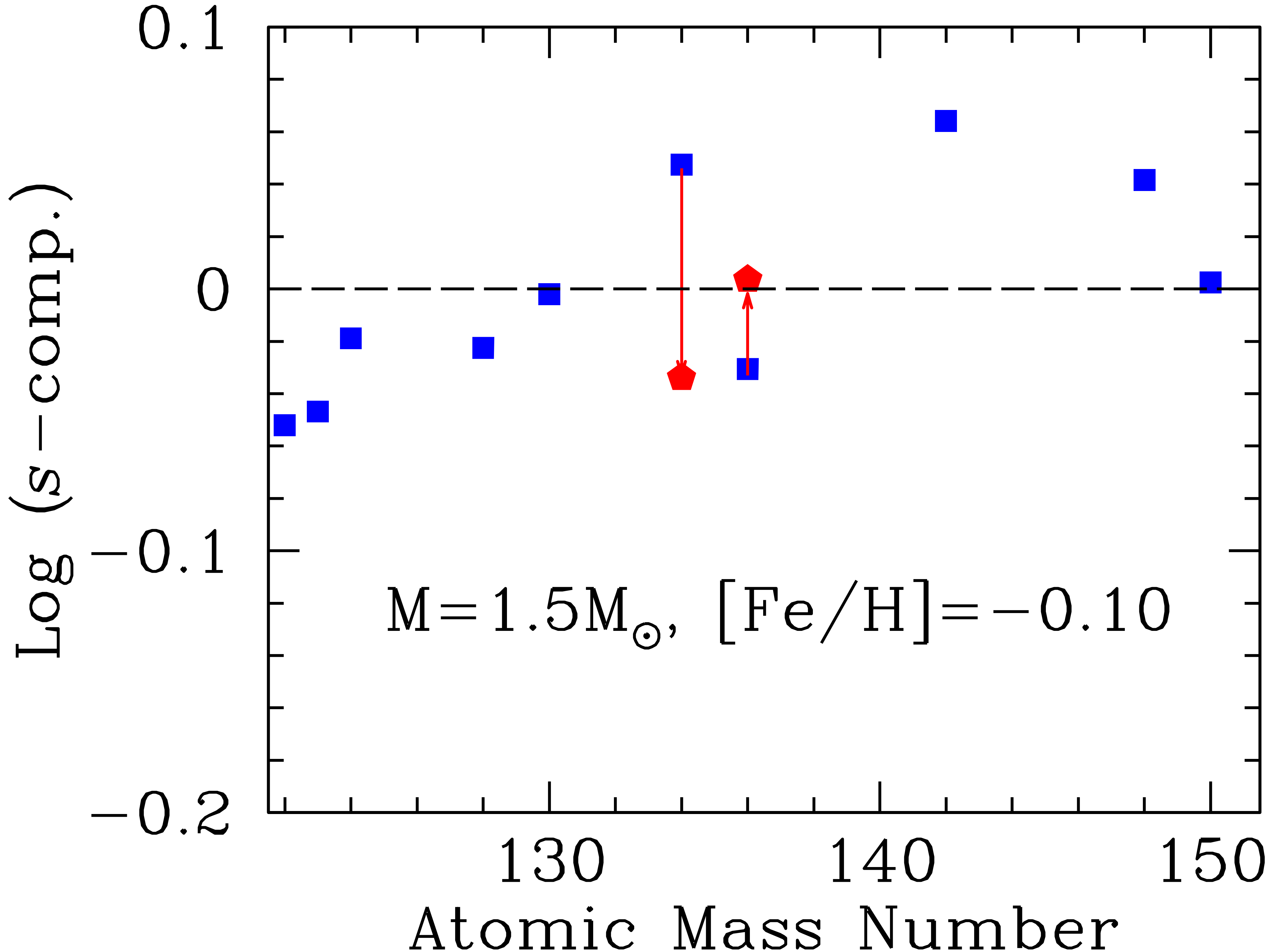
- We calculate directly the nuclear matrix elements that enter the hadronic current from first-principles. To do so, we adopt a mean-field approach, which can of course be systematically improved by using more correlated many-body techniques without modifying the backbone of our method.

- A second substantial difference relies on the treatment of the leptonic current, which is typically neglected or added via a semi-empirical Fermi function. We demonstrate that it may halve the half-life of ^{134}Cs around 10 keV. We include include both bound and continuum channels, the exchange interaction, the non-orthogonality between the parent and daughter electronic orbitals, as a function of plasma density, temperature and charge state distributions, reaching an unprecedented level of accuracy.

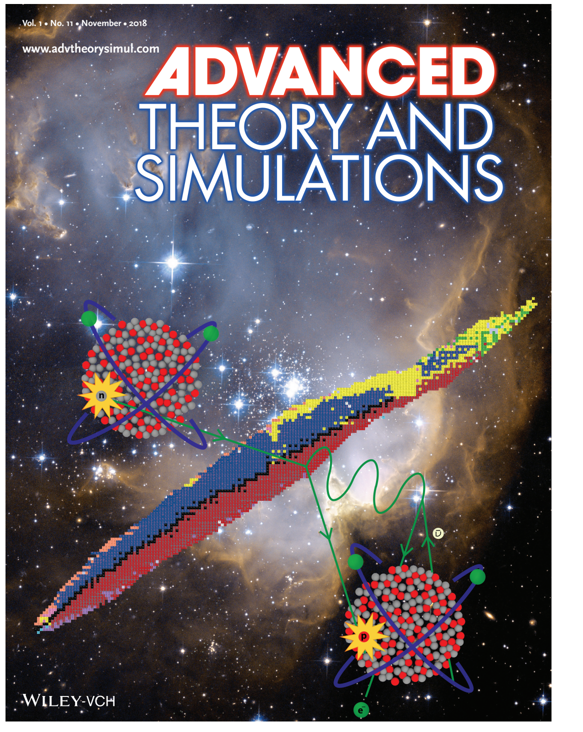
Left panel: isotopic ratios of ^{134}Ba and ^{135}Ba with respect to ^{136}Ba, displayed as part-per-mil deviations (indicated by the symbol δ), with decay rates from TY.
Right panel: the results of the same models, where only the decay rates for ^{134}Cs and ^{135}Cs are changed, using those of the present work. Computations are for 2 $M_\odot$ stars, where magneto-hydrodynamic processes induce the penetration of protons into He-rich layers, producing ^{13}C then releasing neutrons through $^{13}C(\alpha,n)^{16}O$. Abundances are computed in stellar winds, where magnetic blobs further add 5% of C-rich material in flare-like episodes.The symbol [Fe/H] indicates $Log(X_{Fe}/X_{H})_{star}$ - $Log(^{13}C)_{sun}$



Percentage of s-process contributions (blue dots) as computed by M. Busso et al. ApJ 908, 55 (2021) for s-only nuclei near the magic neutron number N= 82.



Conclusions



- A new method for calculating β- and e^--capture decay spectra in medium to heavy nuclei, which extends the standard approach in several ways
- It works also in astrophysical environment by including temperature, density and charge state distribution
- This method can be applied to any nuclear beta decay and include relativistic, many-body screening and post-collisional effects
- Our approach is more accurate than state-of-the-art methods

Outlook

- Inclusion of nuclear dynamic correlation beyond mean-field approximation;
- Estimate of beta-decay rates of different elements (^{176}Lu, ^{94}Nb, any suggestion from the Pandora collaboration)


That's all Folks!