DART WARS

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ISTITUTO NAZIONALE
DI RICERCA METROLOGICA

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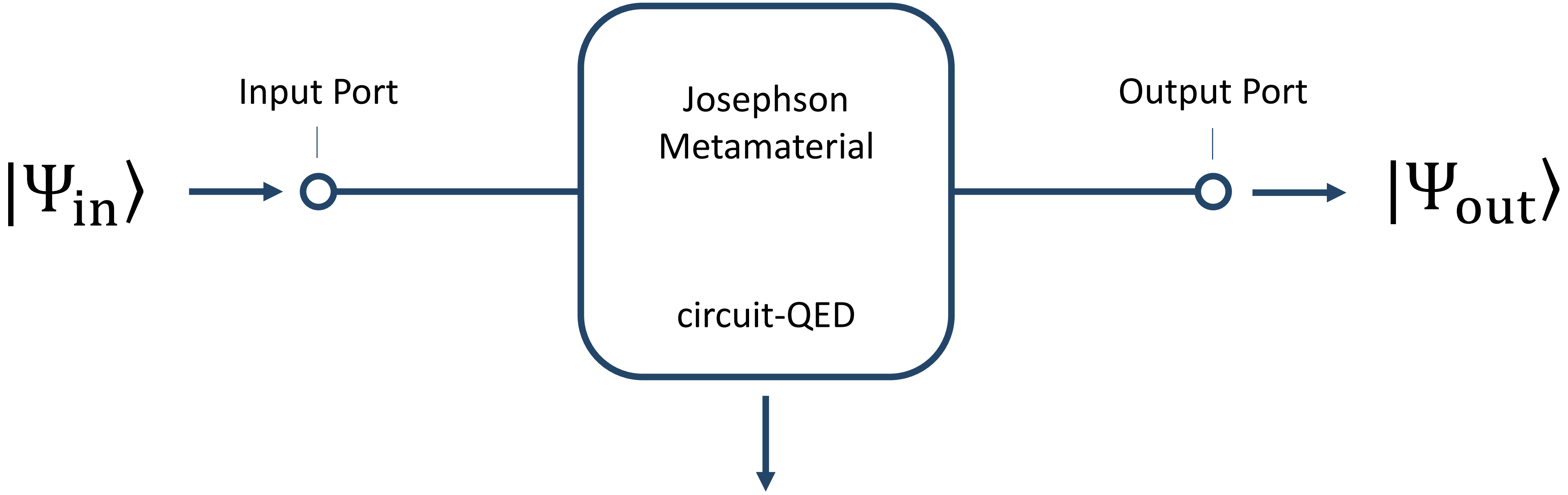
DART WARS Annual Meeting

WP1: Design and Simulation

Emanuele ENRICO

Outline



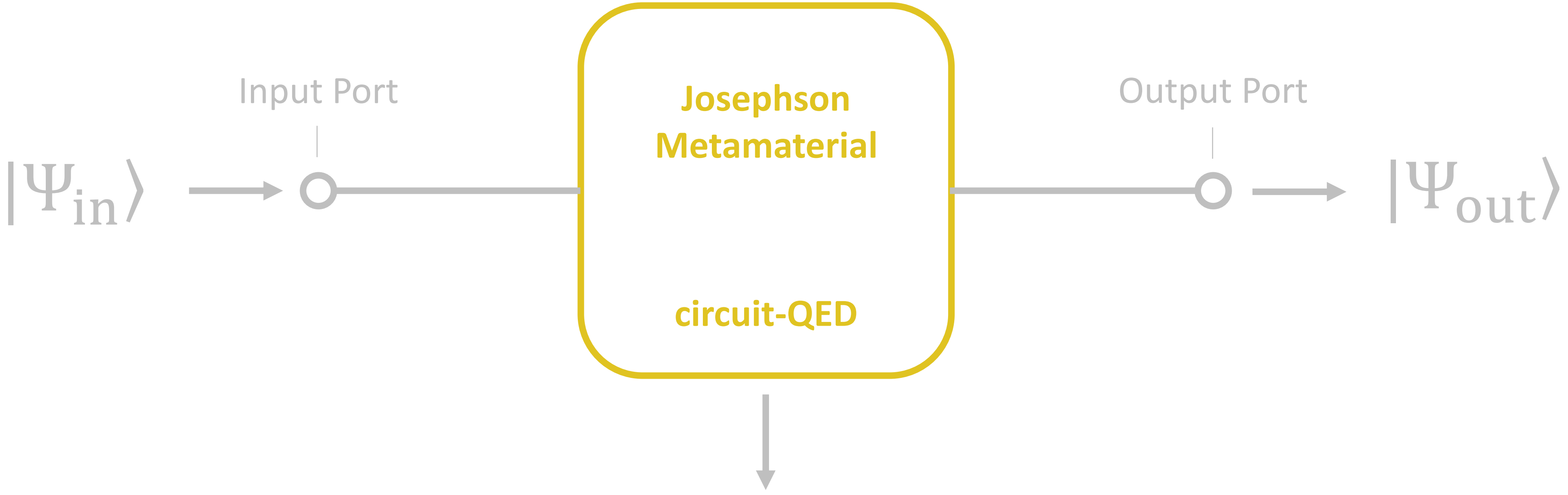
Classic observables:

- Gain
- Noise Figure
- Noise Temperature

Quantum behavior:

- Occupancy probability distribution

Outline



Classic observables:
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Quantum behavior:
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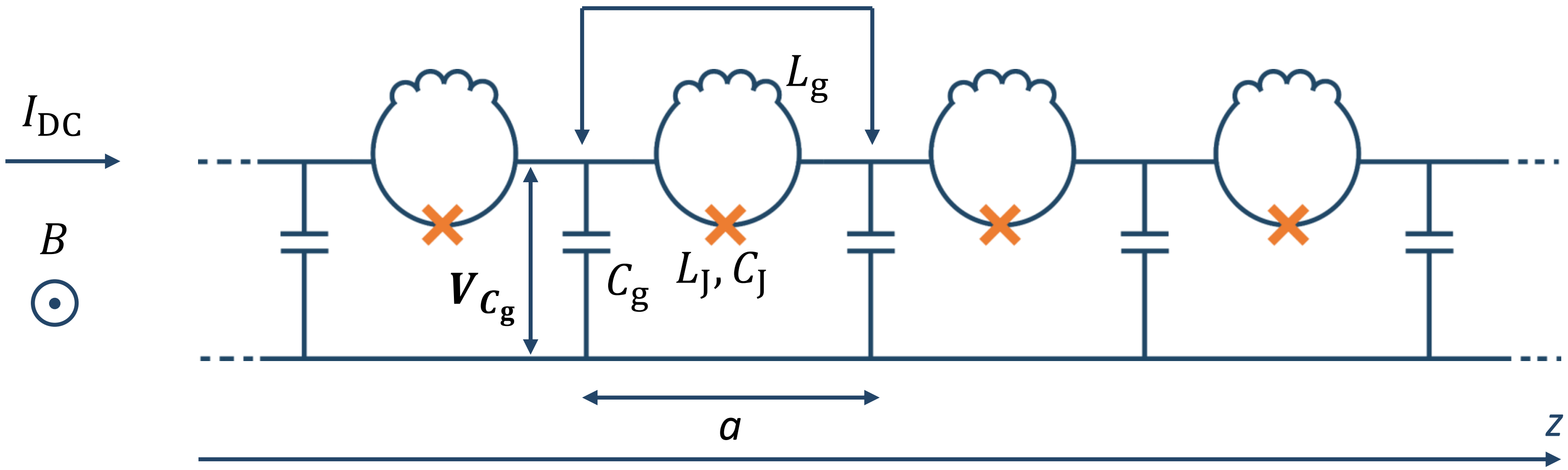
First Quantization Hamiltonian

We adapt and further extend the work presented in [1] to an **rf-SQUID based Josephson Traveling Wave Parametric Amplifier** [2]

$$\Delta\Phi(z,t) = \Delta\Phi_{DC} + \delta\Phi(z,t)$$



$\Delta\Phi_{DC}$ is the **constant** flux difference due to external bias

$\delta\Phi(z,t)$ is the **time-dependent** flux difference induced by the travelling waves

The **first quantization Hamiltonian** can be written as the sum of the electromagnetic energy stored in each component of the transmission line

$$H = \frac{1}{a}\int_0^{aN}\left[\frac{1}{2L_g}\Delta\Phi(z,t)^2 + \varphi_0 I_c\left(1-\cos\left(\frac{\Delta\Phi(z,t)}{\varphi_0}\right)\right) + \frac{C_J}{2}\left(\frac{\partial\Delta\Phi(z,t)}{\partial t}\right)^2 + \frac{C_g}{2}V_{C_g}^2(z,t)\right]dz$$

[1] T. H. A. van der Reep, "Mesoscopic Hamiltonian for Josephson traveling-wave parametric amplifier", Phys. Rev. A **99**, 063838 (2019)
[2] A. B. Zorin, "Josephson Traveling-Wave Parametric Amplifier with Three-Wave Mixing", Phys. Rev. App. **6**, 034006 (2016)

Second Quantization Hamiltonian

$$\boldsymbol{H(\delta\Phi, V_{C_g})} \xrightarrow[V_{C_g} = V_{C_g}(\hat{a}_n, \hat{a}_n^\dagger)]{\delta\Phi = \delta\Phi(\hat{a}_n, \hat{a}_n^\dagger)} \boldsymbol{\hat{H}(\hat{a}_n, \hat{a}_n^\dagger)}$$

First Quantization — Second Quantization

Second Quantization Hamiltonian

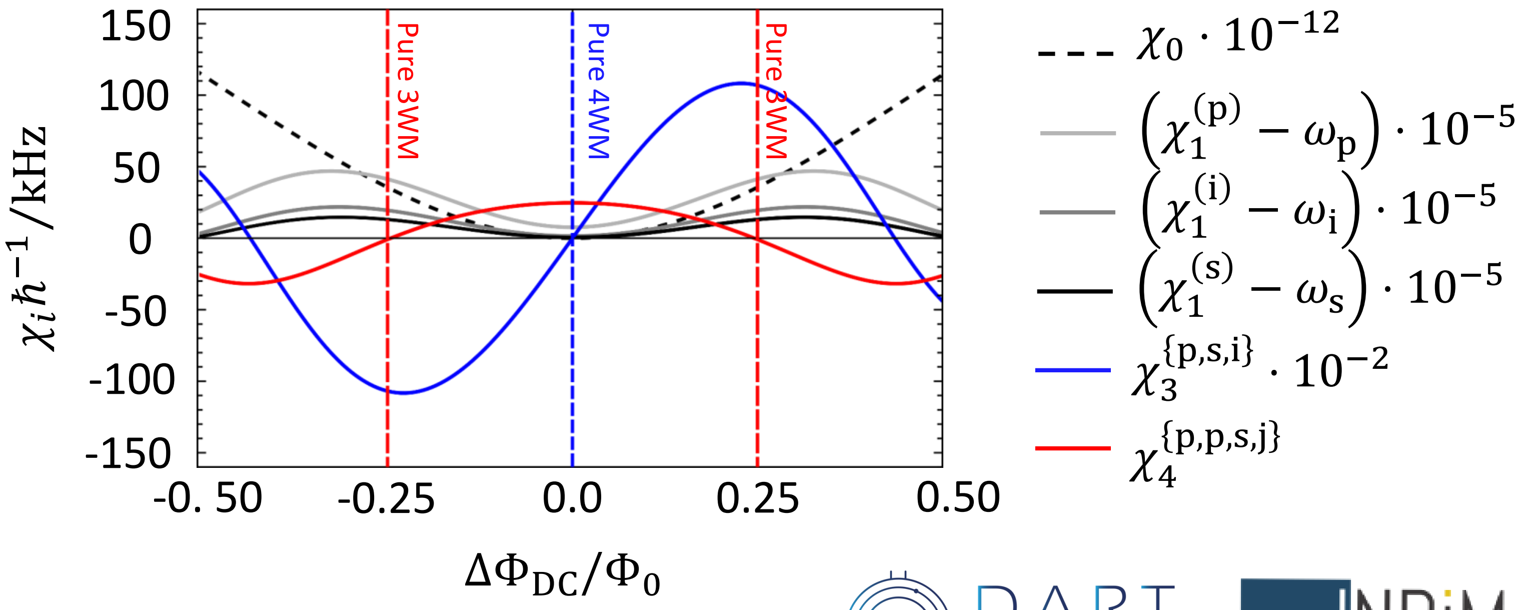
The Hamiltonian describes all the **energy preserving interactions** between 3 or 4 traveling waves (i.e., the **parametric down conversion**, the **sum frequency generation**, the **high order harmonics generation**, etc..)

$$H = \hbar\chi_0 + \sum_n \hbar\chi_1^{(n)}\left(\hat{a}_n^\dagger\hat{a}_n + \frac{1}{2}\right) + \sum_{n,l,m} \hbar\,\chi_3^{(n,l,m)}\{\hat{a}+\hat{a}^\dagger\}_{n,l,m}\cdot\delta_{\Delta_{n,l,m},0} + \sum_{n,l,m,s} \hbar\,\chi_4^{(n,l,m,s)}\{\hat{a}+\hat{a}^\dagger\}_{n,l,m,s}\cdot\delta_{\Delta_{n,l,m,s},0}$$

