

# Two-mode squeezing generation in a traveling wave parametric amplifier

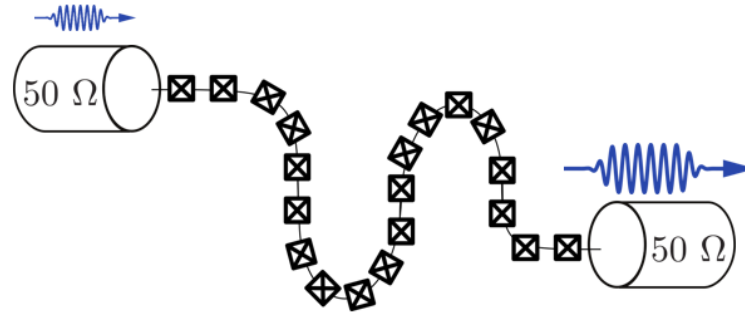
Martina Esposito

CNR SPIN Institute, Naples, Italy



December 1<sup>st</sup> 2021  
DARTWARS Collaboration Meeting

# Grenoble TWPA Team



Giulio Cappelli  
PhD student



Arpit Ranadive  
PhD student



Luca Planat  
Postdoc



Martina Esposito  
Marie Curie IF fellow



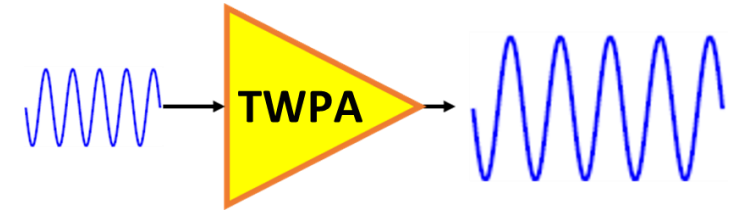
Since Oct. 2021  
permanent researcher at CNR  
Naples, Italy



Nicolas Roch  
Team leader

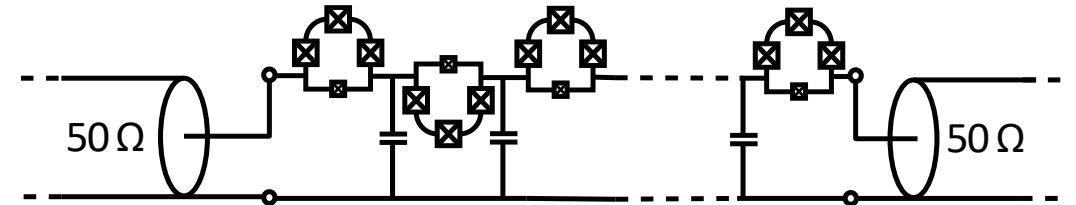


## 1. Josephson Travelling wave parametric amplification



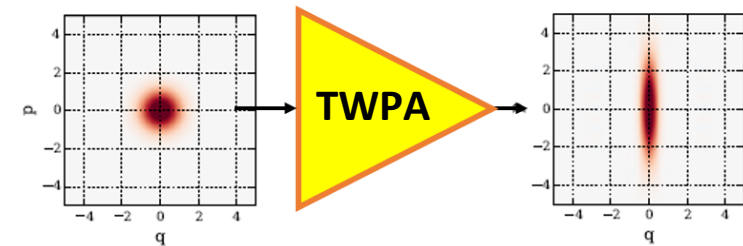
## 2. A reversed Kerr TWPA

[A. Ranadive. et al. arXiv:2101.05815 \(2021\)](#)



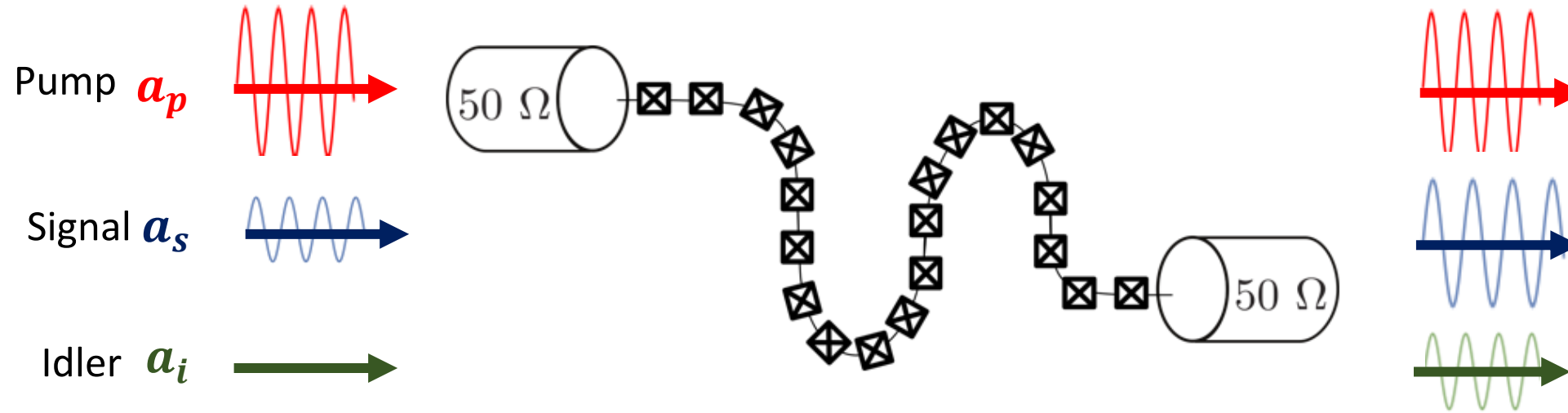
## 3. Two mode squeezing in TWPAs

[M. Esposito et al. arXiv:2111.03696 \(2021\)](#)



# Josephson Travelling Wave Parametric Amplification

## Josephson TWPA: Wave mixing process



4-wave mixing

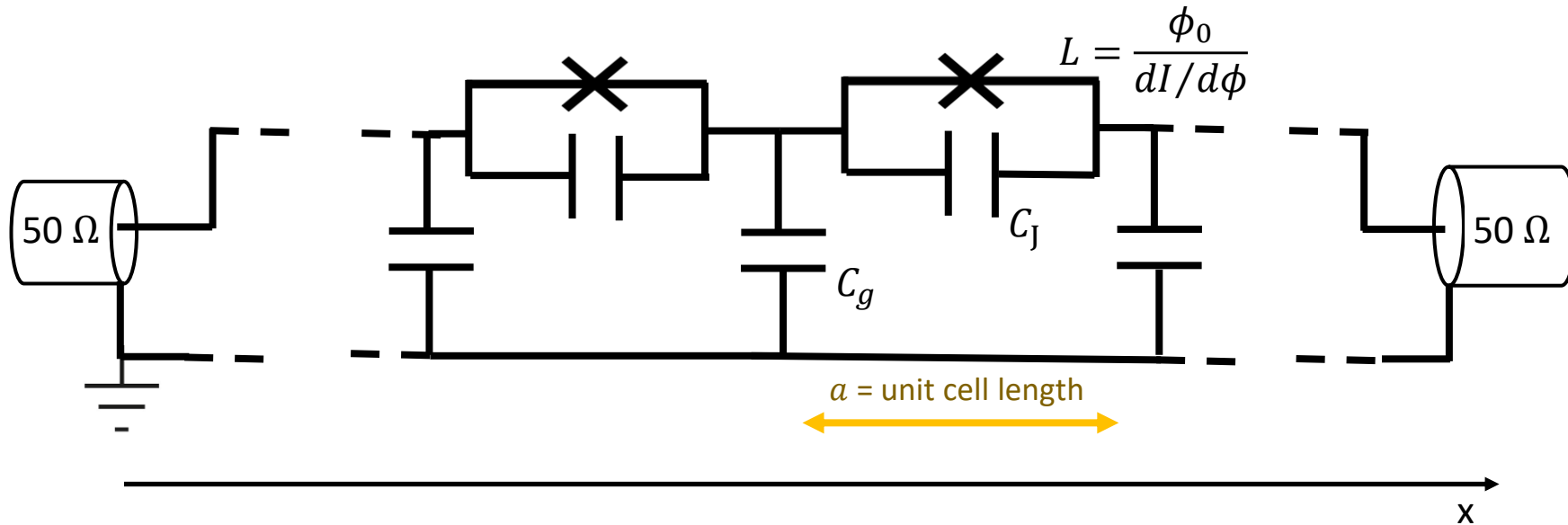
$$H_{\text{int}} = a_i^\dagger a_s^\dagger a_p a_p + hc$$

$$2\omega_p = \omega_i + \omega_s$$

$$2k_p = k_i + k_s$$

Phase mismatch problem in TWPAs!

# Josephson Travelling Wave Parametric Amplification



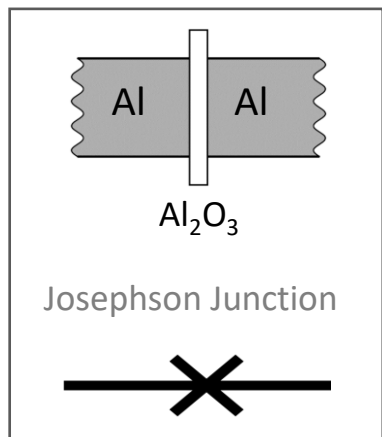
$$n \sim 10^3 \text{ cells}$$

$$a \sim 10 \mu\text{m}$$

$$\lambda \sim 1 \text{ mm}$$

$$\omega_0 = 1/\sqrt{LC_g}$$

$$\omega_J = 1/\sqrt{LC_J}$$



$$I(\phi) = \tilde{\alpha} [I_0 \phi + \gamma I_0 \phi^3 + \dots]$$

$$\chi^{(3)}$$

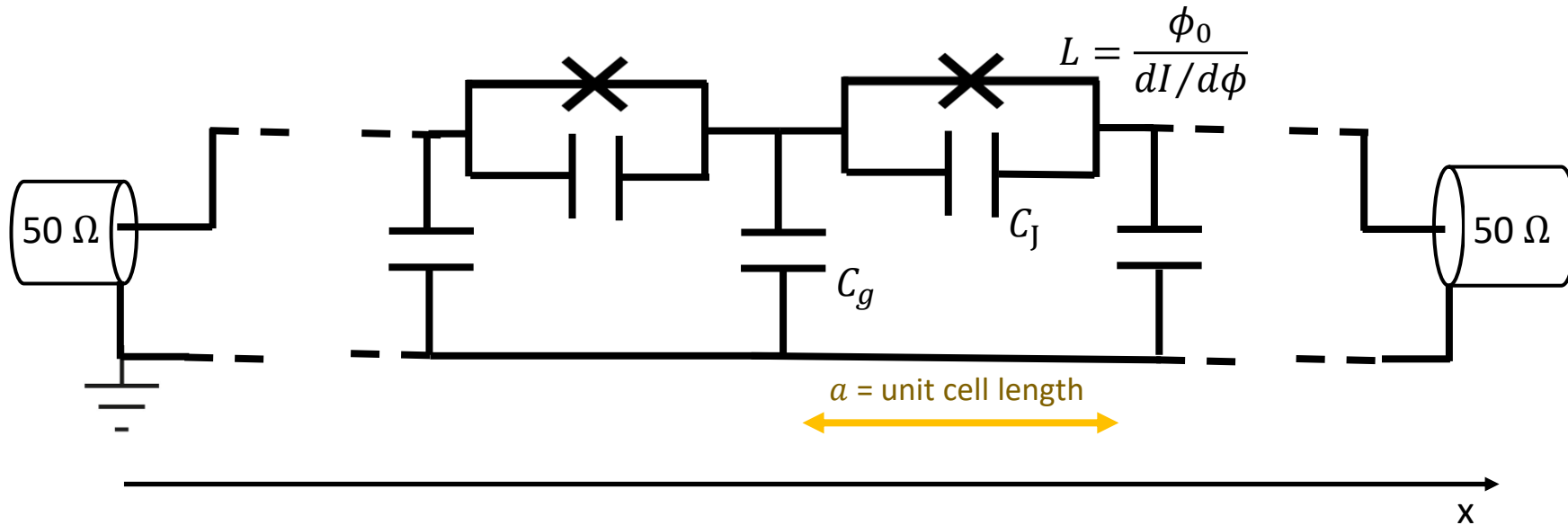
4-wave mixing (Kerr) non linearity

Phase across junction

$$\phi = \int V dt$$

$$\phi(x) \sim \phi_0 e^{ikx}$$

# Josephson Travelling Wave Parametric Amplification



$n \sim 10^3$  cells

$a \sim 10 \mu\text{m}$

$\lambda \sim 1 \text{ mm}$

$\omega_0 = 1/\sqrt{LC_g}$

$\omega_J = 1/\sqrt{LC_J}$

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\omega_0^2} \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{\omega_J^2} \frac{\partial^4 \phi}{\partial t^2 \partial x^2} - \gamma \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)^3 = 0$$

