

Qubit WP1

15/11/2021

Quantum Simulation Module (QSM - NIST)

<https://github.com/Borealis126/QSM>

Chip design

- Chip design and component parameters (inductances) defined using .json files
- Qubit
 - Floating/Grounded
 - Single/Double JJ
 - Only rectangular pads at the moment
- Resonator
 - CPW transmission line
- Transmission lines
 - Feedline
 - Launchpads
 - Qubit-Qubit coupling

Workflow - Layout phase

- *systemParameters.json* → main chip information
 - Number of qubits
 - Number of control lines
 - Material
- *componentParameters.json*:
 - CPW phase velocity
 - Qubits type and inductances
 - Resonators and control lines definition
- *componentGeometries.json*:
 - Position and dimensions of all the components

systemParameters.json

```
{
  "Chip Description": "oneQubit design",
  "Number of Qubits": 1,
  "Number of Readout Resonators": 1,
  "Number of Control Lines": 1,
  "Material": "perfect conductor",
  "Flip Chip?": "No",
  "Chip Markers": "Pappas",
  "Simulate Feedline?": "Yes"
}
```

componentParameters.json

```
1 {
2   "CPW": {
3     "Phase Velocity(um/s)": 1.25e14
4   },
5   "Qubits": {
6     "θ": {
7       "Type": "Floating-rectangularPads-singleJJ",
8       "L_J(H)": 1.0e-8,
9       "L_I(H)": 1.5e-8
10    },
11  },
12  "Readout Resonators": {
13    "θ": {
14      "Pad 1 Type": "T",
15      "Pad 2 Type": "T",
16      "Capacitance to Feedline (F)": 1.2e-10,
17      "Feedline Pad Capacitance to Ground (F)": 1.2e-12
18    }
19  },
20  "Control Lines": {
21    "θ": {
22      "Type": "feedline"
23    }
24  }
25 }
```

componentGeometries.json

```
1 {
2   "Qubits": {
3     "0": {
4       "Angle": 0,
5       "Center X": -1500,
6       "Center Y": 0,
7       "Pad Spacing": 30,
8       "Pad 1 Width": 300,
9       "Pad 1 Length": 100,
10      "Pad 1 Height": 0.1,
11      "Pad 1 Side 1 Boundary": 20,
12      "Pad 1 Side 2 Boundary": 20,
13      "Pad 1 Side 3 Boundary": 20,
14      "Pad 1 Side 4 Boundary": 20,
15      "Pad 2 Width": 300,
16      "Pad 2 Length": 100,
17      "Pad 2 Height": 0.1,
18      "Pad 2 Side 1 Boundary": 20,
19      "Pad 2 Side 2 Boundary": 20,
20      "Pad 2 Side 3 Boundary": 20,
21      "Pad 2 Side 4 Boundary": 20,
22      "JJ Location": "[0:0]",
23      "JJ Stem Boundary": 30,
24      "JJ Stem Width": 4,
25      "JJ Patch Width": 4,
26      "JJ Patch Length": 8,
27      "JJ Top Electrode Width": 0.52,
28      "JJ Bottom Electrode Width": 0.4
29    }
30  },
```

Qubit

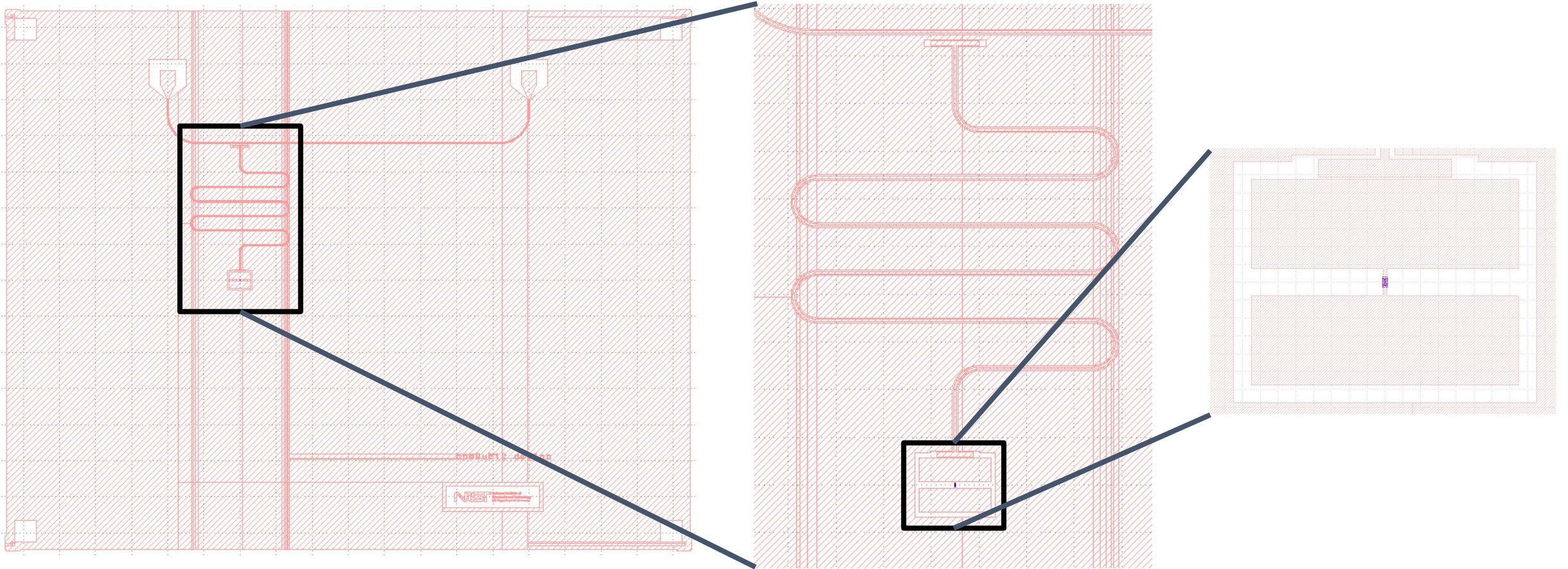
```
31   "Readout Resonators": {
32     "0": {
33       "Length": 8000,
34       "Angle": 0,
35       "Center X": -1500,
36       "Center Y": 990,
37       "Pad Separation": 1725,
38       "Meander Turn Radius": 100,
39       "Pad T Stem Length": 50,
40       "Meander To Pad Minimum Distance": 100,
41       "Pad 1 Curve Angle": 0,
42       "Pad 2 Curve Angle": 0,
43       "Pad 1 Width": 200,
44       "Pad 1 Length": 20,
45       "Pad 1 Height": 0.1,
46       "Pad 1 Side 1 Boundary": 30,
47       "Pad 1 Side 2 Boundary": 5,
48       "Pad 1 Side 3 Boundary": 30,
49       "Pad 1 Side 4 Boundary": 5,
50       "Pad 2 Width": 150,
51       "Pad 2 Length": 20,
52       "Pad 2 Height": 0.1,
53       "Pad 2 Side 1 Boundary": 30,
54       "Pad 2 Side 2 Boundary": 5,
55       "Pad 2 Side 3 Boundary": 30,
56       "Pad 2 Side 4 Boundary": 5
57     }
58   },
```

