

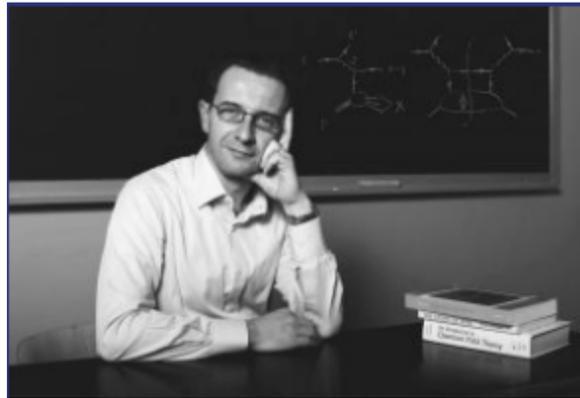
PROGRESS IN THE EXTRACTION OF UNPOLARIZED TMDS FROM GLOBAL DATA SETS

Matteo Cerutti

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RESULTS OBTAINED WITH CONTRIBUTIONS FROM

Alessandro Bacchetta



Marco Radici



Andrea Signori



Valerio Bertone



Chiara Bissolotti



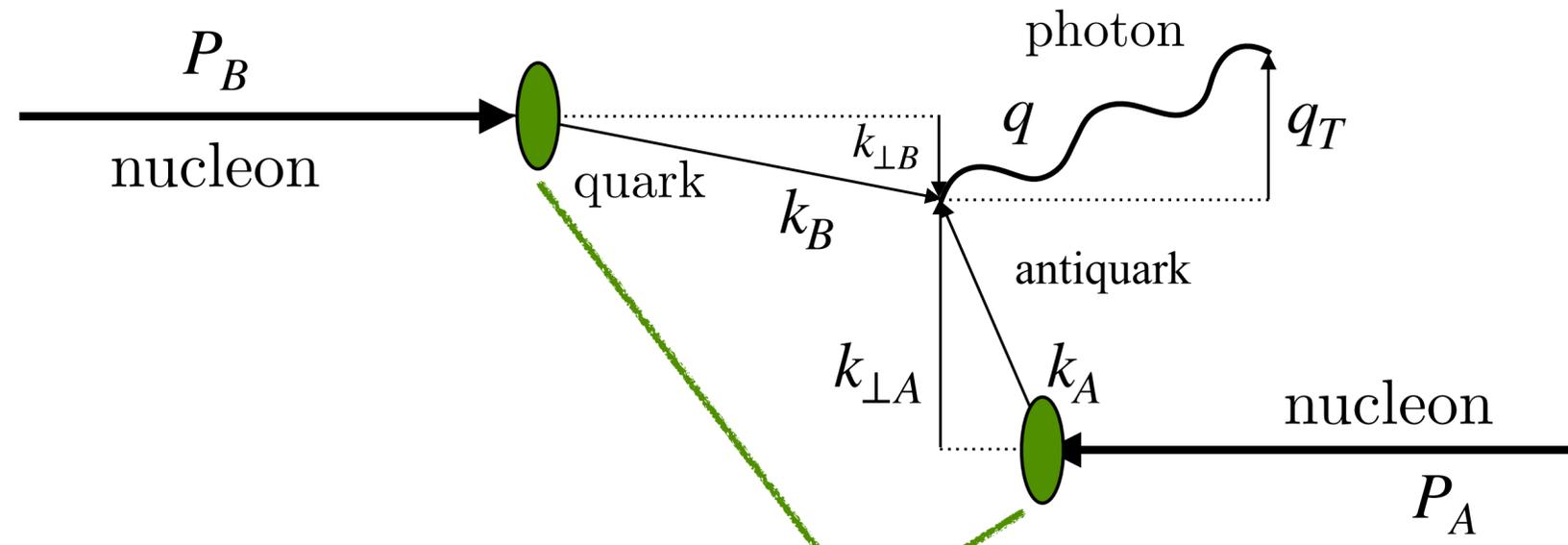
Giuseppe Bozzi



Fulvio Piacenza



TMDS IN DRELL-YAN PROCESSES



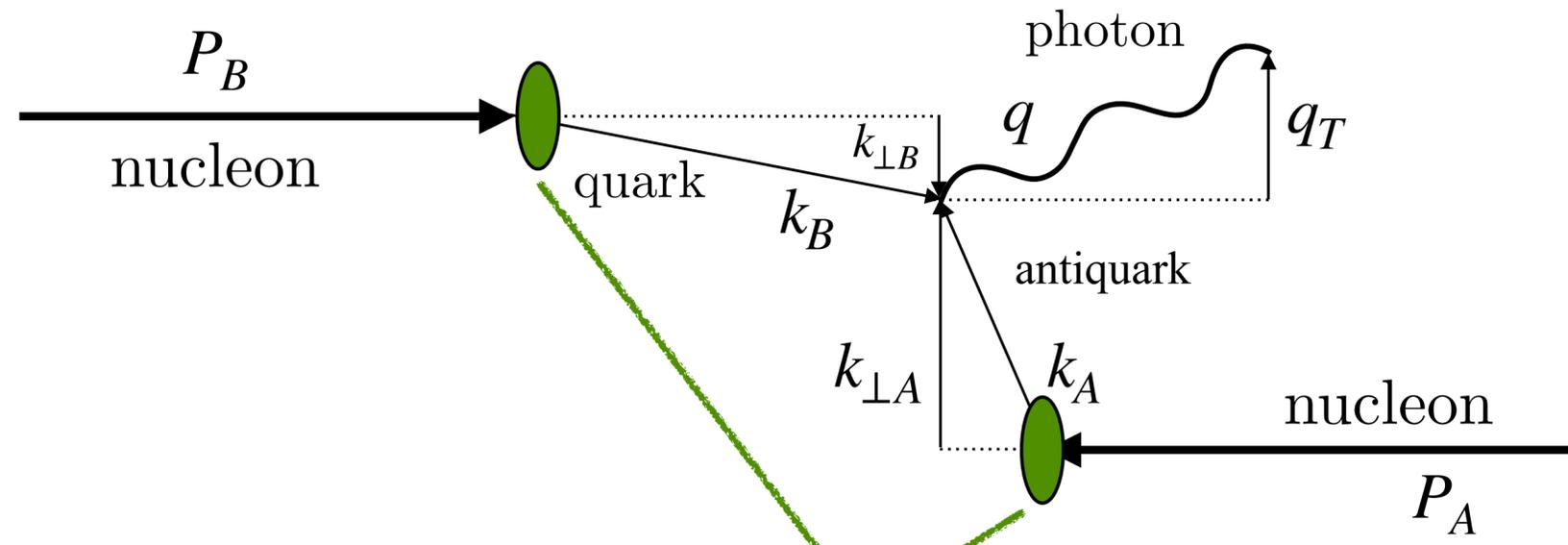
$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp A} d^2 \mathbf{k}_{\perp B} f_1^a(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

W term

$$+ Y_{UU}^1(Q^2, \mathbf{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

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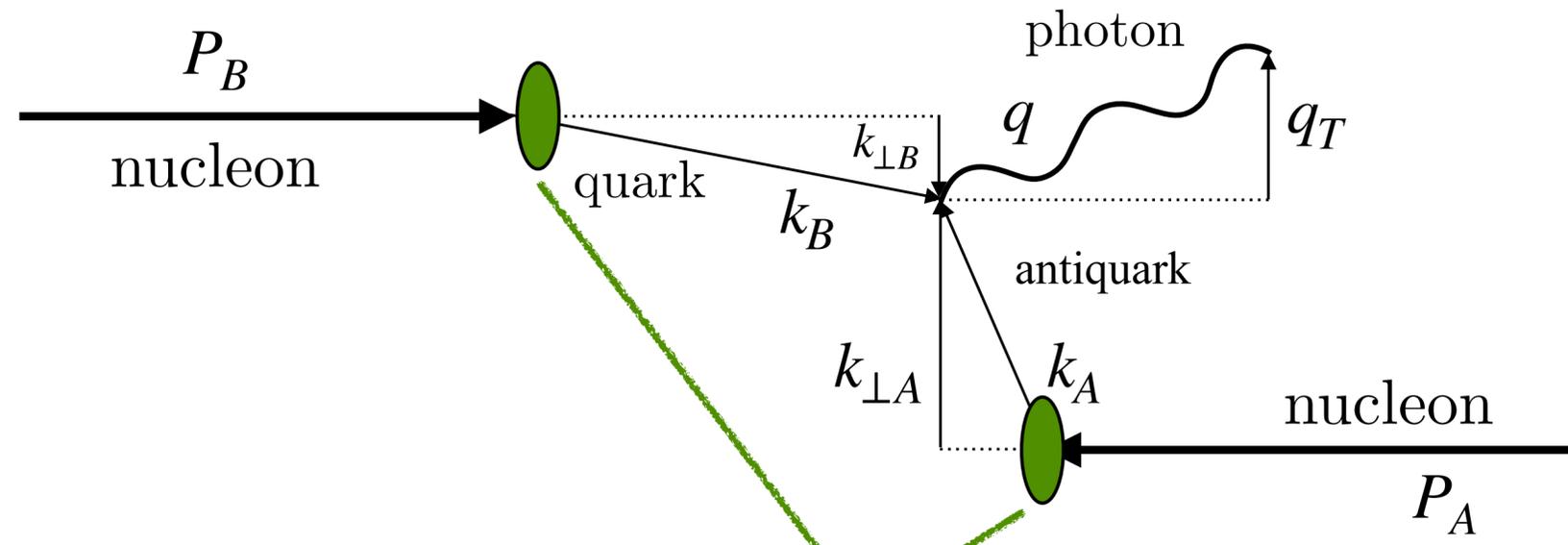
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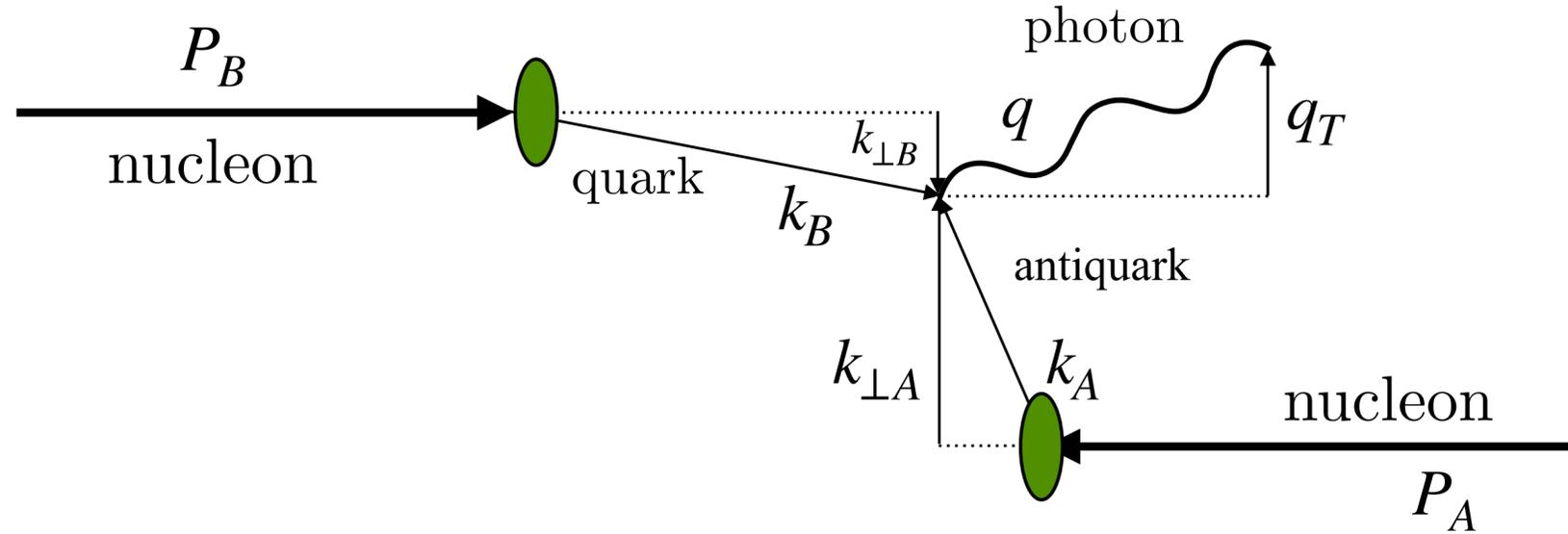
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- Y term has been excluded in the Pavia analyses

