The physics of flocking: building a model from data

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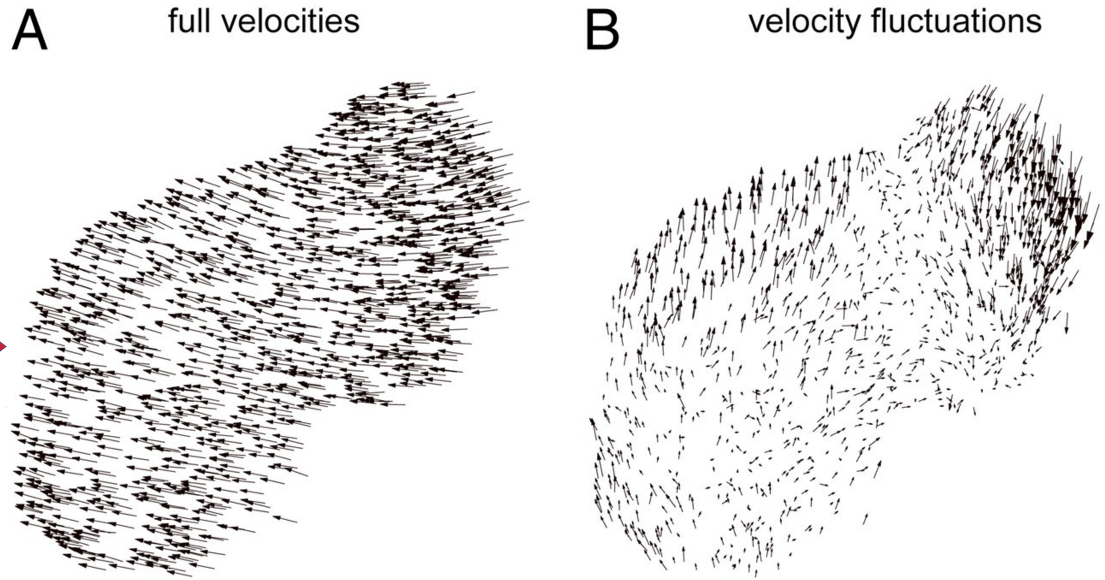
Our system

Tracking



Basic experimental results

Polarization

$$\phi = \frac{1}{N} \left| \sum_{i=1}^{N} \frac{\vec{v}_i}{|\vec{v}_i|} \right|$$