- Energy from **external bias**
- Energy of the **propagating waves**
- All 3-wave mixing energy-preserving processes
- All 4-wave mixing energy-preserving processes

For **parametric down-conversion**

- **3WM** $n, l, m = \{\mathrm{p}, \mathrm{s}, \mathrm{i}\}$
  with $\omega_\mathrm{p} = \omega_\mathrm{s} + \omega_\mathrm{i}$
- **4WM** $n, l, m, s = \{\mathrm{p}, \mathrm{s}, \mathrm{j}\}$
  with $2\omega_\mathrm{p} = \omega_\mathrm{s} + \omega_\mathrm{j}$



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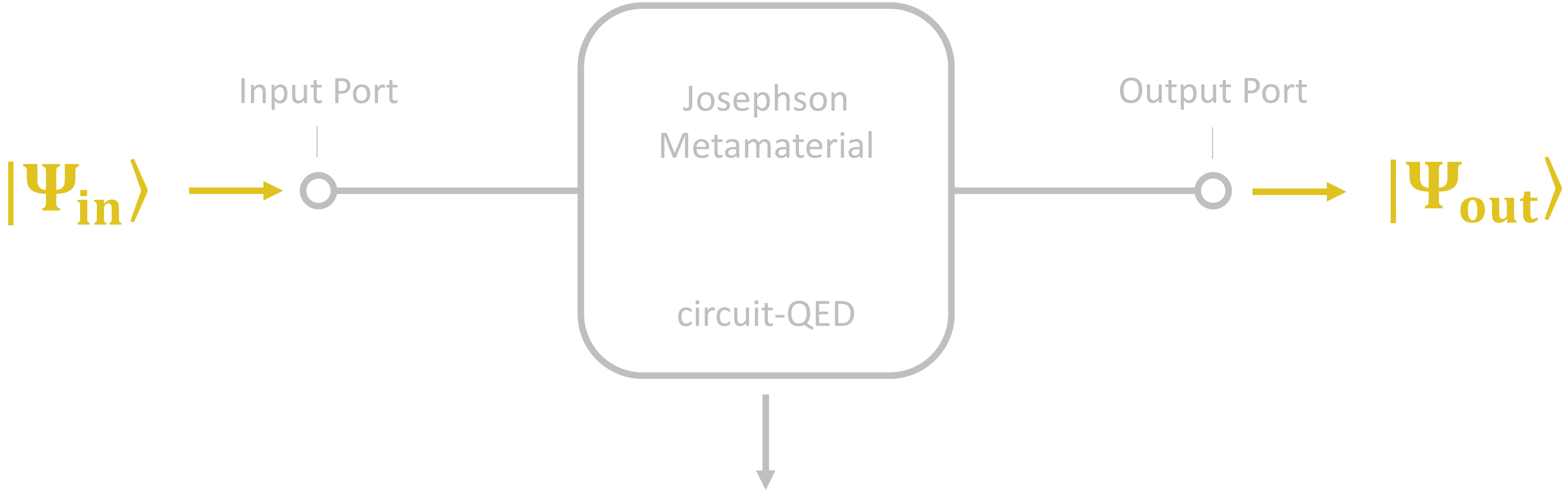

Classic observables:

- Gain
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Quantum behavior:

- Occupancy probability distribution

CMEs from Heisenberg Equation

Selecting a proper bias condition, the amplifier can work as a pure 3-Wave Mixer or 4-Wave Mixer ($H_{3\text{WM}}$, $H_{4\text{WM}}$).
In this condition the **evolution of propagating modes** can be derived solving the **Heisenberg equation**:

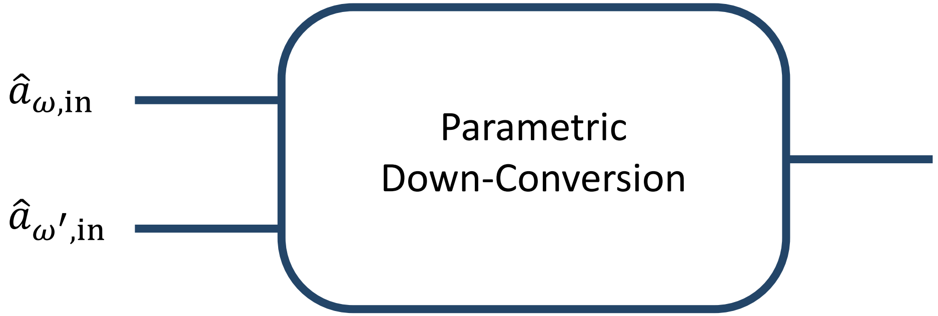
$$\frac{d\hat{a}_n}{dt} = \frac{i}{\hbar}\left[H_{3\text{WM}(4\text{WM})}, \hat{a}_n\right] + \frac{\partial \hat{a}_n}{\partial t}$$

where $n = \{\text{p}, \text{s}, \text{i}, \text{j}\}$

Under the **undepleted and classical pump** approximation, the output field at the **signal frequency ω** is:

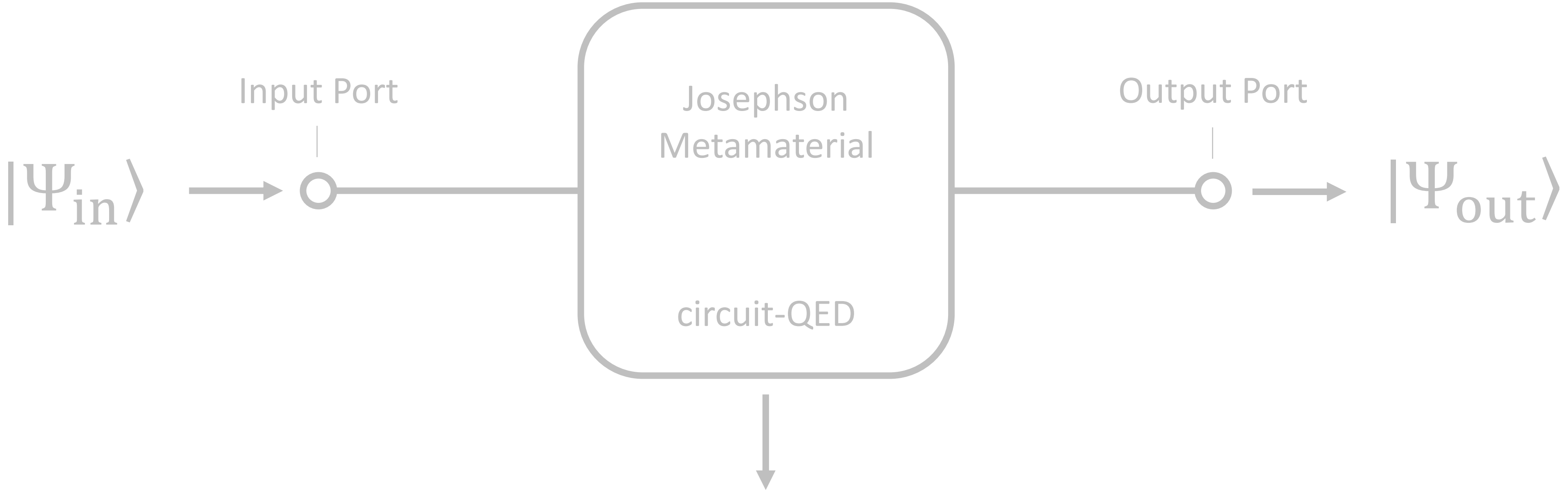
$$\hat{a}_\omega(t) = \left[\left(\cosh(gt) + \frac{i\Psi}{2g}\sinh(gt)\right)\hat{a}_{\omega,\text{in}} - \left(\frac{i\Upsilon}{g}\sinh(gt)\right)\hat{a}^{\dagger}_{\omega',\text{in}}\right] e^{-i\left(\frac{\Psi}{2}\right)t}$$

where g is the ***complex gain factor***, Ψ is the ***density phase mismatch*** and Υ is the ***interaction parameter***.



$$\hat{a}_{\omega,\text{out}} = u(\omega)\hat{a}_{\omega,\text{in}} - v(\omega)\hat{a}^{\dagger}_{\omega',\text{in}}$$

Outline



Classic observables:

- **Gain**
- **Noise Figure**
- **Noise Temperature**

Quantum behavior:

- Occupancy probability distribution

Signal photons at the output port

$$\langle \hat{n}_{\omega,out}\rangle = \langle \hat{a}^{\dagger}_{\omega,\,\mathrm{out}}\,\hat{a}_{\omega,\mathrm{out}}\rangle =$$

$$= |u(\omega)|^2\langle \hat{n}_{\omega,\,\mathrm{in}}\rangle + |v(\omega)|^2\langle \hat{n}_{\omega',\mathrm{in}}\rangle + i\left(u^*(\omega)v(\omega)\left\langle \hat{a}^{\dagger}_{\omega,\mathrm{in}}\,\hat{a}^{\dagger}_{\omega',\mathrm{in}}\right\rangle - u(\omega)v^*(\omega)\langle \hat{a}_{\omega',\mathrm{in}}\,\hat{a}_{\omega,\mathrm{in}}\rangle\right) + |v(\omega)|^2$$

Amplification of the input field at ω frequency

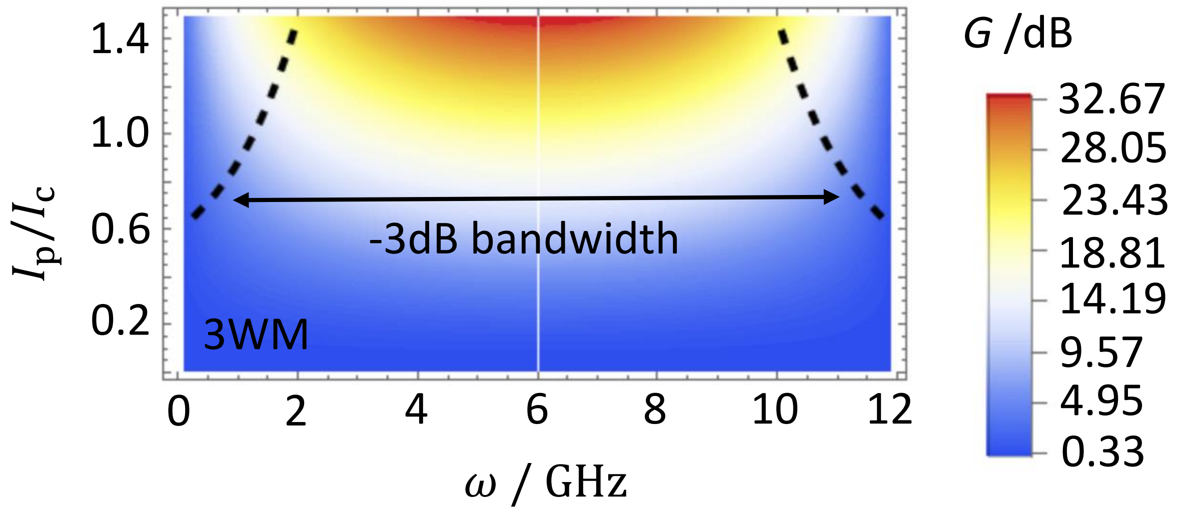
Contribution to the amplification of the **input field at ω** frequency due to the presence of an input signal at **ω' frequency**

Contribution from the **spontaneous annihilation** of a pump photon

Contribution from the **spontaneous creation** of a pump photon

Added noise photons $(\langle \mathrm{vac}|\hat{n}_{\omega,\mathrm{out}}|\mathrm{vac}\rangle)$

$$G(\omega) \equiv \frac{\langle \hat{n}_{\omega,\mathrm{out}}\rangle}{\langle \hat{n}_{\omega,\mathrm{in}}\rangle} = |u(\omega)|^2$$