TMDS IN DRELL-YAN PROCESSES

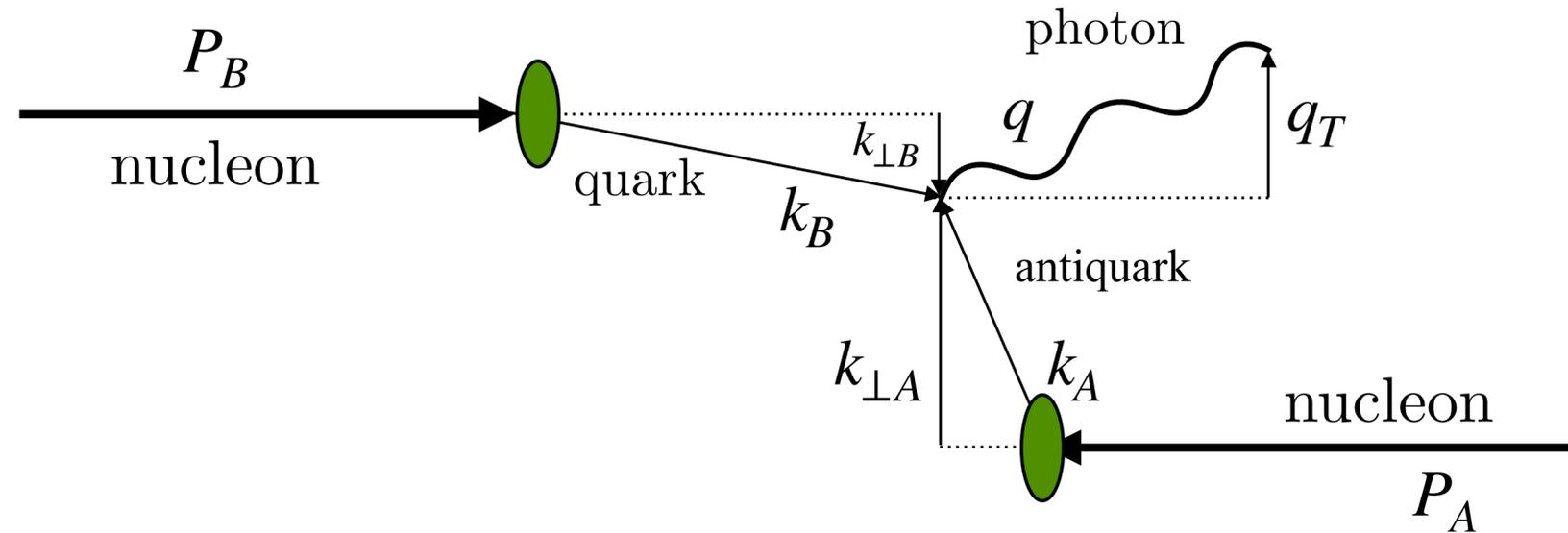


$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

TMDS IN DRELL-YAN PROCESSES



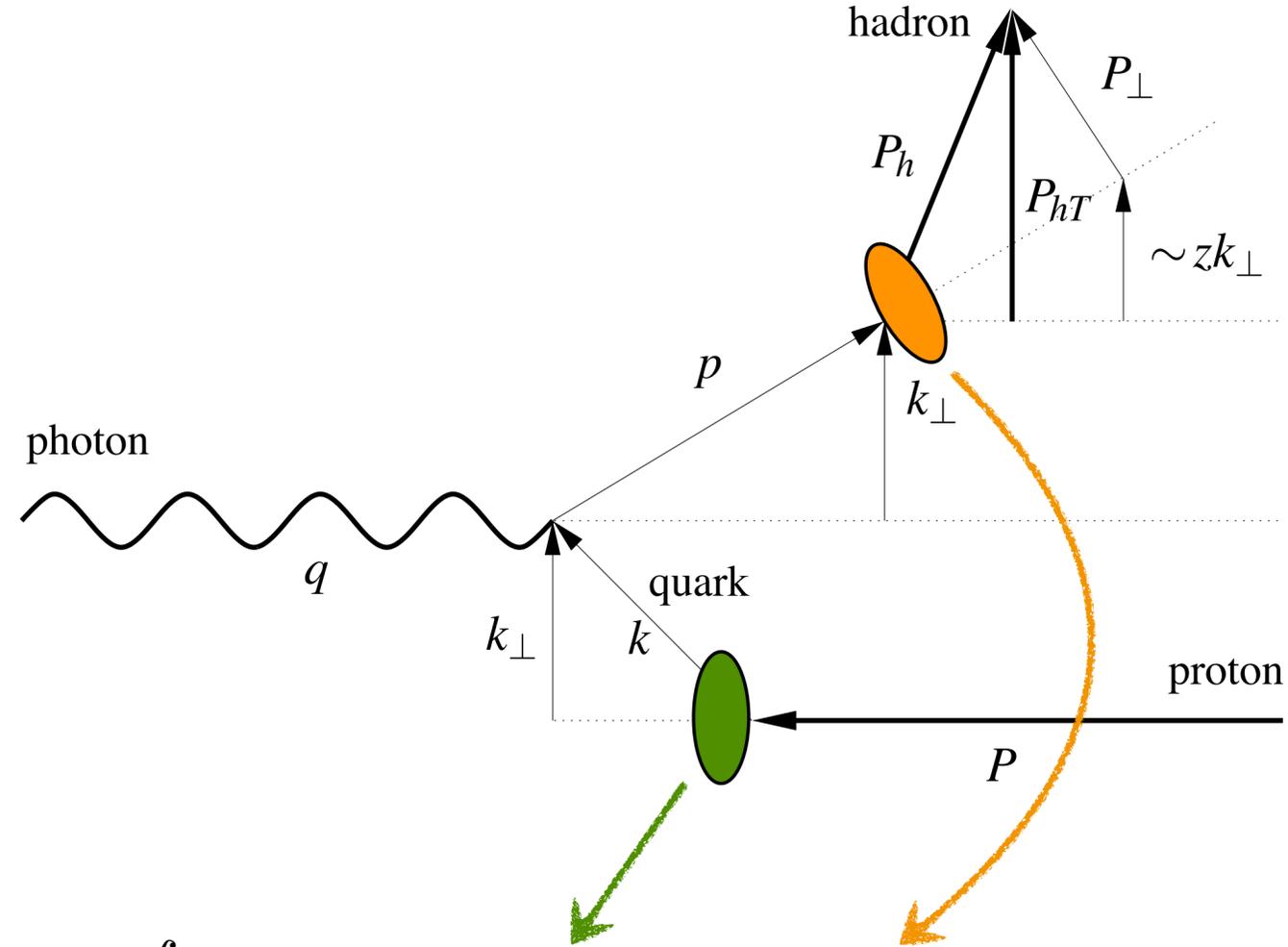
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- At small q_T the dominant part is given by TMDs
- Fourier-transformed space to avoid convolutions
- TMDs formally depend on two scales, but we set them equal.

TMDS IN SEMI-INCLUSIVE DIS PROCESS



$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2\mathbf{k}_\perp d^2\mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z\mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

$$= x \sum_a \mathcal{H}_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_\perp^2; \mu^2)$$

TMD STRUCTURE

$$\hat{f}_1^q(x, b_T; \mu^2) = \int d^2 \mathbf{k}_\perp e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, \mathbf{k}_\perp^2; \mu^2)$$

see, e.g.,
Collins, "Foundations of Perturbative QCD" (11)

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collinear PDF

nonperturbative part of evolution

nonperturbative part of TMD

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PERTURBATIVE ORDER OF EACH INGREDIENT

Orders in powers of α_S

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Accuracy	Hard factor and matching coefficient	Ingredients in perturbative Sudakov form factor		PDF and α_S evol.
	H and C	K and γ_F	γ_K	
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO

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Collinear fragmentation functions not available beyond NLO!!

RECENT GLOBAL FITS OF UNPOLARIZED TMD DATA

	Framework	HERMES	COMPASS	DY	Z production	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2017 arXiv:1706.01473	NNLL'	✗	✗	✓	✓	309	1.23
BSV 2019 arXiv:1902.08474	NNLL'	✗	✗	✓	✓	457	1.17
SV 2019 arXiv:1912.06532	N ³ LL ⁻	✓	✓	✓	✓	1039	1.06
Pavia 2019 arXiv:1912.07550	N ³ LL	✗	✗	✓	✓	353	1.02

OUR WORK IN THE LAST TWO YEARS

New Global Fit

Simultaneously extraction of unpolarized TMD PDFs and FFs

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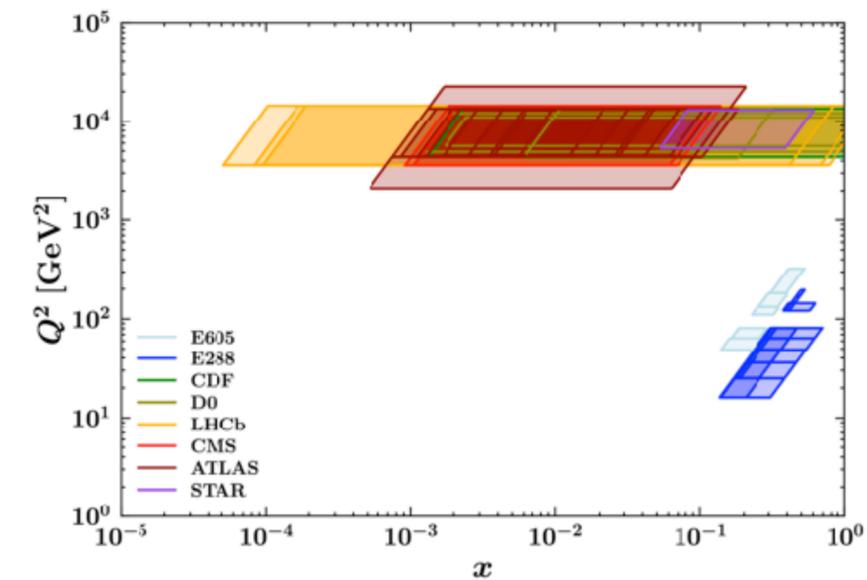
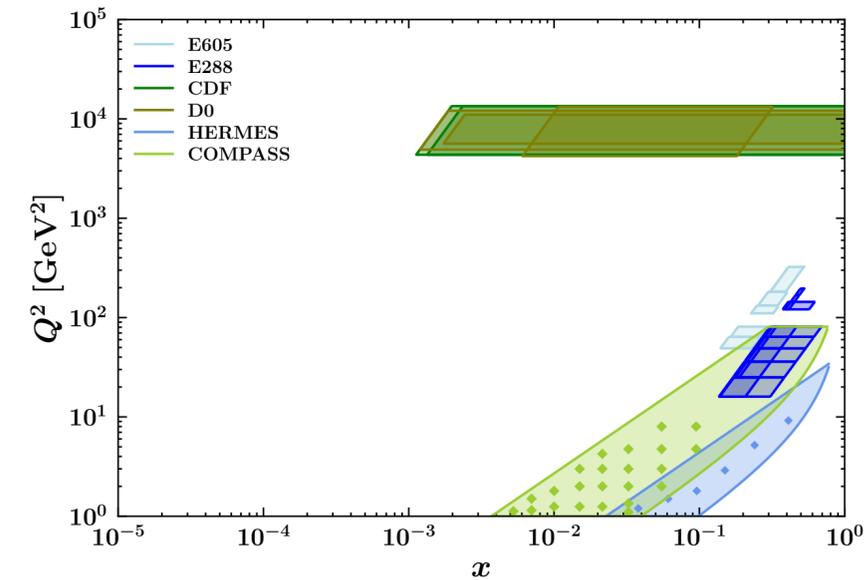
- SIDIS + Drell Yan

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- Integrated variables

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Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

