Geometrical Origin of CP Violation and CKM and MNS Matrices in SUSY SU(5) x T´

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Phys. Lett. B681, 444 (2009)
work in progress

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Motivation: Tri-bimaximal Neutrino Mixing

- Neutrino Oscillation Parameters $P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t\rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Latest Global Fit [GS98, Bari group, AGSS09] (1σ) Gonzalez-Garcia, Maltoni, Salvado (2010)

$$\sin^2\theta_{23} = 0.463(0.415\text{–}0.530), \quad \sin^2\theta_{12} = 0.319(0.303\text{–}0.335), \quad \sin\theta_{13} = 0.127(0.072\text{–}0.165)$$

$$\Delta m^2_{21} = 7.59 \pm 0.20 \ \times 10^{-5}\ \mathrm{eV}^2 \qquad \Delta m^2_{31} = \begin{cases} -2.36 \pm 0.11 & \times 10^{-3}\ \mathrm{eV}^2 \\ +2.46 \pm 0.12 & \times 10^{-3}\ \mathrm{eV}^2 \end{cases}$$ (Global Minima)

- Tri-bimaximal Mixing Pattern Wolfenstein (1978); Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

Best fit value using atm data only
⇒ $\sin\theta_{13} = 0.077$
(SuperK, Neutrino2010)

data getting closer to TBM

Theoretical Challenges

(i) Absolute mass scale: Why $m_\nu \ll m_{u,d,e}$?

- seesaw mechanism: most appealing scenario ⇒ Majorana
  - GUT scale (type-I, II) vs TeV scale (type-III, double seesaw)
- TeV scale new physics (extra dimension, extra U(1)) ⇒ Dirac or Majorana

(ii) Flavor Structure: Why neutrino mixing large while quark mixing small?

- seesaw doesn't explain entire mass matrix w/ 2 large, 1 small mixing angles
- neutrino anarchy: no parametrically small number Hall, Murayama, Weiner (2000)
  - near degenerate spectrum, large mixing
  - predictions strongly depend on choice of statistical measure
- family symmetry: there's a structure, expansion parameter (~~symmetry~~ effect)
  - leptonic symmetry (normal or inverted)
  - quark-lepton connection ↔ GUT (normal, inverted possible)

- In this talk: assume 3 generations, no LSND
  - MiniBoone anti-neutrino mode: excess in low energy region consistent with LSND
  - 4th generation model: (3+3) consistent with experiments including MiniBoone Hou, Lee, arXiv:1004.2359

Origin of Flavor Mixing and Mass Hierarchy

- SM: 22 arbitrary parameters in Yukawa sector
- No fundamental origin found or suggested
- Reduce number of parameters
  - Grand Unification
    - seesaw scale ~ GUT scale
    - quarks and leptons unified
    - 1 coupling for entire multiplet

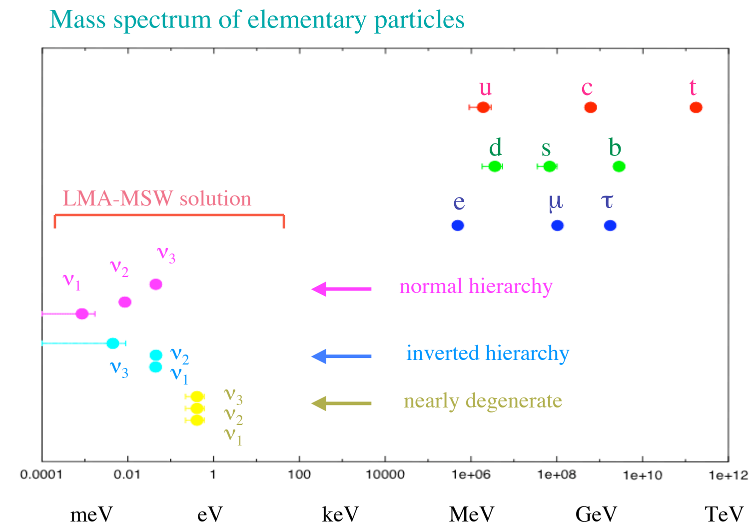
  ⇒ intra-family relations (e.g. SO(10))