$$\langle \phi \rangle = 0.96$$

Single particle speed

$$s_i = |\vec{v}_i|$$

Average speed

$$s = \frac{1}{N} \sum_{i=1}^{N} s_i$$

$$\langle s \rangle \simeq 11.9 \text{ m/s}$$

$$\sigma_s \simeq 2.3 \text{ m/s}$$

Correlation functions

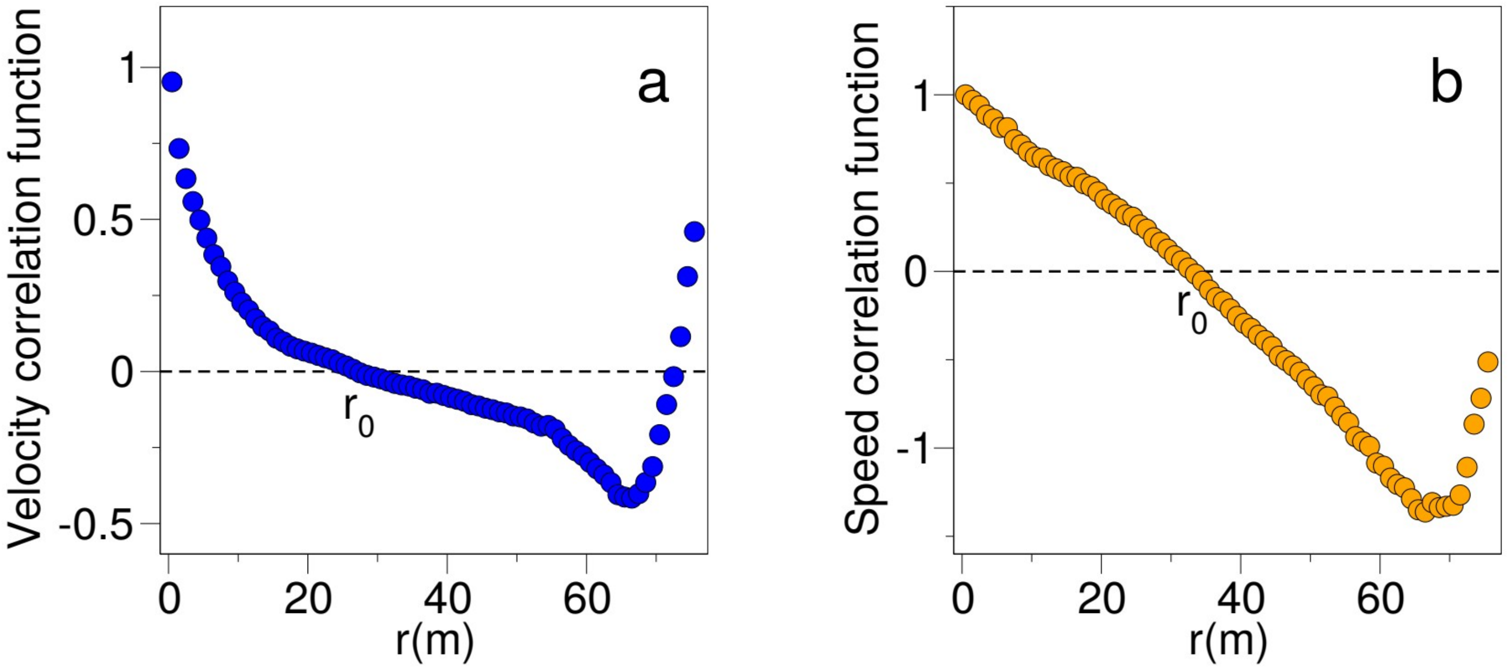
$$C(r) = \frac{\sum_{i,j} \delta\vec{v}_i \cdot \delta\vec{v}_j \;\delta(r - r_{ij})}{\sum_{i,j} \delta(r - r_{ij})}$$

$$\delta\vec{v}_i \equiv \vec{v}_i - \frac{1}{N}\sum_k \vec{v}_k$$

$$C_s(r) = \frac{\sum_{i,j} \delta s_i \delta s_j \delta(r - r_{ij})}{\sum_{i,j} \delta(r - r_{ij})}$$

$$\delta s_i = s_i - \frac{1}{N}\sum_k s_k$$

Experiments – correlation functions



Correlation lengths

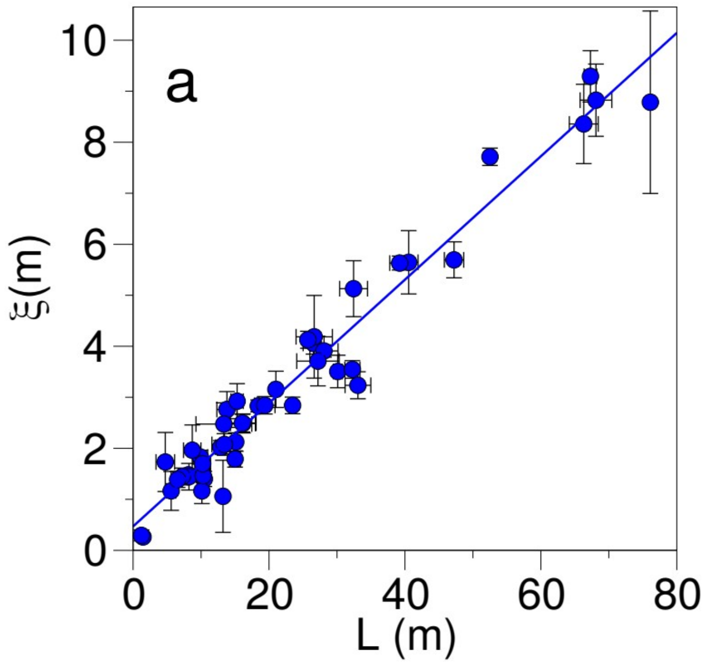
$$\xi = \frac{\int_0^{r_0} \mathrm{d}r \; r \; C(r)}{\int_0^{r_0} \mathrm{d}r \; C(r)}$$