$\chi^{(3)}$

4-wave mixing (Kerr) non linearity

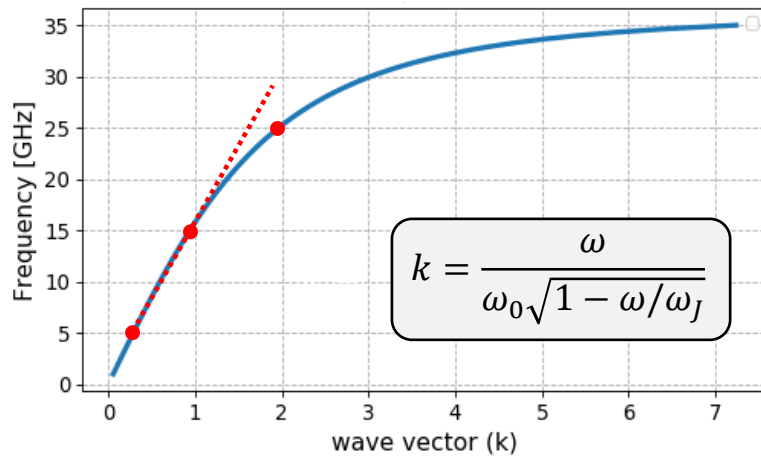
## Single-junction TWPA

$$\underbrace{\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\omega_0^2} \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{\omega_J^2} \frac{\partial^4 \phi}{\partial t^2 \partial x^2}}_{\Delta k_0} - \underbrace{\gamma \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)^3}_{\Delta k_{Kerr}} = 0$$

$$\Delta k_0 = k_s + k_i - 2k_p > 0$$

$$\Delta k_{Kerr} \propto \gamma |A_p|^2$$

dispersion relations



Phase mismatch problem:

$$\Delta k = \Delta k_0 + \Delta k_{Kerr} \neq 0$$

Power gain (Maximal when  $\Delta k = 0$ )

$$G = |\cosh(gN) + i\Delta k/2g \sinh(gN)|^2$$

$$g = \sqrt{K_{si} - \Delta k^2/4}$$

Possible solution: Dispersion engineering

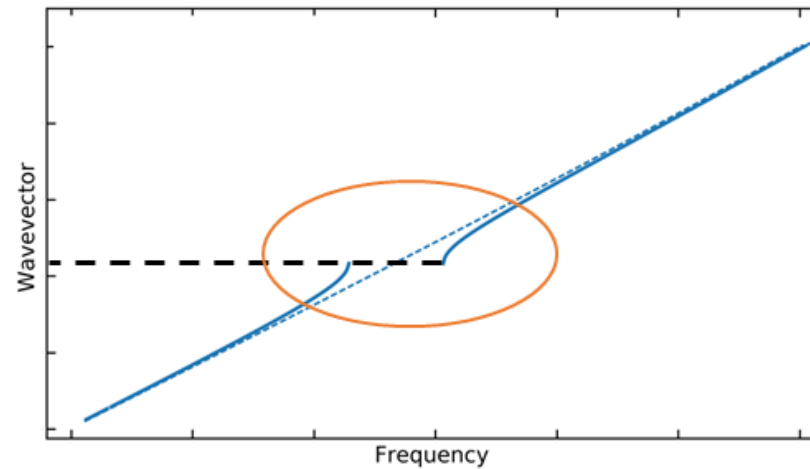
# Josephson Travelling Wave Parametric Amplification

## Dispersion engineering Phase Matching

$$\Delta k = \Delta k_0 + \Delta k_{Kerr} \rightarrow 0$$

$$\Delta k_0 < 0$$

dispersion relations

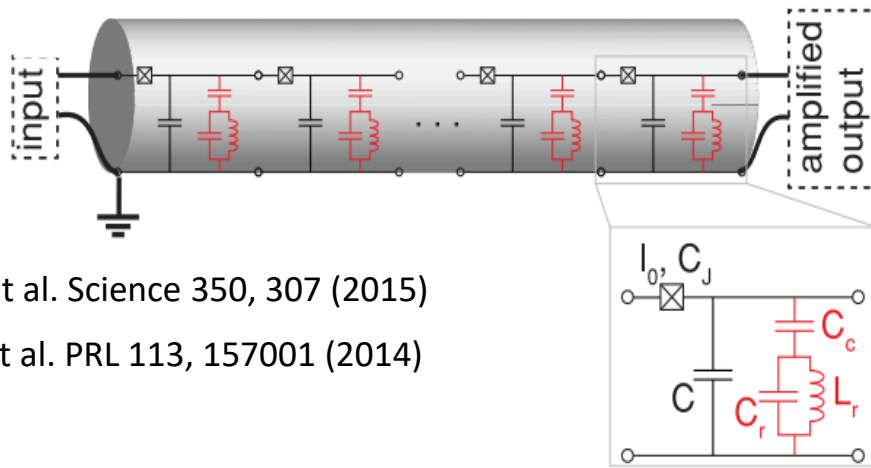




# Josephson Travelling Wave Parametric Amplification

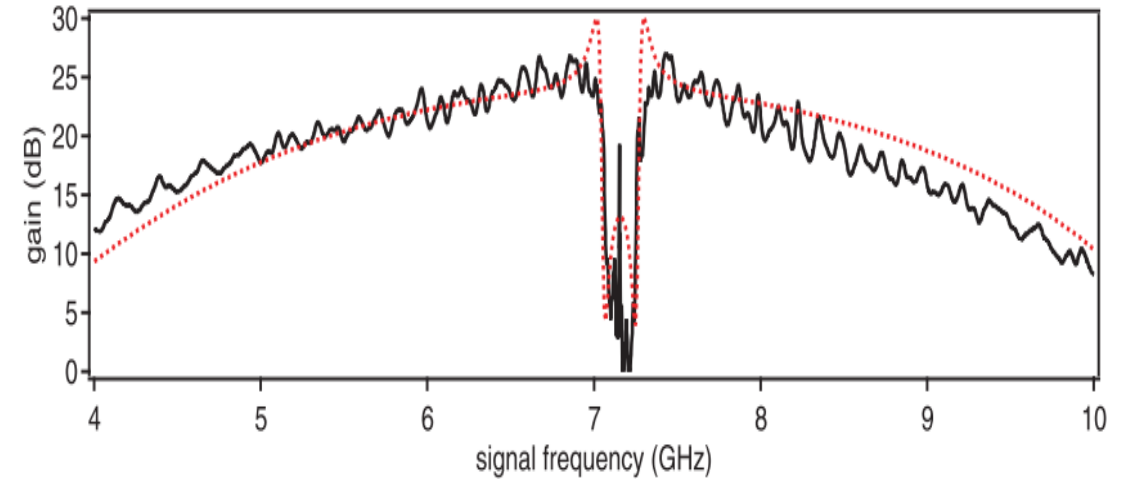
## Dispersion engineering Phase Matching

### 1. Berkeley/MIT/Lincoln Lab : Resonant Phase matching

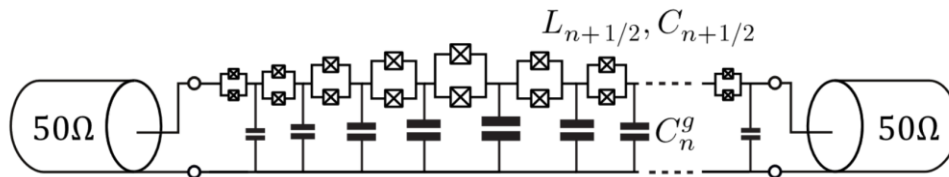


Macklin et al. Science 350, 307 (2015)

O'Brien et al. PRL 113, 157001 (2014)

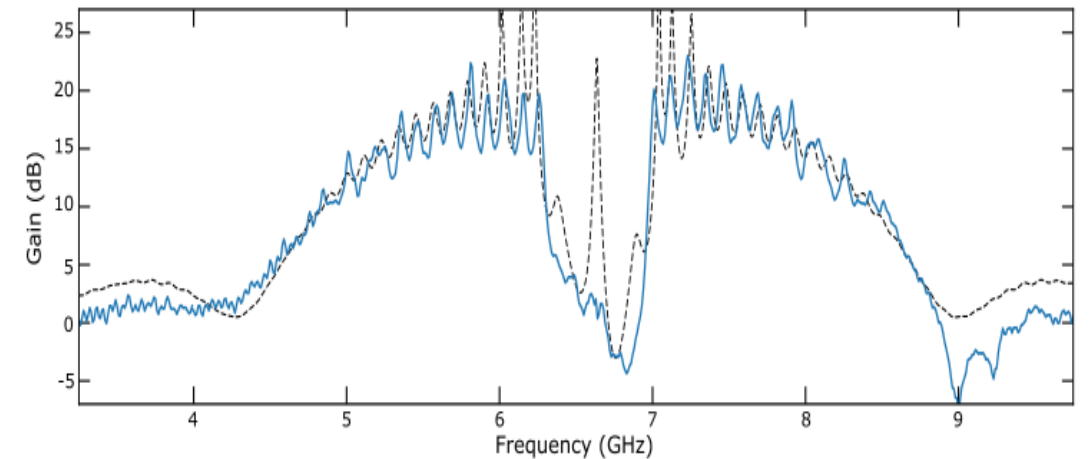


### 2. Grenoble: Photonic gap by modulation

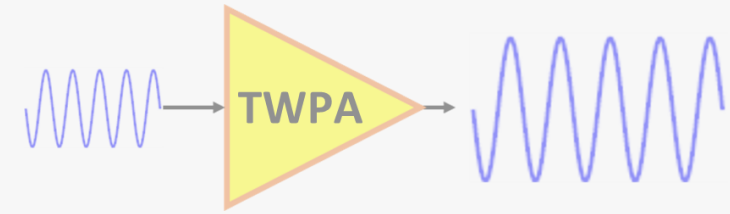


Planat et al. Physical Review X 10, 021021 (2020)

Planat et. al. Phys. Rev. Applied 12, 064017 (2019)