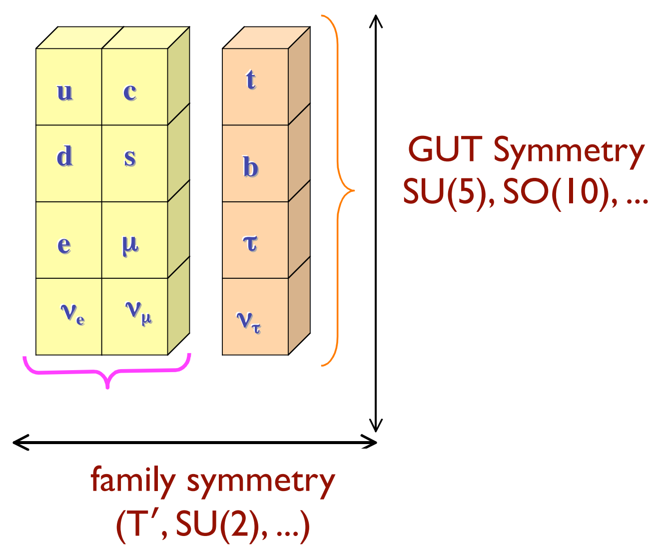
  Up-type quarks ⇔ Dirac neutrinos

  Down-type quarks ⇔ charged leptons

  - Family Symmetry

  ⇒ inter-family relations (flavor structure)





Tri-bimaximal Neutrino Mixing

- Neutrino mass matrix

$$M = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \longrightarrow \qquad \sin^2 2\theta_{23} = 1 \qquad \theta_{13} = 0$$

solar mixing angle NOT fixed

- If A+B = C + D $\longrightarrow$ $\tan^2\theta_{12} = 1/2$ TBM pattern

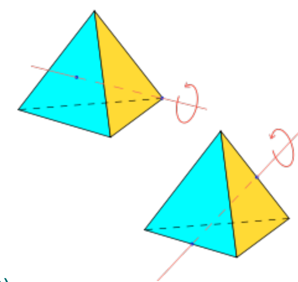
  - Mass Matrix M diagonalized by TBM matrix

$$U_{TBM}^T \, M \, U_{TBM} = diag(m_1, m_2, m_3) \qquad U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

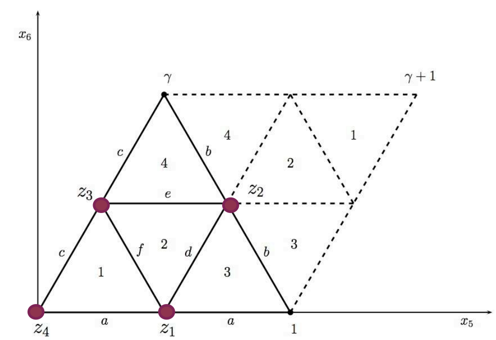
Double Tetrahedral T´ Symmetry

- Smallest Symmetry to realize TBM ⇒ Tetrahedral group A_4 Ma, Rajasekaran (2004)
  - even permutations of 4 objects

    S: (1234) → (4321), T: (1234) → (2314)



  - invariance group of tetrahedron
  - can arise from extra dimensions: 6D → 4D Altarelli, Feruglio (2006)
  - does NOT give quark mixing



- Double Tetrahedral Group T´ Frampton, Kaphart (1995); M.-C.C., K.T. Mahanthappa PLB652, 34 (2007); 681, 444 (2009)
  - inequivalent representations

    A4: $1,\ 1',\ 1'',\ 3$ (vectorial) → TBM for neutrinos

    other: $2,\ 2',\ 2''$ (spinorial) → 2 +1 assignments for quarks and charged leptons