<https://github.com/vbertone/NangaParbat/releases>

For the last development branch you can clone the master code:

```
git clone git@github.com:vbertone/NangaParbat.git
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If you instead want to download a specific tag:

<https://github.com/MapCollaboration>

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Simultaneously extraction of unpolarized TMD PDFs and FFs

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- Up to N^2LL/N^3LL



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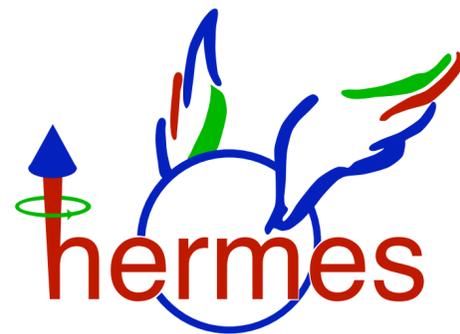
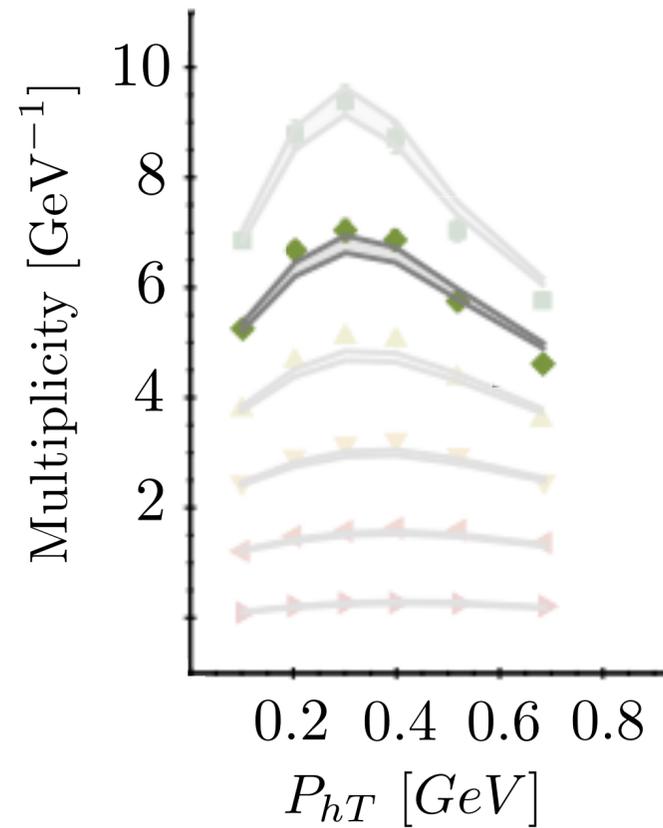
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RESULTS AT NLL: SIDIS (MULTIPLICITIES)

What we expected

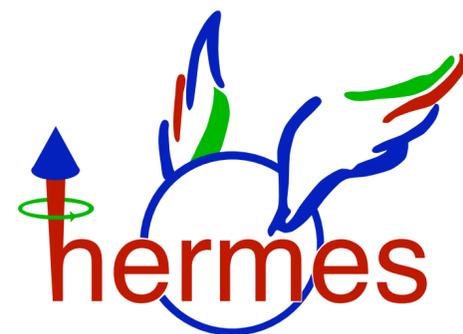
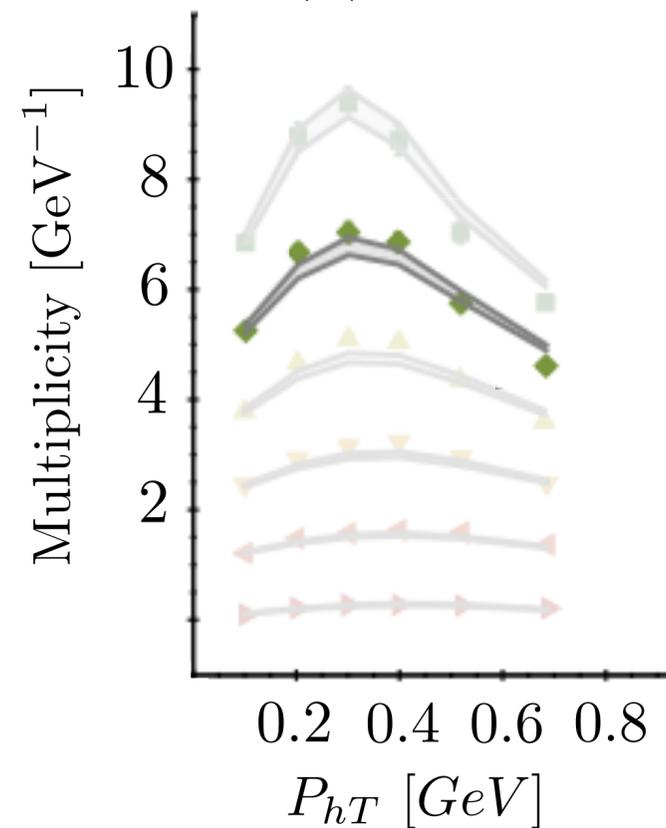
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$$\langle x \rangle = 0.15$$



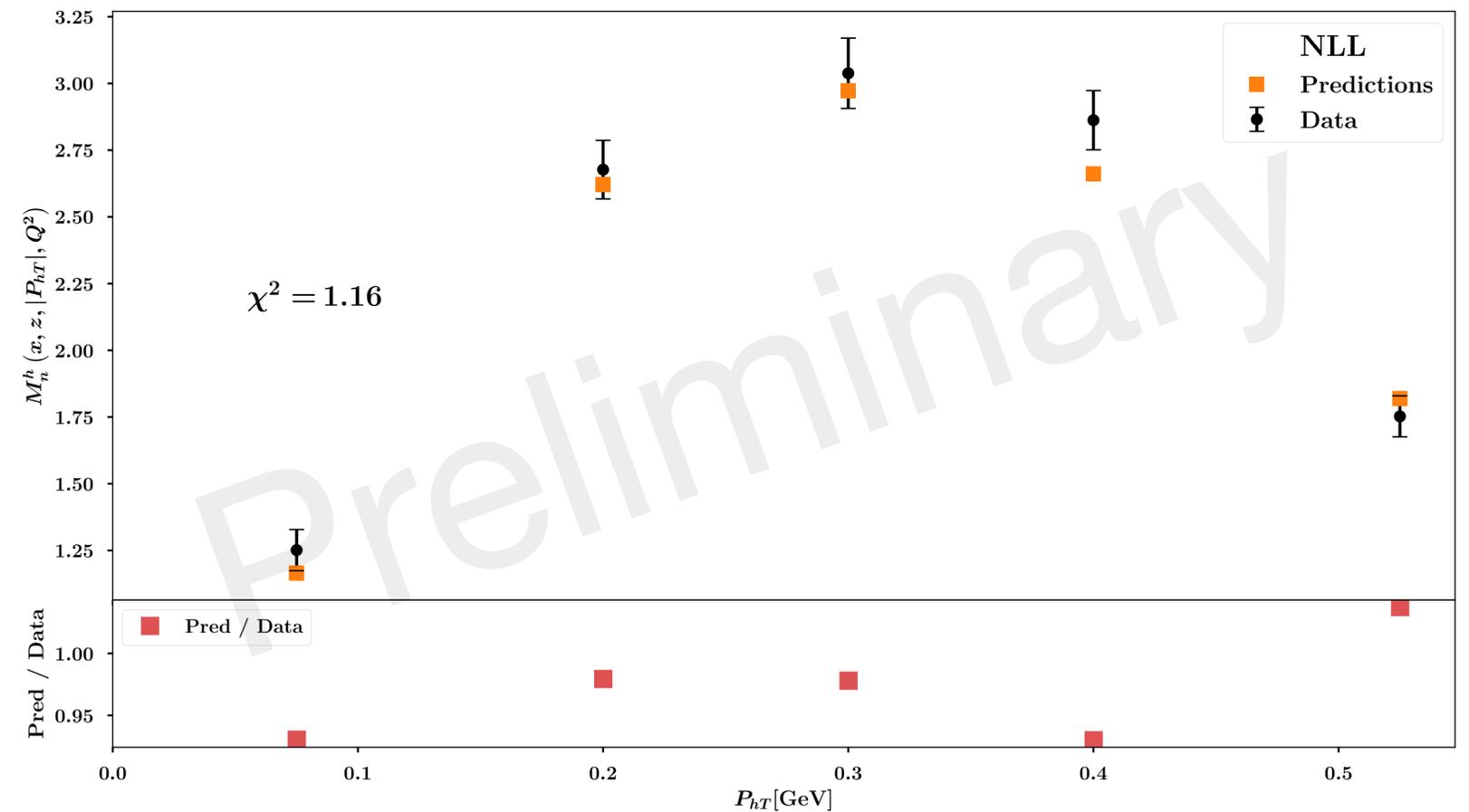