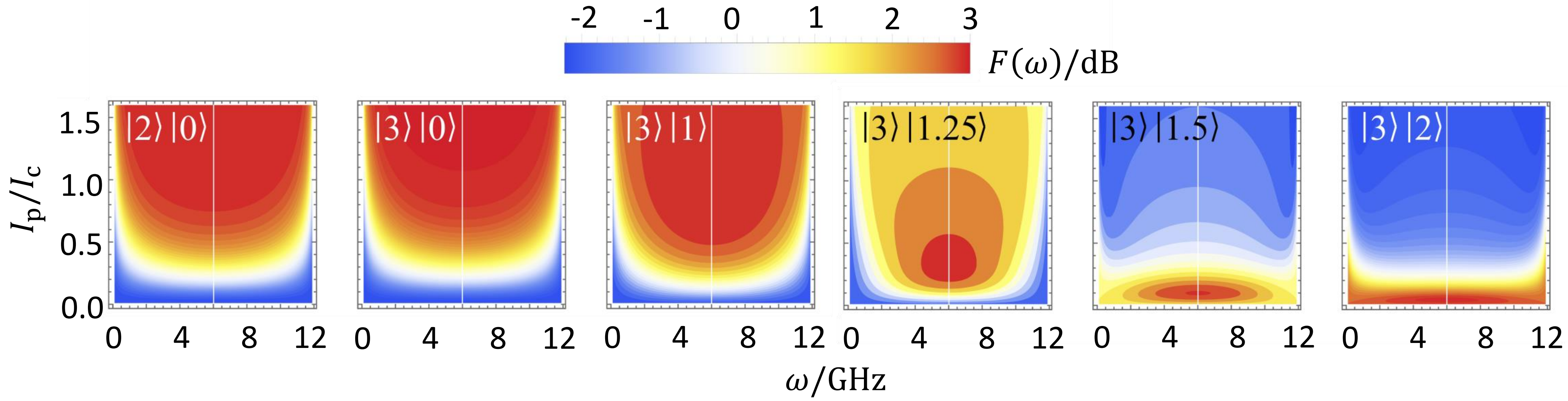


Noise Figure

$$F(\omega) \equiv \frac{\mathrm{SNR_{in}}(\omega)}{\mathrm{SNR_{out}}(\omega)} \qquad \mathrm{SNR_{in}}(\omega) \equiv \frac{\langle \hat{n}_{\omega,\mathrm{in}} \rangle^2}{\langle (\Delta\hat{n}_{\omega,in})^2 \rangle} \qquad \mathrm{SNR_{out}}(\omega) \equiv \frac{\langle \hat{n}_{\omega,\mathrm{out}} \rangle^2 - \langle \hat{n}^{\mathrm{vac}}_{\omega,\mathrm{out}} \rangle^2}{\langle (\Delta\hat{n}_{\omega,\mathrm{out}})^2 \rangle}$$ [4]

Variance of the input/output photon number

Supposing at the input a **Bimodal Coherent state:** $|\Psi_C\rangle = D(\alpha_\omega)|0_\omega\rangle D(\alpha_{\omega'})|0_{\omega'}\rangle = |\alpha_\omega\rangle|\alpha_{\omega'}\rangle$



An **idler** tone at the input port **changes the Noise figure** $F(\omega)$ of the amplifier

[4] Z. Shi *et al.*, "Quantum noise properties of non-ideal optical amplifiers and attenuators", *J. Opt.* **13** (2011)

Noise Temperature

The **effective temperature $T_{\mathrm{eff}}(\omega)$** of the amplifier is the temperature that a **Bose-Einstein distribution** should have to equal the output ω mode occupancy generated by a vacuum input state [5]:

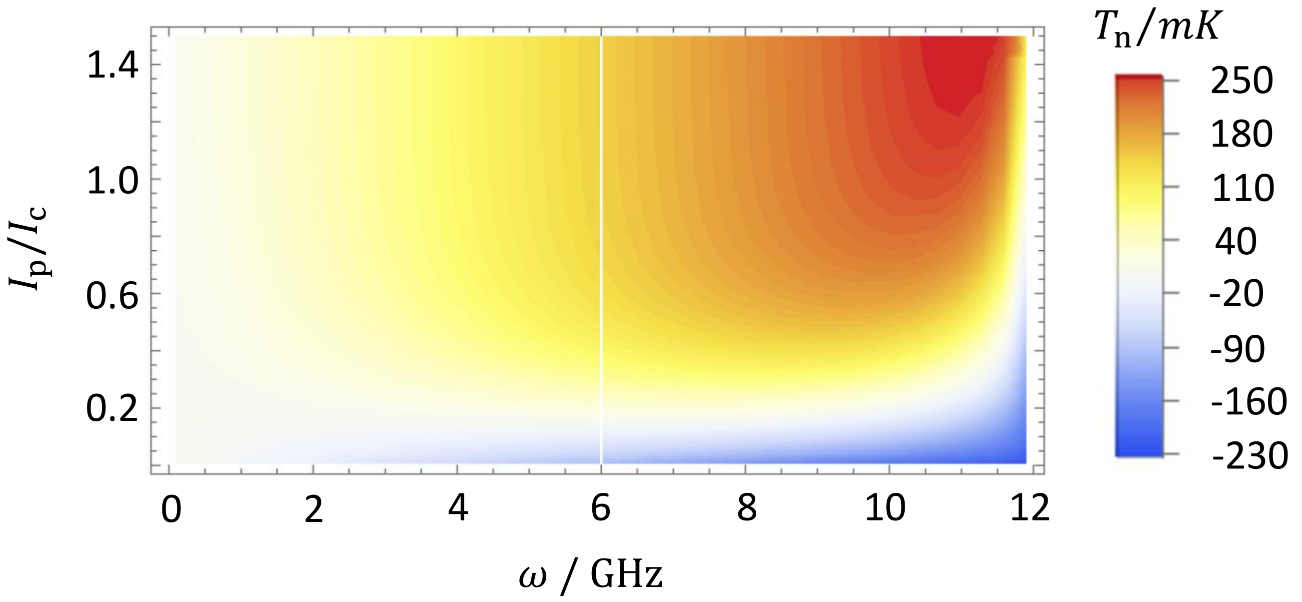
$$\frac{1}{e^{\hbar\omega/k_B T_{\mathrm{eff}}(\omega)}-1} = |v(\omega)|^2$$

The **noise temperature $T_n(\omega)$** is the effective temperature normalized on the gain minus the contribution given by the fluctuation of the input vacuum state:

$$T_n(\omega) = \frac{T_{\mathrm{eff}}(\omega)}{G(\omega)} - \frac{1}{2}\frac{\hbar\omega}{k_B}$$

For high gain $T_n(\omega)$ approaches the **Standard Quantum Limit**:

$$T_{n,SQL} = \frac{1}{2}\frac{\hbar\omega}{k_B}$$

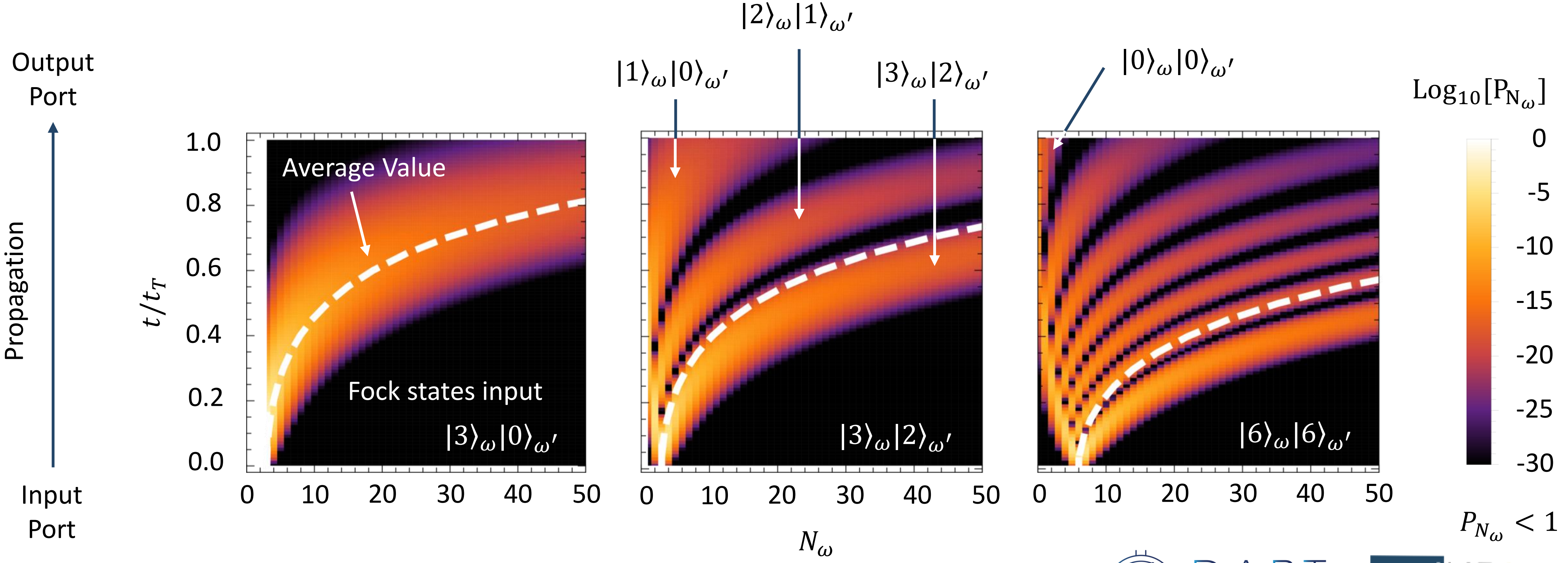


[5] A. A. Clerk *et al.*, “Introduction to quantum noise, measurement, and amplification”, *Rev. Mod. Phys.* **82,** 1155 (2010)

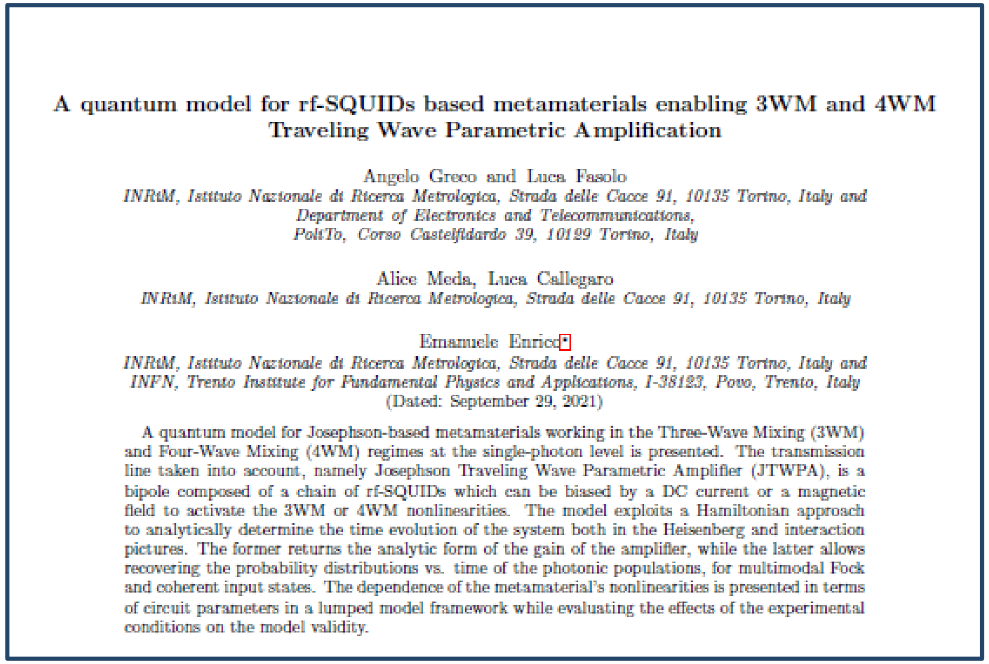
Time-evolution of bimodal Fock states

The average photon number at frequency ω at a certain time *t* can be written in terms of the **probability P_{N_ω}** to find N_ω photons at a given time

$$\langle n_\omega(t)\rangle = \sum_{N_\omega} P_{N_\omega}(t) \cdot N_\omega$$



For more details…



A quantum model for rf-SQUIDs based metamaterials enabling 3WM and 4WM Traveling Wave Parametric Amplification

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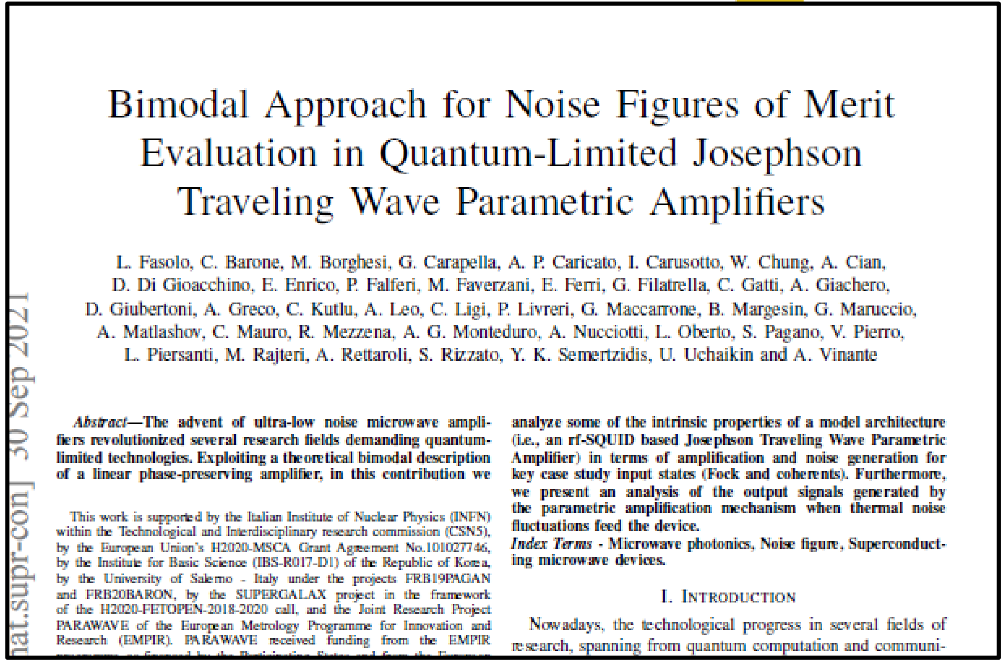
(Dated: September 29, 2021)

A quantum model for Josephson-based metamaterials working in the Three-Wave Mixing (3WM) and Four-Wave Mixing (4WM) regimes at the single-photon level is presented. The transmission line taken into account, namely Josephson Traveling Wave Parametric Amplifier (JTWPA), is a bipole composed of a chain of rf-SQUIDs which can be biased by a DC current or a magnetic field to activate the 3WM or 4WM nonlinearities. The model exploits a Hamiltonian approach to analytically determine the time evolution of the system both in the Heisenberg and interaction pictures. The former returns the analytic form of the gain of the amplifier, while the latter allows recovering the probability distributions vs. time of the photonic populations, for multimodal Fock and coherent input states. The dependence of the metamaterial's nonlinearities is presented in terms of circuit parameters in a lumped model framework while evaluating the effects of the experimental conditions on the model validity.