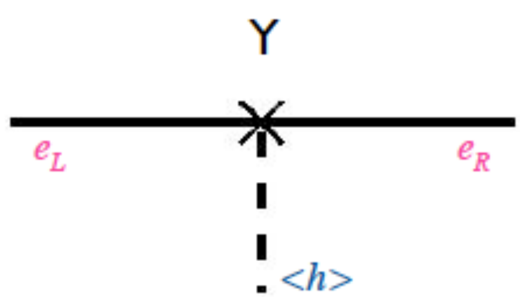
  - complex CG coefficients when spinorial representations are involved

CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>

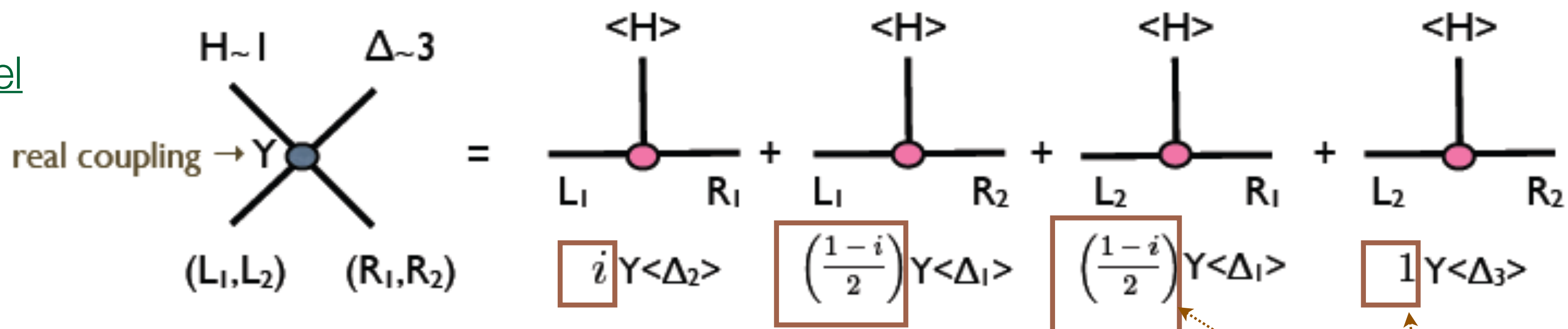


A Novel Origin of CP Violation

M.-C.C, K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

- Complex CG coefficients in T′ ⇒ explicit CP violation
  - real Yukawa couplings, real scalar VEVs
  - CPV in quark and lepton sectors purely from complex CG coefficients
  - no additional parameters needed ⇒ extremely predictive model!

a toy model



- scalar potential: Z_3 symmetry ⇒ $\langle\Delta_1\rangle = \langle\Delta_2\rangle = \langle\Delta_3\rangle \equiv \langle\Delta\rangle$ real
- complex effective mass matrix

CGs of T′

$$\begin{array}{ccc} & L_1 & L_2 \\ M = & \left(\begin{array}{cc} i & \frac{1-i}{2} \\ \frac{1-i}{2} & 1 \end{array}\right) Y\langle\Delta\rangle & \begin{array}{c} R_1 \\ R_2 \end{array} \end{array}$$

The Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009)

- Symmetry: SUSY SU(5) x T´ x Z12 x Z12

SU(5)		T´	
$10(Q,\, u^c,\, e^c)_L$	:	$(T_1, T_2) \sim 2,\ \ T_3 \sim 1$	1: (N1,N2,N3) ~ 3
$\overline{5}(d^c, \ell)_L$	:	$(F_1, F_2, F_3) \sim 3$	