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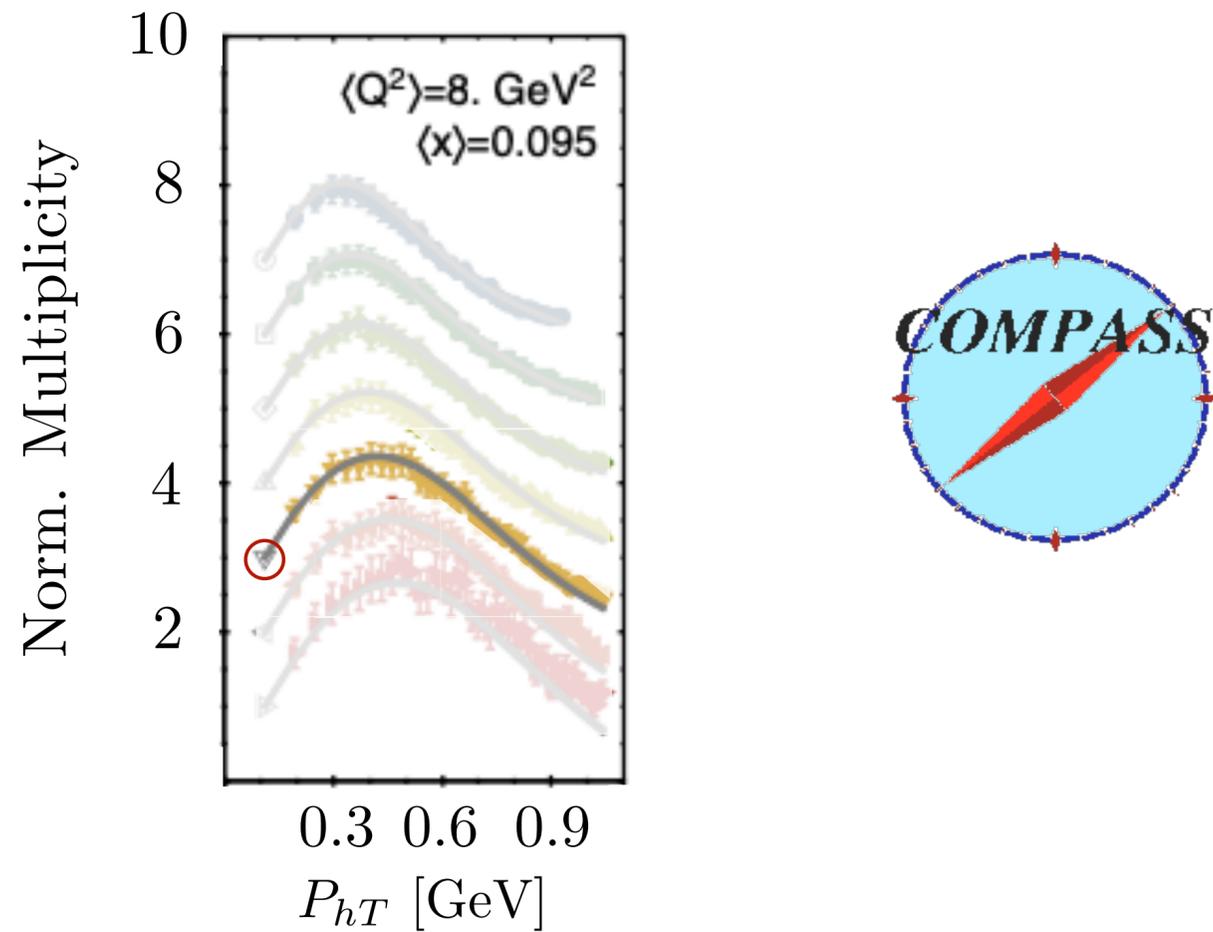


What we found

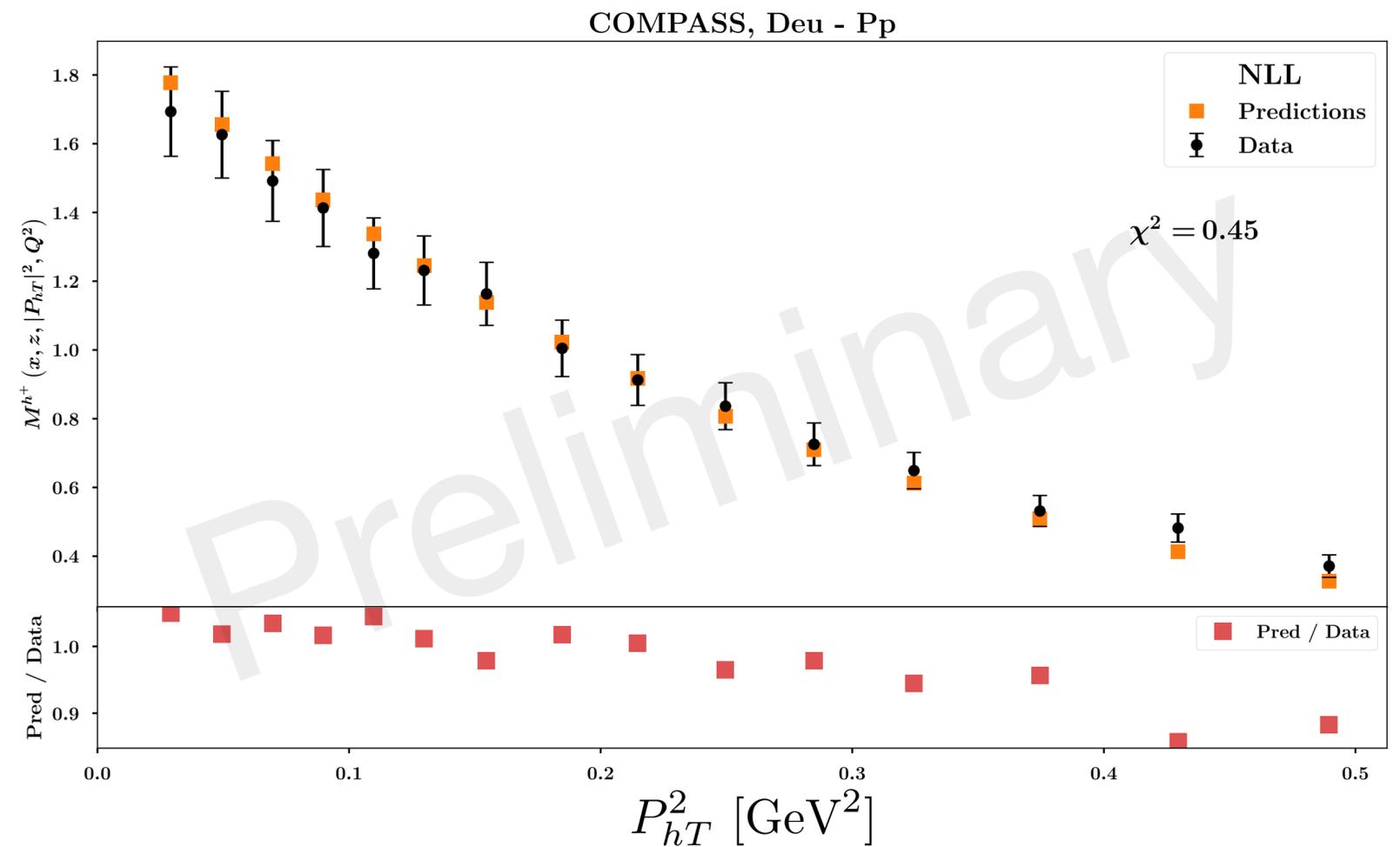


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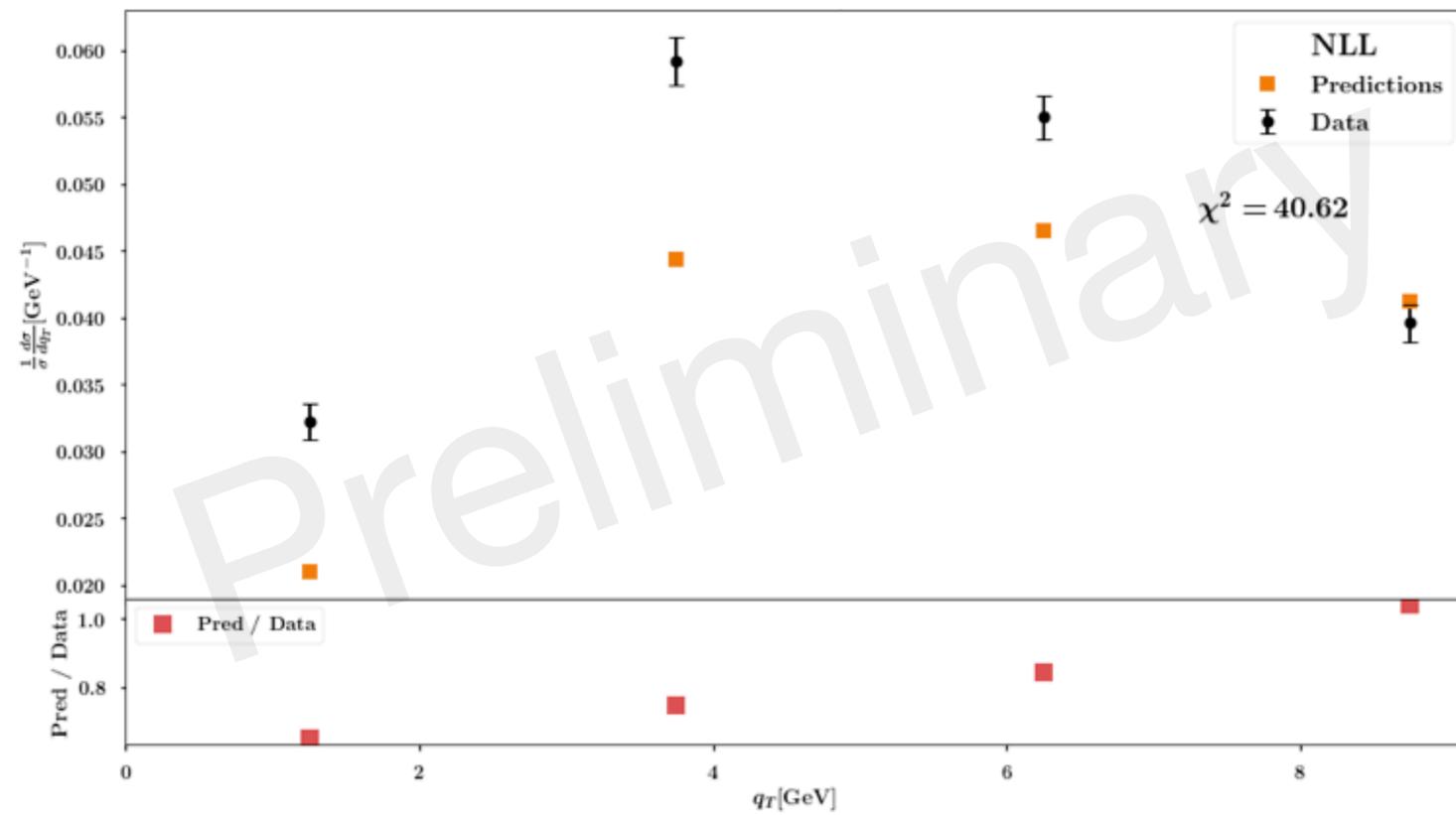
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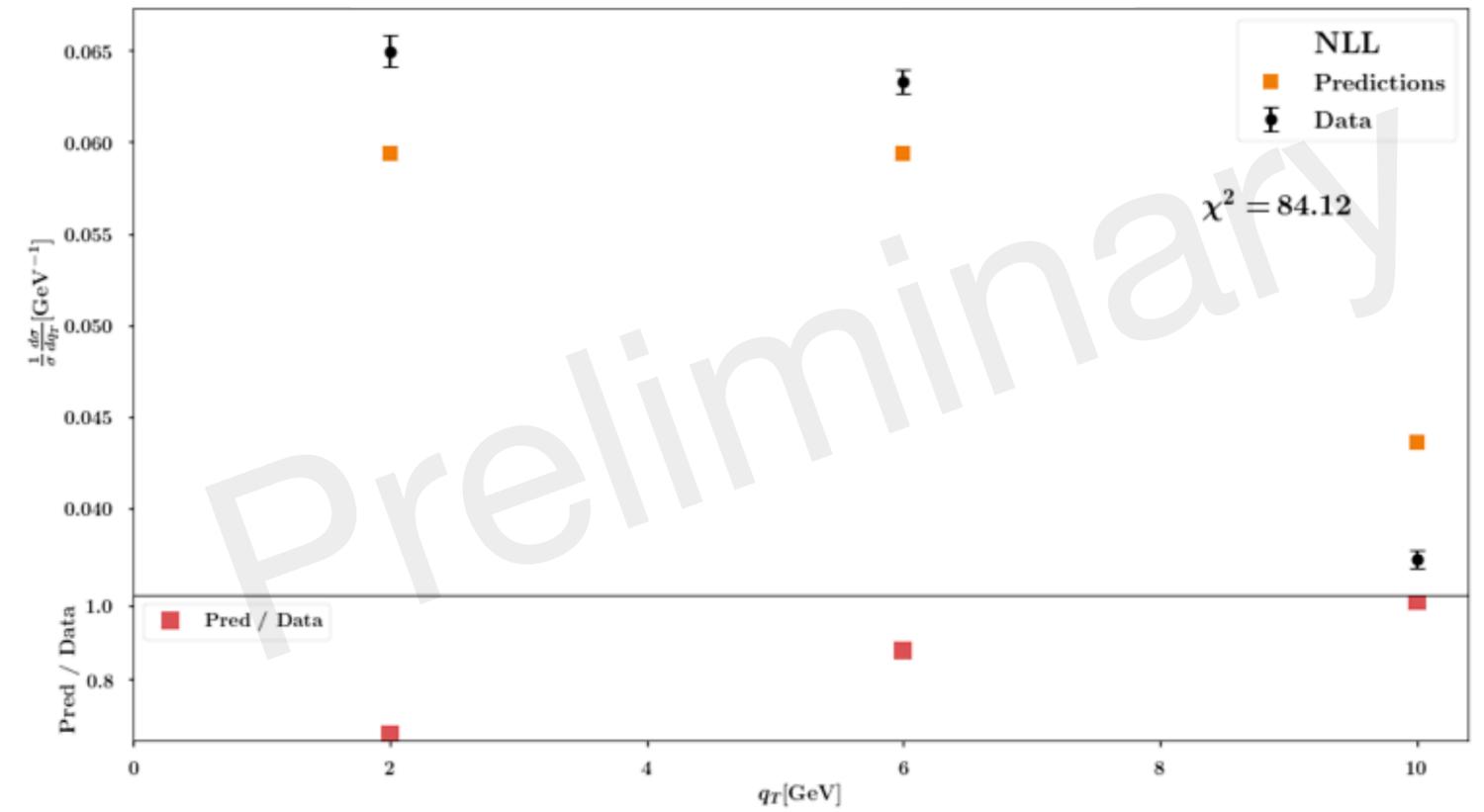
RESULTS AT NLL: DRELL YAN

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CMS 7 TeV

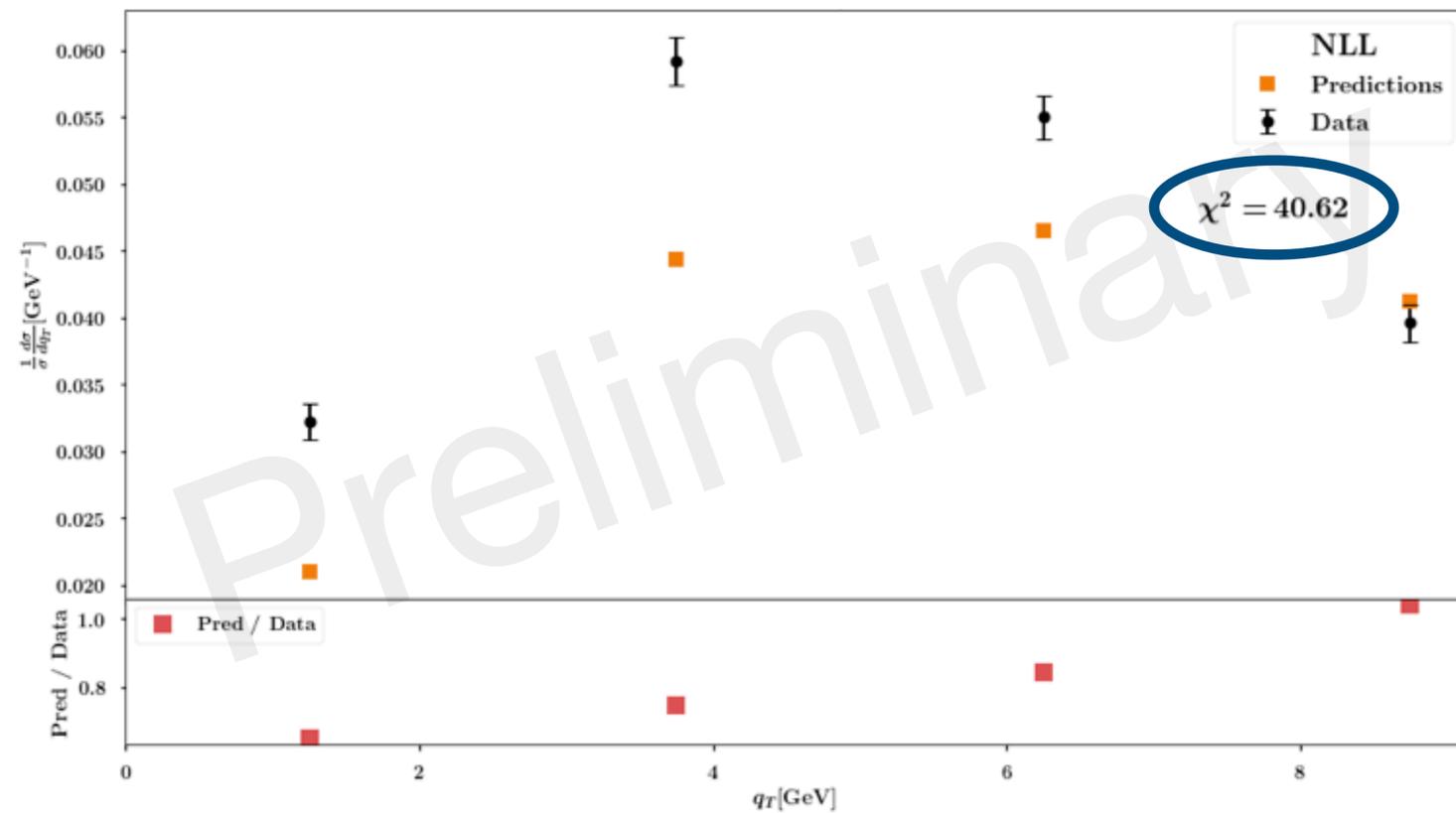


D0 Run II muons

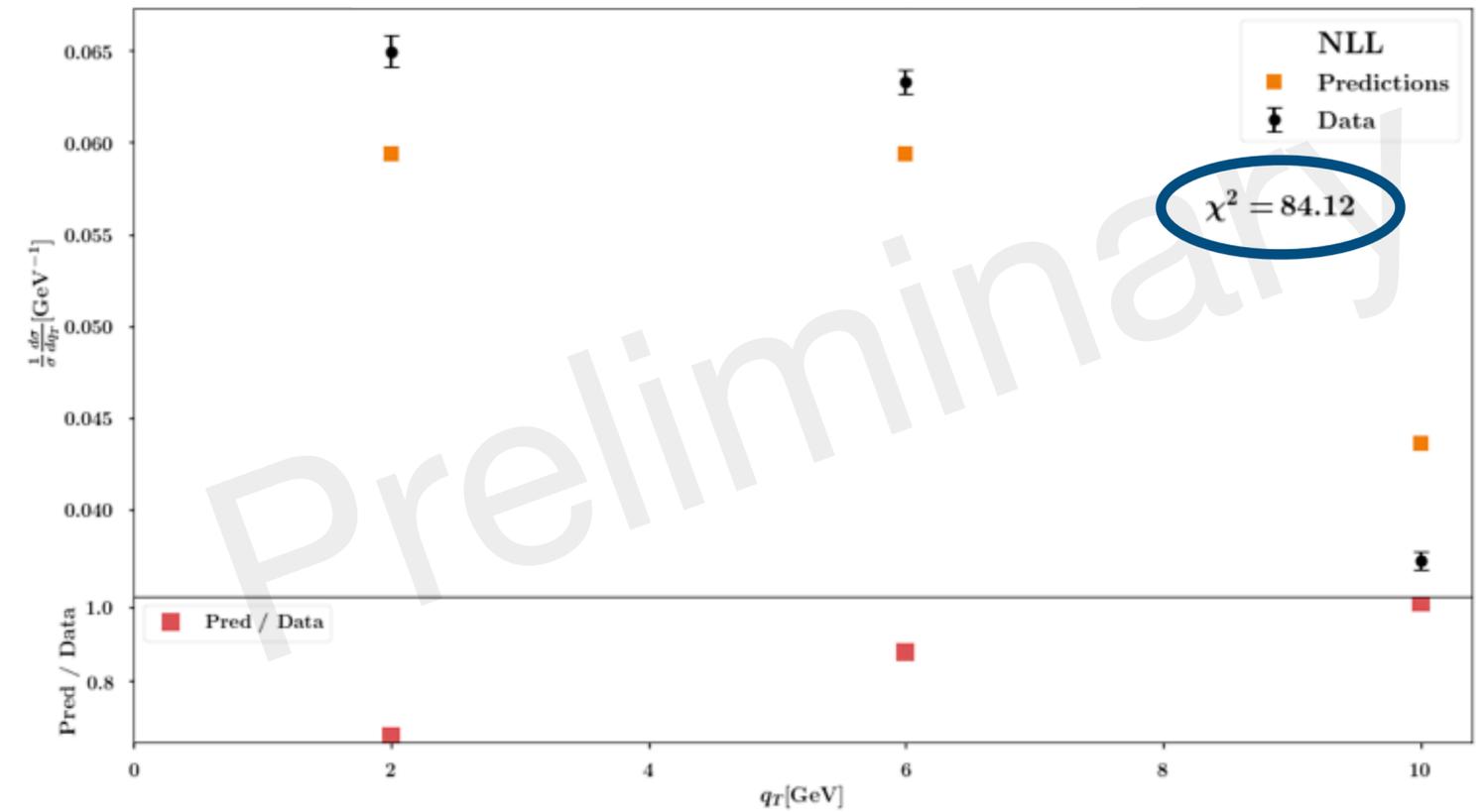


RESULTS AT NLL: DRELL YAN

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D0 Run II muons



We need to increase the accuracy!!

COMPARISON OF DIFFERENT ORDERS

Accuracy at N²LL and N³LL

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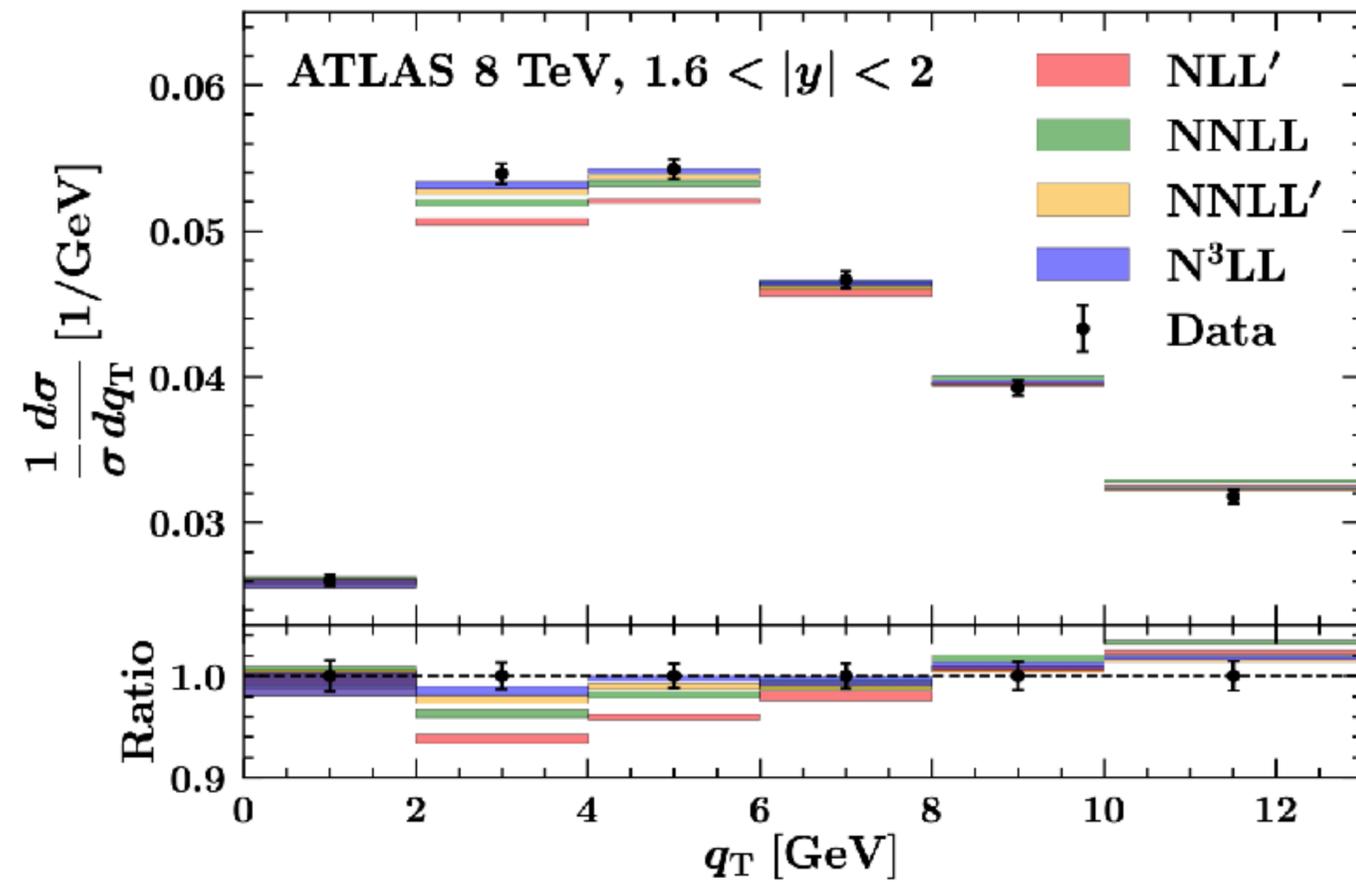
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What we expected

$Q \sim 100$ GeV



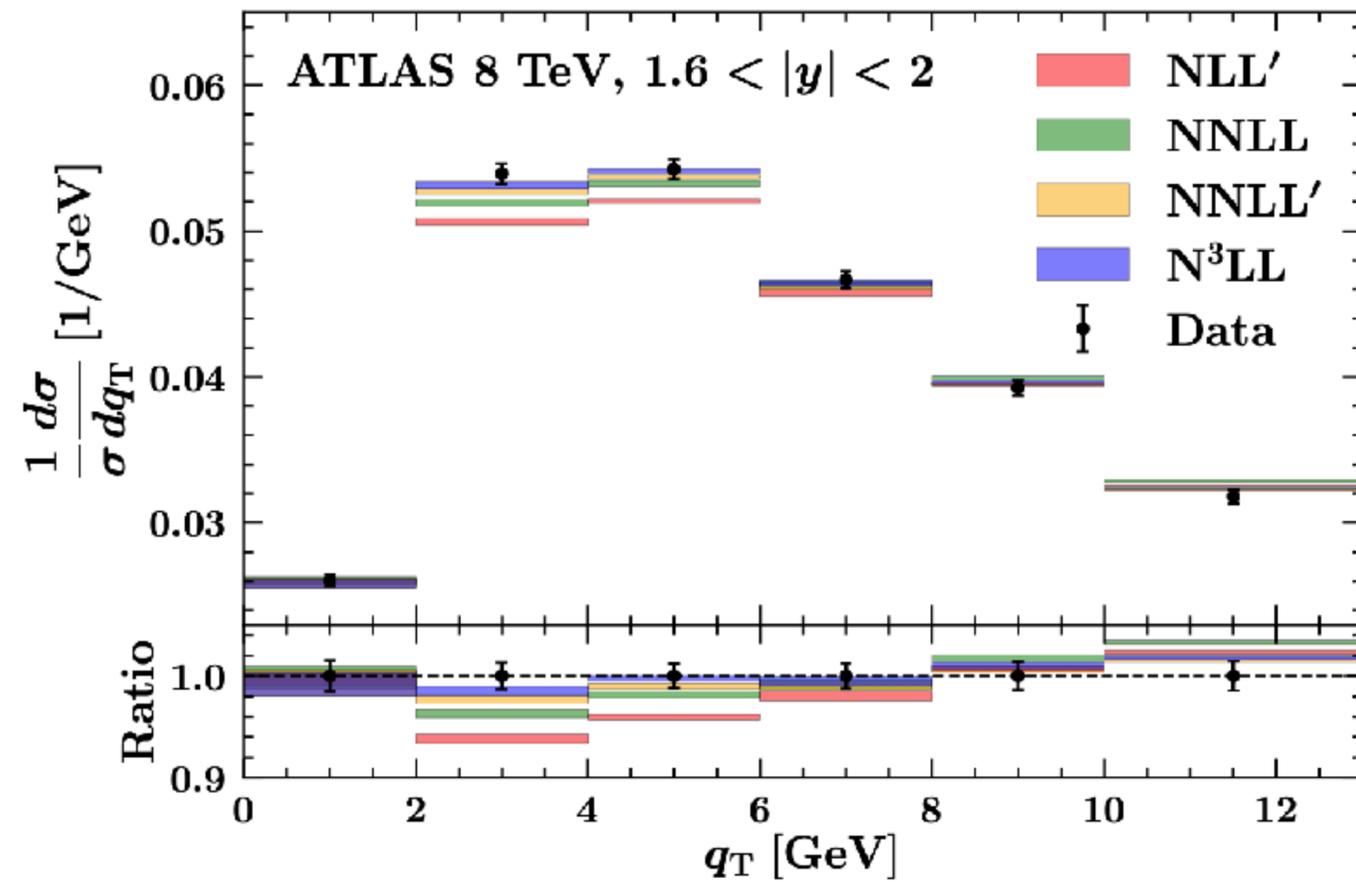
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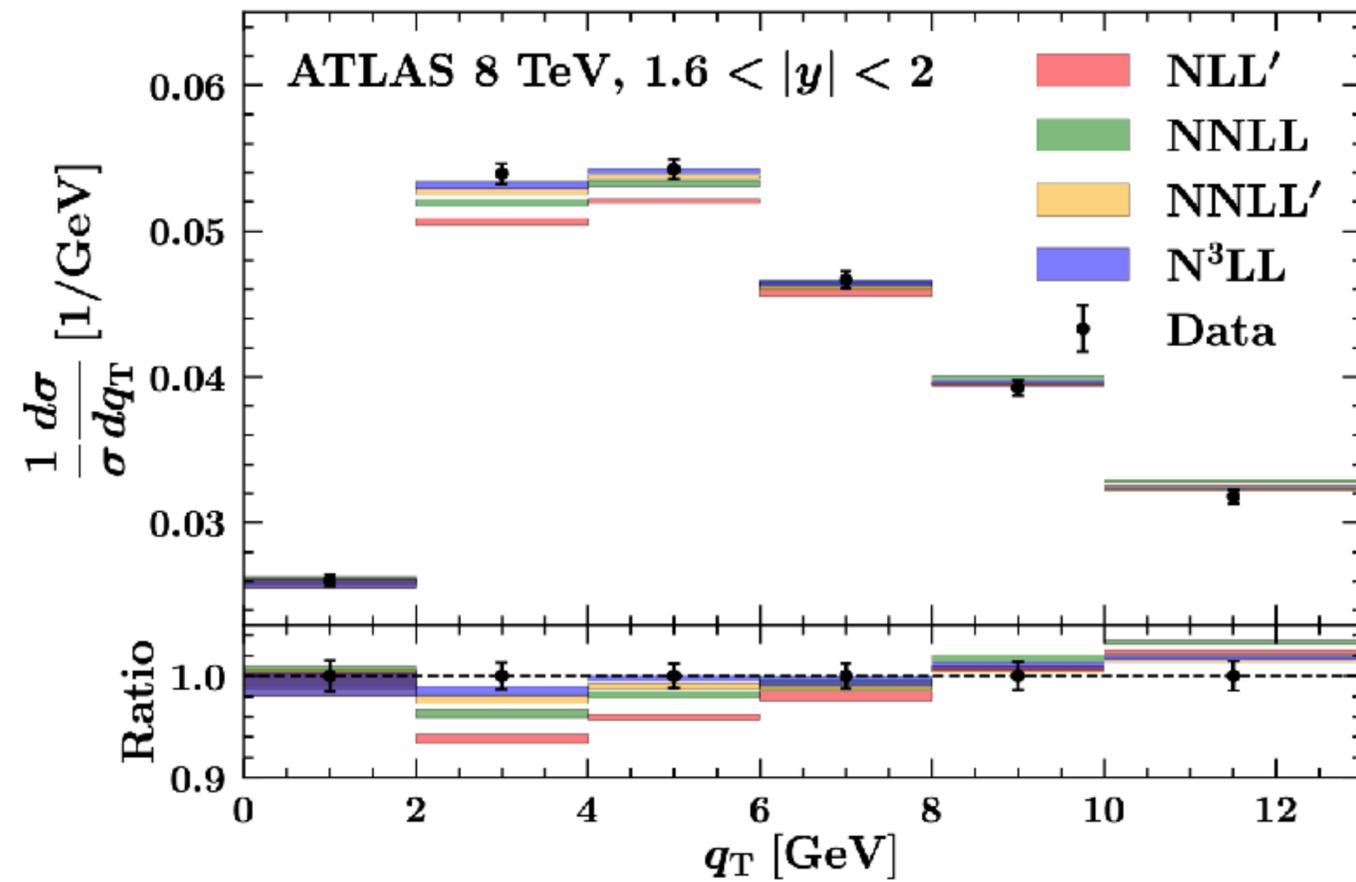