Bimodal Approach for Noise Figures of Merit Evaluation in Quantum-Limited Josephson Traveling Wave Parametric Amplifiers

L. Fasolo, C. Barone, M. Borghesi, G. Carapella, A. P. Caricato, I. Carusotto, W. Chung, A. Cian, D. Di Gioacchino, E. Enrico, P. Falferi, M. Faverzani, E. Ferri, G. Filatrella, C. Gatti, A. Giachero, D. Giubertoni, A. Greco, C. Kutlu, A. Leo, C. Ligi, P. Livreri, G. Maccarrone, B. Margesin, G. Maruccio, A. Matlashov, C. Mauro, R. Mezzena, A. G. Monteduro, A. Nucciotti, L. Oberto, S. Pagano, V. Pierro, L. Piersanti, M. Rajteri, A. Rettaroli, S. Rizzato, Y. K. Semertzidis, U. Uchaikin and A. Vinante

***Abstract*—The advent of ultra-low noise microwave amplifiers revolutionized several research fields demanding quantum-limited technologies. Exploiting a theoretical bimodal description of a linear phase-preserving amplifier, in this contribution we analyze some of the intrinsic properties of a model architecture (i.e., an rf-SQUID based Josephson Traveling Wave Parametric Amplifier) in terms of amplification and noise generation for key case study input states (Fock and coherents). Furthermore, we present an analysis of the output signals generated by the parametric amplification mechanism when thermal noise fluctuations feed the device.**

***Index Terms* - Microwave photonics, Noise figure, Superconducting microwave devices.**

This work is supported by the Italian Institute of Nuclear Physics (INFN) within the Technological and Interdisciplinary research commission (CSN5), by the European Union's H2020-MSCA Grant Agreement No.101027746, by the Institute for Basic Science (IBS-R017-D1) of the Republic of Korea, by the University of Salerno - Italy under the projects FRB19PAGAN and FRB20BARON, by the SUPERGALAX project in the framework of the H2020-FETOPEN-2018-2020 call, and the Joint Research Project PARAWAVE of the European Metrology Programme for Innovation and Research (EMPIR). PARAWAVE received funding from the EMPIR

I. INTRODUCTION

Nowadays, the technological progress in several fields of research, spanning from quantum computation and communi-

at.supr-con] 30 Sep 2021

arXiv:2009.01002v5

arXiv:2109.14924v1

A. Greco *et al.*, "Quantum model for rf-SQUID based metamaterials enabling 3WM and 4WM Traveling Wave Parametric Amplification". Phys. Rev. B 104, 184517 (2021).

L. Fasolo *et al.*, "Bimodal Approach for Noise Figures of Merit Evaluation in Quantum-Limited Josephson Traveling Wave Parametric Amplifiers". *IEEE TAS (under major revisions)*.

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DART WARS Annual Meeting

WP2: JTWPA Devices fabrication

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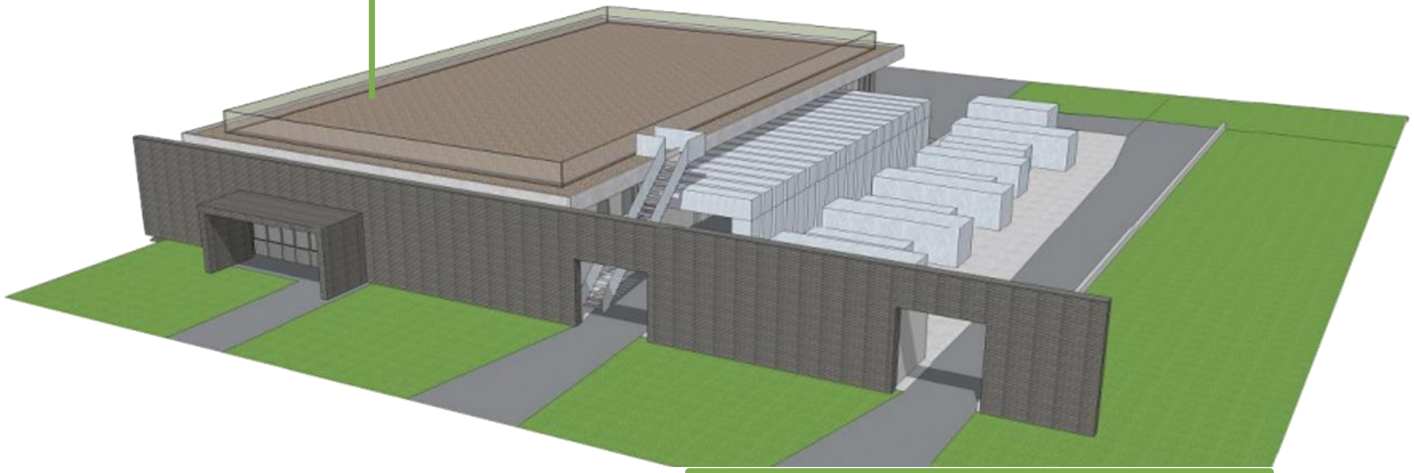
PiQuET Laboratory

PiQuET
Piemonte Quantum Enabling technology



500 m² Cleanroom

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POLITECNICO DI TORINO

UNIVERSITA DEGLI STUDI DI TORINO

REGIONE PIEMONTE

POR Piemonte FESR 2014/2020

DART WARS

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PiQuET Laboratory

October 2017
01
Project submission

March 2018
02
Obtainment of funding

August 2019
03
Public tender

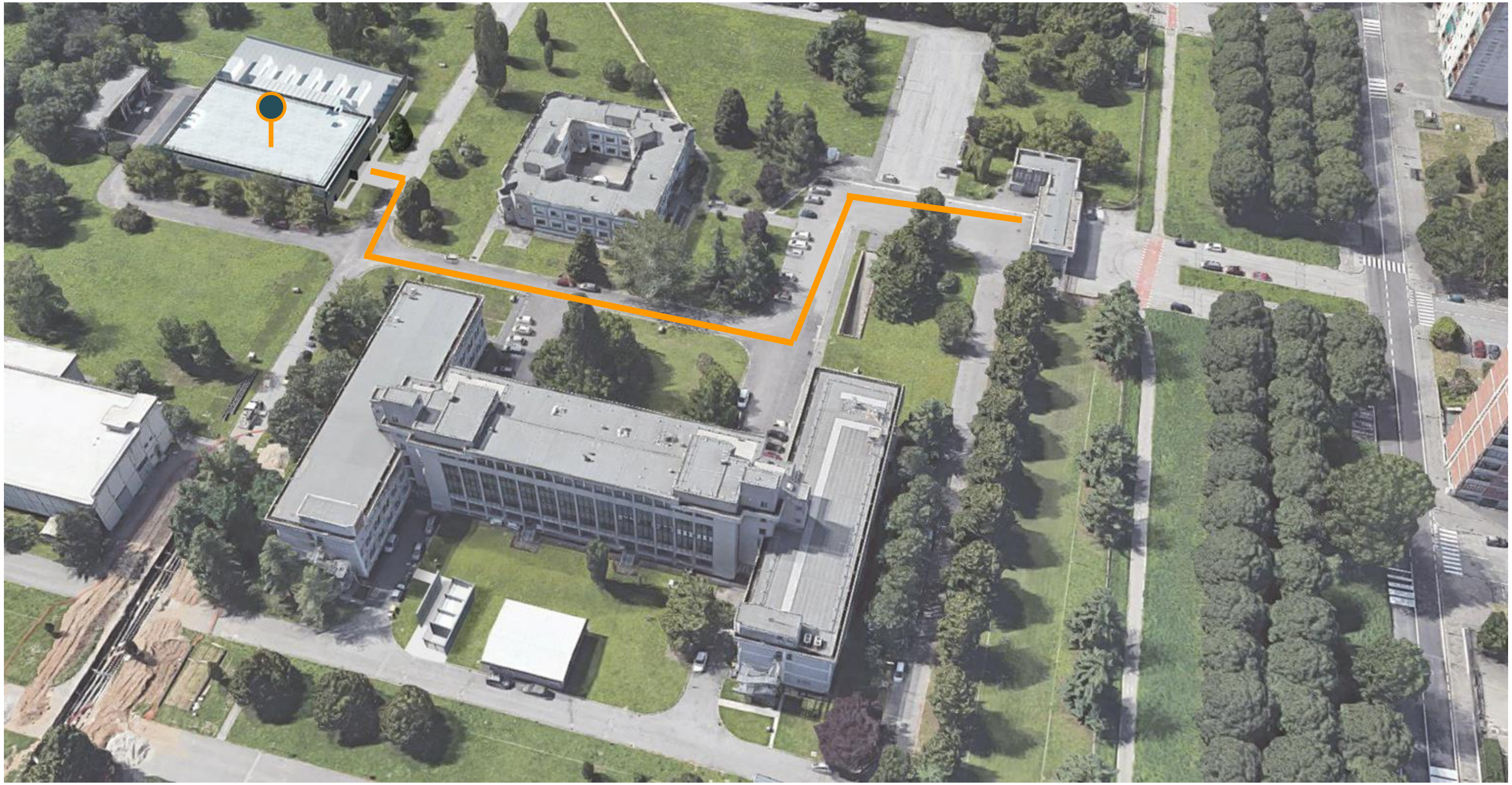
October 2019
04
Reliance of the supply

Decembe r 2019
05
Starting of building work

June 2020
06
Cleanroom supply

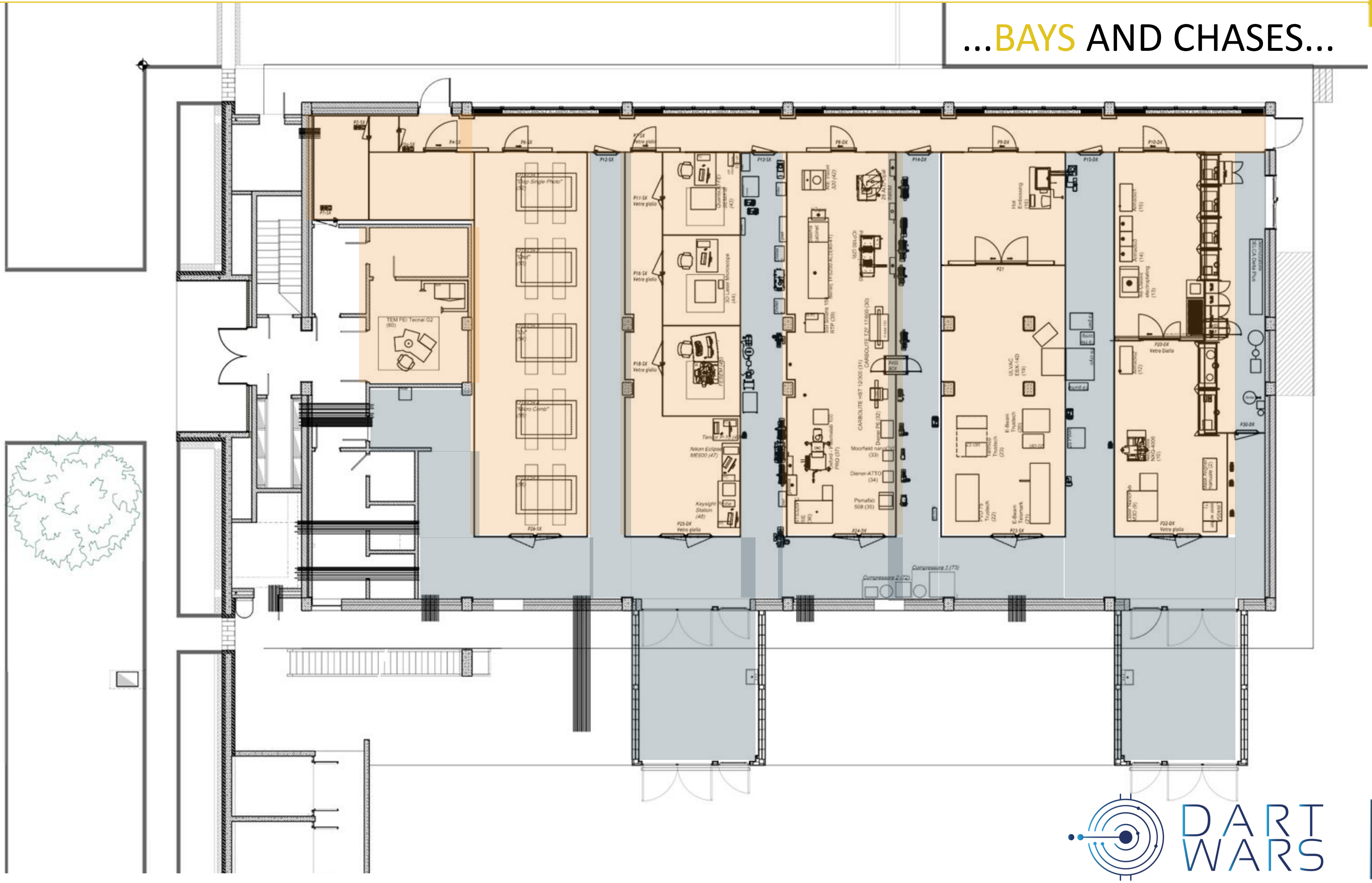
January 2021
07
End of project: Start-up!