- Superpotential: only 10 operators allowed

(7+2) parameters fit 22 masses, mixing angles, CPV measures

$$\mathcal{W}_{\rm Yuk} = \mathcal{W}_{TT} + \mathcal{W}_{TF} + \mathcal{W}_{\nu}$$

spinorial representations ⇒ complex CGs

⇒ CPV in quark & charged lepton sector

$$\mathcal{W}_{TT} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} H_5 \left[y_{ts} T_3 T_a \psi \zeta + y_c T_a T_b \phi^2 \right] + \frac{1}{\Lambda^3} y_u H_5 T_a T_b \phi'^3$$

up type quarks

$$\mathcal{W}_{TF} = \frac{1}{\Lambda^2} y_b H_{\overline{5}}' \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \overline{F} T_a \phi \psi \zeta' + y_d H_{\overline{5}'} \overline{F} T_a \phi^2 \psi' \right]$$

down type quarks & charged leptons

$$\mathcal{W}_{\nu} = \lambda_1 NNS + \frac{1}{\Lambda^3} \left[H_5 \overline{F} N \zeta \zeta' \left(\lambda_2 \xi + \lambda_3 \eta \right) \right]$$

neutrino masses

Λ : scale above which T' is exact

Reality of Yukawa couplings: ensured by degrees of freedom in field redefinition

Model Predictions

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007); Phys. Lett. B681, 444 (2009)

- Resulting neutrino mass matrices

only vector representations
$\Rightarrow$ all CG are real
$\Rightarrow$ Majorana phases: 0 or π

$$M_{RR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} S_0 \qquad M_D = \begin{pmatrix} 2\xi_0 + \eta_0 & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & -\xi_0 + \eta_0 \\ -\xi_0 & -\xi_0 + \eta_0 & 2\xi_0 \end{pmatrix} \zeta_0 \zeta_0' v_u$$

- seesaw mechanism: effective neutrino mass matrix

$$U_{TBM}^T M_\nu U_{TBM} = \mathrm{diag}((3\xi_0 + \eta_0)^2, \eta_0^2, -(-3\xi_0 + \eta_0)^2) \frac{(\zeta_0 \zeta_0' v_u)^2}{S_0}$$

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

- mass sum rule among 3 masses:

$$\left| |\sqrt{m_1}| + |\sqrt{m_3}| \right| = 2|\sqrt{m_2}| \quad \text{for} \quad (3\xi_0 + \eta_0)(3\xi_0 - \eta_0) > 0$$

$$\left| |\sqrt{m_1}| - |\sqrt{m_3}| \right| = 2|\sqrt{m_2}| \quad \text{for} \quad (3\xi_0 + \eta_0)(3\xi_0 - \eta_0) < 0$$

can accommodate both normal
& inverted mass spectrum

Model Predictions

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- Charged Fermion Sector (7 parameters)

spinorial representations ⇒ complex CGs
⇒ CPV in quark sector

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+(1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u$$

V_{cb}

$$M_d,\ M_e^T = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & (1,-3)c & 0 \\ b & b & 1 \end{pmatrix} y_b v_d \phi_0$$

V_{ub}

$$\theta_c \simeq \left|\sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c}\right| \sim \sqrt{m_d/m_s},$$

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

Georgi-Jarlskog relations ⇒ $V_{d,L} \neq I$
SU(5) ⇒ $M_d = (M_e)^T$
⇒ corrections to TBM related to θ_c

- Neutrino Sector (2 parameters)

$$U_{\rm MNS} = V_{e,L}^\dagger U_{\rm TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\boxed{\theta_{13} \simeq \theta_c/3\sqrt{2}}$$

CGs of SU(5) & T´

$$\boxed{\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta}$$

neutrino mixing angle

1/2

quark mixing angle

complex CGs: leptonic Dirac CPV
(the only non-zero leptonic CPV phase)

prediction for Majorana phases: 0, π

⇒ connection between leptogenesis & CPV in neutrino oscillation

correction accounts for discrepancy between exp best fit value and TBM prediction for solar angle