Resonator

```
59   "Control Lines": {
60     "0": {
61       "Start X": -2500,
62       "Start Y": 2500,
63       "Start Angle": -1.57,
64       "Section Code": "(S:200)(R:1.57:400)(S:4200)(R:1.57:400)(S:200)"
65     }
66   },
67   "CPW": {
68     "Width": 10,
69     "Gap": 6,
70     "Trench": 0.1,
71     "Height": 0.1
72   },
73   "Ground(s)": {
74     "0": {
75       "Height": 0.1,
76       "Mesh Boundary": 40,
77       "Mesh Spacing": 50,
78       "Mesh Size": 5
79     }
80   },
81   "Substrate(s)": {
82     "0": {
83       "Material": "silicon",
84       "Thickness": 500,
85       "Width": 9500,
86       "Length": 7500
87     }
88   },
```

**Control
lines**

Result of the layout phase → GDS file



Workflow - Simulations and analysis

1) Capacitance matrix extracted with Ansys Q3D

$$\begin{pmatrix} C_{1,1} & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & C_{i,i} & -C_{i,j} & \dots \\ \dots & \dots & -C_{j,i} & \dots & \dots \\ \dots & \dots & \dots & \dots & C_{n,n} \end{pmatrix}$$

$$C_{i,i} = C_{i,g} + \sum_{i \neq j}^N C_{i,j}$$

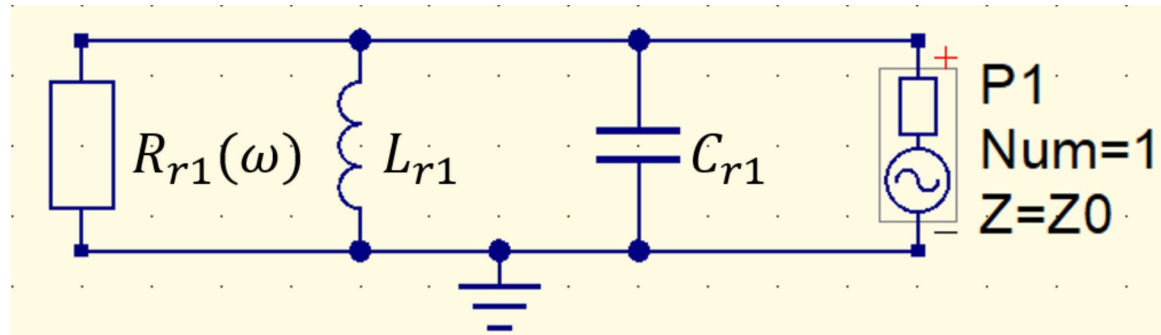
$$C_{i,j} < 0, i \neq j$$

$i, j = \text{resonators and qubits pads}$

| | Q0Pad1 | Q0Pad2 | R0Pad1 | R0Pad2 |
|--------|----------|----------|---------|----------|
| Q0Pad1 | 0.09411 | -0.01927 | 0 | -0.01935 |
| Q0Pad2 | -0.01927 | 0.08391 | 0 | -0.00039 |
| R0Pad1 | 0 | 0 | 0.05146 | 0 |
| R0Pad2 | -0.01935 | -0.00039 | 0 | 0.04416 |