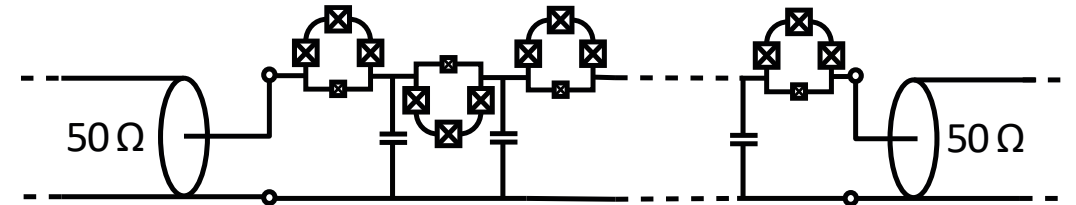


## 1. Josephson Travelling wave parametric amplification



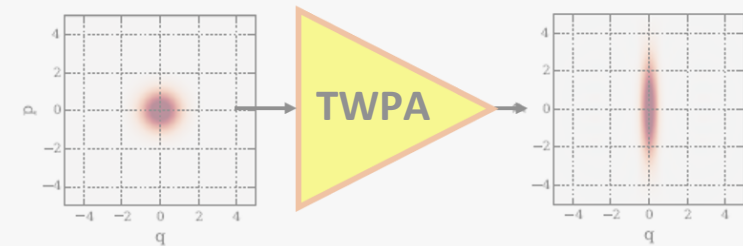
## 2. A reversed Kerr TWPA

[A. Ranadive. et al. arXiv:2101.05815 \(2021\)](https://arxiv.org/abs/2101.05815)

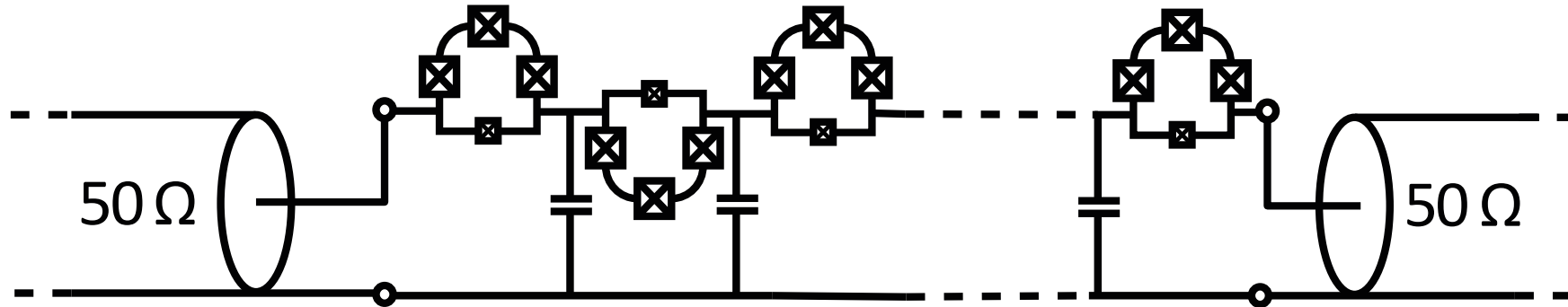


## 3. Two mode squeezing in TWPAs

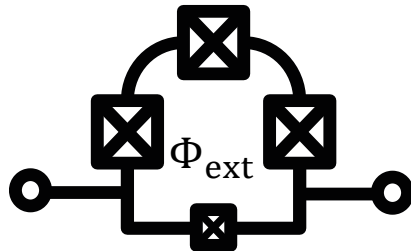
[M. Esposito et al. arXiv:2111.03696 \(2021\)](https://arxiv.org/abs/2111.03696)



## Our new solution: Reversed Kerr Phase Matching



SNAIL  
(Superconducting Nonlinear Asymmetric Inductive element)



Frattini et al. Appl. Phys. Lett. **110**, 222603 (2017)

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\omega_0^2} \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{\omega_J^2} \frac{\partial^4 \phi}{\partial t^2 \partial x^2} - \gamma \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)^3 = 0$$



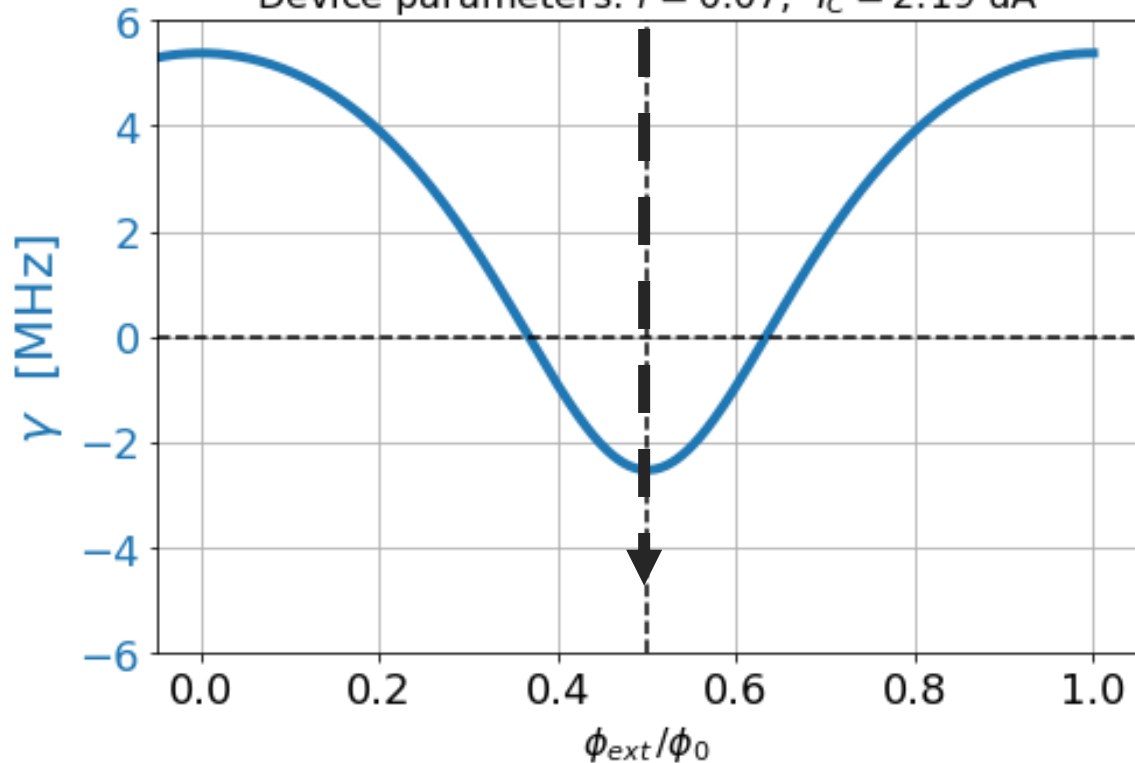
4-wave mixing

$\gamma$  is flux dependent!

Our new solution: **Reversed Kerr** Phase Matching

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\omega_0^2} \frac{\partial^2 \phi}{\partial t^2} + \frac{1}{\omega_J^2} \frac{\partial^4 \phi}{\partial t^2 \partial x^2} - \gamma \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)^3 = 0$$

Device parameters:  $r = 0.07$ ,  $I_C = 2.19 \mu\text{A}$

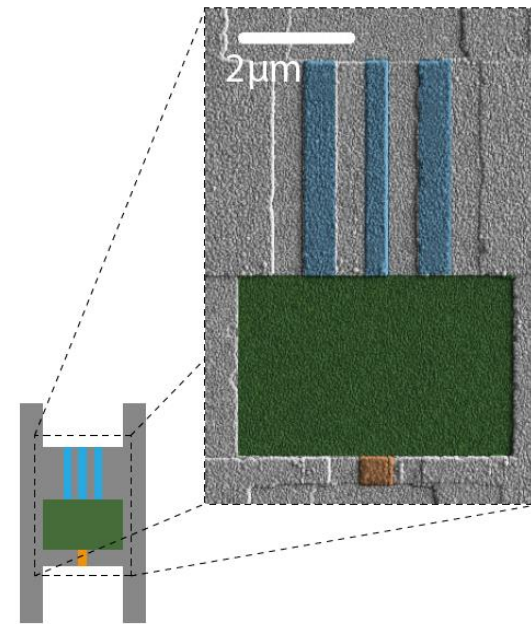
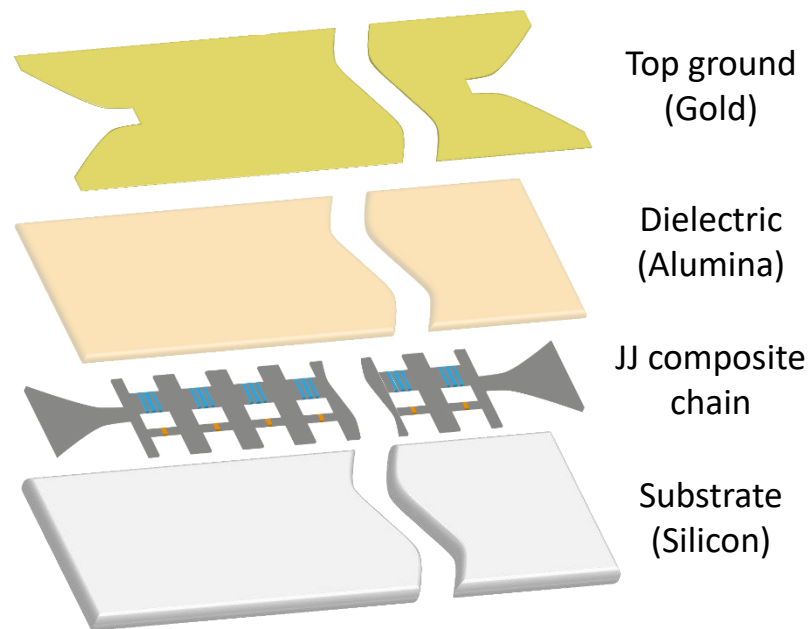
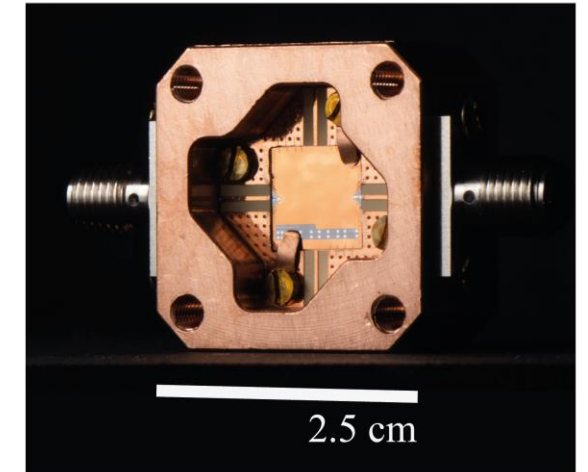
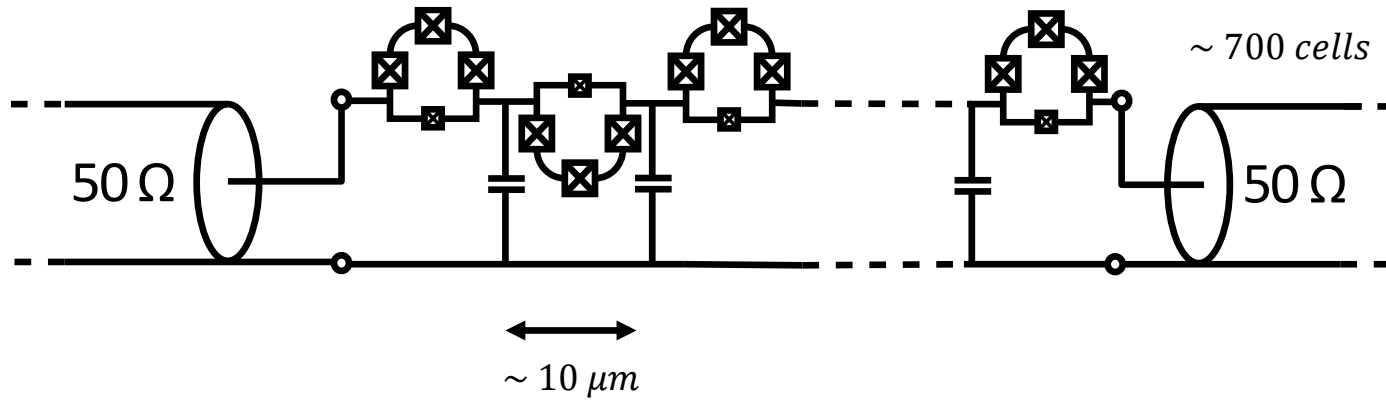