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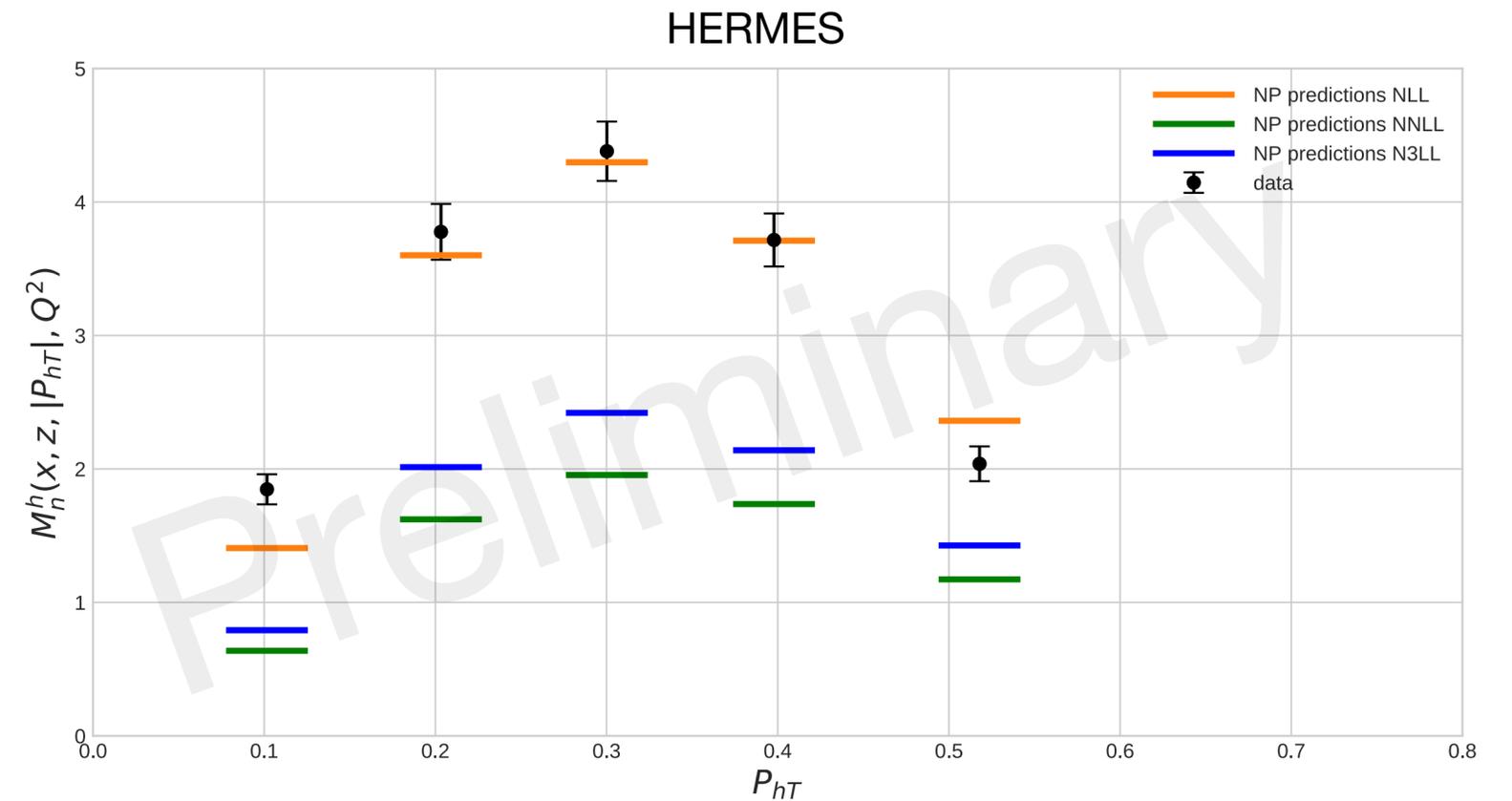
What we expected

$Q \sim 100 \text{ GeV}$



What we get

$Q \sim 2 \text{ GeV}$

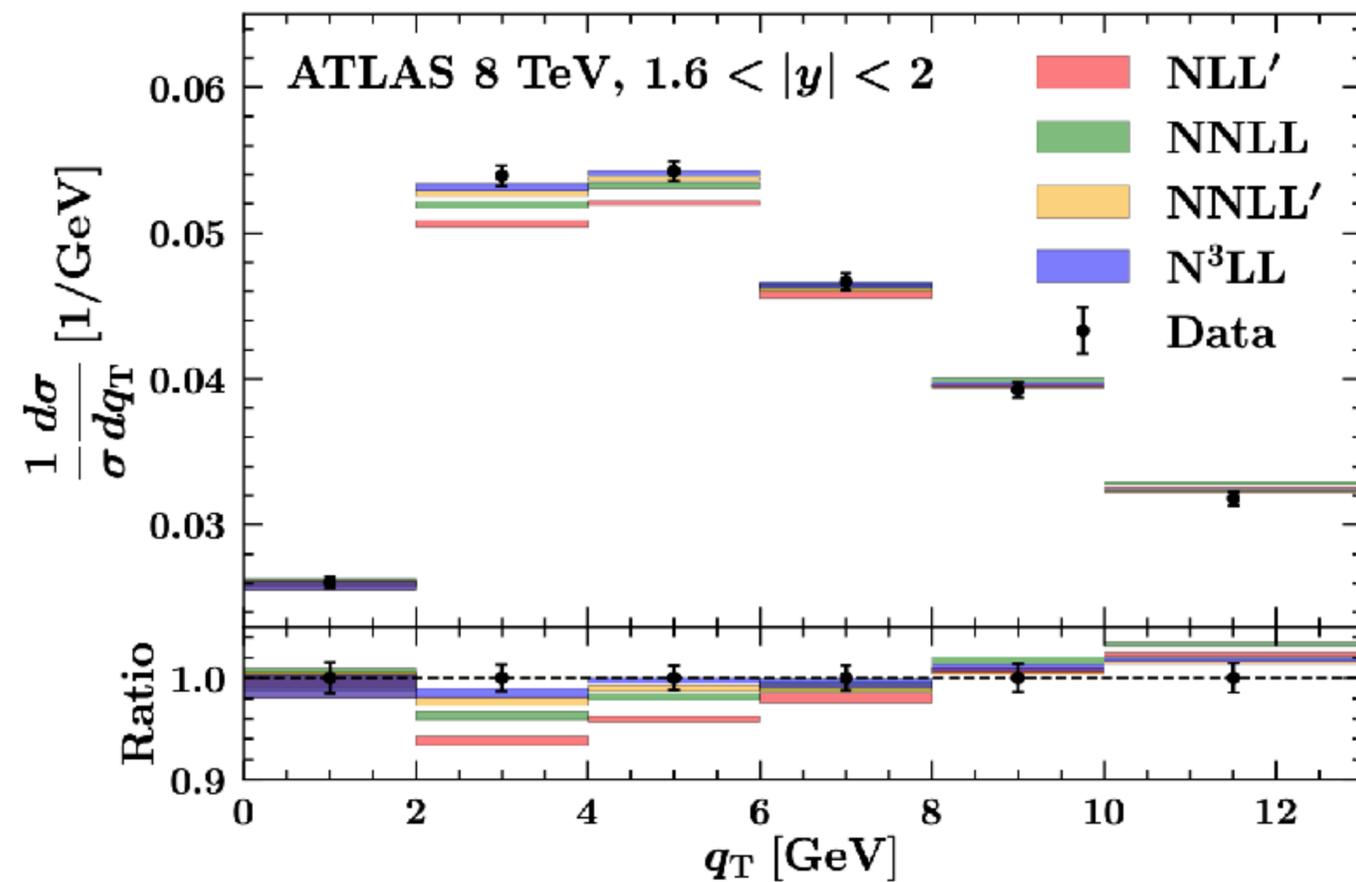


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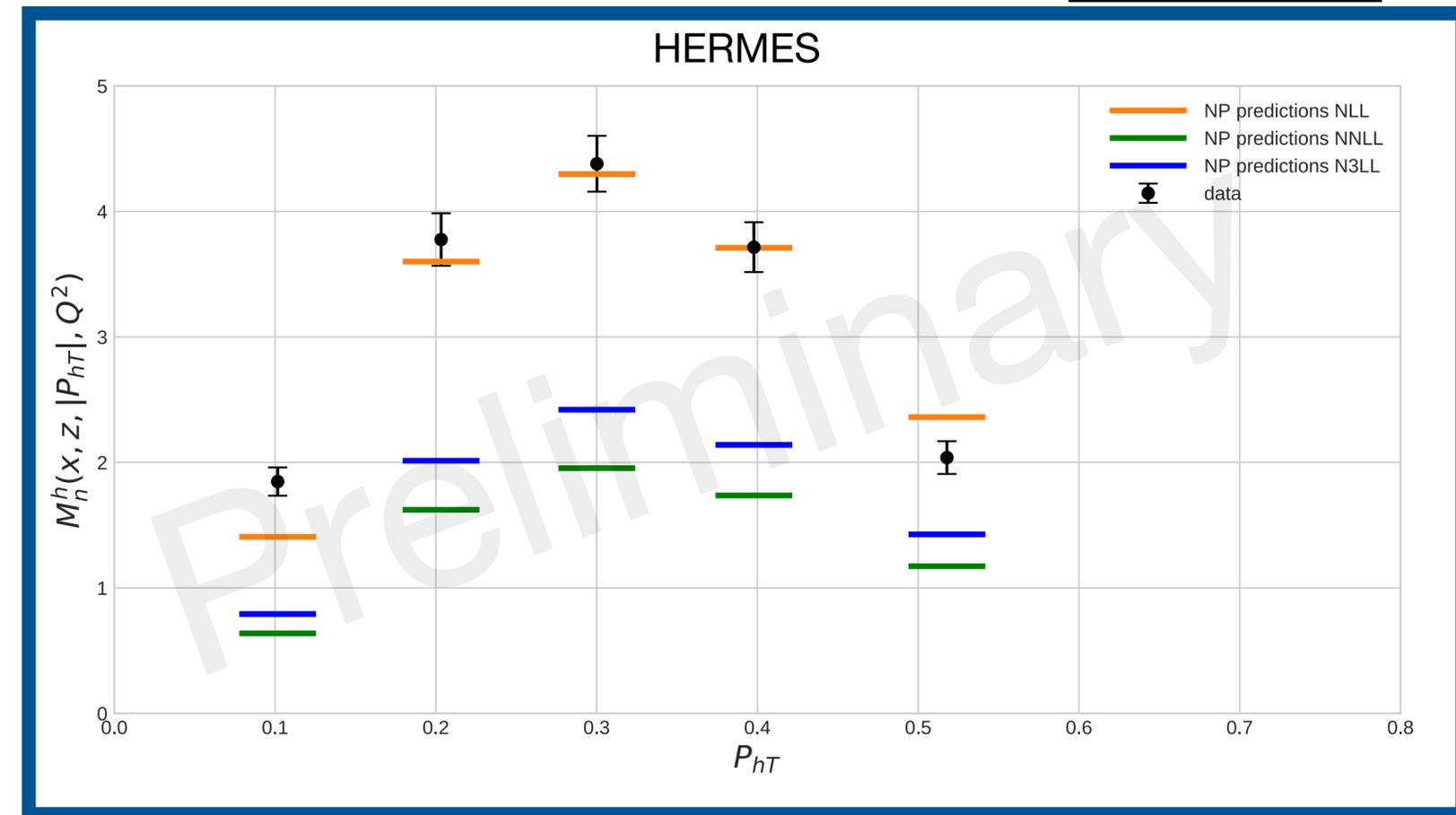
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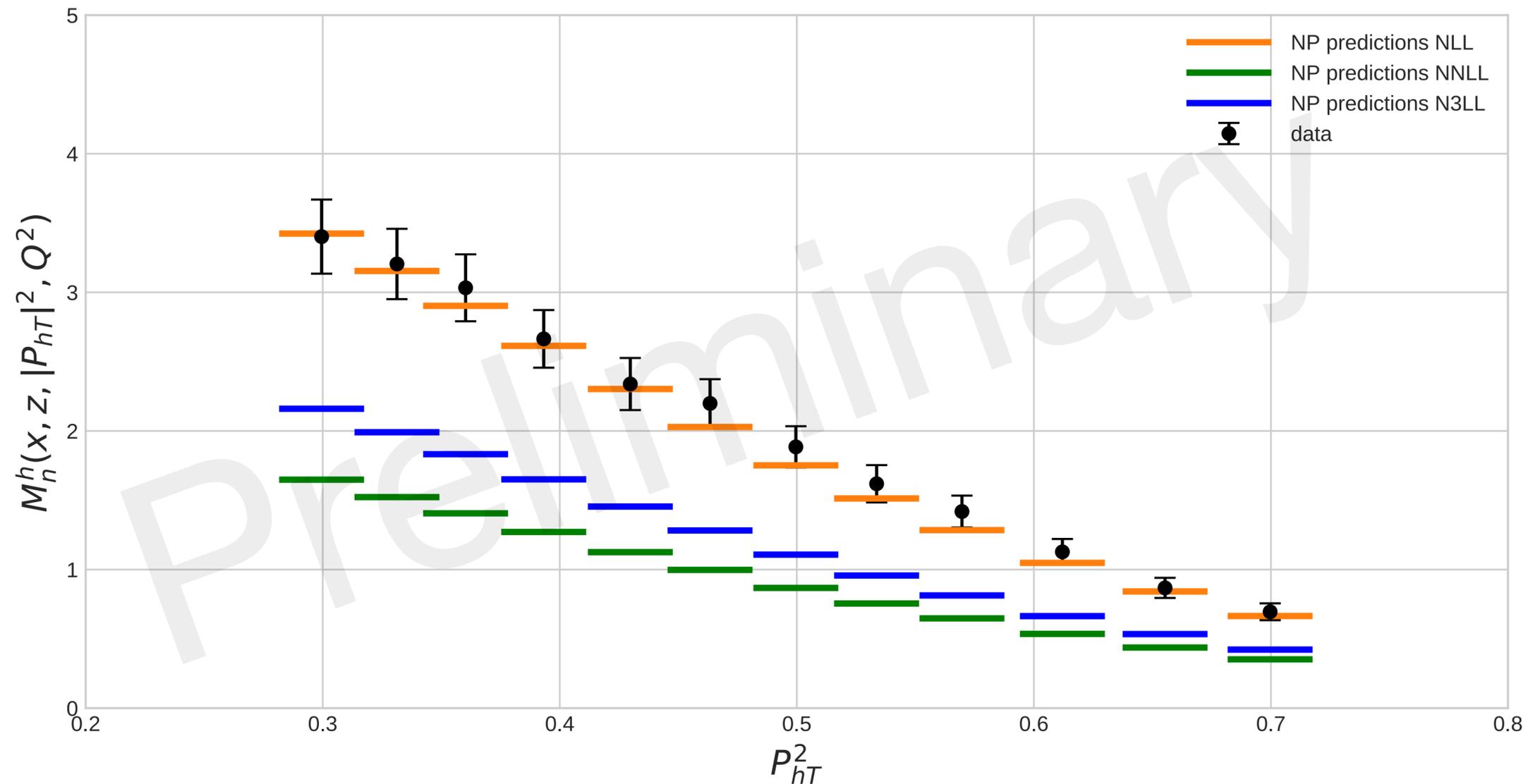


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

The description considerably worsens at higher orders!!

COMPARISON OF DIFFERENT ORDERS – SIDIS

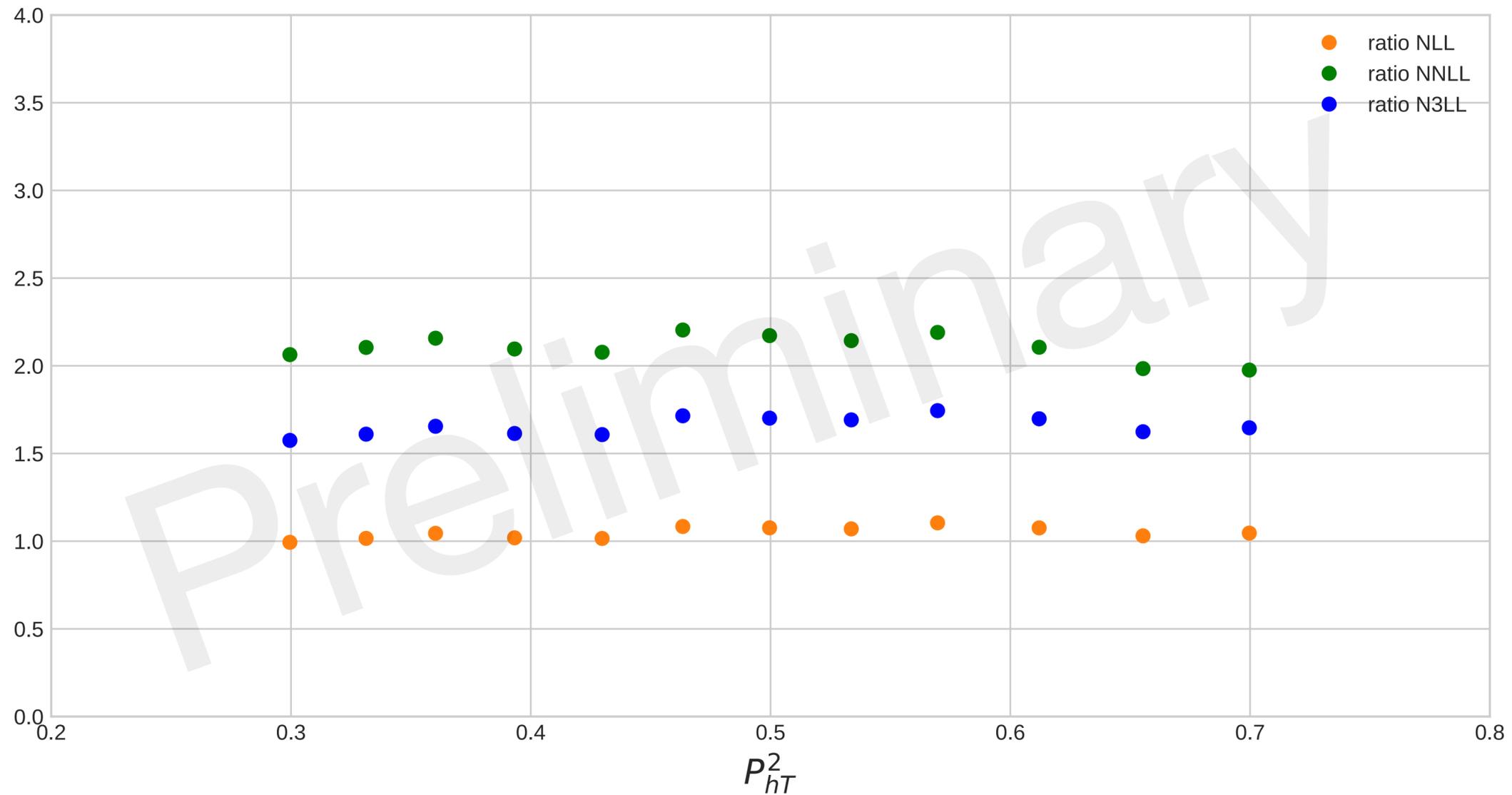
COMPASS multiplicities (one of many bins)



The description considerably worsens at higher orders!!

RATIO DATA/PREDICTIONS: SIDIS

COMPASS multiplicities (one of many bins)



The discrepancy amounts to an almost constant factor!!

OUR TENTATIVE SOLUTION

Introduction of a normalization prefactor