Workflow - Simulations and analysis

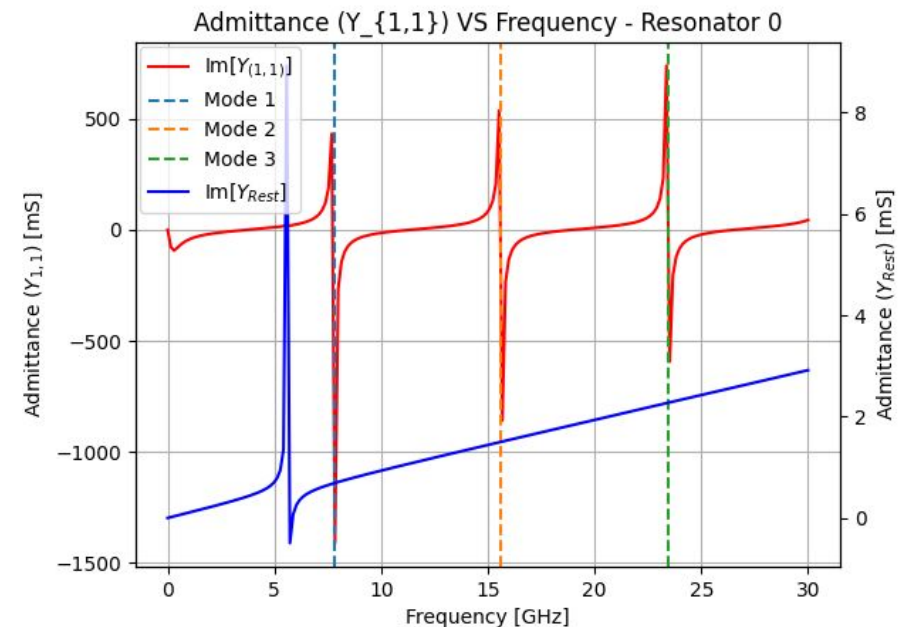
2) Predict resonator frequency using admittance



$$\omega_n = n \times \frac{\pi v_p}{d} \quad \text{for } \lambda/2 \text{ resonator}$$

Directly probing each readout resonator

$$Y_{1,1}(\omega) = \frac{1}{j\omega L_r} + j\omega C_r + \frac{1}{R_r(\omega)}$$



Workflow - Simulations and analysis

3) E_C and E_J for qubits

$$\frac{E_J}{E_C} \approx 45$$

$$E_C^{(Q)} = \frac{e^2}{2 \left[C_{i,i}^{-1} \right]^{-1}}$$

$C_{i,i}^{-1}$ i^{th} diagonal element
of the inverted capacitance matrix

$$E_J^{(Q)} = \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_J}$$

with $\Phi_0 = \frac{\hbar\pi}{e}$, magnetic flux quantum

L_J defined in componentParameters.json

Workflow - Quantization

4) Quantization

$$\omega_q \approx 6.48 \text{ GHz}$$

$$\Phi = \sqrt{\frac{\hbar Z}{2}} (a^\dagger + a)$$

$$Q = j\sqrt{\frac{\hbar}{2Z}} (a^\dagger - a)$$

$$H = \frac{1}{2} Q^T C^{-1} Q - \sum_i^{N_{\text{qubits}}} \left[E_j^{(i)} \cos \left(\frac{\Phi_i}{\Phi_0} \right) \right] + \sum_i^{N_{\text{resonators}}} \left[\frac{\Phi_i^2}{2L_i} \right]$$