Numerical Results

- Experimentally: $m_u : m_c : m_t \;=\; \theta_c^{7.5} : \theta_c^{3.7} : 1 \qquad m_d : m_s : m_b \;=\; \theta_c^{4.6} : \theta_c^{2.7} : 1$
- Model Parameters at M_{GUT}:

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g + (1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u$$

$$\frac{M_d}{y_b v_d \phi_0 \zeta_0} = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & c & 0 \\ b & b & 1 \end{pmatrix}$$

$$\begin{aligned} b &\equiv \phi_0 \psi_0' / \zeta_0 = 0.00304 \\ c &\equiv \psi_0 \zeta_0' / \zeta_0 = -0.0172 \\ k &\equiv y' \psi_0 \zeta_0 = -0.0266 \\ h &\equiv \phi_0^2 = 0.00426 \\ g &\equiv \phi_0'^3 = 1.45 \times 10^{-5} \end{aligned}$$

$$y_t = 1.25$$

$$y_b \phi_0 \zeta_0 \simeq m_b / m_t \simeq 0.011$$

7 parameters in charged fermion sector

predicting: 9 masses, 3 mixing angles, 1 CP Phase; all agree with exp within 3σ

- CKM Matrix and Quark CPV measures:

CPV entirely from CG coefficients

$$|V_{CKM}| = \begin{pmatrix} 0.974 & 0.227 & 0.00412 \\ 0.227 & 0.973 & 0.0412 \\ 0.00718 & 0.0408 & 0.999 \end{pmatrix}$$

$$\begin{aligned} A &= 0.798 \\ \overline{\rho} &= 0.299 \\ \overline{\eta} &= 0.306 \end{aligned}$$

$$\beta \equiv \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 23.6^o, \; \sin 2\beta = 0.734 \, ,$$

$$\alpha \equiv \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 110^o \, ,$$

$$\boxed{\gamma \equiv \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \delta_q = 45.6^o} \, ,$$

$$J \equiv \mathrm{Im}(V_{ud}V_{cb}V_{ub}^*V_{cs}^*) = 2.69 \times 10^{-5} \, ,$$

Direct measurements @ 3σ (ICHEP2010)

$$\begin{aligned} \sin 2\beta &= 0.672^{+0.069}_{-0.07} \\ \gamma\,(\mathrm{deg}) &= 71^{+46}_{-45} \\ \alpha\,(\mathrm{deg}) &= 89^{+21}_{-13} \end{aligned}$$

Numerical Results

- Diagonalization matrix for charged leptons $\begin{pmatrix} 0.997e^{i177^o} & 0.0823e^{i131^o} & 1.31\times10^{-5}e^{-i45^o} \\ 0.0823e^{i41.8^o} & 0.997e^{i176^o} & 0.000149e^{-i3.58^o} \\ 1.14\times10^{-6} & 0.000149 & 1 \end{pmatrix}$

- MNS Matrix   Note that these predictions do NOT depend on η_0 and ξ_0

$$|U_{MNS}| = \begin{pmatrix} 0.838 & 0.542 & 0.0583 \\ 0.362 & 0.610 & 0.705 \\ 0.408 & 0.577 & 0.707 \end{pmatrix}$$

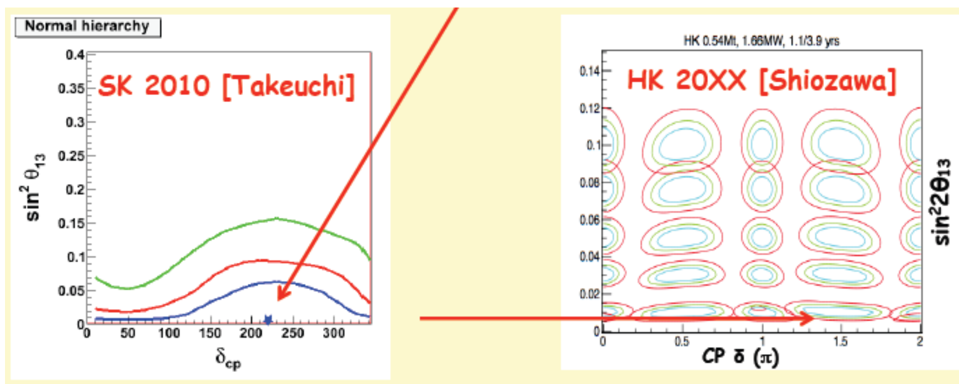
prediction for Dirac CP phase: δ = 227 degrees