Velocity correlation length

$$\xi_s = \frac{\int_0^{r_0} \mathrm{d}r \; r \; C_s(r)}{\int_0^{r_0} \mathrm{d}r \; C_s(r)}$$

Speed correlation length

Scale-free correlations – full velocity

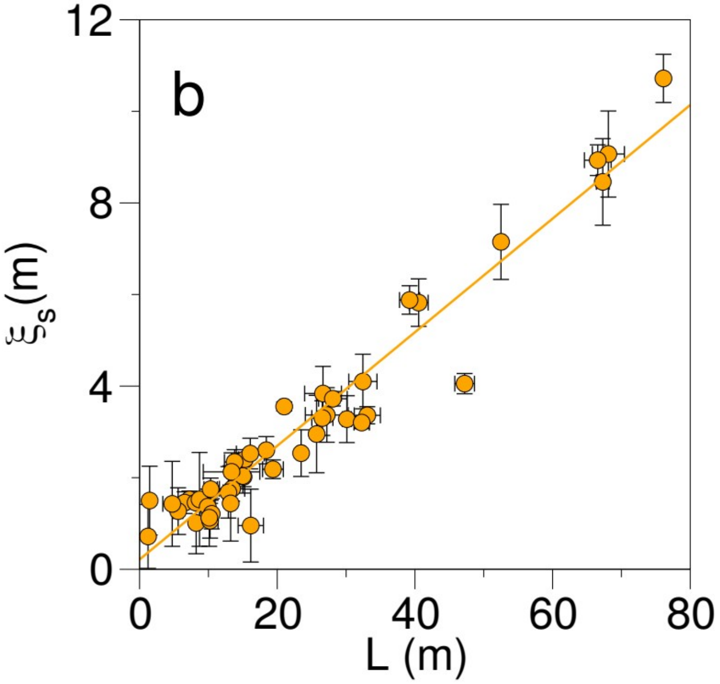


$$\langle \phi \rangle \simeq 1$$

Spontaneous symmetry breaking of a continuous symmetry

Goldstone modes imply scale-free correlations for orientation's fluctuations

Scale-free correlations - speed



?

Stochastic equations and Hamiltonian

$$\begin{cases} \dfrac{\mathrm{d}\vec{r}_i}{\mathrm{d}t} = \vec{v}_i \\ \dfrac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = -\dfrac{\partial \mathcal{H}}{\partial \vec{v}_i} + \vec{\xi}_i \end{cases}$$

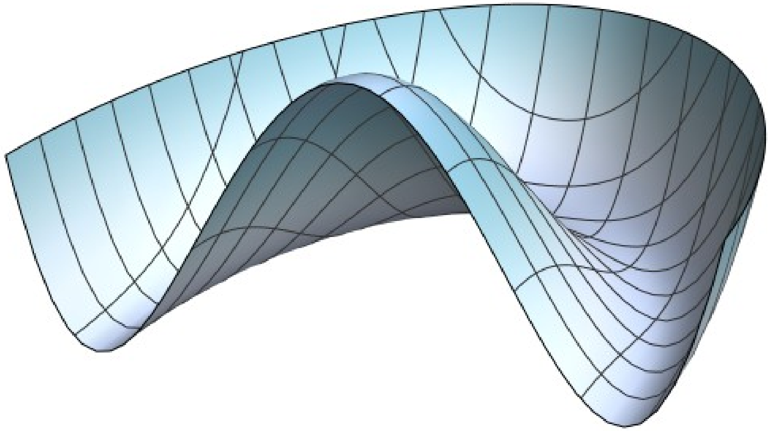
$$\left\langle \xi_i^{(\alpha)}(t) \right\rangle = 0$$

$$\left\langle \xi_i^{(\alpha)}(t)\xi_j^{(\beta)}(t') \right\rangle = \frac{2T}{v_0^2}\delta_{ij}\delta(t-t')\delta_{\alpha\beta}$$