$$\text{PREFACTOR}(x, z, Q) = \frac{\frac{d\sigma^h}{dx dQ^2 dz} \Big|_{\text{nonmix.}}}{\int W d^2 q_T}$$

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$$\left. \frac{d\sigma^h}{dx dQ^2 dz} \right|_{O(\alpha_S)} = \sigma_0 \sum_{f, f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} \left\{ [D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}] (x, z, Q) \right\} \Big|_{\text{nonmix.}}$$
$$\int W \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f, f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} [D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N}] (x, z, Q)$$

OUR TENTATIVE SOLUTION

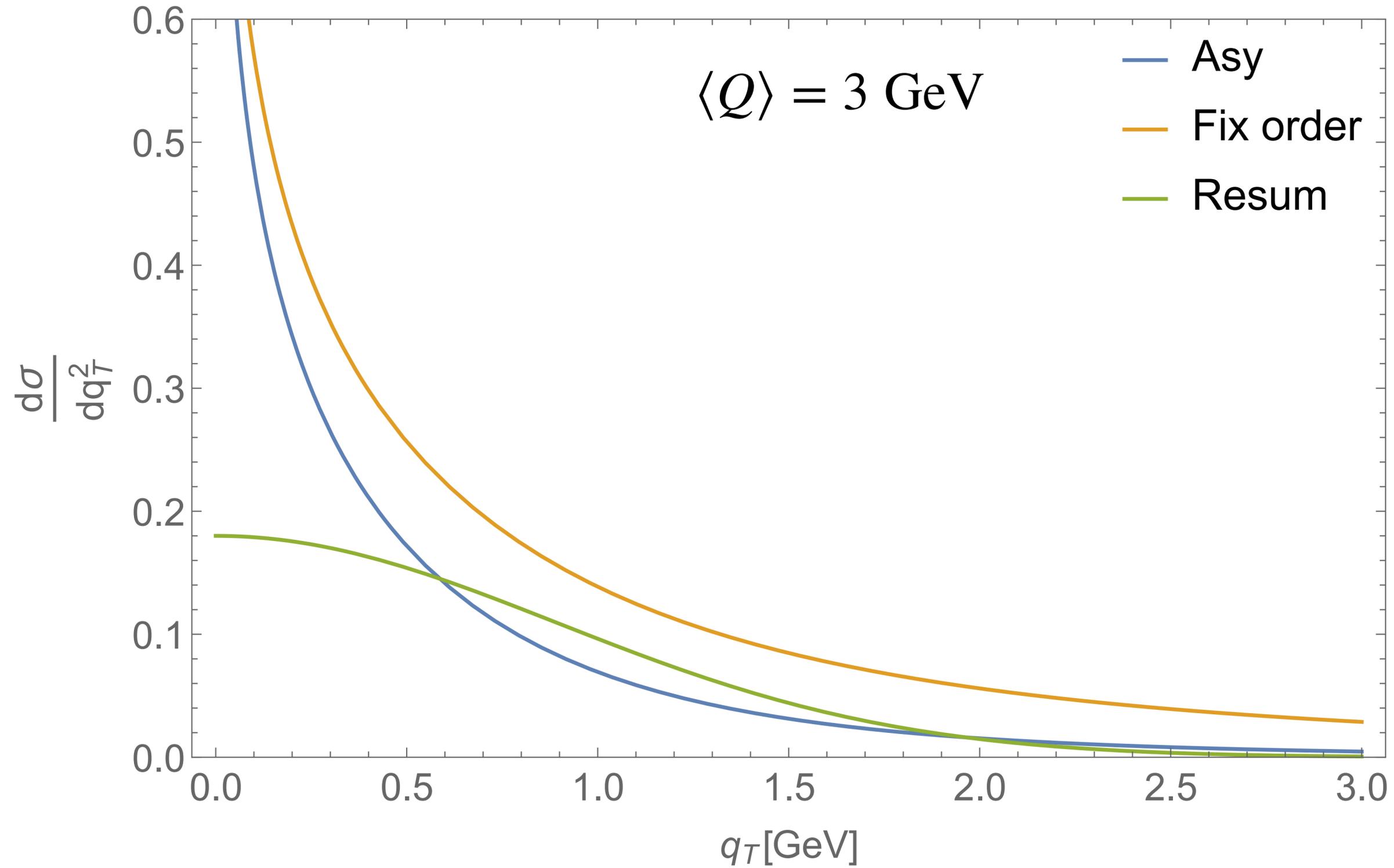
Introduction of a normalization prefactor

$$\text{PREFACTOR}(x, z, Q) = \frac{\left. \frac{d\sigma^h}{dx dQ^2 dz} \right|_{\text{nonmix.}}}{\int W d^2 q_T}$$

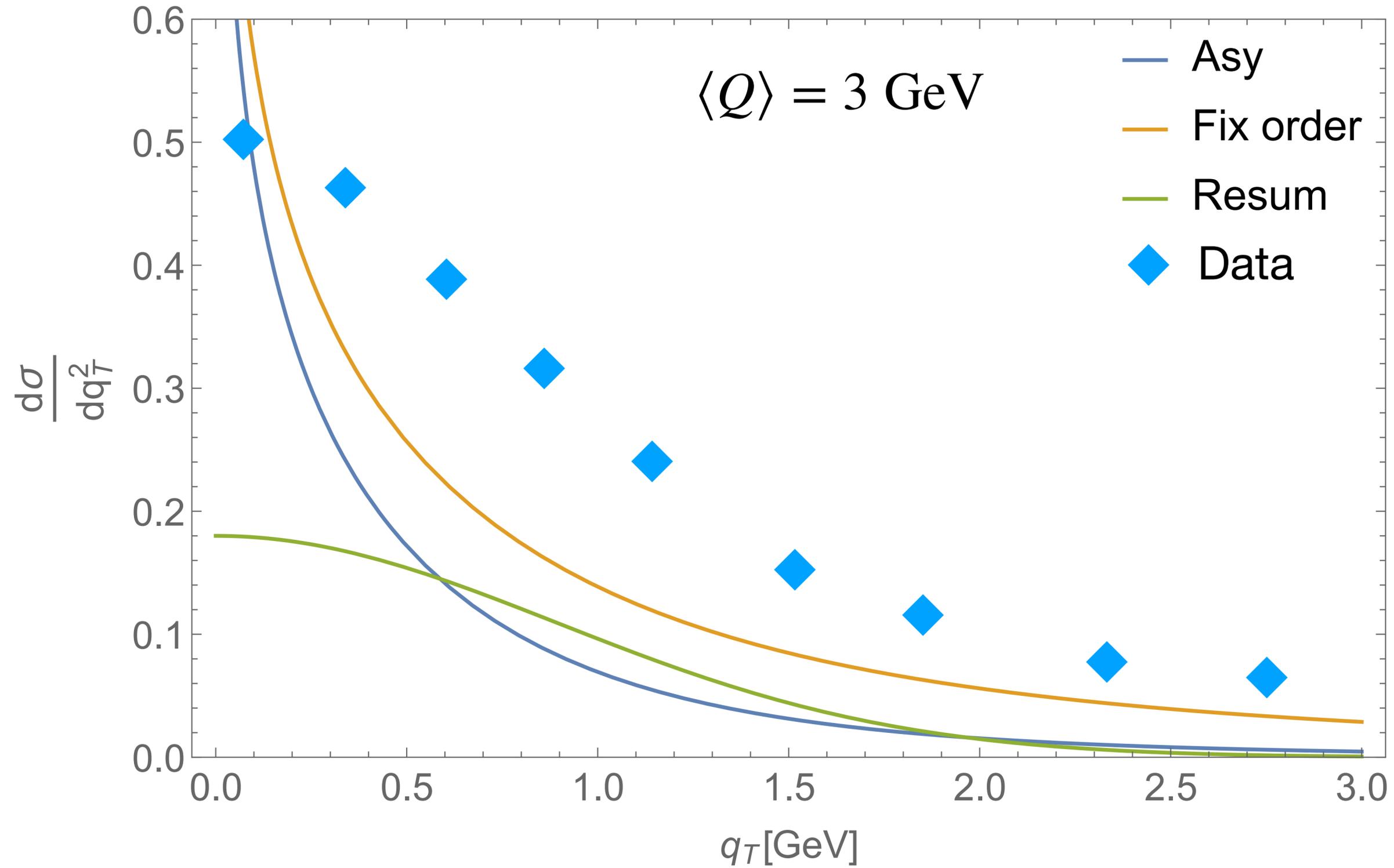
$$\left. \frac{d\sigma^h}{dx dQ^2 dz} \right|_{O(\alpha_S)} = \sigma_0 \sum_{f, f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} \left\{ [D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}](x, z, Q) \right\} \Big|_{\text{nonmix.}}$$