Calculations

Anharmonicity

$$\alpha = (E_2 - E_1) - (E_1 - E_0) \approx -390 \text{ MHz}$$

with $E_q \rightarrow q = \text{qubit state}$

Dispersive shift

$$\chi = (E_{1,1} - E_{1,0}) - (E_{0,1} - E_{0,0}) \approx -1.79 \text{ MHz}$$

with $E_{q,r} \rightarrow q = \text{qubit state}; r = \text{resonator state}$

Qubit-Qubit coupling

$$ZZ_{\text{coupling}} = E_{1,1} - E_{0,1} - E_{1,0}$$

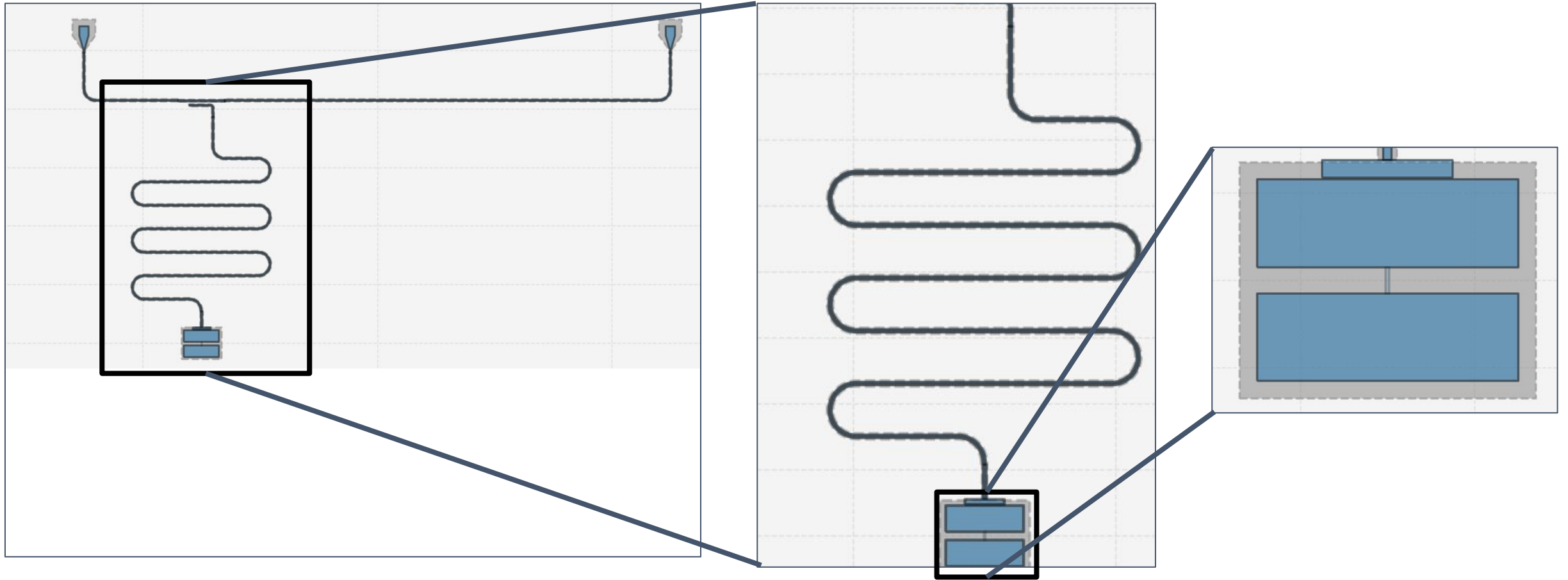
with $E_{q1,q2}$

Qiskit Metal (IBM)

Chip design

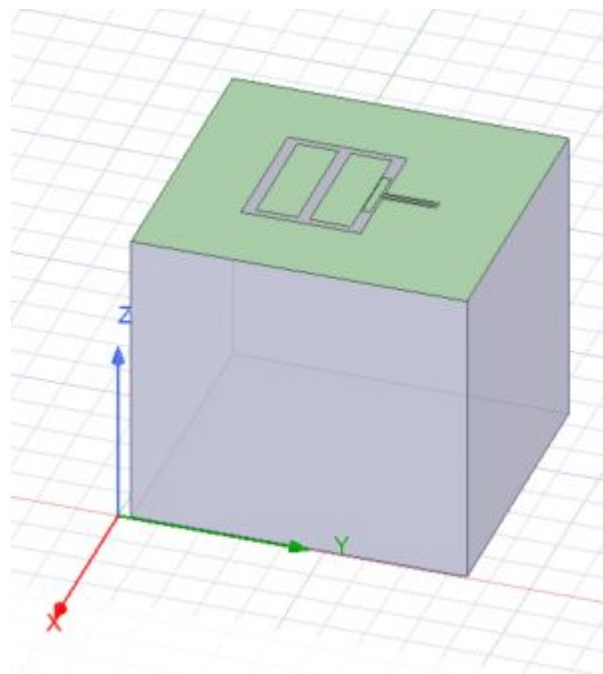
- Each component is an object with dedicated design options and pins
- Qubits
 - Simple transmon
 - Interdigitated transmon
 - Xmon
 - Concentric transmon
- Resonator
 - CPW transmission line
- Transmission lines
 - Semi-automatic management → Path finder from pin to pin

Result of the layout phase



Analysis - Lumped element analysis

- Capacitance matrix \rightarrow lumped oscillator model (LOM) simulation



```
Transmon Properties
f_Q 6.451516 [GHz]
EC 356.734261 [MHz]
EJ 16.339560 [GHz]
alpha -413.005775 [MHz]
dispersion 29.403922 [KHz]
Lq 9.995968 [nH]
Cq 54.298760 [fF]
Tl 145.957291 [us]
```

```
**Coupling Properties**
tCqbus1 -9.030674 [fF]
gbus1_in_MHz -317.445256 [MHz]
chi_bus1 -1.725205 [MHz]
1/Tlbus1 1090.421326 [Hz]
Tlbus1 145.957291 [us]
Bus-Bus Couplings
```

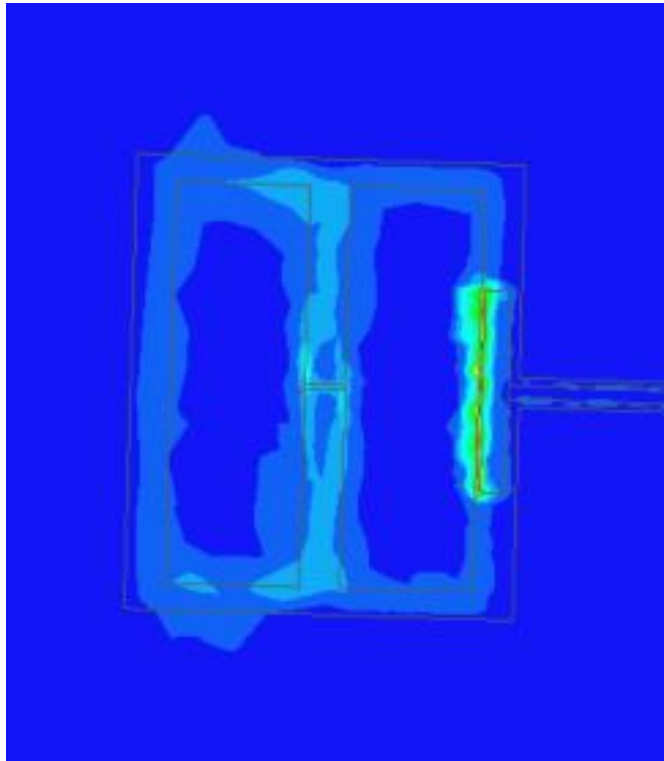
Analysis - Finite Element Analysis and Energy Participation Ratio (EPR)