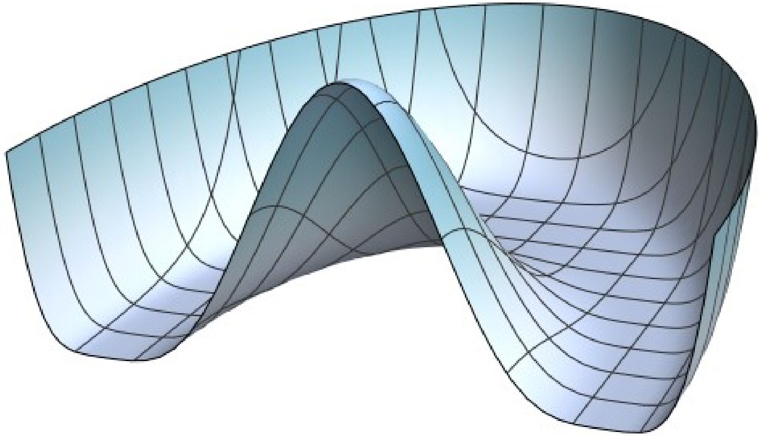
$$\mathcal{H}(\{\vec{v}_i\}) = \frac{J}{2}\sum_{i,j} n_{ij}\left(t\right)\left(\vec{v}_i - \vec{v}_j\right)^2 + \sum_i V\left(\vec{v}_i\right)$$