$$\Delta k = \Delta k_0 + \Delta k_{\text{Kerr}} \rightarrow 0$$

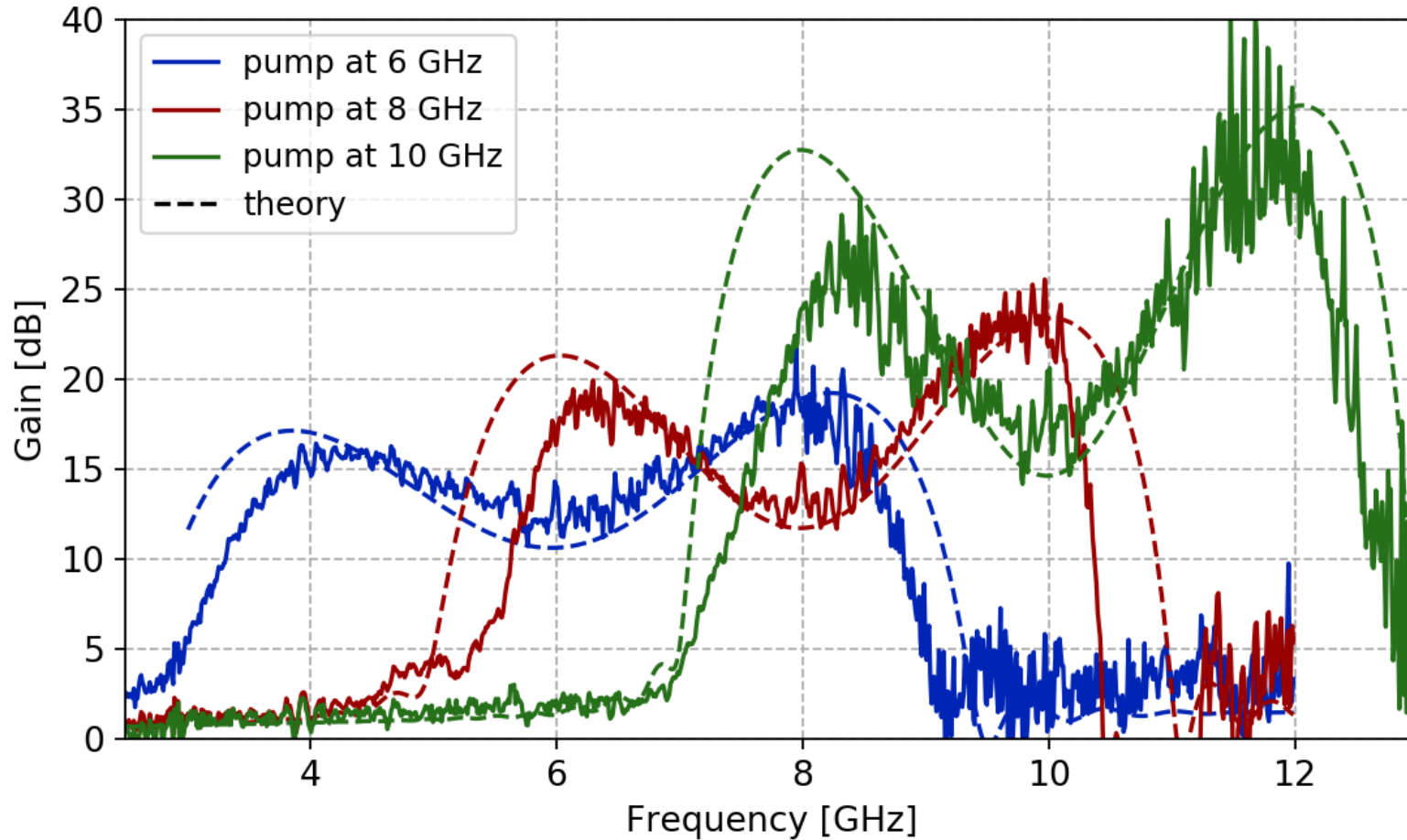
$$\Delta k_{\text{Kerr}} \propto \gamma |A_p|^2$$

When  $\gamma < 0 \rightarrow \Delta k = 0$  is possible!

# A Reversed Kerr TWPA - Fabrication

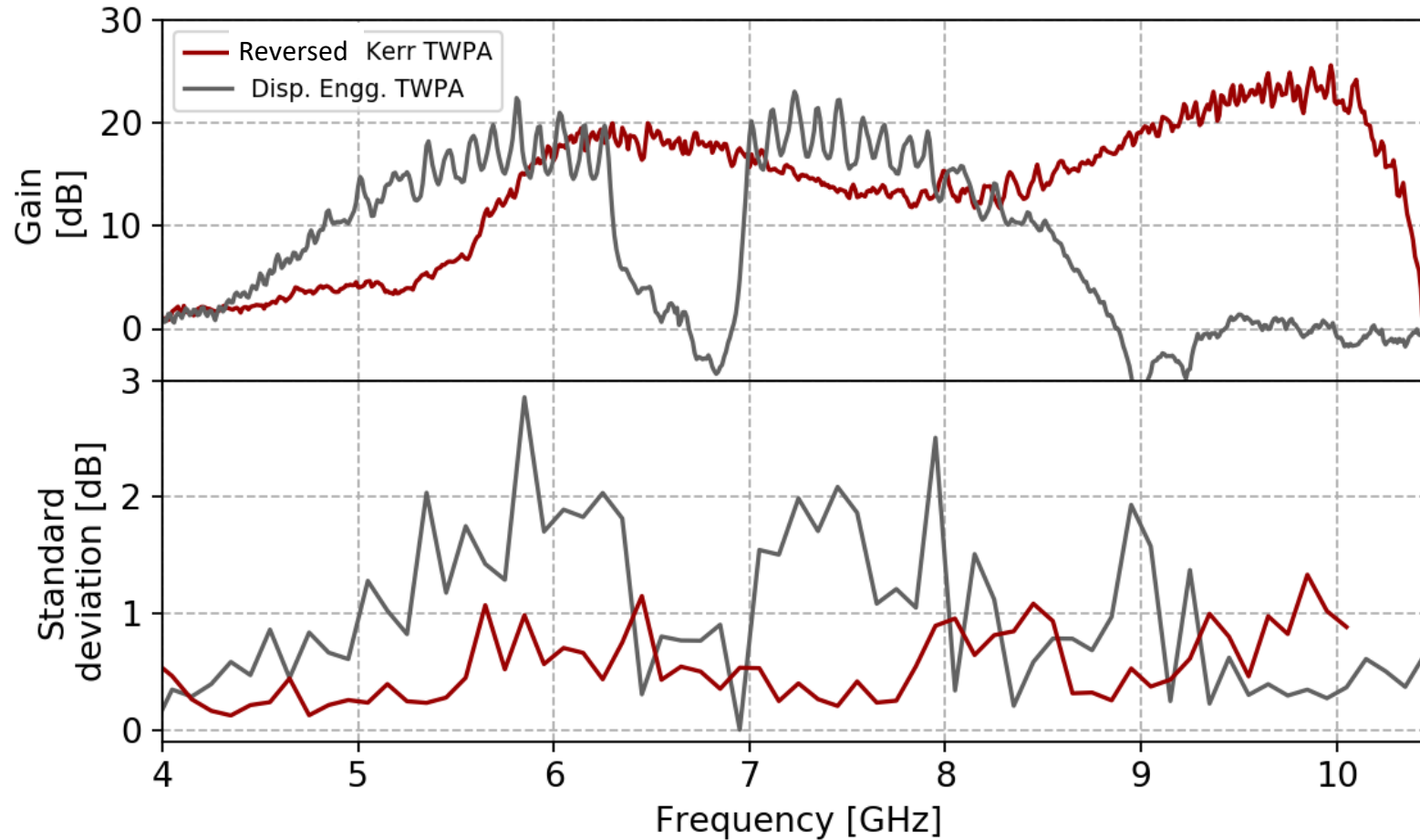


# A Reversed Kerr TWPA – Gain performance



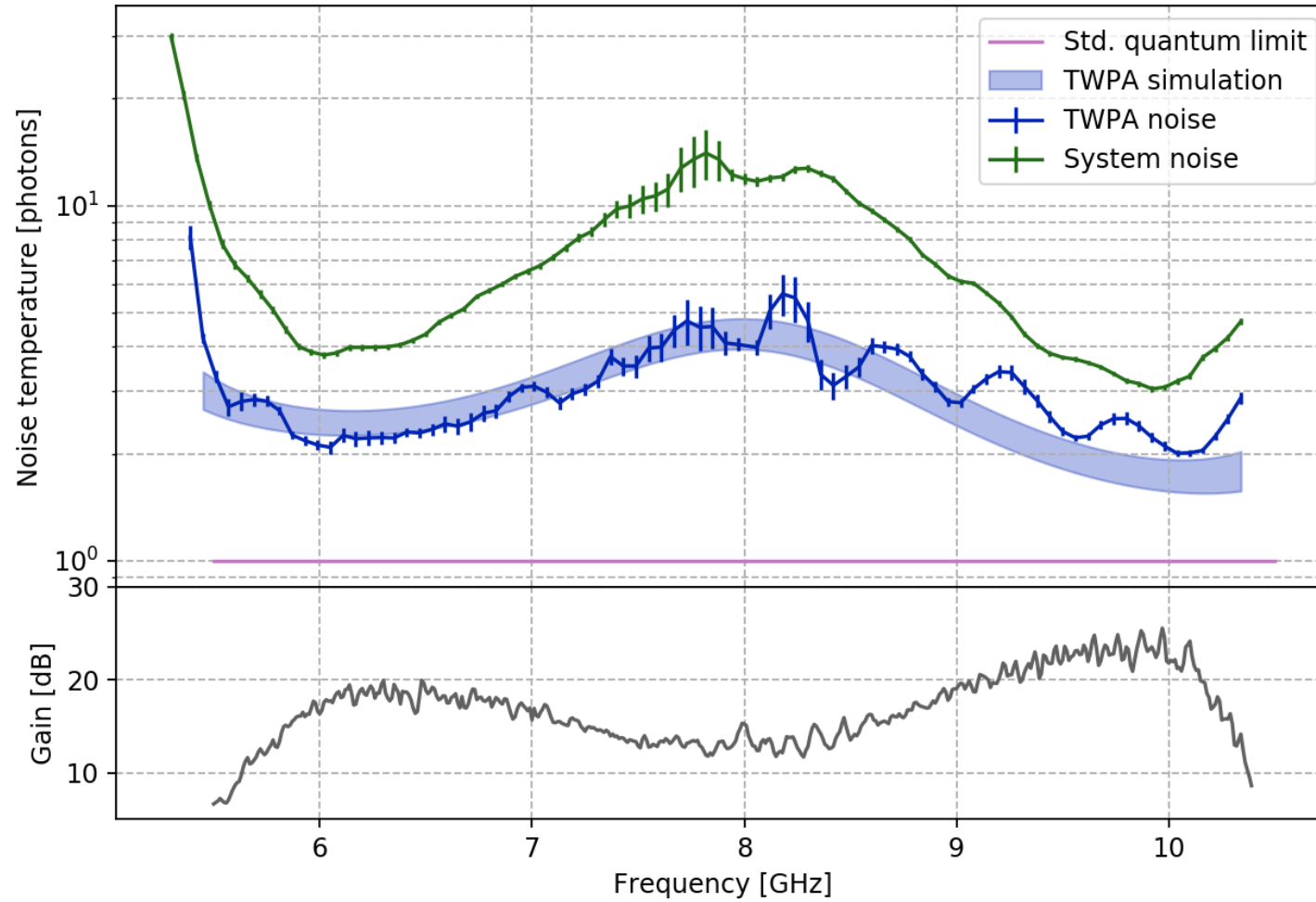
**BW > 3.5 GHz**

## Reduction in gain ripples



↖  
Evaluated over  
100 MHz bins

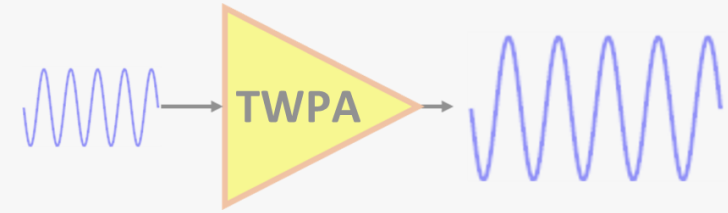
# A Reversed Kerr TWPA – Noise performance