Participation ratio $p_m := \frac{\text{Inductive energy stored in the junction}}{\text{Total inductive energy of mode } m}$

$$\frac{\phi_{mj}^2}{\hbar} = p_{mj} \frac{\omega_m}{E_J} \quad \text{j=junction and m=mode}$$

$$\alpha_m = p_m^2 \frac{\hbar \omega_m^2}{8E_J} \quad \chi_{qc} = p_q p_c \frac{\hbar \omega_q \omega_c}{4E_J}$$

Analysis - Finite Element Analysis and Energy Participation Ratio (EPR)

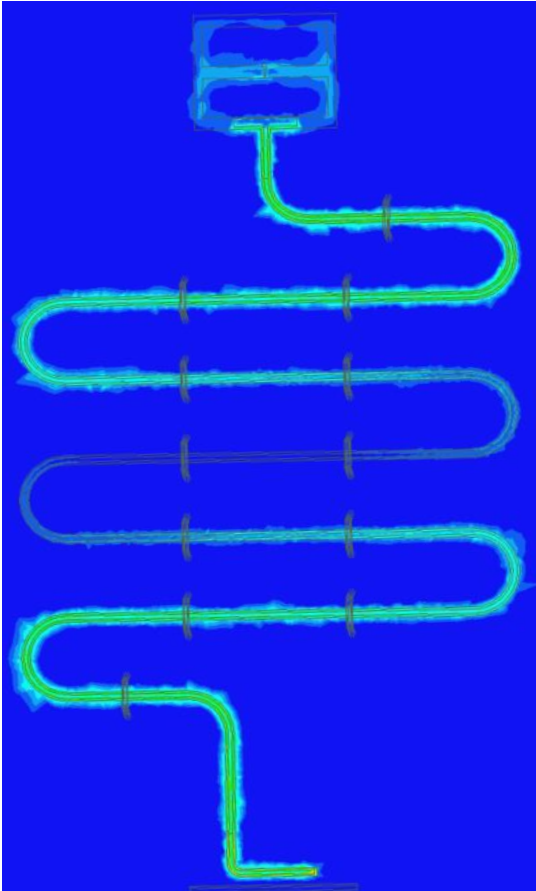


```
Mode 0 at 6.52 GHz [1/4]
Calculating  $\bar{\epsilon}_{\text{magnetic}}, \bar{\epsilon}_{\text{electric}}$ 
( $\bar{\epsilon}_E - \bar{\epsilon}_H$ ) /  $\bar{\epsilon}_E$        $\bar{\epsilon}_E$        $\bar{\epsilon}_H$ 
              79.3%   3.471e-25  7.182e-26

Calculating junction energy participation ration (EPR)
method=`line_voltage`. First estimates:
junction      EPR p_0j   sign s_0j   (p_capacitive)
Energy fraction (Lj over Lj&Cj)= 96.75%
jj            0.792782 (+)      0.0266327
              (U_tot_cap-U_tot_ind)/mean=1.33%
Calculating Qdielectric_main for mode 0 (0/3)
p_dielectric_main_0 = 0.9155999424602178
```

<https://www.nature.com/articles/s41534-021-00461-8>

Analysis - Finite Element Analysis and Energy Participation Ratio (EPR)



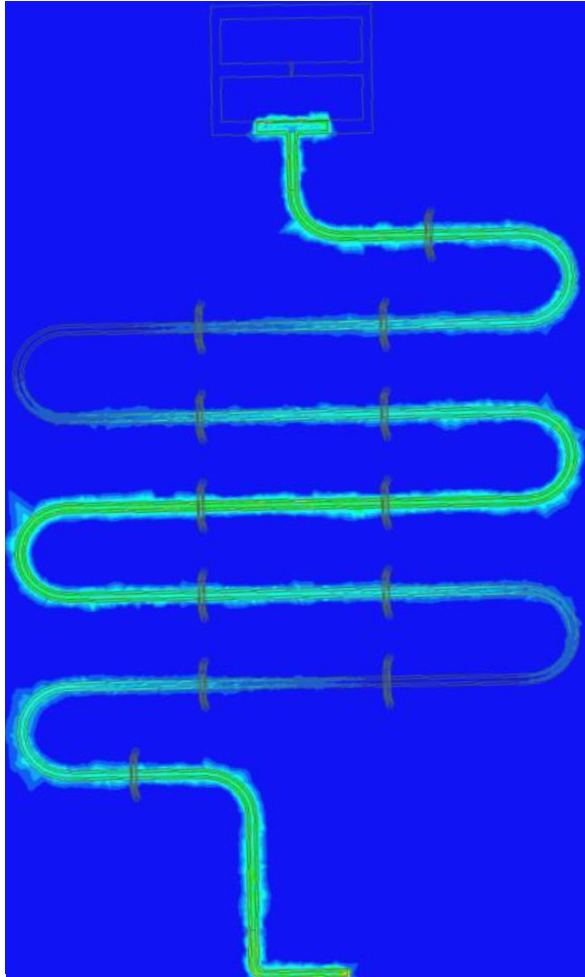
```
Mode 1 at 6.93 GHz [2/4]
Calculating  $\bar{\mathcal{E}}_{\text{magnetic}}, \bar{\mathcal{E}}_{\text{electric}}$ 
( $\bar{\mathcal{E}}_E - \bar{\mathcal{E}}_H$ ) /  $\bar{\mathcal{E}}_E$        $\bar{\mathcal{E}}_E$        $\bar{\mathcal{E}}_H$ 
                20.1%      1.64e-24  1.311e-24

Calculating junction energy participation ration (EPR)
method=`line_voltage`. First estimates:
junction      EPR p_lj   sign s_lj   (p_capacitive)
Energy fraction (Lj over Lj&Cj)= 96.34%
jj            0.200683 (+)      0.00761676
(U_tot_cap - U_tot_ind) / mean = 0.38%

Calculating Qdielectric_main for mode 1 (1/3)
p_dielectric_main_1 = 0.9195616301087681
```