INRiM Campus, Torino



INRiM Campus, Torino

...BAYS AND CHASES...



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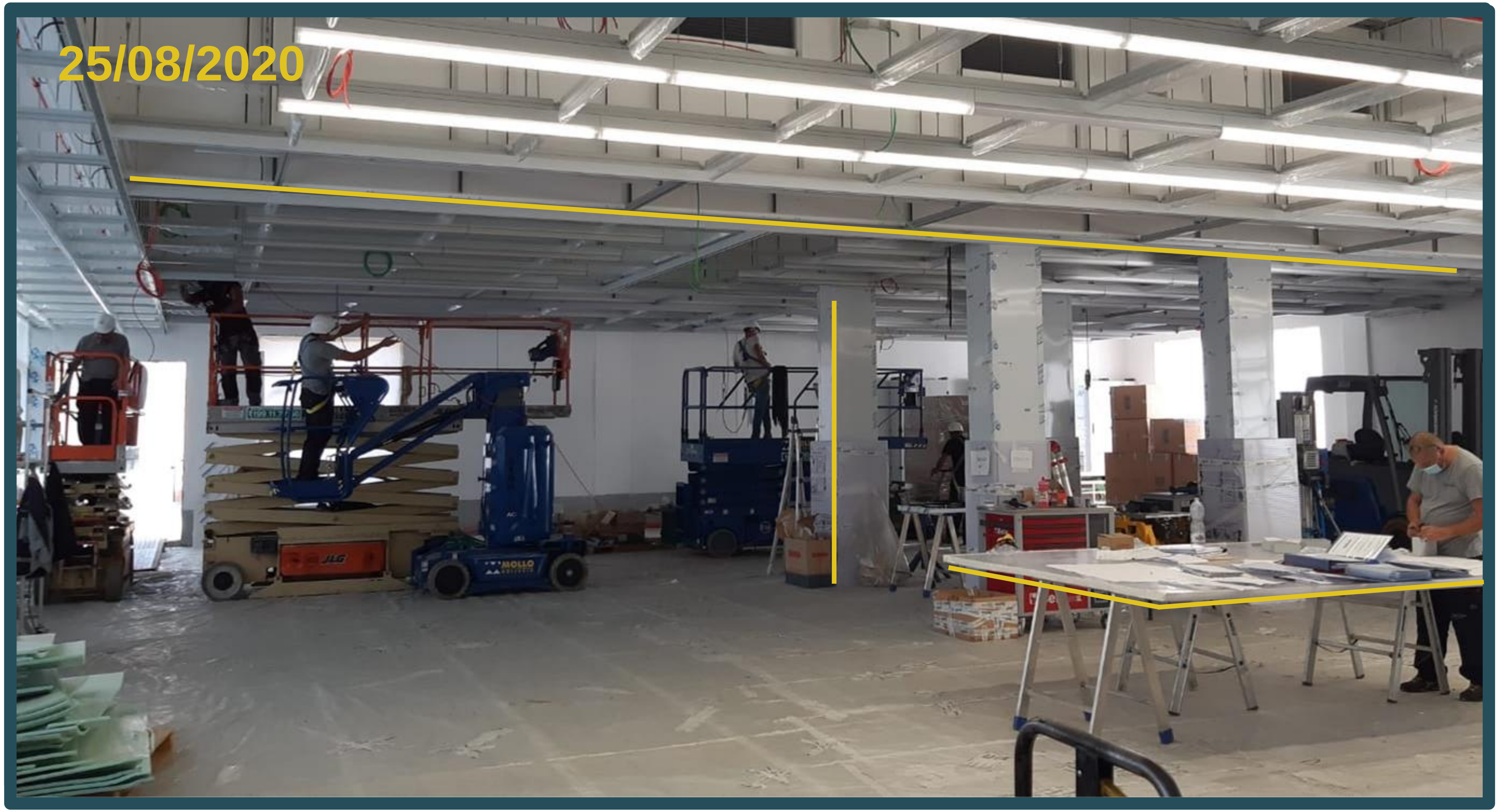

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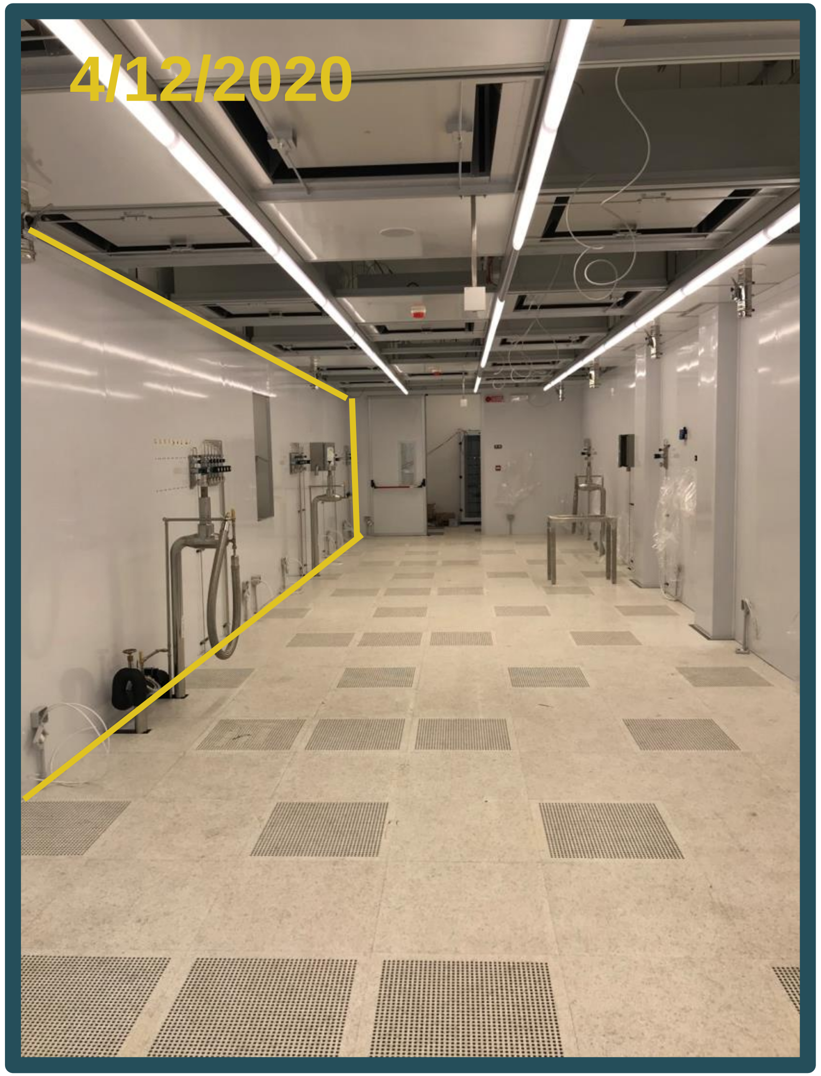

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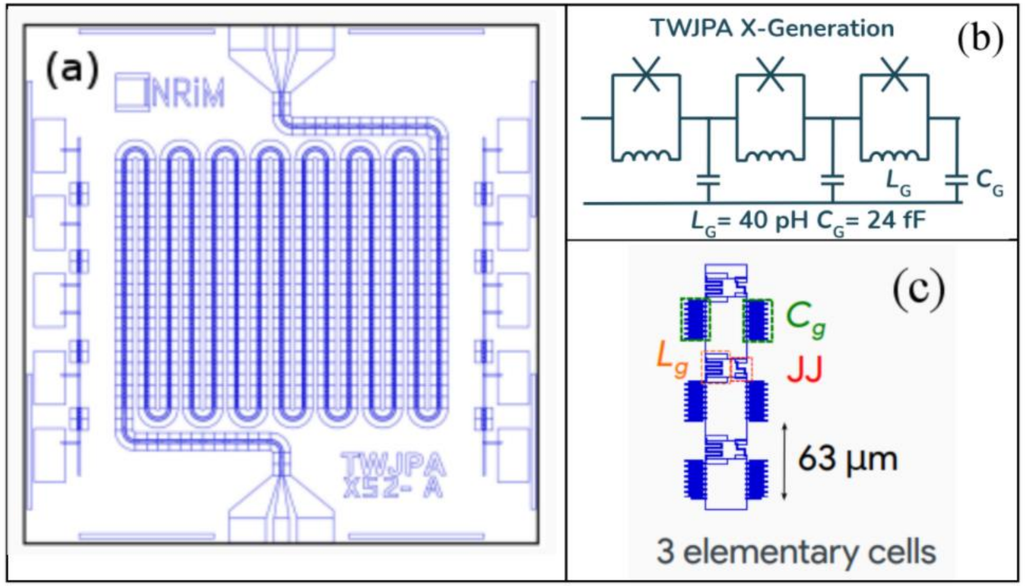
PiQuET Laboratory

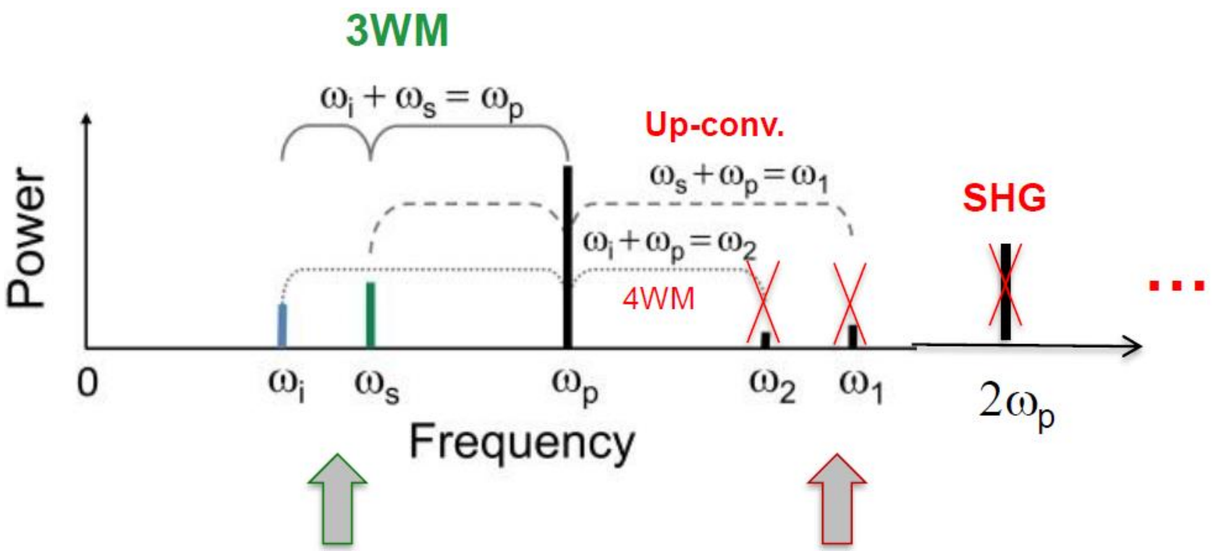
PiQuET Laboratory

- 2 inches wafer standardization of the whole fabrication chain
- 12 chips per wafer (1x1 cm^2)
- PiQuET wet processes migration
- PiQuET E-Beam evaporator installed and running (from may 2021)
- PiQuET Installation of Ebeam and Optical Litho expected by March 2022 (currently back-and-forth from the QR Lab@INRiM)

Unwanted high-tone waves generation



First generation (X) preliminary tested at LNF & IBS



Good phase matching is needed!