$$N_{\text{TWPA}} \sim 2 \text{ SQL}$$

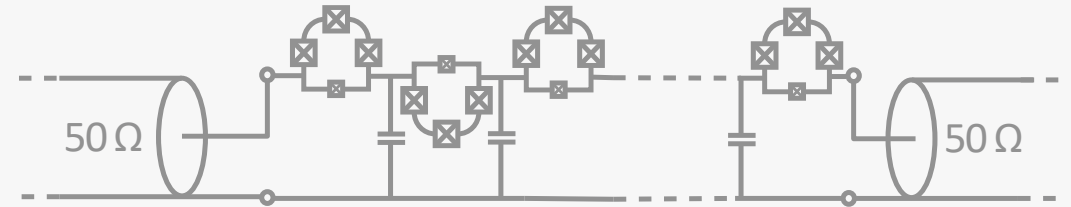


## 1. Josephson Travelling wave parametric amplification



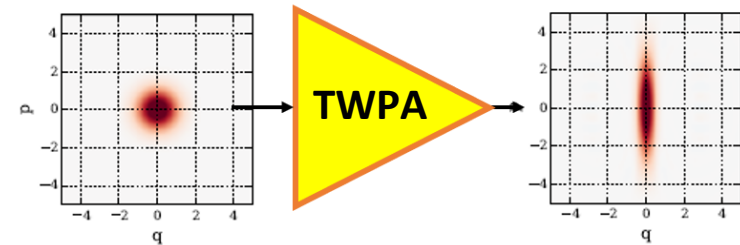
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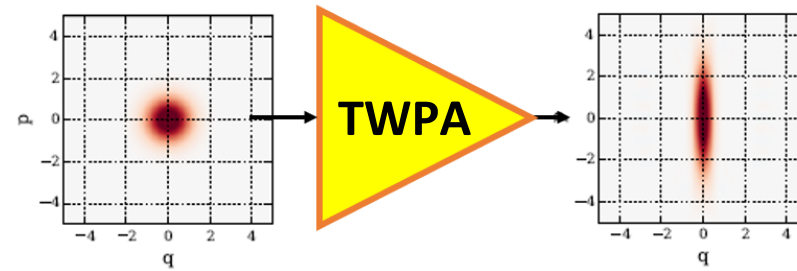
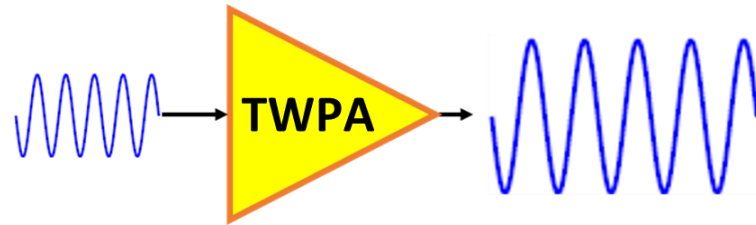


## 3. Two mode squeezing in TWPAs

[M. Esposito et al. arXiv:2111.03696 \(2021\)](#)



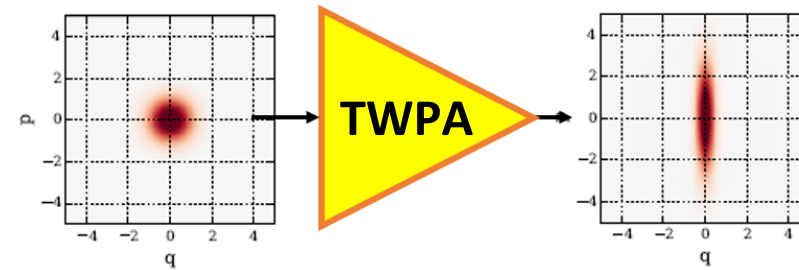
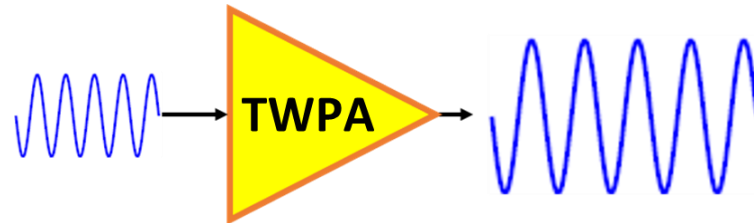
**Not only amplification.....**



**Quantum Microwave Photonics**

# Two mode squeezing in TWPAs - Motivations

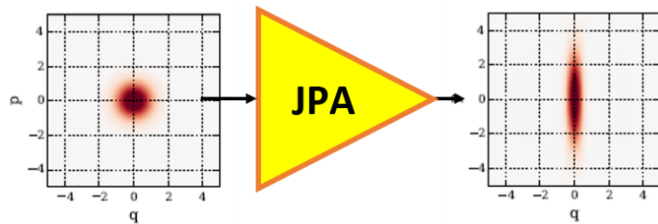
Not only amplification.....



Quantum Microwave Photonics

Microwave frequencies:

Single-mode and Two-mode squeezing in JPAs



- E. Flurin. Et al. Phys. Rev. Lett., 114(090503):1–5 (2015)
- L Zhong et al. New Journal of Physics, 15(12):125013 (2013)
- C. Eichler et al. Physical Review Letters, 107(11):1–5 (2011)

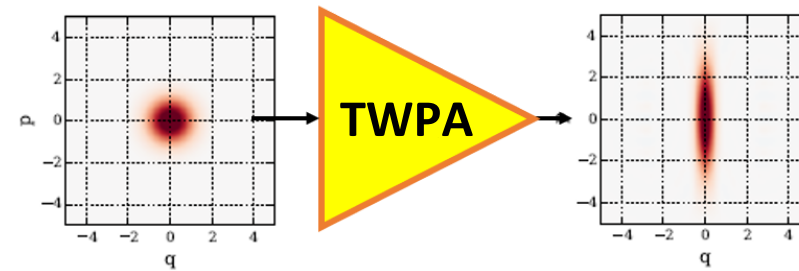
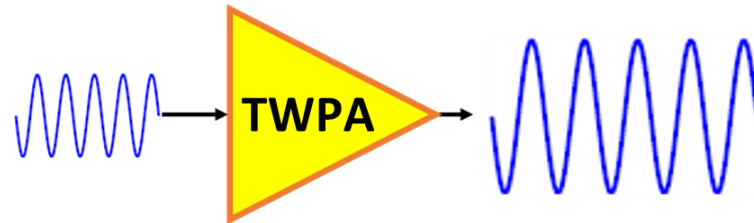
Narrow-band  
Resonant JPAs

- B. H. Schneider et al. Phys.Rev. Lett., 124:140503 (2020)
- C. M. Wilson et al. Nature 479, 376, (2011)

Dynamical Casimir Effect  
in transmission lines

# Two mode squeezing in TWPAs - Motivations

Not only amplification.....



Quantum Microwave Photonics

Microwave frequencies:

Single-mode and Two-mode squeezing in JPAs

Quantum enhanced detection

Squeezed Vacuum Used to Accelerate the Search for a Weak Classical Signal

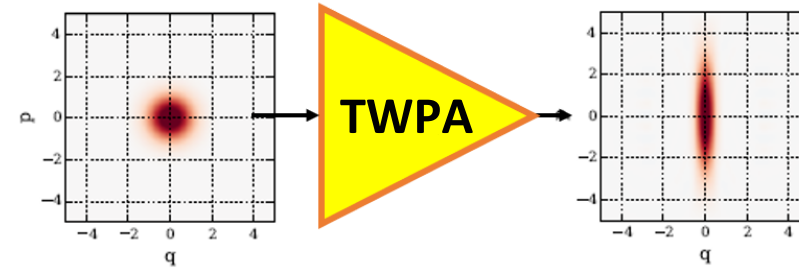
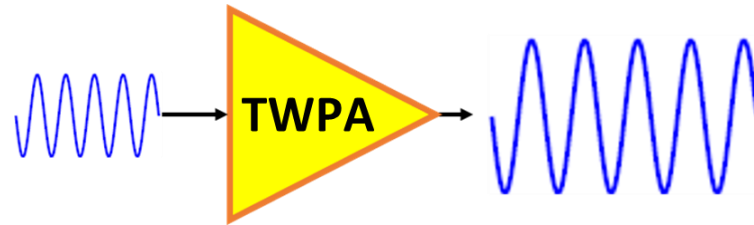
M. Malnou, D. A. Palken, B. M. Brubaker, Leila R. Vale, Gene C. Hilton, and K. W. Lehnert  
Phys. Rev. X **9**, 021023 – Published 3 May 2019; Erratum Phys. Rev. X **10**, 039902 (2020)

Article nature

**A quantum enhanced search for dark matter axions** 2021

K. M. Backes<sup>1,6</sup>✉, D. A. Palken<sup>2,3,6</sup>, S. Al Kenany<sup>4</sup>, B. M. Brubaker<sup>2,3</sup>, S. B. Cahn<sup>1</sup>, A. Droster<sup>4</sup>, Gene C. Hilton<sup>5</sup>, Sumita Ghosh<sup>1</sup>, H. Jackson<sup>4</sup>, S. K. Lamoreaux<sup>1</sup>, A. F. Leder<sup>4</sup>, K. W. Lehnert<sup>2,3,5</sup>, S. M. Lewis<sup>4</sup>, M. Malnou<sup>2,5</sup>, R. H. Maruyama<sup>1</sup>, N. M. Rapidis<sup>4</sup>, M. Simanovskaia<sup>4</sup>, Sukhman Singh<sup>1</sup>, D. H. Speller<sup>1</sup>, I. Urdinarian<sup>4</sup>, Leila R. Vale<sup>5</sup>, E. C. van Assendelft<sup>1</sup>, K. van Bibber<sup>4</sup> & H. Wang<sup>1</sup>

Not only amplification.....



Quantum Microwave Photonics

Microwave frequencies:

Can we generate squeezing in TWPAs ?