<https://www.nature.com/articles/s41534-021-00461-8>

Analysis - Finite Element Analysis and Energy Participation Ratio (EPR)



```
Mode 2 at 13.72 GHz [3/4]
Calculating  $\bar{\epsilon}_{\text{magnetic}}$ ,  $\bar{\epsilon}_{\text{electric}}$ 
( $\bar{\epsilon}_E - \bar{\epsilon}_H$ ) /  $\bar{\epsilon}_E$        $\bar{\epsilon}_E$        $\bar{\epsilon}_H$ 
                0.1%  9.423e-25  9.414e-25

Calculating junction energy participation ration (EPR)
method=`line_voltage`. First estimates:
junction      EPR p_2j  sign s_2j  (p_capacitive)
Energy fraction (Lj over Lj&Cj)= 87.07%
jj            0.000943155 (+)      0.000140086
(U_tot_cap-U_tot_ind)/mean=0.01%
Calculating Qdielectric_main for mode 2 (2/3)
p_dielectric_main_2 = 0.9199757953971986
```

<https://www.nature.com/articles/s41534-021-00461-8>

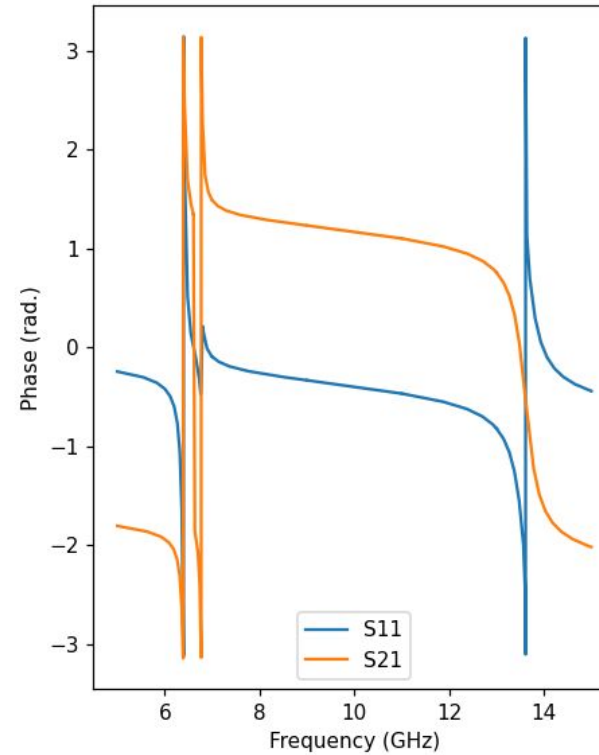
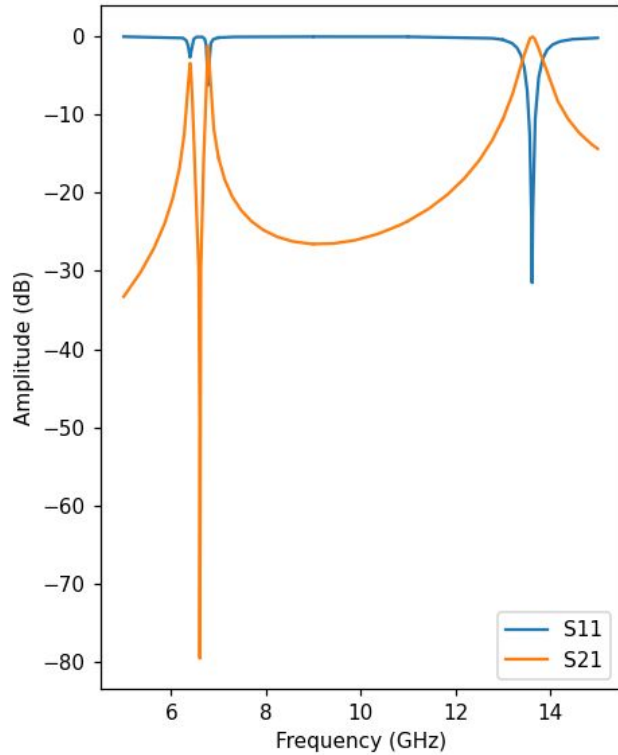
Analysis - Finite Element Analysis and Energy Participation Ratio (EPR)

```
*** Chi matrix ND (MHz)
Anharmonicity 348      31.1      1.27      0.553
               31.1      1.26      0.0907     0.0379
Dispersive shift 1.27    0.0907    0.00118   0.00116
               0.553     0.0379    0.00116   0.000327
```

```
*** P (participation matrix, normalized.)
      0.8
      0.2
0.00094
0.00031
```