$$\sin^2 2\theta_{atm} = 1, \quad \tan^2\theta_{\odot} = 0.419, \quad |U_{e3}| = 0.0583$$

$$J_\ell = -0.00967$$

Dirac phase the only
non-vanishing leptonic CPV phase

⇒ connection between leptogenesis
& CPV in neutrino oscillation



SuperK best fit: δ = 220 degrees

- Neutrino Masses: using best fit values for Δm^2

$$\xi_0 = -0.0791\ , \quad \eta_0 = 0.1707\ , \qquad S_0 = 10^{12}\ \text{GeV}$$

$$|m_1| = 0.00134\ \text{eV}, \ |m_2| = 0.00882\ \text{eV}, \ |m_3| = 0.0504\ \text{eV}$$

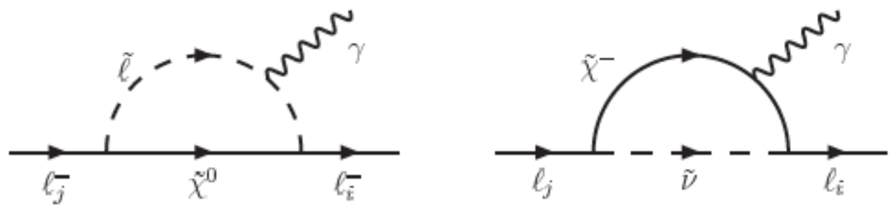
2 independent parameters in neutrino sector

- Majorana phases: $\alpha_{21} = \pi \qquad \alpha_{31} = 0.$

predicting: 3 masses, 3 angles, 3 CP Phases;
both θ_{sol} & θ_{atm} agree with exp

Predictions for LFV Radiative Decay

- SUSY GUTs: slepton-neutralino and sneutrino-chargino loop: Borzumati, Masiero (1986)



- CMSSM: at M_{GUT}, slepton mass matrices flavor blind
- RG evolution: generate off diagonal elements in slepton mass matrices
- dominant contribution: LL slepton mass matrix Hisano, Moroi, Tobe, Yamaguichi (1995)

$$BRji = \frac{\alpha^3}{G_F^2 m_s^8} |(m_{LL}^2)_{ji}|^2 \tan^2\beta$$

$$(m_{LL}^2)_{ji} = -\frac{1}{8\pi^2} m_0^2 (3 + A_0^2/m_0^2) Y_{jk}^\dagger \log\left(\frac{M_G}{M_k}\right) Y_{ki}$$

very model dependent

good approximation to full evolution effects:

$$m_s^8 \simeq 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$$

Petcov, Profumo, Takanishi, Yaguna (2003)

Predictions for LFV Radiative Decay

- in SUSY SU(5) x T' model (normal hierarchy case):
  - degenerate RH masses
  - ratios of branching fractions depend on mixing & light neutrino masses

$$Y^{+}Y = \begin{pmatrix} 0.000122635 & 0.0000589172 & 0.000131458 \\ 0.0000589172 & 0.000941119 & 0.000720549 \\ 0.000131458 & 0.000720549 & 0.000936627 \end{pmatrix}$$