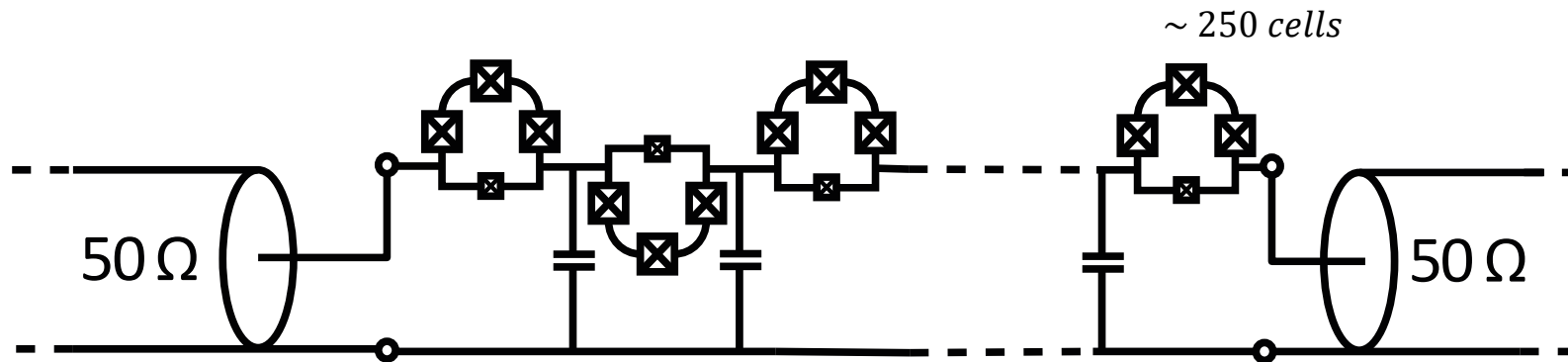
**Advantages:**

- Broadband nature
- Customization and flux tunability of the non-linearities

**Difficulties:**

- Losses in TWPAs
- Spurious non-linear processes

# Two mode squeezing in TWPAs - Device



4-wave mixing

$$H_{\text{int}} = \gamma a_s^\dagger a_i^\dagger a_p a_p + \text{h. c.}$$

4-wave mixing



Two-mode squeezing

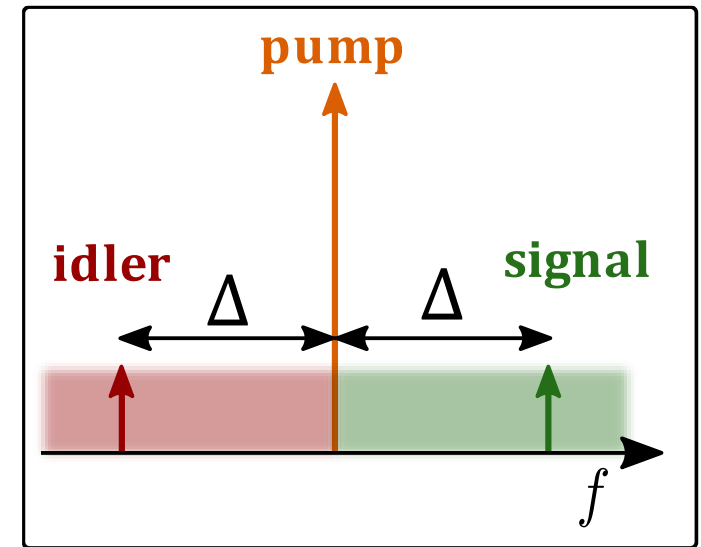
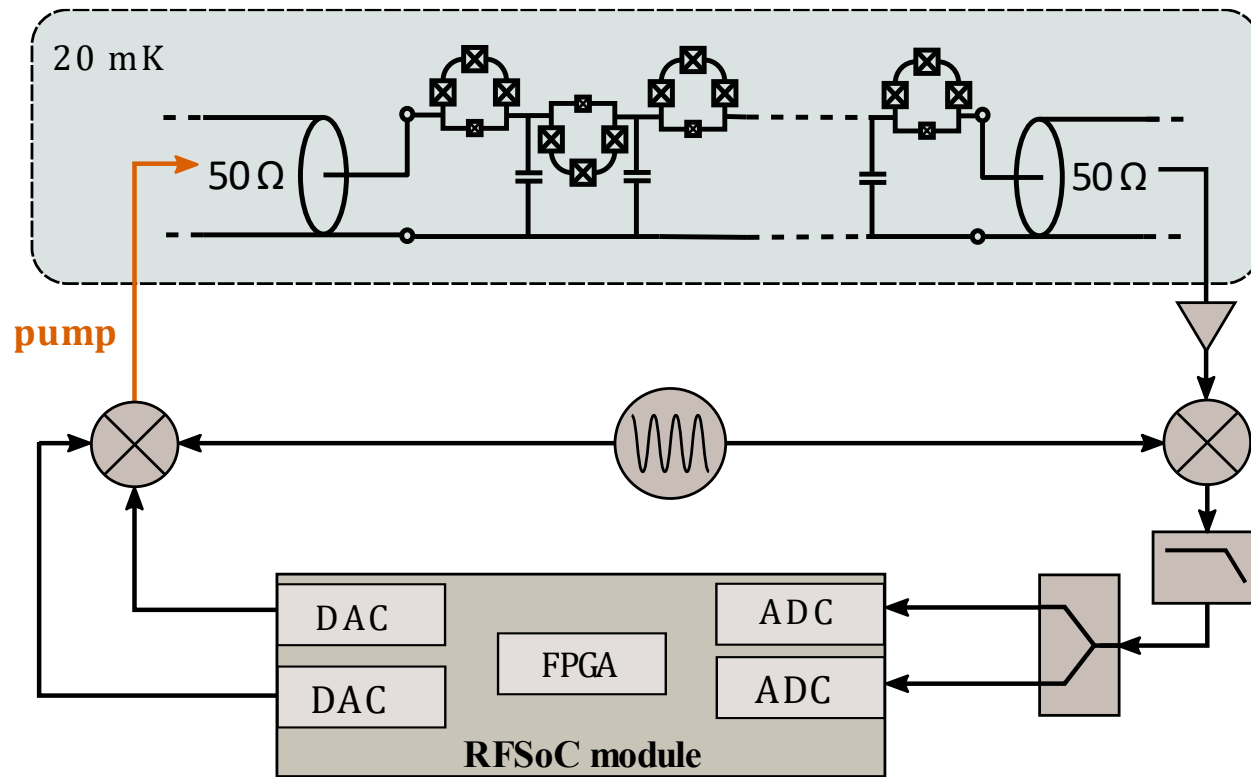
$$H_{\text{int}} = \gamma |A_p|^2 a_s^\dagger a_i^\dagger + \text{h.c.} \quad \longrightarrow \quad \mathbf{S}(\xi) = \exp \left[ \frac{1}{2} (\xi a_i^\dagger a_s^\dagger - \xi^* a_i a_s) \right]$$

$$\xi = r e^{i\phi} \quad \text{Squeezing parameter}$$

$$G_{\text{TWPA}} = \cosh^2(r)$$

$$|\text{Sq}\rangle = \mathbf{S} |0\rangle_s |0\rangle_i = \frac{1}{\cosh r} \sum_n (\tanh r)^n |n\rangle_s |n\rangle_i$$

# Two-mode squeezing in TWPAs

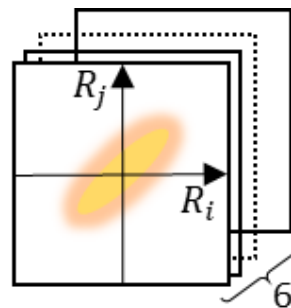


4-wave mixing

$$H_{\text{int}} = \gamma |A_p|^2 a_s^\dagger a_i^\dagger + \text{h. c.}$$

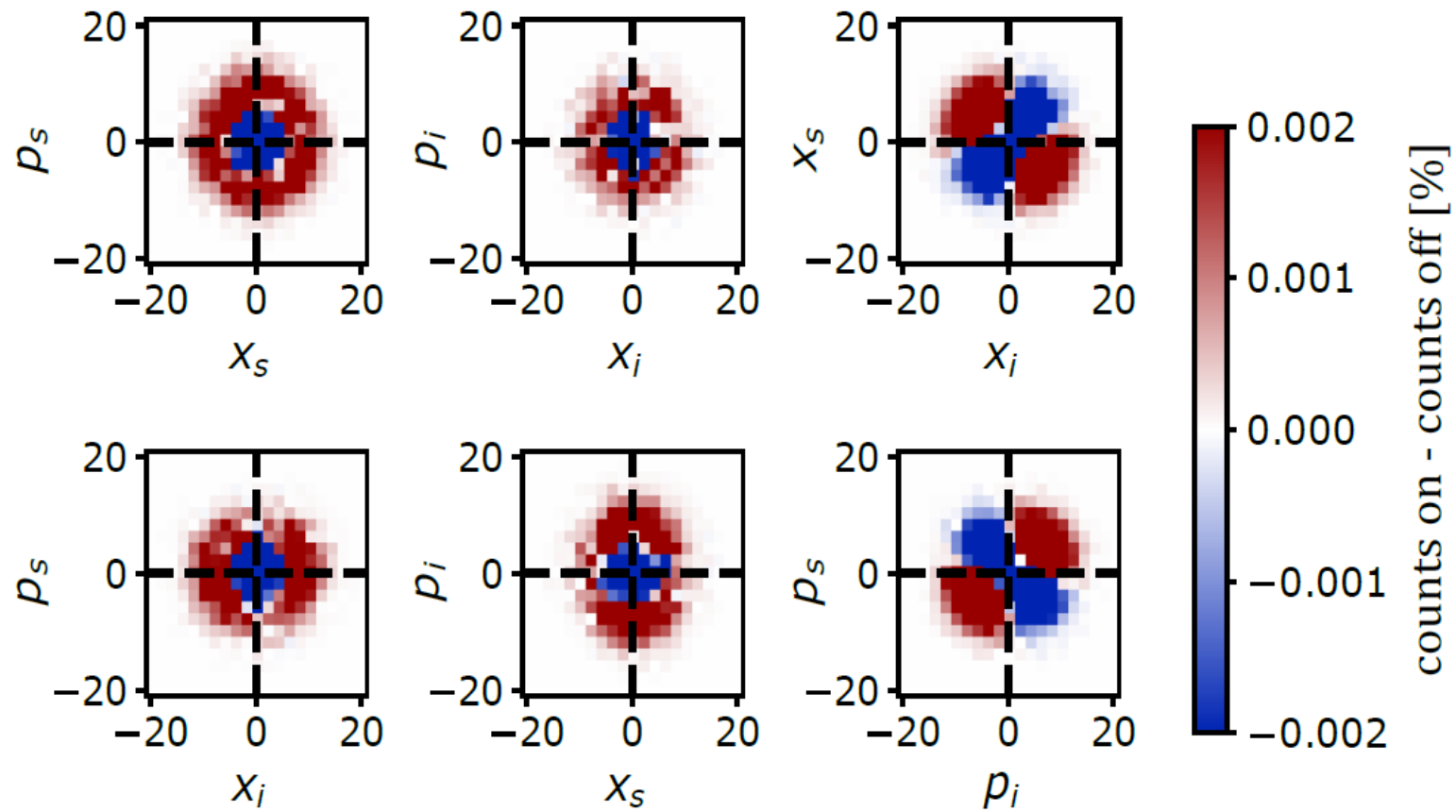
$$\mathbf{R} = (\hat{x}_s, \hat{p}_s, \hat{x}_i, \hat{p}_i)$$

$$\hat{x}_{s,i} = \frac{1}{2} (\hat{A}_{s,i} + \hat{A}_{s,i}^\dagger) \quad \hat{p}_{s,i} = \frac{1}{2i} (\hat{A}_{s,i} - \hat{A}_{s,i}^\dagger)$$