$$\int W \Big|_{O(\alpha_S)} = \sigma_0 \sum_{f, f'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_S}{\pi} [D_1^{h/f'} \otimes C_{\text{TMD}}^{f'f} \otimes f_1^{f/N}](x, z, Q)$$

Independent of the fitting parameters!!

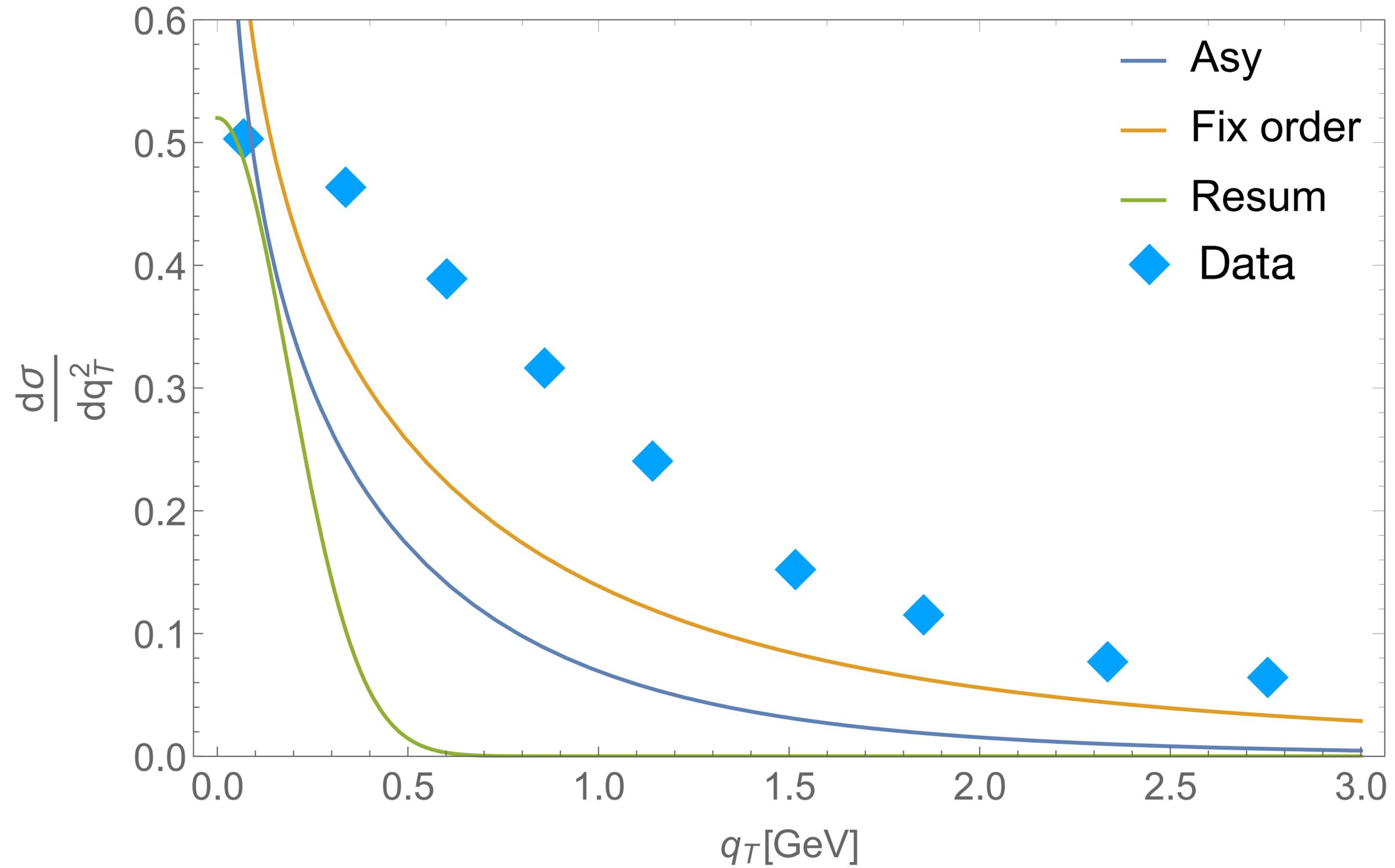
SOME JUSTIFICATION: INITIAL SITUATION



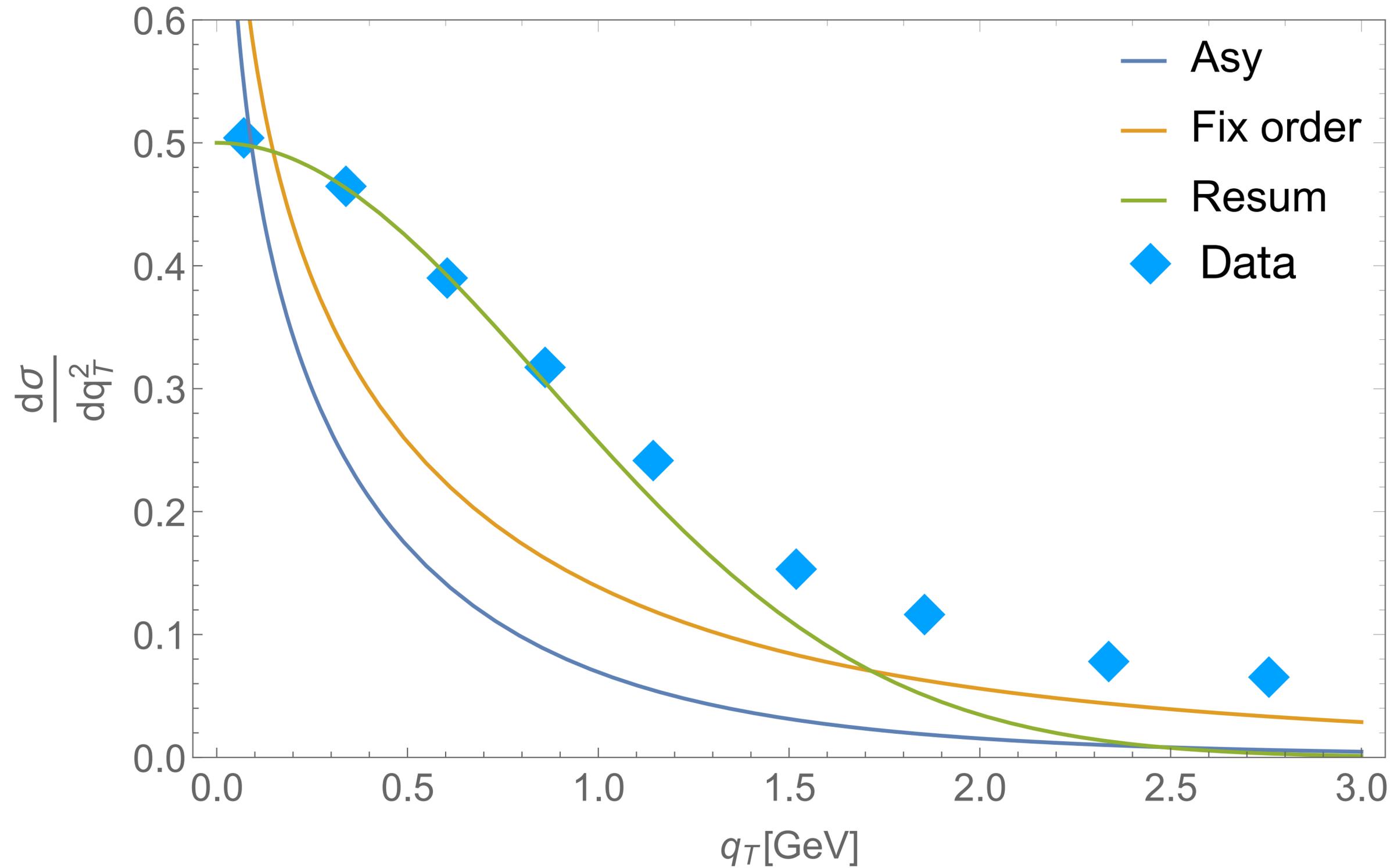
SOME JUSTIFICATION: INITIAL SITUATION



SOLUTION1: RESTRICT TMD REGION



SOLUTION2: ENHANCE TMD CONTRIBUTIONS



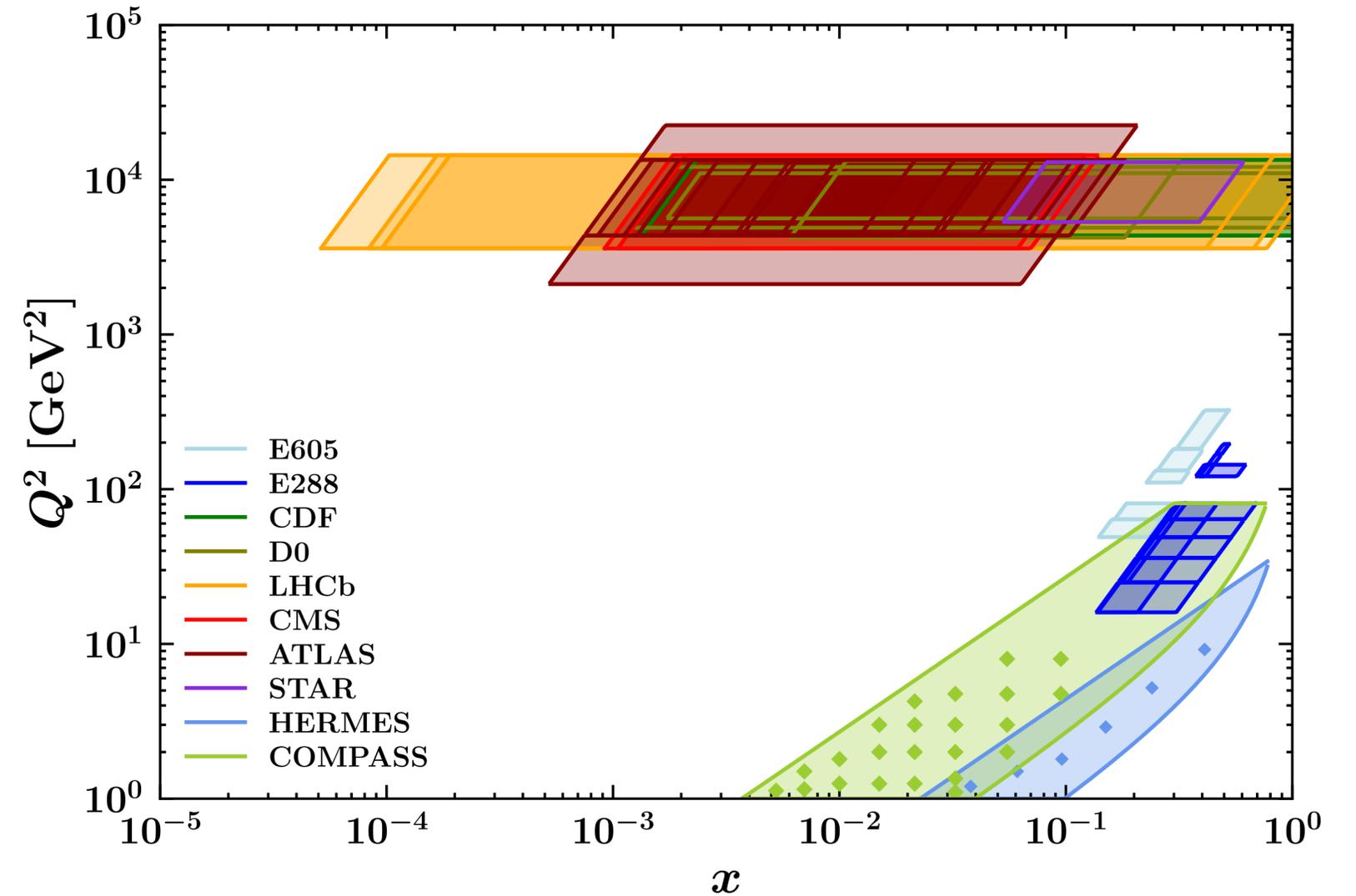
MAP21 TMD FIT CHOICES (PRELIMINARY)

$$\langle Q \rangle > 1.3 \text{ GeV}$$

$$0.2 < z < 0.6$$

$$q_T < 0.2 Q \quad (\text{DY})$$

$$P_{hT} < \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ] \quad (\text{SIDIS})$$



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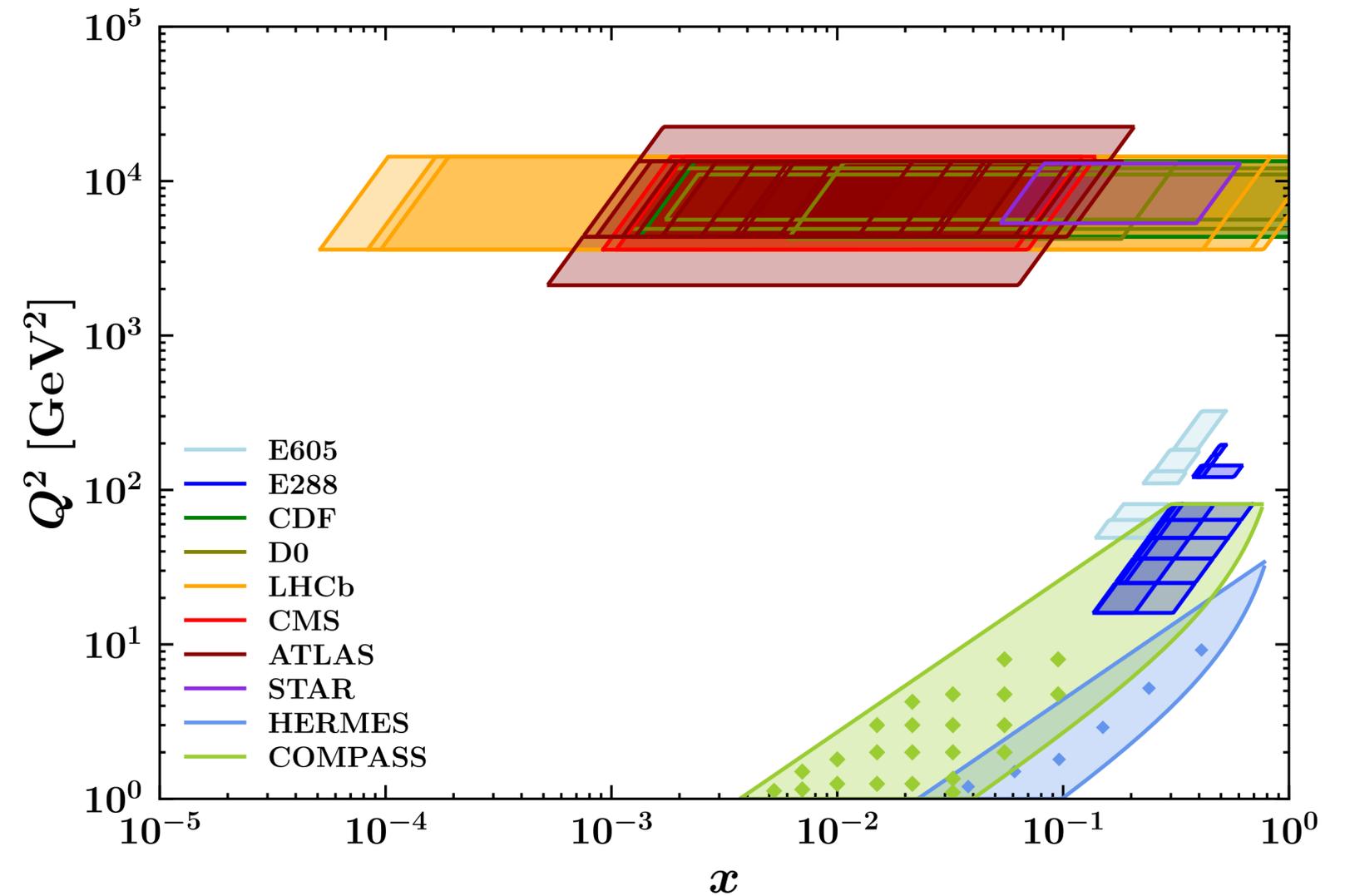
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Number of points > 1500