The marginal potential

$$V(\vec{v}_i) = \lambda(v_0^2 - v_i^2)^4$$

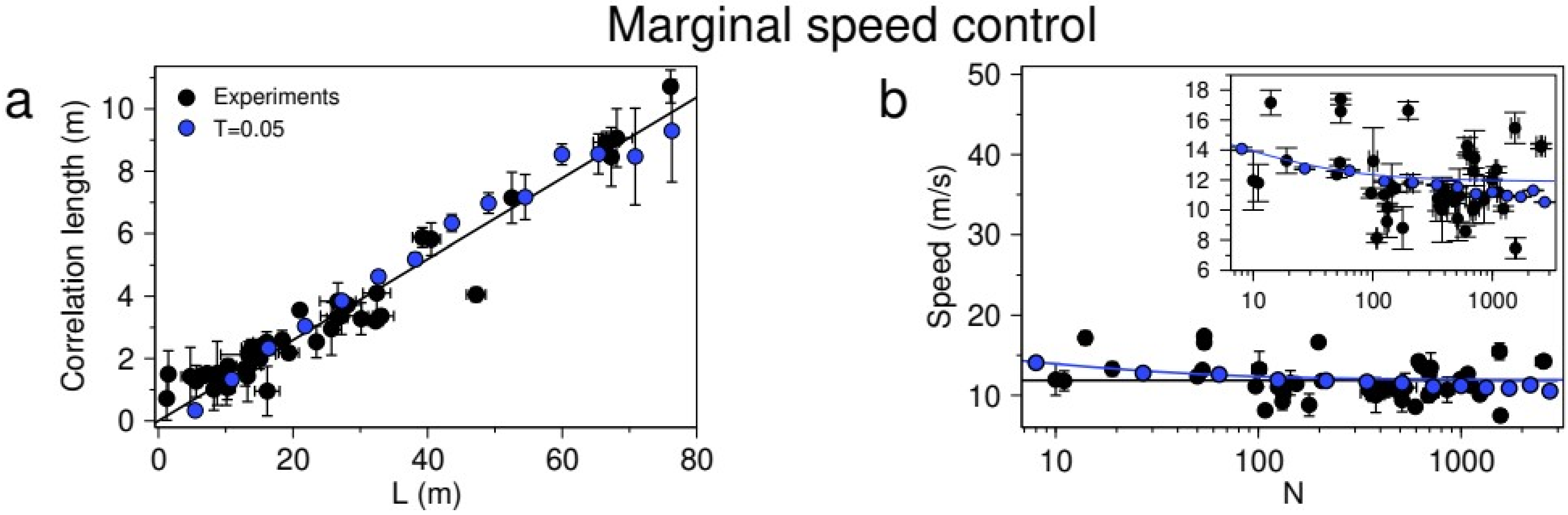


Standard quadratic potential



Marginal potential

Data VS model



The marginal model – Mean field and RG analysis

- Mean Field approximation
- Renormalization Group analysis

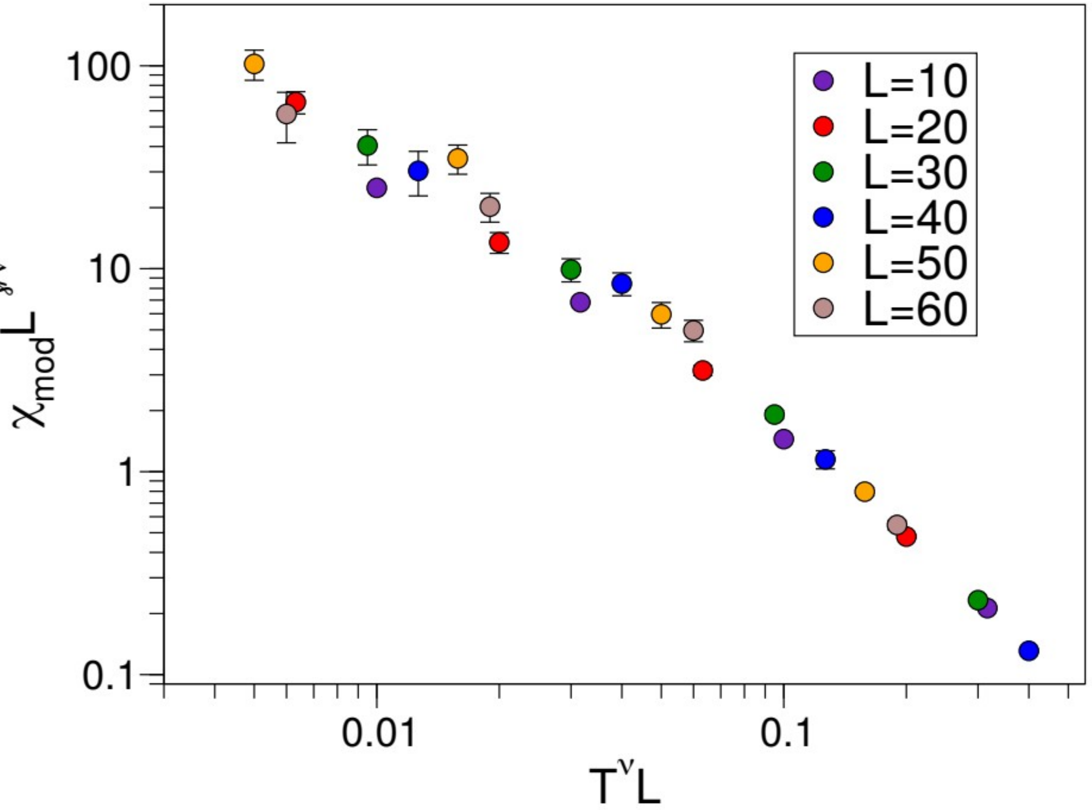
$$\chi_{\mathrm{mod}} \sim T^{-\gamma}$$

$$\xi_{\mathrm{mod}} \sim T^{-\nu}$$

Critical exponents

$$\gamma = 1 \qquad \nu = 0.5$$

The marginal model – on-lattice simulations



Conclusions & outlook

- We reproduced data!

- A new theory in the field of critical phenomena

- We need a more accurate description of the dynamics

- Can we generalize our model?

- Are there any other systems with scale-free speed correlations?

References

- Image
  https://www.isc.cnr.it/groups/cobbs/
- Tracking algorithm

A. Attanasi *et al*., "GReTA-A Novel Global and Recursive Tracking Algorithm in Three Dimensions" in *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1 Dec. 2015.

- Experimental findings

A. Cavagna *et al.*, “Scale-free correlations in starling flocks,” Proc Natl Acad Sci USA, vol. 107, no. 26, pp. 11 865–11 870, Jun 2010. [Online]. Available: https://doi.org/10.1073/pnas.1005766107

References

- Marginal model

A. Cavagna *et al.*, “Low-temperature marginal ferromagnetism explains anomalous scale-free correlations in natural flocks,” Comptes Rendus Physique, vol. 20, pp. 319–328, May-Jun 2019. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1631070519300374

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