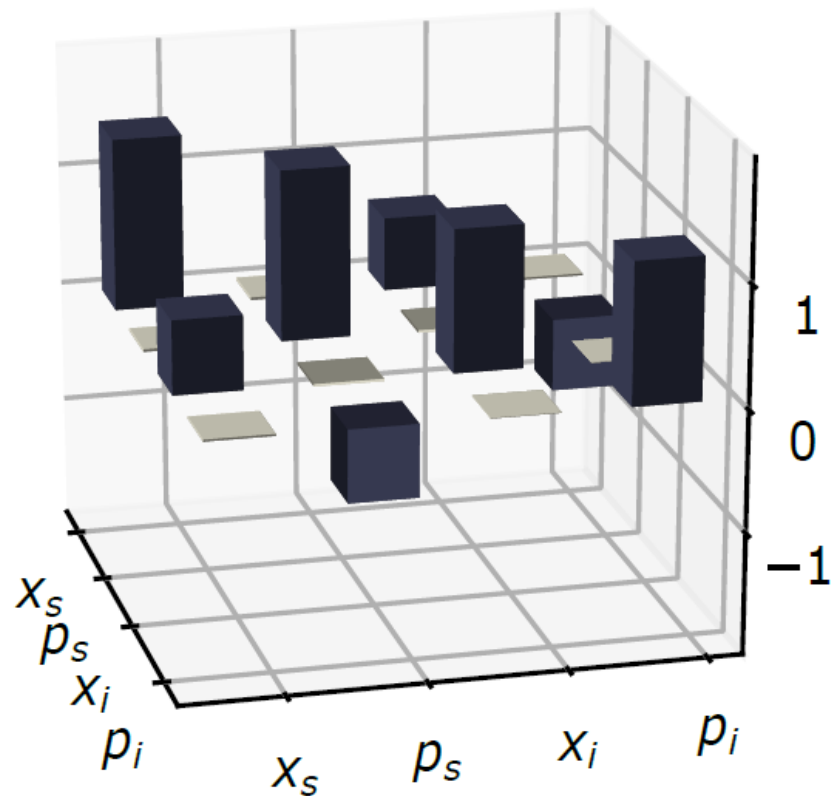


## Phase- space histogram distribution



## Covariance Matrix reconstruction

$$\sigma = \sigma^{\text{meas,on}} - \sigma^{\text{meas,off}} + \mathbb{1}_4$$



$$\mathbf{R} = (\hat{x}_s, \hat{p}_s, \hat{x}_i, \hat{p}_i)$$

$$\sigma_{jk}^{\text{meas}} = 4 \left[ \frac{1}{2} \langle R_j R_k + R_k R_j \rangle - \langle R_j \rangle \langle R_k \rangle \right]$$

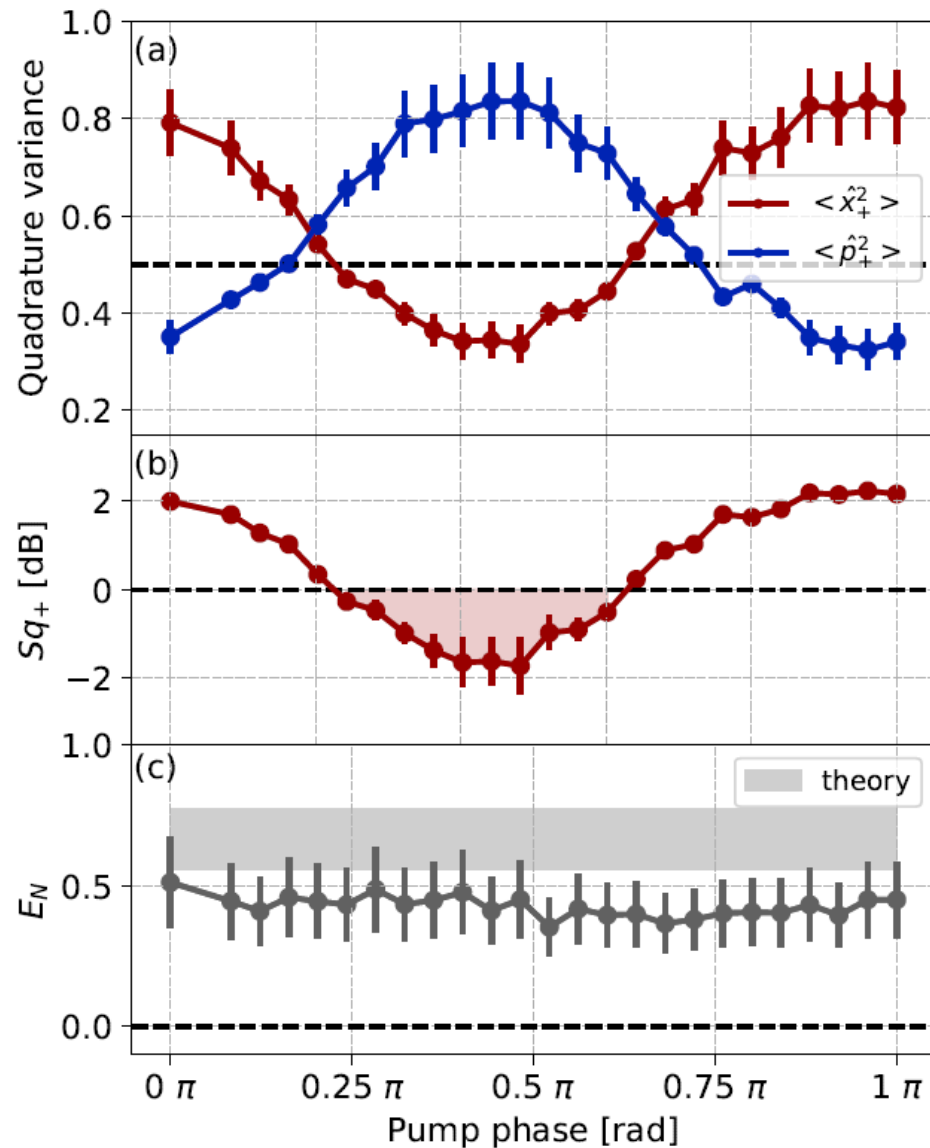


- Entanglement : Logarithmic negativity  $E_N$
- Squeezing of the collective quadrature

$$x_+ = (x_s + x_i)$$

$$\text{Sq} = 10 \log \left( \frac{\langle x_+^2 \rangle}{0.5} \right)$$

# Two-mode squeezing in TWPAs

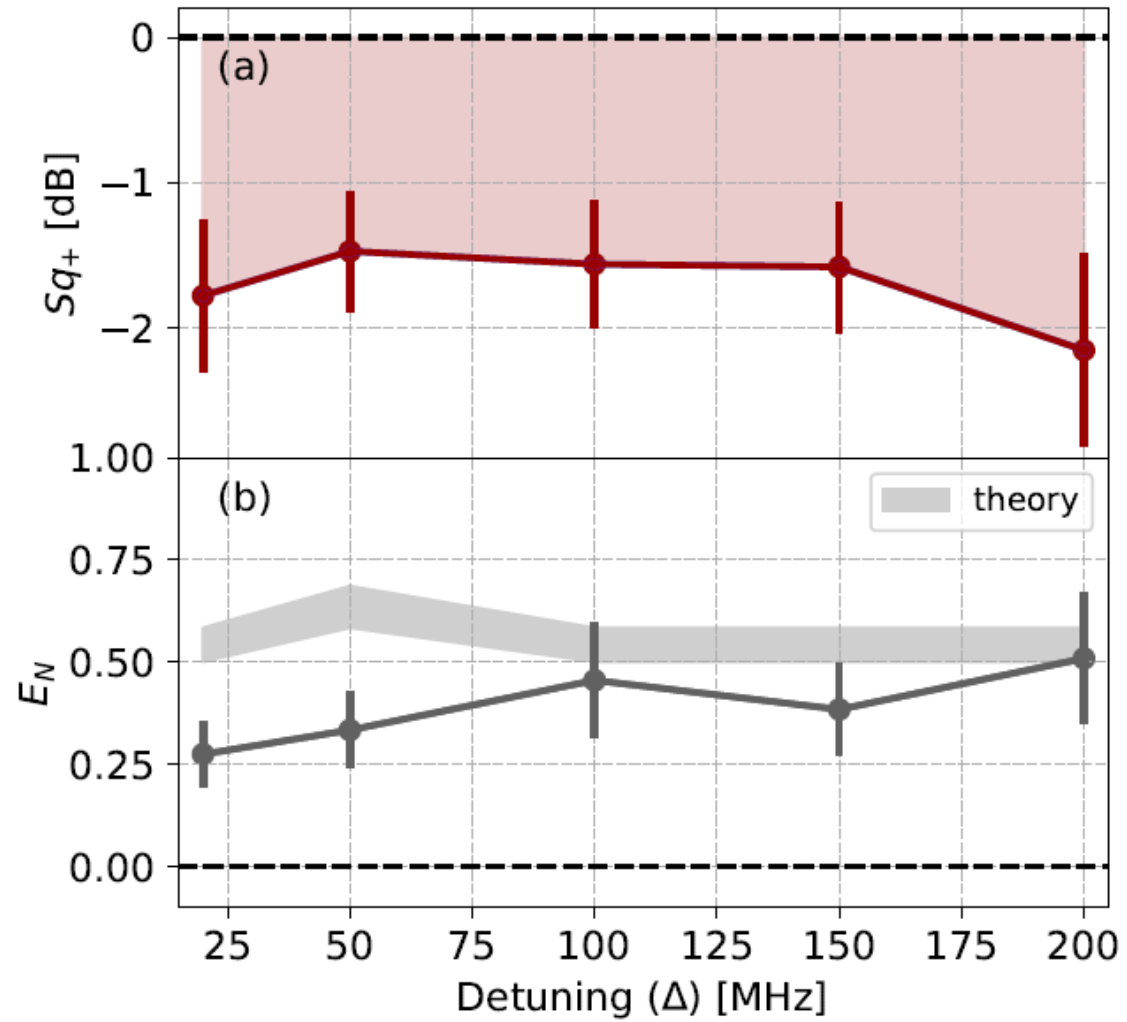


## Demonstration of two-mode squeezing in TWPAs

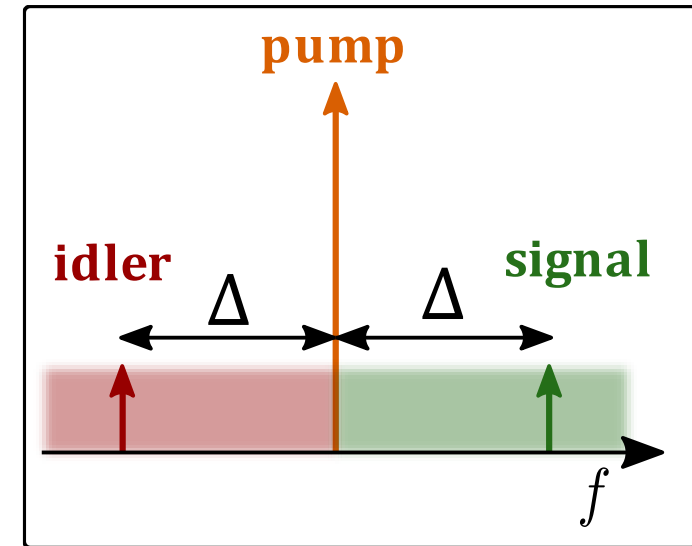
$$\hat{x}_+ = (\hat{x}_s + \hat{x}_i), \quad \hat{p}_+ = (\hat{p}_s + \hat{p}_i)$$