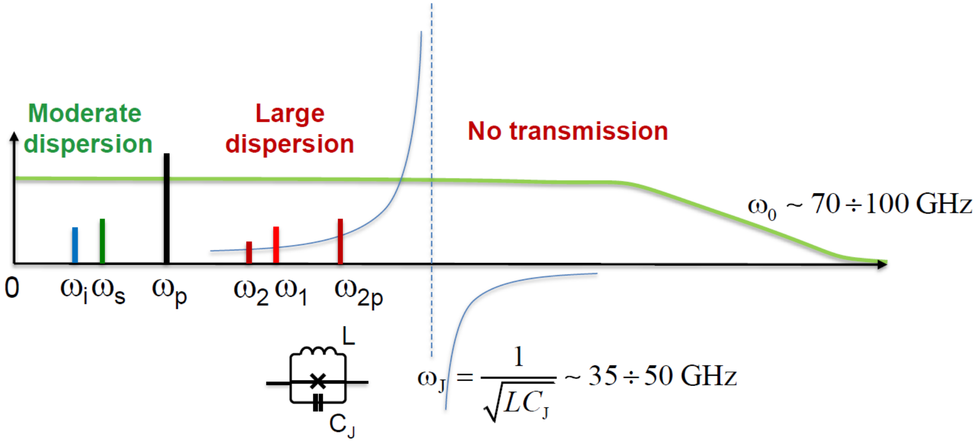
Efficient suppression of propagation is needed!

Resonant Phase Matching Scheme

Destroy phase matching condition for up conversion :

$$k_{sum}=k_p+k_s+\Delta k$$

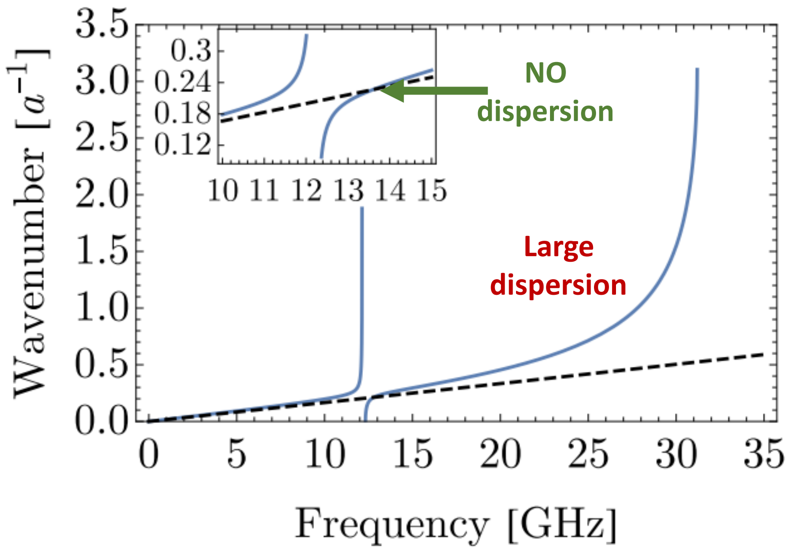
- By decreasing the plasma frequency

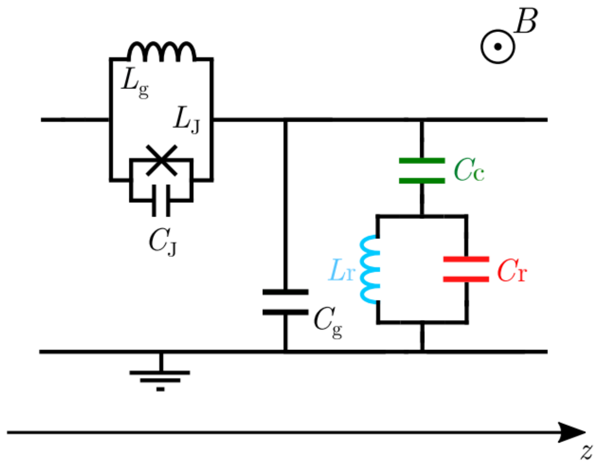


Repair phase matching condition for down conversion:

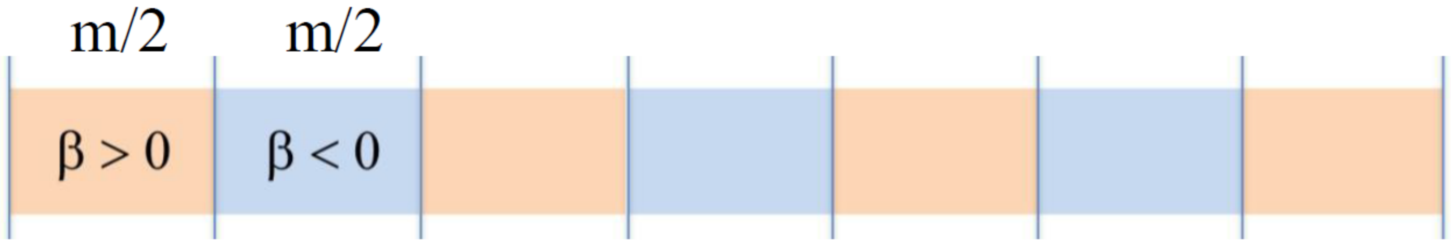
$$k_p=k_s+k_i+\Delta k-\Delta k$$

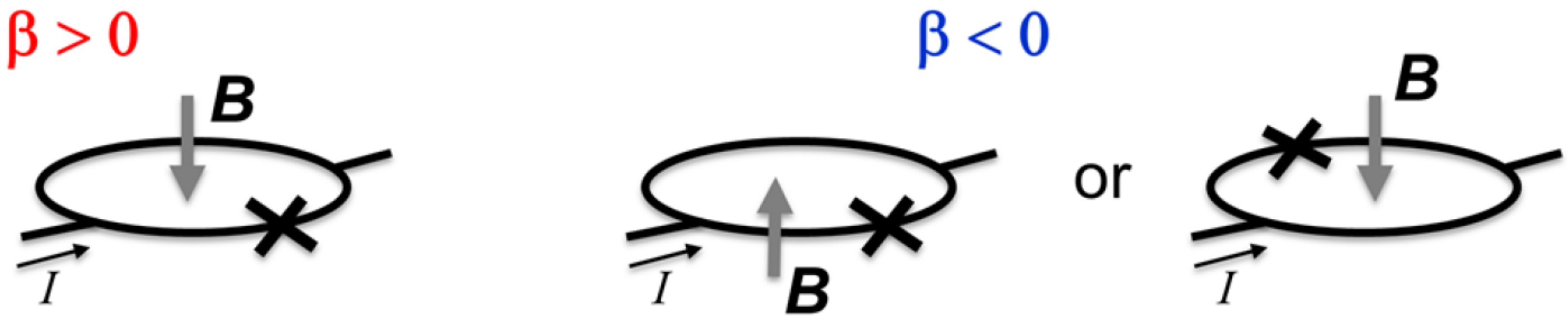
- By resonant phase matching (RPM) [O'Brien 2014]
- Resonators can be either lumped element [Macklin 2015] or distributed resonators [White 2015]





The poled Josephson transmission line





A.B. Zorin, "Quasi-phasematching in a poled Josephson traveling-wave parametric amplifier with three-wave mixing". Appl. Phys. Lett. 118, 222601 (2021).

RPM and QPM – Classical CMEs [1]

$$\frac{dA_i}{dx} = \frac{\beta_L}{4}\left(k_p k_s A_p A_s^* e^{i(k_p-k_s)x} + k_p k_{p+i} A_p A_{p+i}^* e^{i(k_p-k_{p+i})x} + k_{2p} k_{p+s} A_{2p} A_{p+s}^* e^{i(k_{2p}-k_{p+s})x}\right) e^{-ik_i x}$$

$$\frac{dA_s}{dx} = \frac{\beta_L}{4}\left(k_p k_i A_p A_i^* e^{i(k_p-k_i)x} + k_p k_{p+s} A_p A_{p+s}^* e^{i(k_p-k_{p+s})x} + k_{2p} k_{p+i} A_{2p} A_{p+i}^* e^{i(k_{2p}-k_{p+i})x}\right) e^{-ik_s x}$$

$$\frac{dA_p}{dx} = \frac{\beta_L}{4}\left(-k_s k_i A_s A_i^* e^{i(k_s+k_i)x} + k_s k_{p+s} A_s A_{p+s}^* e^{i(k_s-k_{p+s})x} + k_i k_{p+i} A_i A_{p+i}^* e^{i(k_i-k_{p+i})x} + \right.$$
$$\left. + k_{2p} k_p A_{2p} A_p^* e^{i(k_{2p}-k_p)x}\right) e^{-ik_p x}$$

$$\frac{dA_{p+i}}{dx} = \frac{\beta_L}{4}\left(-k_p k_i A_p A_i^* e^{i(k_p+k_i)x} + k_{2p} k_s A_{2p} A_s^* e^{i(k_{2p}-k_s)x}\right) e^{-ik_{p+i} x}$$

$$\frac{dA_{p+s}}{dx} = \frac{\beta_L}{4}\left(-k_p k_s A_p A_s^* e^{i(k_p+k_s)x} + k_{2p} k_i A_{2p} A_i^* e^{i(k_{2p}-k_i)x}\right) e^{-ik_{p+s} x}$$

$$\frac{dA_{2p}}{dx} = \frac{\beta_L}{4}\left(-\frac{k_p^2 A_p^2}{2} e^{i(k_p+k_p)x} - k_{p+i} k_s A_{p+i} A_s^* e^{i(k_{p+i}+k_s)x} - k_{p+s} k_i A_{p+s} A_i^* e^{i(k_{p+s}+k_i)x}\right) e^{-ik_{2p} x}$$

[1] T. Dixon *et. al.*, “Capturing Complex Behavior in Josephson Traveling-Wave Parametric Amplifiers”, Phys. Rev. Applied 14, 034058