  - predicting

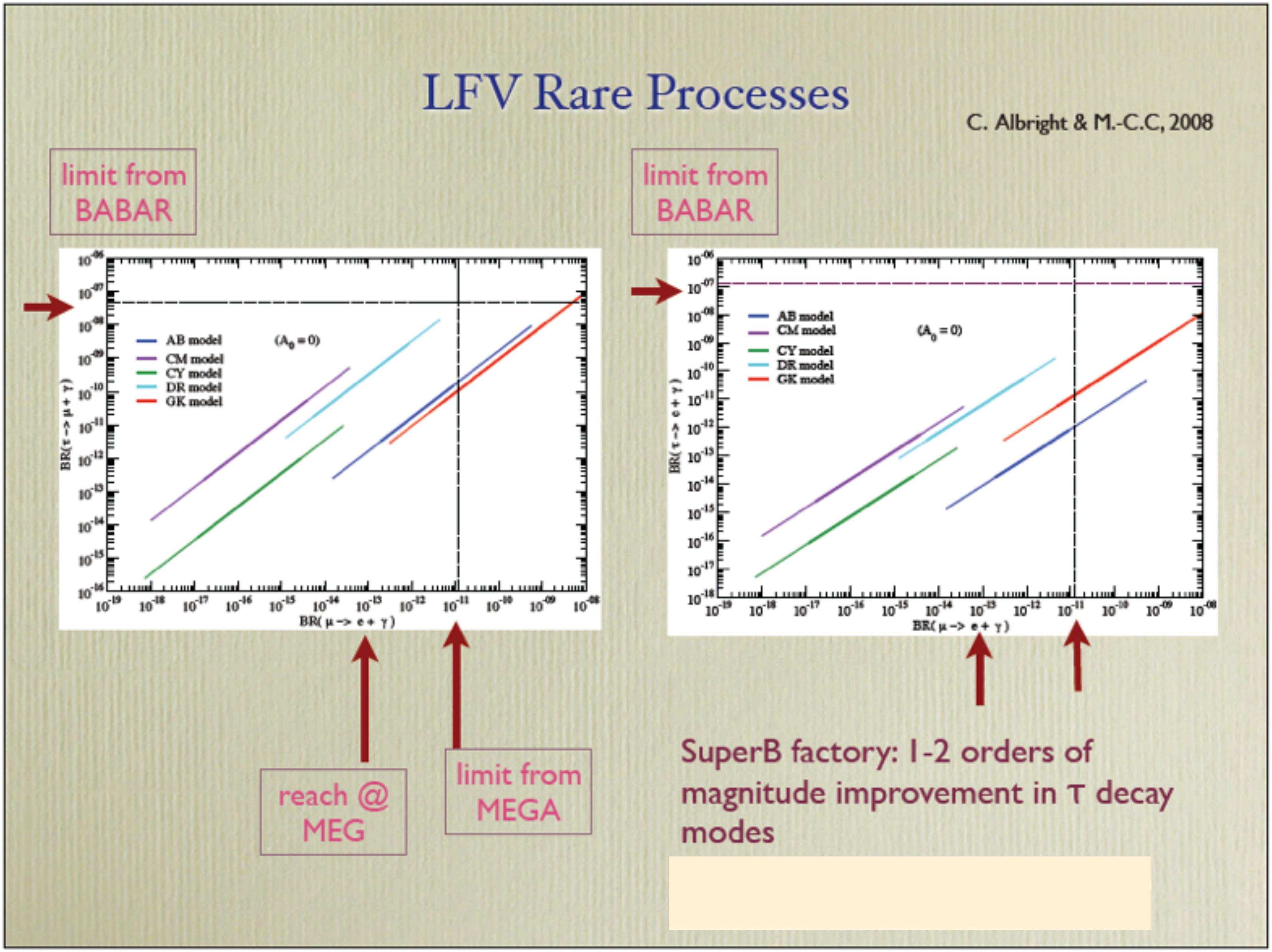
$$Br(\mu \to e\gamma) < Br(\tau \to e\gamma) < Br(\tau \to \mu\gamma)$$