$$Sq = 10 \log \left( \frac{\langle \hat{x}_+^2 \rangle}{0.5} \right)$$

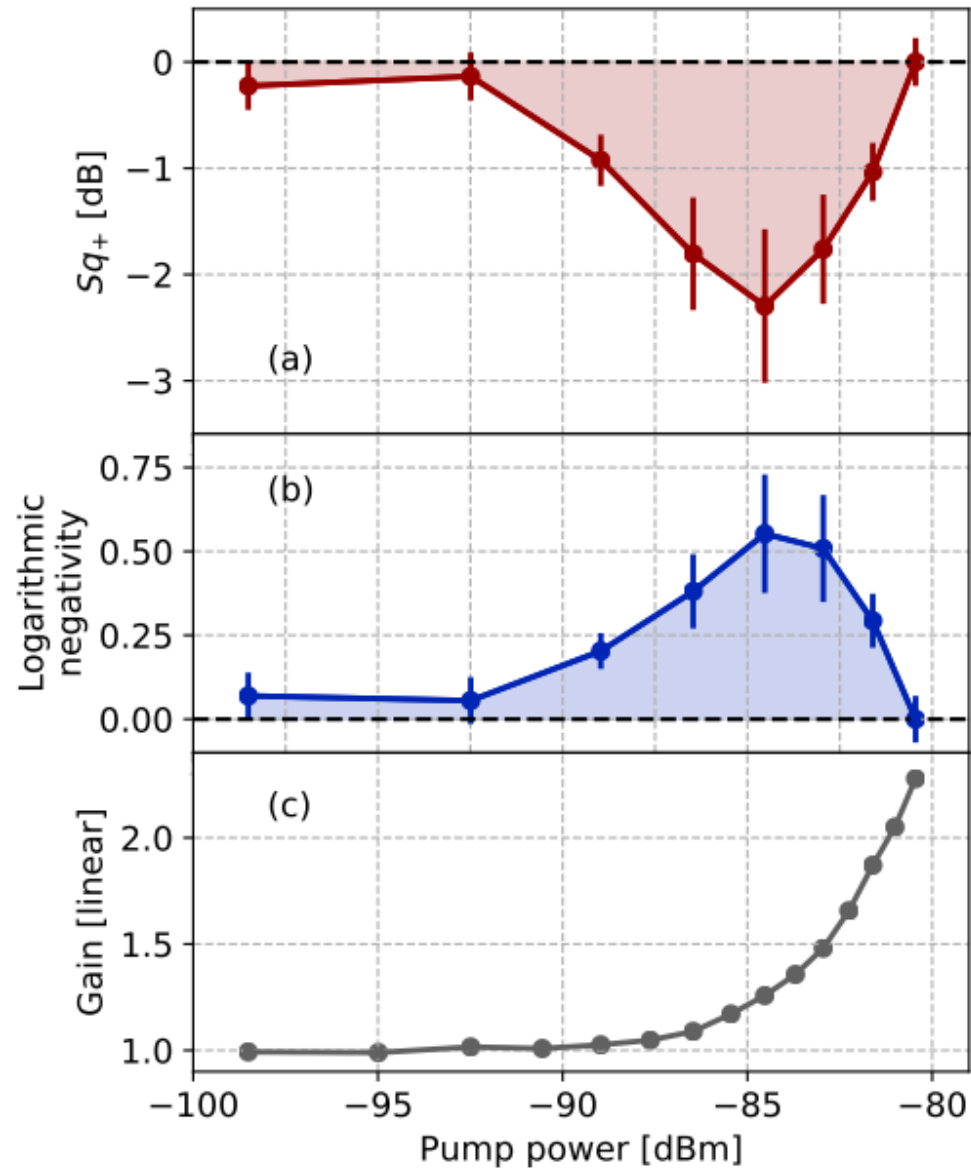
# Two-mode squeezing in TWPAs



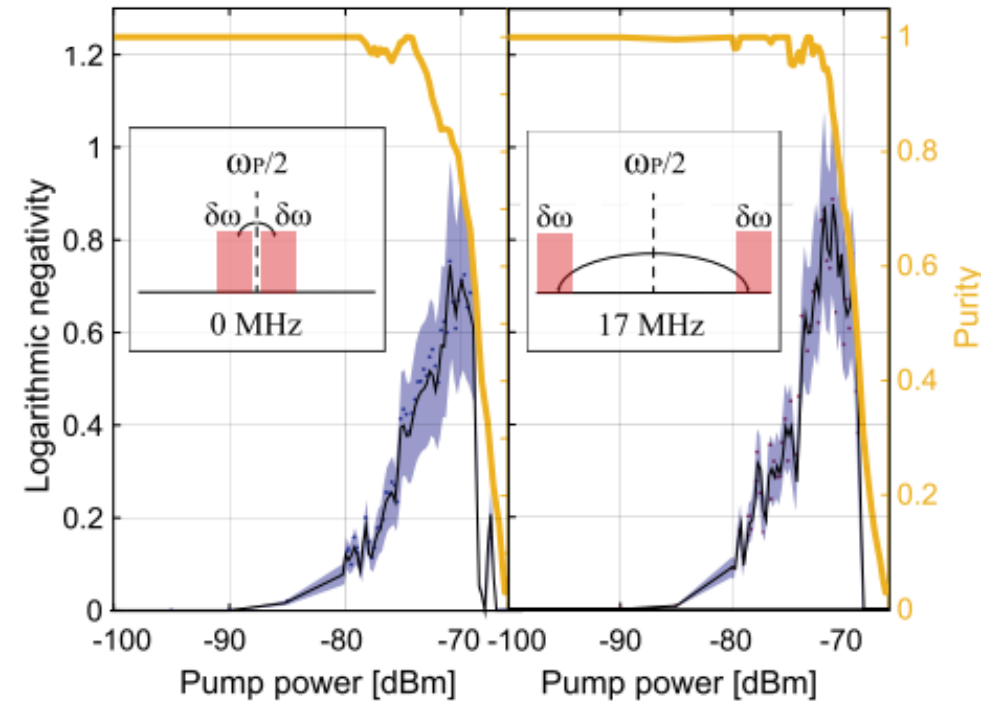
## Broadband squeezing



# Perspectives: Pump power dependence study



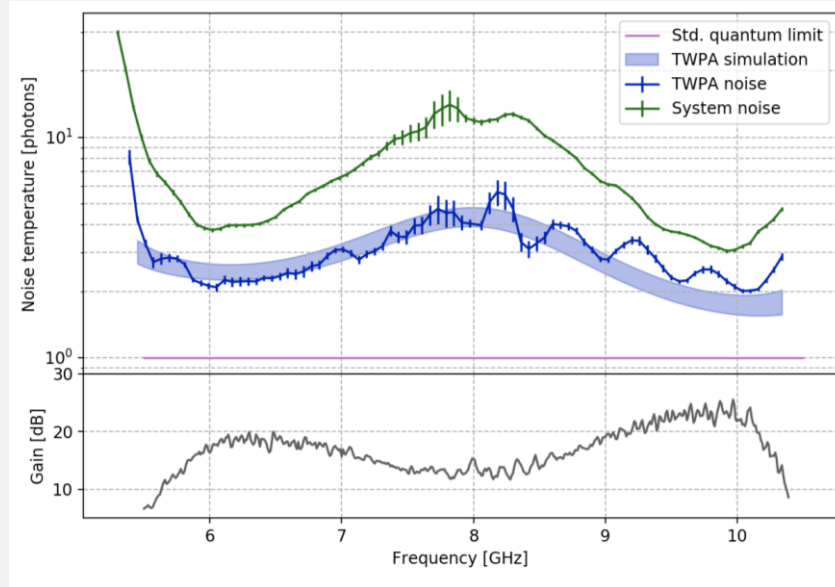
Similar Results for a 3WM Two-mode squeezing



M. Perelshtein et al. <https://arxiv.org/abs/2111.06145> (2021)  
Aalto University / VTT Finland

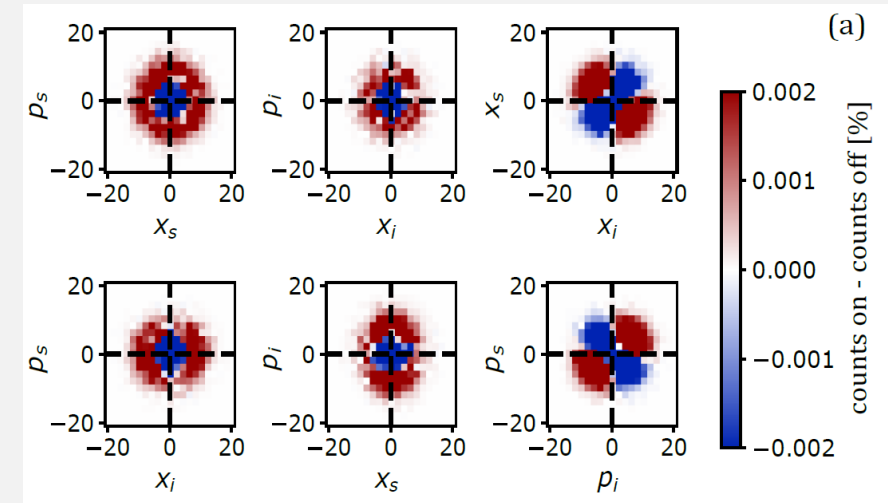
- Detailed study of role of spurious wave-mixing processes i.e.: harmonics generation, up and down conversions...

## Reversed Kerr amplification



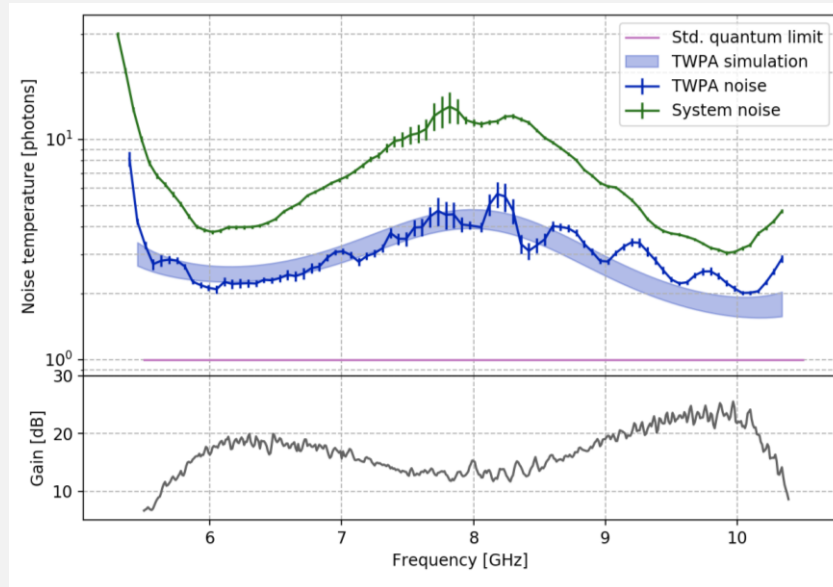
[A. Ranadive. et al. arXiv:2101.05815 \(2021\)](#)

## Two-mode squeezing generation



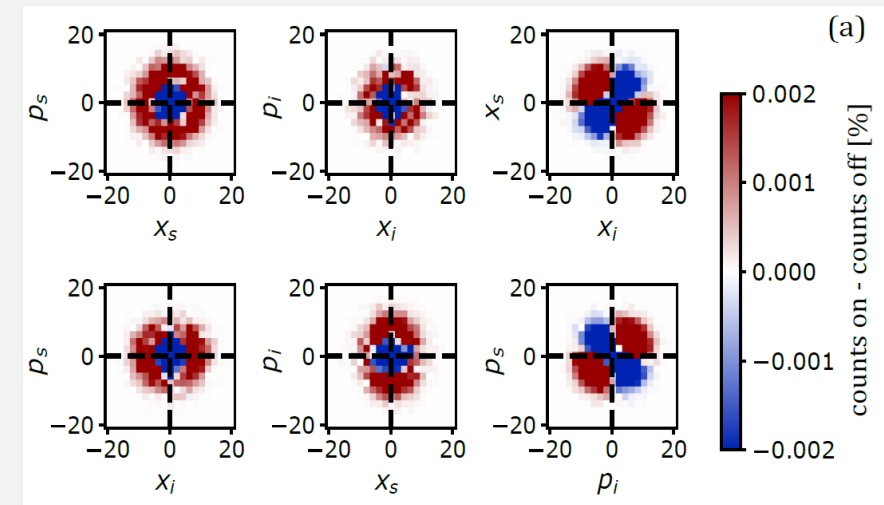
[M. Esposito et al. arXiv:2111.03696 \(2021\)](#)

## Reversed Kerr amplification



[A. Ranadive. et al. arXiv:2101.05815 \(2021\)](#)

## Two-mode squeezing generation



[M. Esposito et al. arXiv:2111.03696 \(2021\)](#)



Giulio Cappelli



Arpit Ranadive



Luca Planat



Martina Esposito



Nicolas Roch