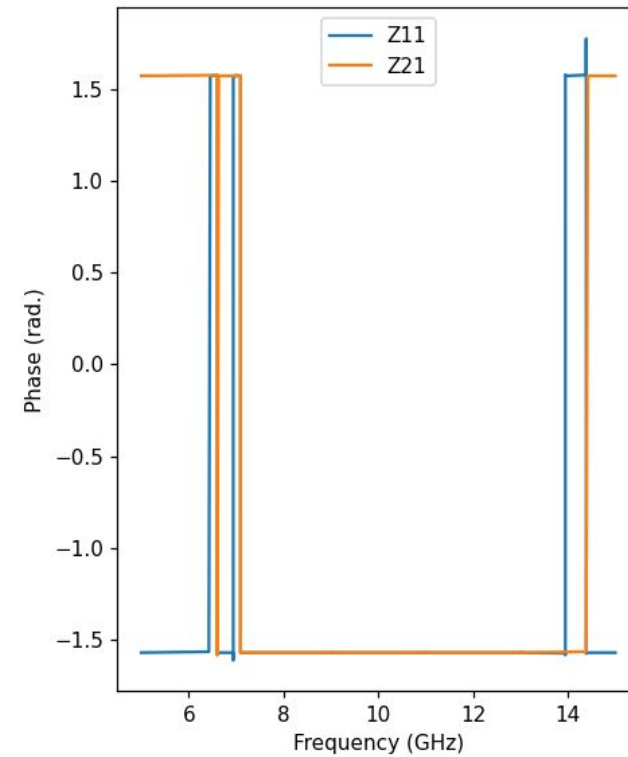
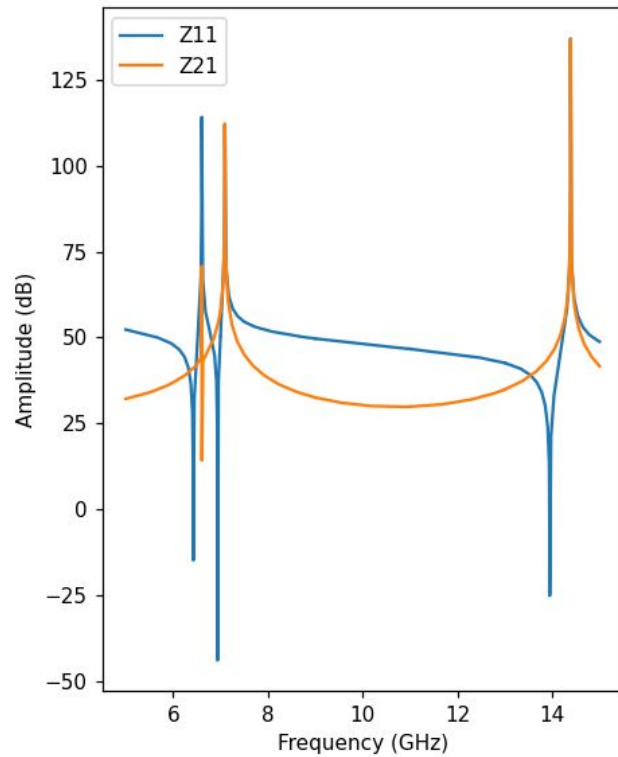
Analysis - Impedance, Scattering and Admittance

- Circuit analysis



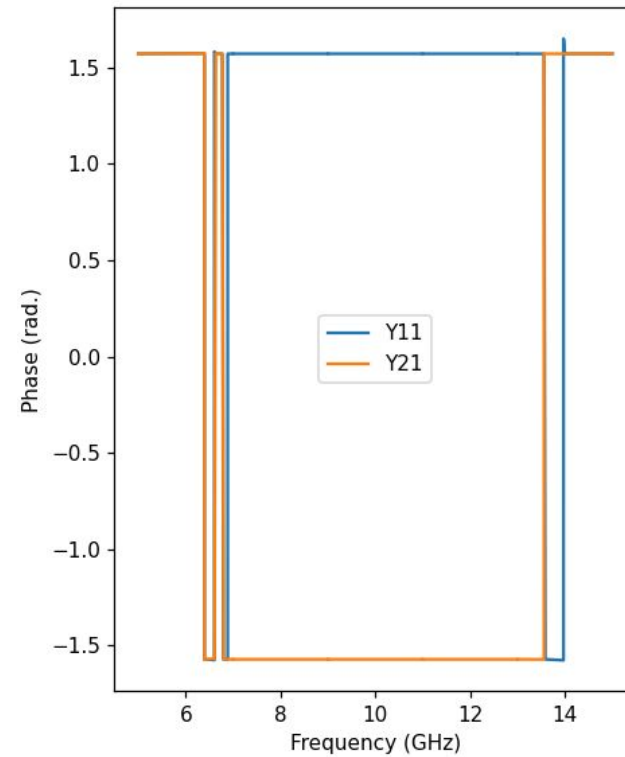
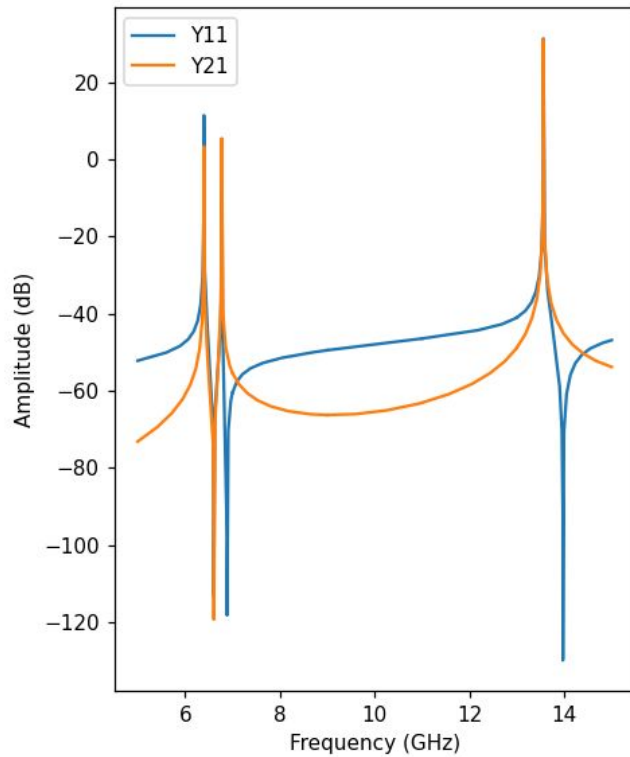
Analysis - Impedance, Scattering and Admittance