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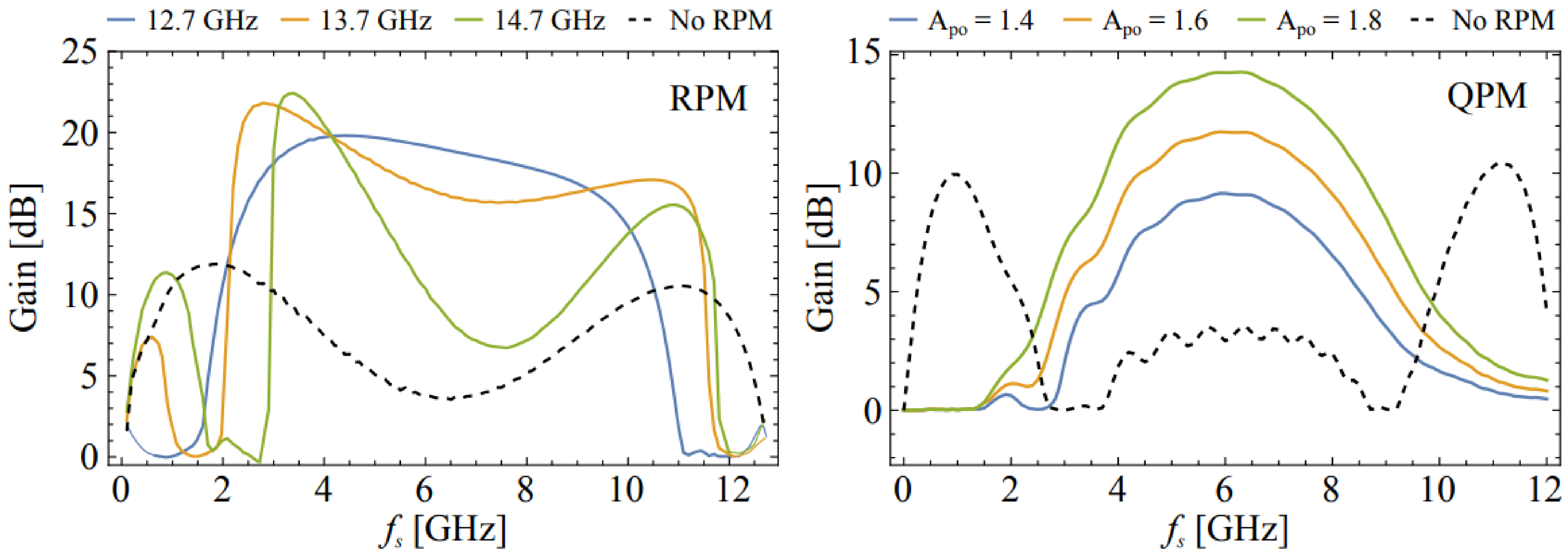
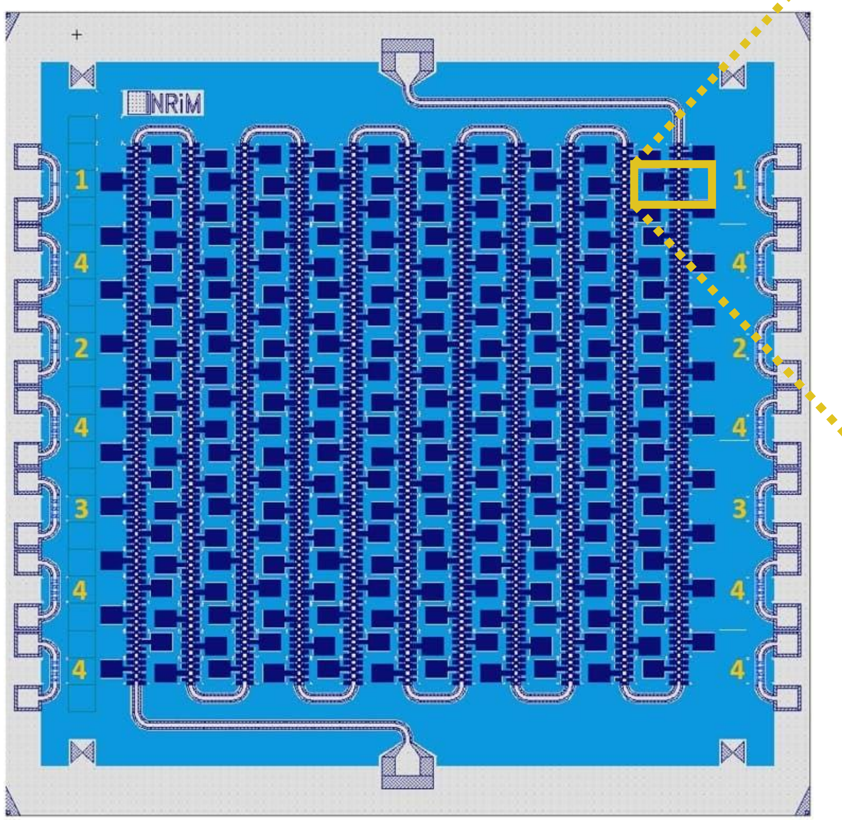
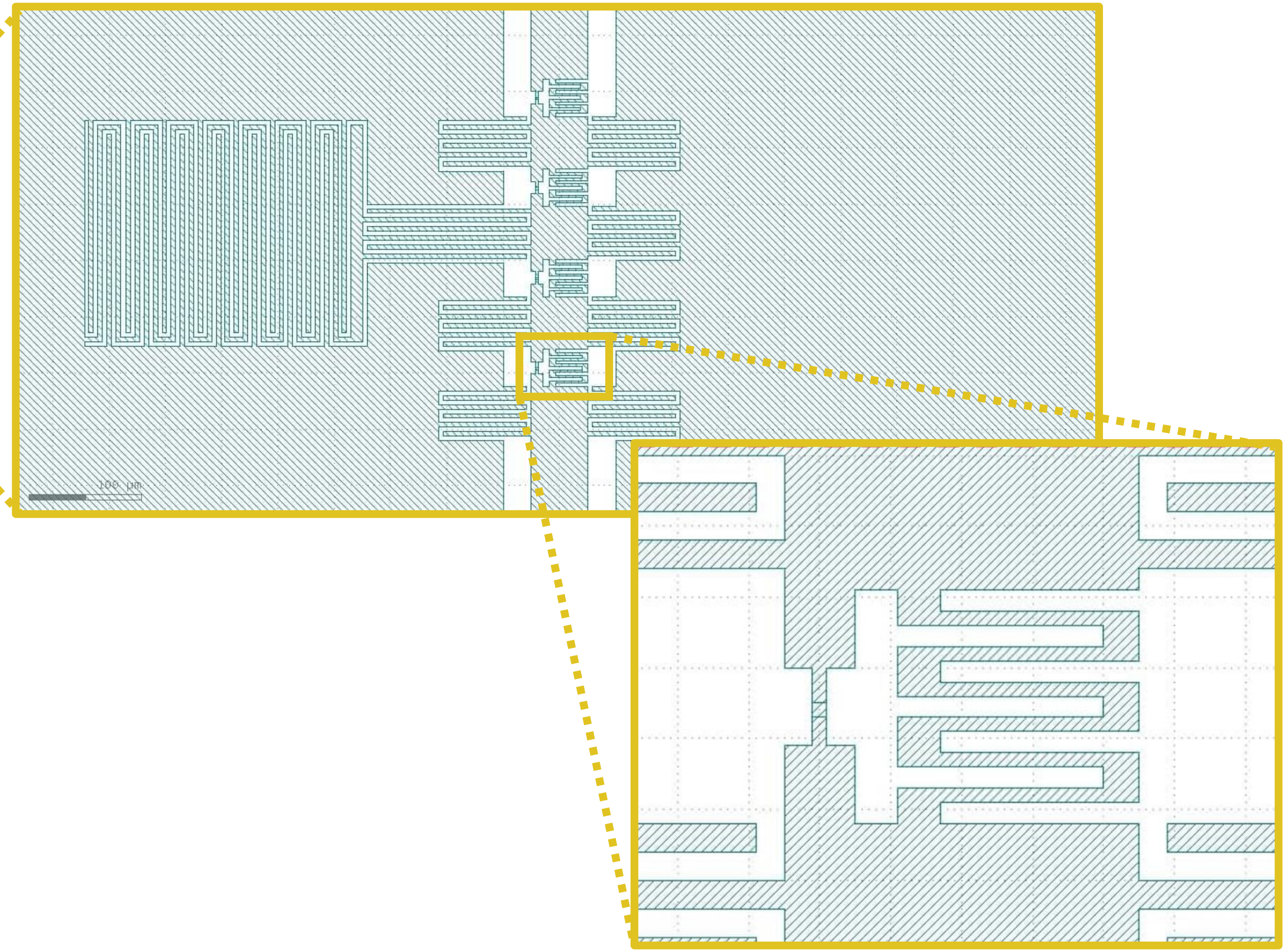
RPM and QPM – Classical CMEs