  - m_0 = 50 GeV, $M_{1/2}$ = 200 GeV, $A_0 = 7m_0$:
    - Br($\tau \to \mu + \gamma$) =1.38E-9
    - Br($\tau \to e + \gamma$) = 4.59E-11
    - Br($\mu \to e + \gamma$) = 9.23E-12

inverted light neutrino mass pattern under investigation

M.-C.C., Mahanthappa, Petcov, and a student at SISSA, under preparation

Distinguishing Different Models:
SO(10) SUSY GUTs example with DM constraints

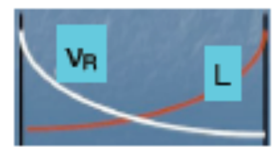


Curing FCNC Problem: Family Symmetry vs MFV

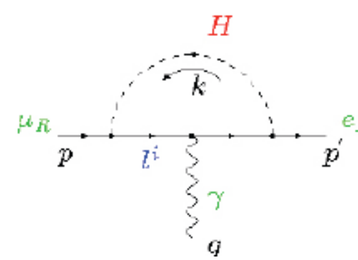
- low scale new physics severely constrained by flavor violation
- Minimal Flavor Violation

  D'Ambrosio, Giudice, Isidori, Strumia (2002); Cirigliano, Grinstein, Isidori, Wise (2005)

  - assume Yukawa couplings the only source of flavor violation
- Example: Warped Extra Dimension

  UV  SM (us)

  - wave function overlap ⇒ naturally small Dirac mass $\psi_{(0)} \sim e^{(1/2-c)ky}$
  - non-universal bulk mass terms (c) ⇒ FCNCs at tree level ⇒ Λ > O(10) TeV
    - FCNCs: present even in the limit of massless neutrinos
      - tree-level: μ-e conversion, μ→3e, etc
    - charged current
      - one-loop: μ→e+γ, τ→e+γ, τ→μ+γ



  - fine-tuning to get large mixing and mild mass hierarchy for neutrinos

Curing FCNC Problem: Family Symmetry vs MFV

- Two approaches:
  - Minimal Flavor Violation in RS

    quark sector: A. Fitzpatrick, G. Perez, L. Randall (2007)
    lepton sector: M.-C.C., H.B. Yu (2008)

$$C_e = aY_e^\dagger Y_e, \quad C_N = dY_\nu^\dagger Y_\nu, \quad C_L = c(\xi Y_\nu Y_\nu^\dagger + Y_e Y_e^\dagger)$$

  - T´ symmetry in the bulk for quarks & leptons:

    M.-C.C., K.T. Mahanthappa, F. Yu (PLB2009);
    A4: Csaki, Delaunay, Grojean, Grossmann

    - TBM neutrino mixing: common bulk mass term, no tree-level FCNCs
    - TBM mixing and masses decouple: no fine-tuning
    - realistic masses and mixing angles in quark sector
    - no tree-level FCNCs in lepton sector and 1-2 family of quark sector

- Family Symmetry: alternative to MFV to avoid FCNCs in TeV scale new physics
  - many family symmetries violate MFV ⇒ possible new FV contributions

Summary

- SUSY SU(5) x T′ symmetry: near TBM lepton mixing & realistic CKM matrix
  - deviations from TBM calculable due to GUT relations
- complex CG coefficients in T′: origin of CPV both in quark and lepton sectors
  - quark sector

    quark CP phase: γ = 45.6 degrees

  - Leptonic Dirac CP phase:
    - the only non-vanishing leptonic CPV phase
    - sufficient leptogenesis can be generated

    leptonic Dirac CP phase:
    δ = 227°
    (SuperK best fit: 220°)

- interesting quark-lepton complementarity sum rules:

$$\boxed{\theta_{13} \simeq \theta_c/3\sqrt{2} \sim 0.05}$$

$$\boxed{\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta}$$

- predictions for LFV charged lepton decay related to light neutrino mass pattern