- Circuit analysis



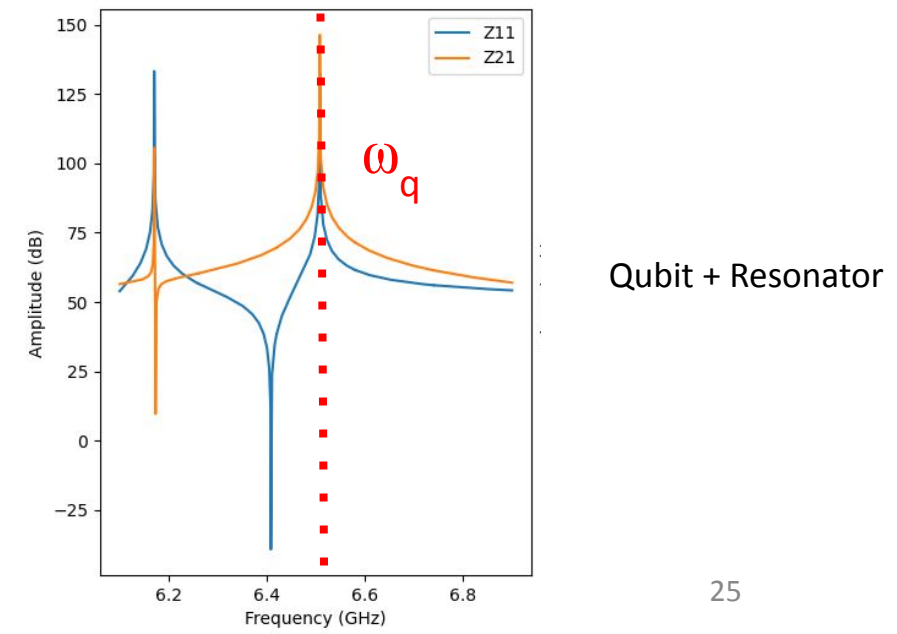
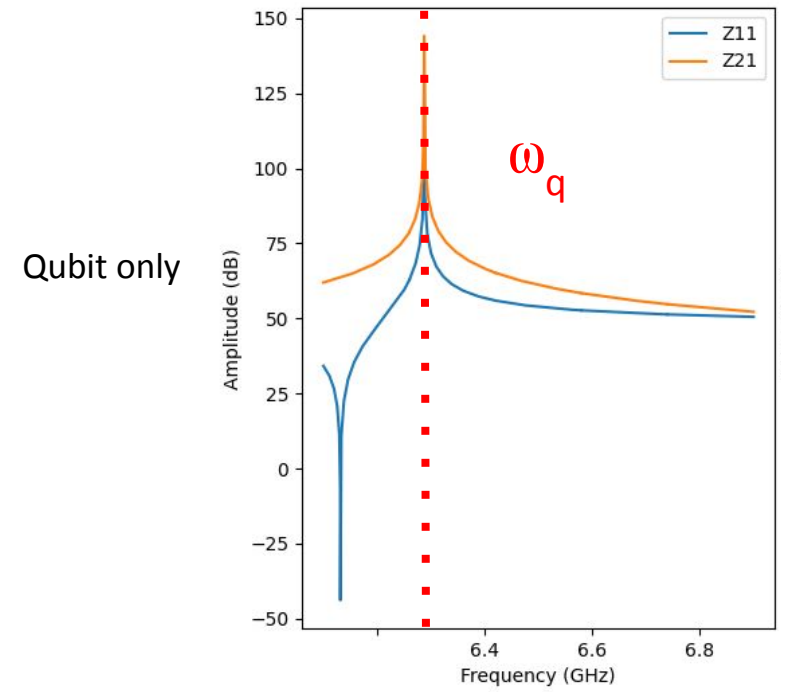
Analysis - Impedance, Scattering and Admittance

- Circuit analysis



Comparison QSM-Qiskit

| Parameters | QSM | Qiskit-Metal | |
|-------------------------|-------------------|---------------------------------|------------|
| | | LOM | EPR |
| ω_q [GHz] | 6.48 | 6.45 | 6.52 |
| α [MHz] | -390 | -413 | -348 |
| χ [MHz] | -1.79 | -1.73 | -1.27 |
| E_C [MHz] | 359.77 | 356.73 | / |
| E_J [GHz] | 16.35 | 16.34 | / |
| E_J/E_C | 45.44 | 45.80 | / |
| T_1 | To be implemented | 146 | / |
| Computational cost | Low | Low | High |
| User defined parameters | L_i, L_j | $L_i, C_j,$ readoutFrequency | L_j, C_j |



Conclusions and next steps

- QSM is still far from Qiskit-Metal in terms of possibilities
- On the other hand, for QSM it's easy to add custom simulations and calculations
- QSM also aims to be totally open-source (e.g. using FasterCap <https://www.fastfieldsolvers.com/fastercap.htm>)
- Qiskit-Metal offers more options for the design and also custom qubits
- Next step: simulation of other designs (e.g. trying to recreate the design shown <https://www.nature.com/articles/nphys1710.pdf>)