A. Giachero *et al.*, “Detector Array Readout with Traveling Wave Amplifiers”. **Submitted to *JLTP***.

JTWPA by 2-steps optical lithography

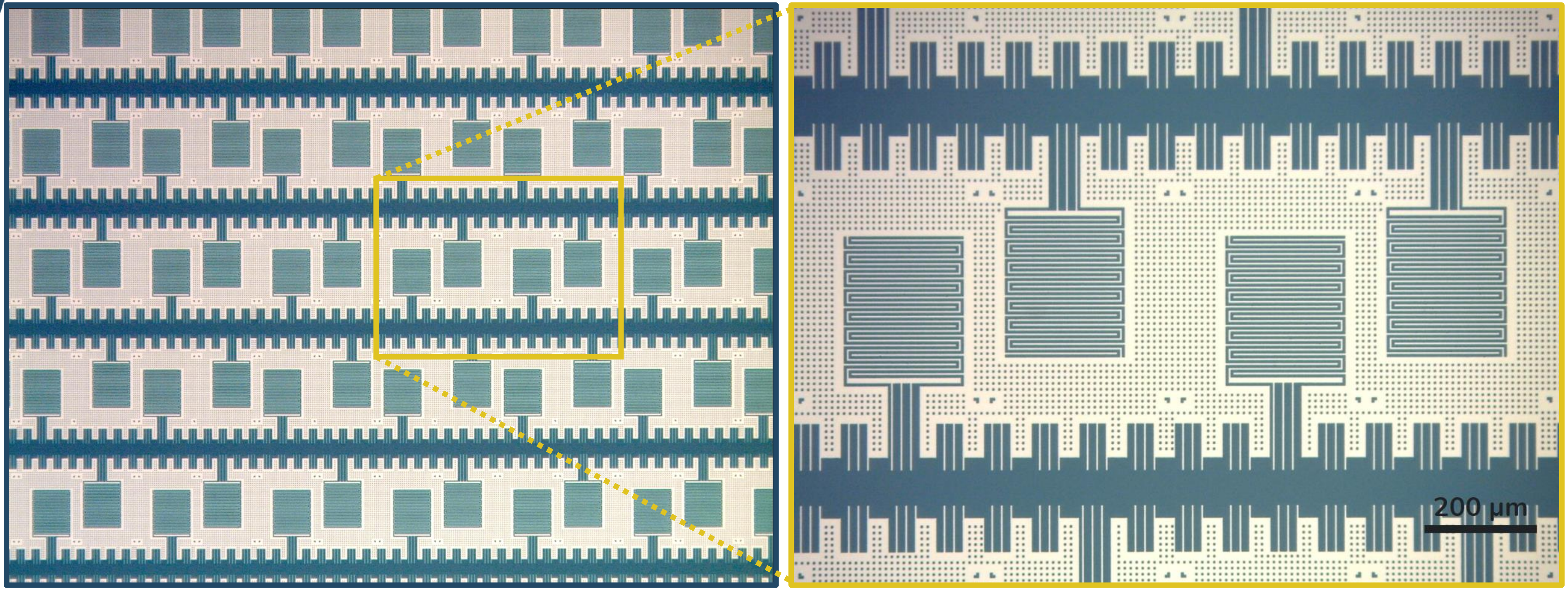




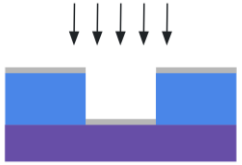
DART WARS

INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

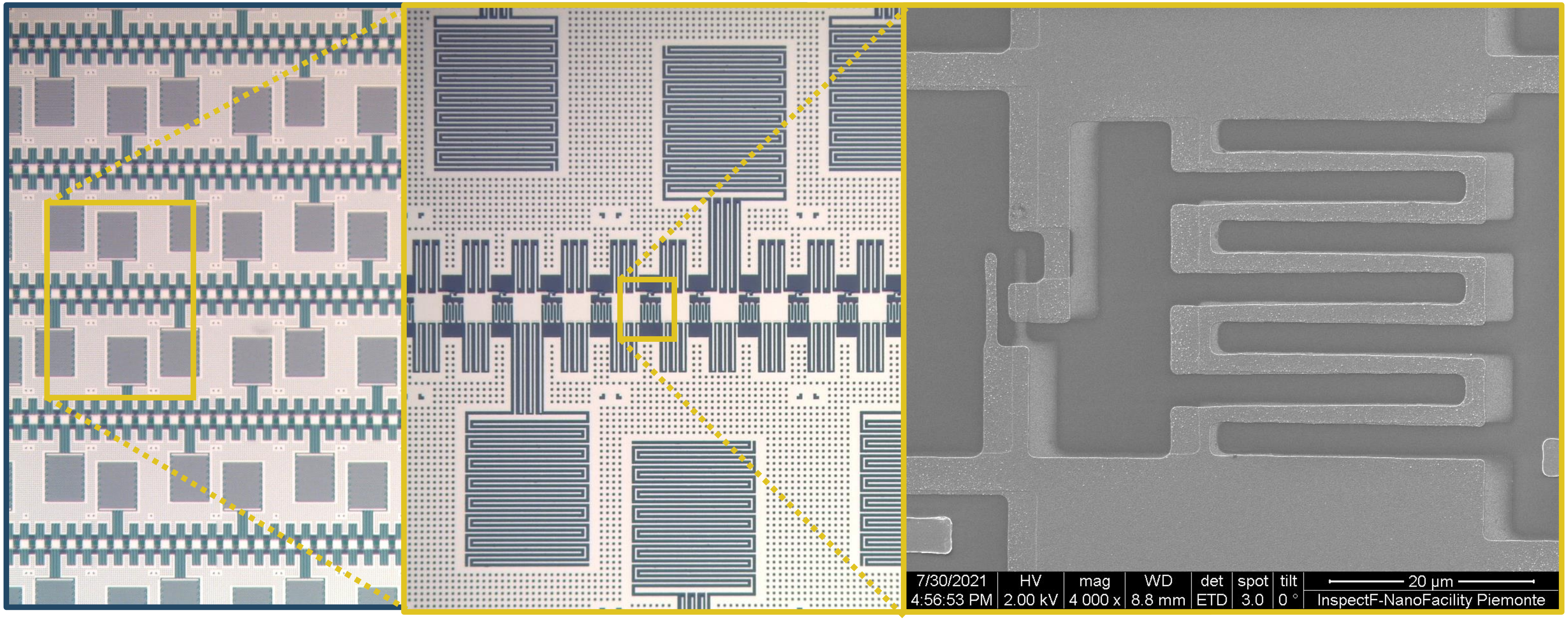
JTWPA by 2-steps optical lithography



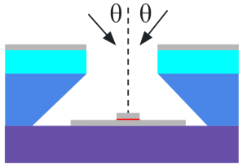
Single angle evaporation



JTWPA by 2-steps optical lithography



Double angle evaporation



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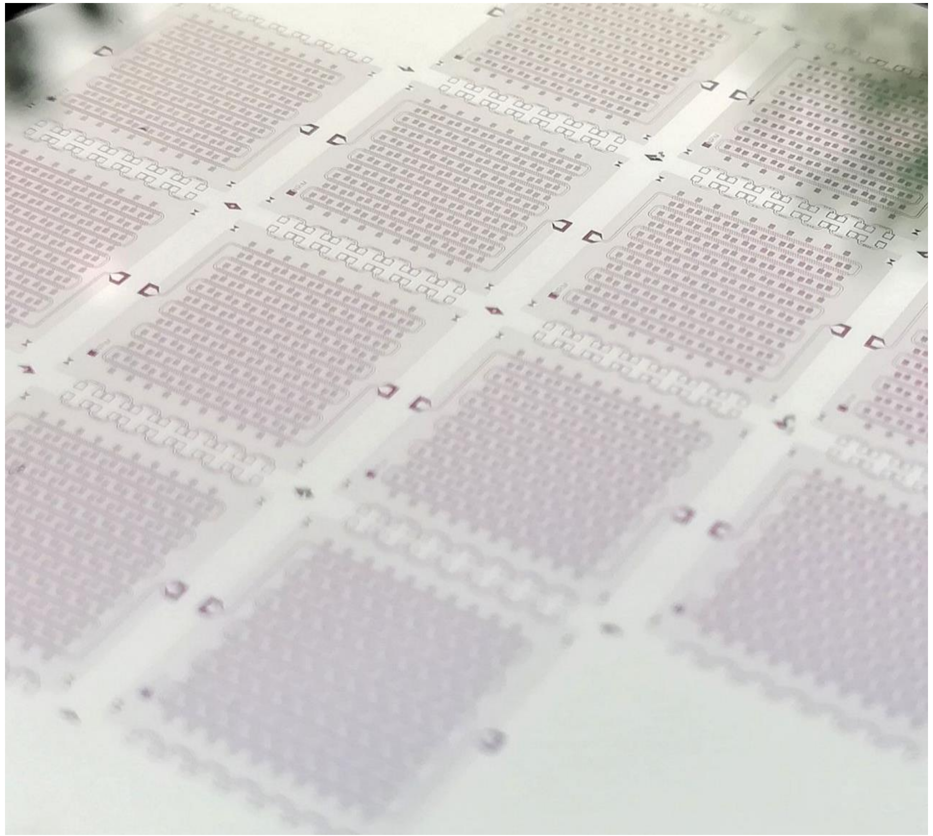
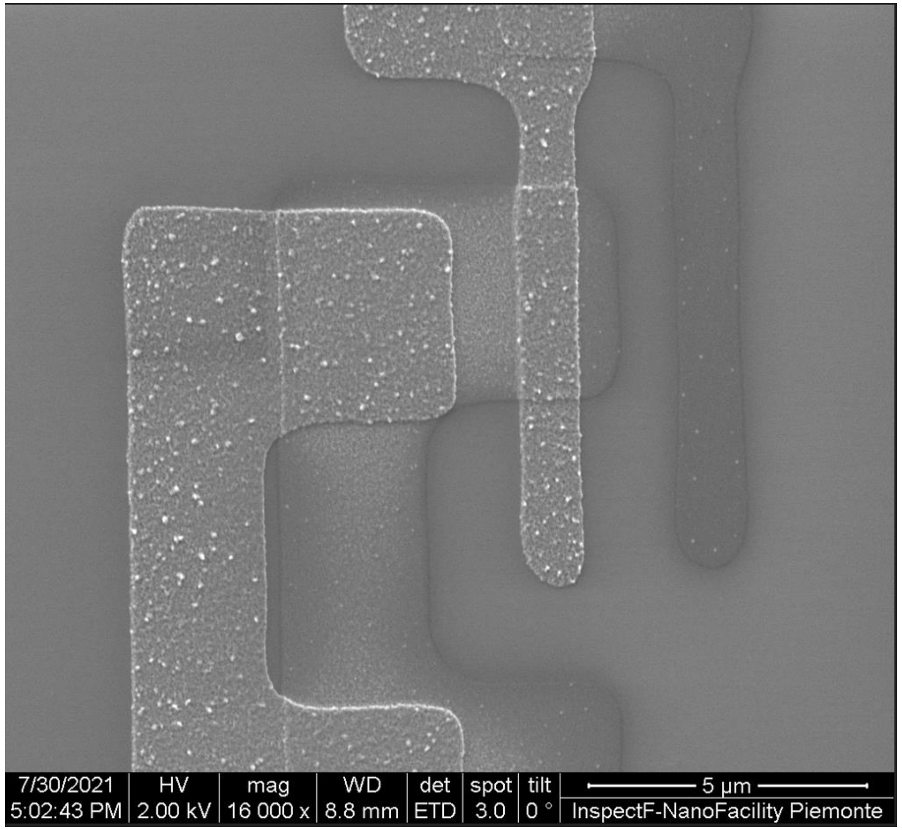
JTWPA Architectures and Generations

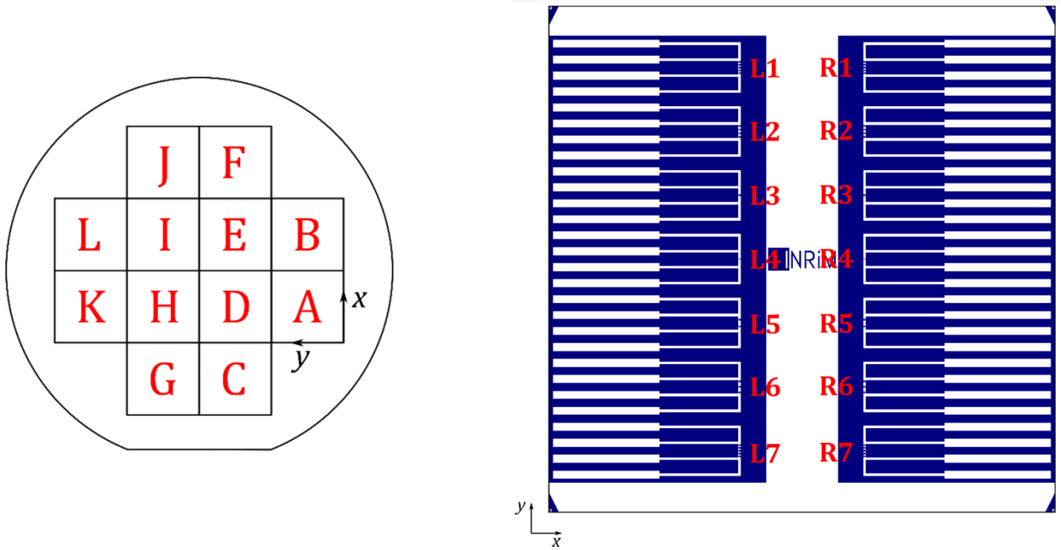


- JTWPA_X
  - **_RPM_DielectricLess_4uA_1**
  - _RPM_200SiO2_2-5uA_1
  - _RPM_100aSiH_2-5uA_1
- JTWPA_C
  - **_RPM_DielectricLess_4uA_1**
  - **_QPM_DielectricLess_2uA_1**
  - _RPM_200SiO2_2-5uA_1
  - _RPM_100aSiH_2-5uA_1
- JTWPA_LS
  - _RPM_200SiO2_2-5uA_1
  - _RPM_100aSiH_2-5uA_1

JJs reproducibility



ID	Device	Main Properties
Substrate 02	SQE_JJs_ Reproducibility_Test_Array_4uA_225fF_1	Dynamic Oxidation/Diced
Substrate 05	SQE_JJs_ Reproducibility_Test_Array_4uA_225fF_2	Static Oxidation/Not Diced
Substrate 06	SQE_JJs_ Reproducibility_Test_Array_4uA_225fF_3	Test Wafer

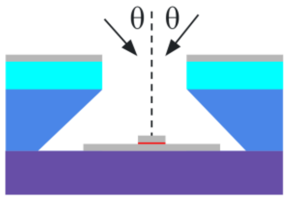


For further information (next talk):

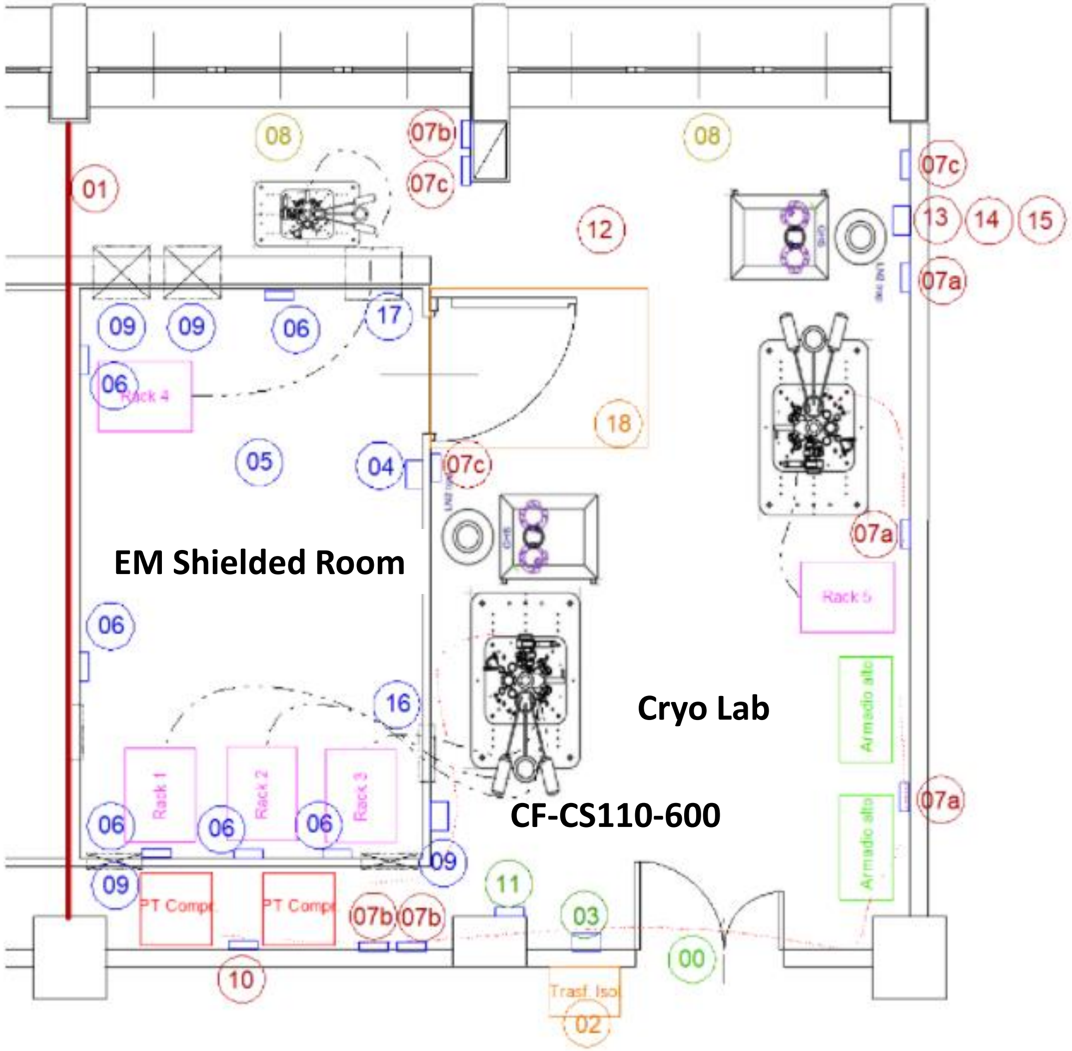
WP4: Measurement at room temperature on the Josephson junctions

Labranca Danilo and Origo Luca

Double angle evaporation



Dry fridge – Expected January 2022



DART WARS

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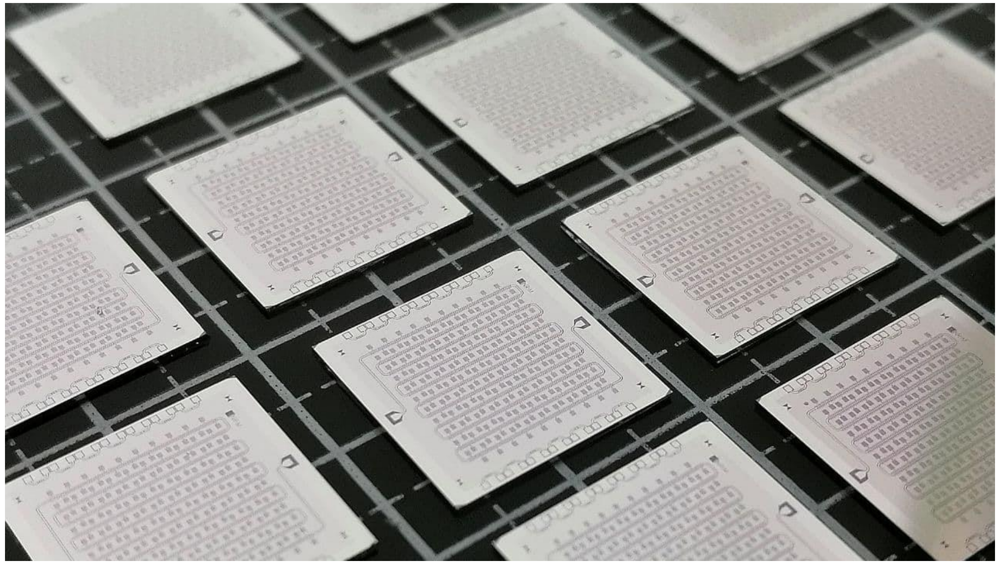
Conclusions



- JJs still to be refined
  - New RT measurement run in December
  - Then -> JJ test array ready to be delivered with proper I_c (beginning of January)
- Circuit parameters to be fine-tuned accordingly
  - SONNET simulations (in collaboration with INFN-UniMiB)
- JTWPA_X and JTWPA_C
  - New fabrication runs by the end of January 2022
- Chip Packaging
  - Flexible packaging still under development and charachterization