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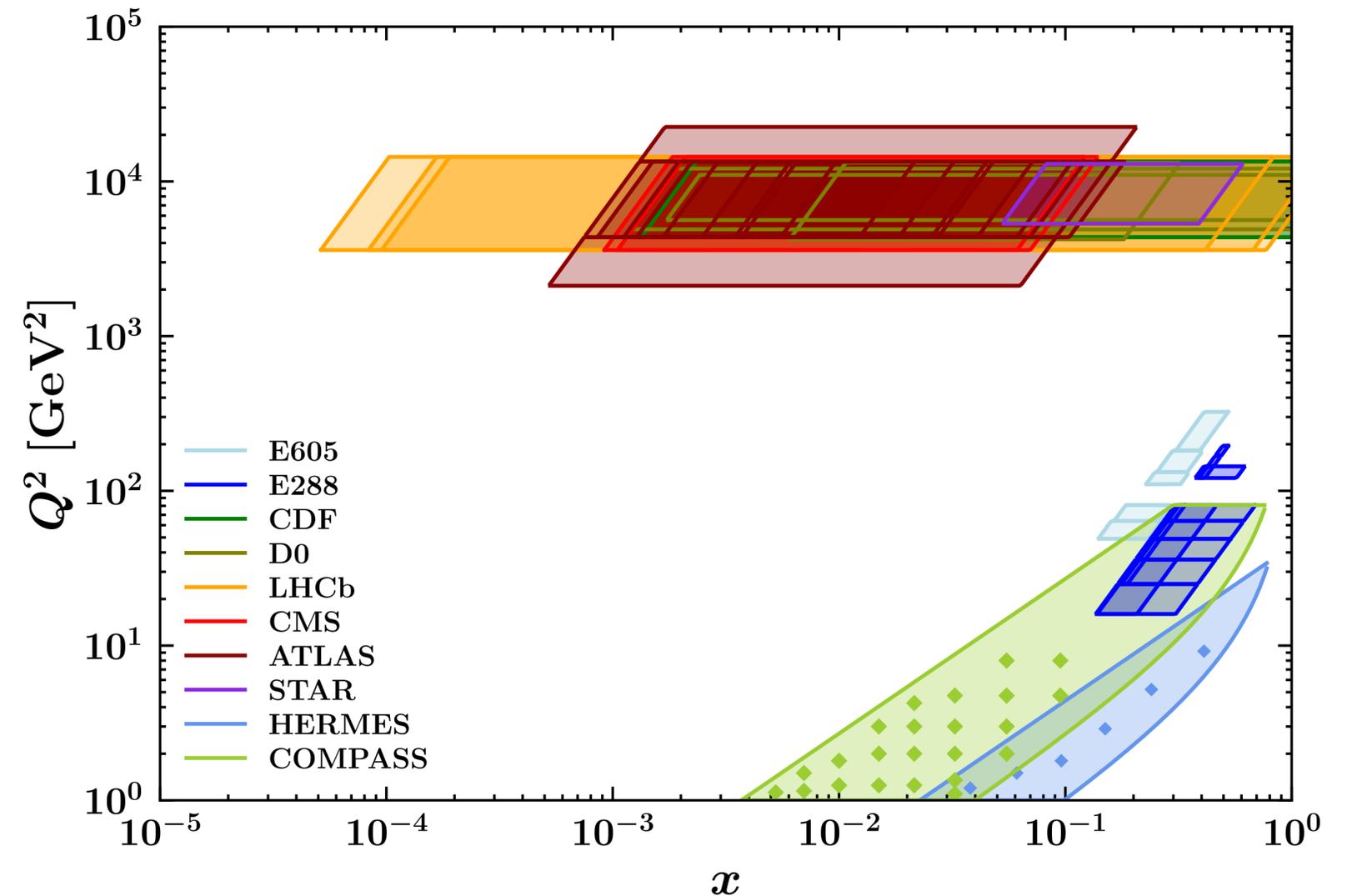
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$$0.2 < z < 0.6 \rightarrow 0.7$$

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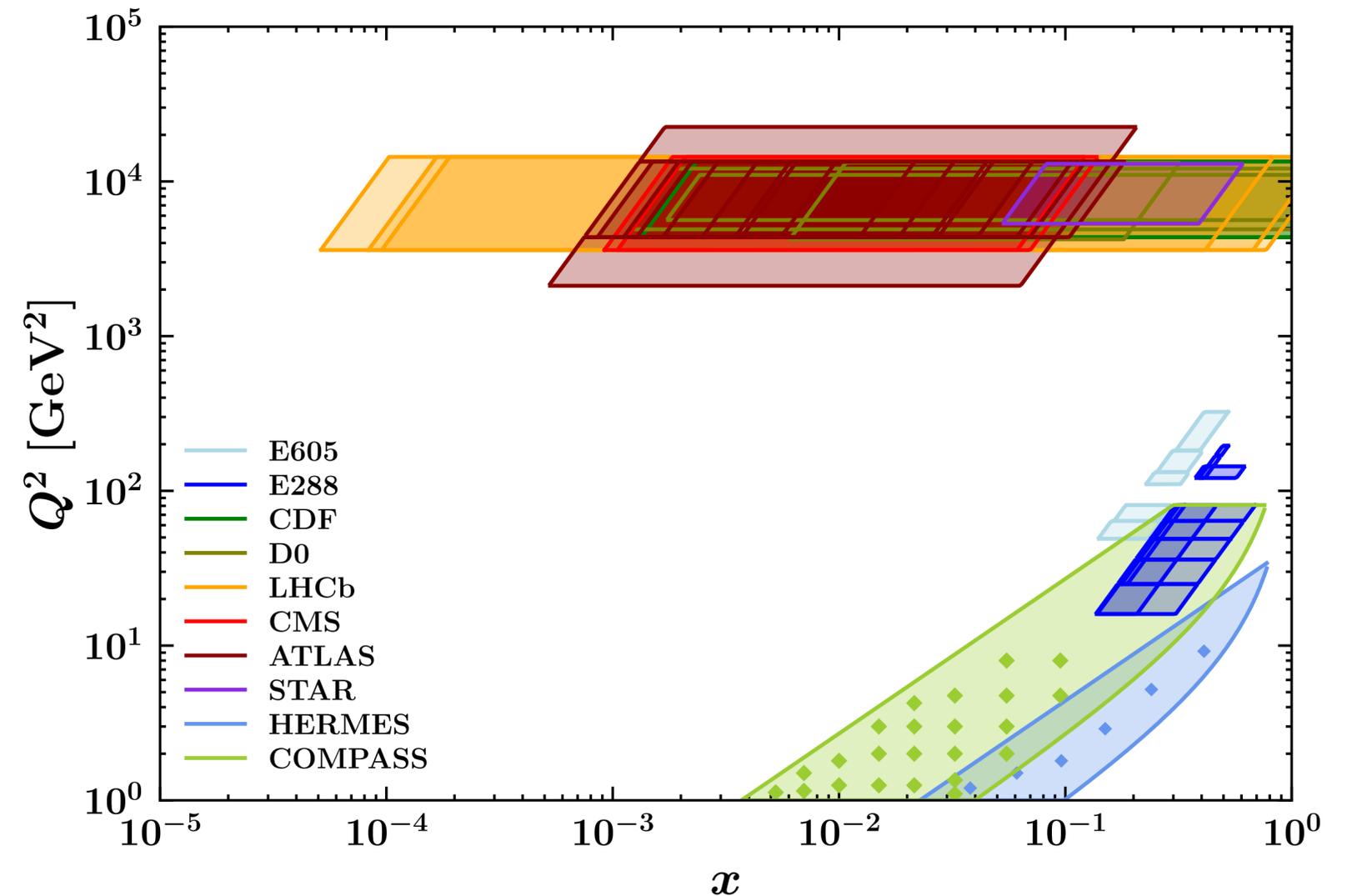
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$$\text{Number of points} > 1500 \rightarrow 2000 ?$$



FUNCTIONAL FORM (PRELIMINARY)

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

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Still working on the flexibility
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$$g_1(x) = N_1 \frac{(1-x)^{\alpha} x^{\sigma}}{(1-\hat{x})^{\alpha} \hat{x}^{\sigma}}$$

$$g_K(b_T^2) = -\frac{g_{2A}}{2} b_T^2 - \frac{g_{2B}}{2} b_T^4$$

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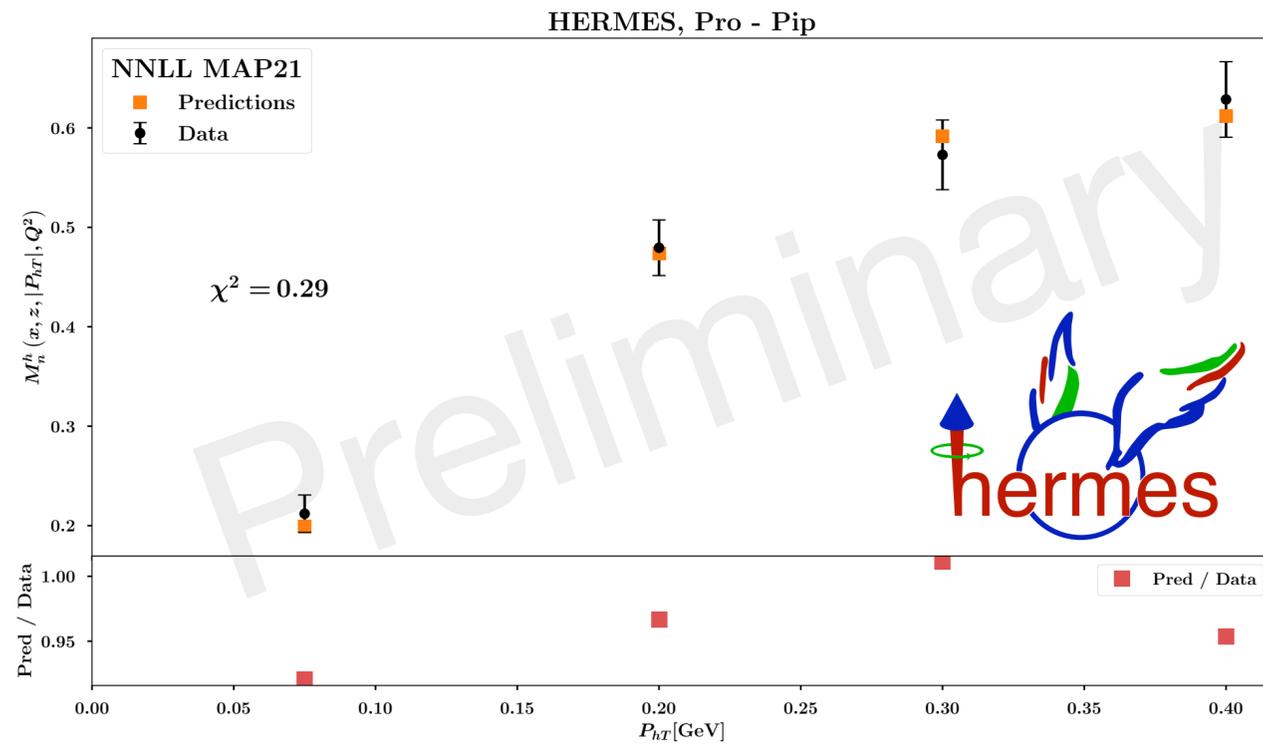
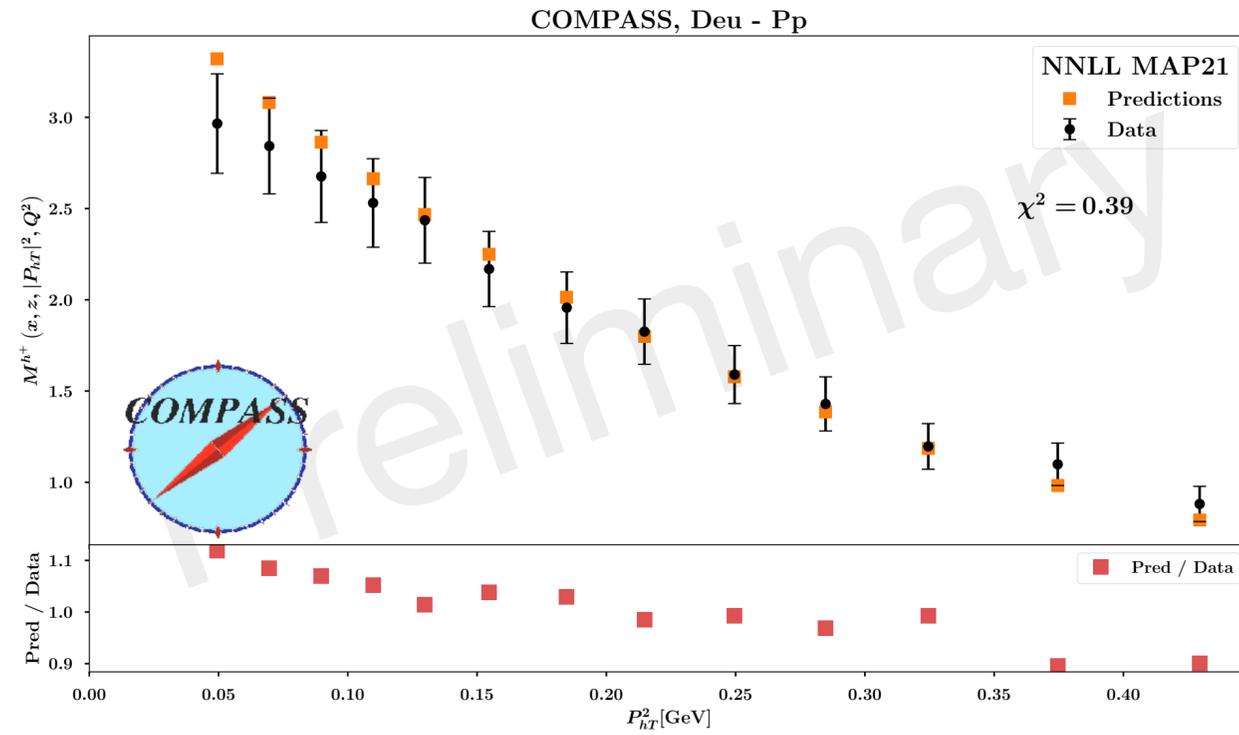
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Still working on the flexibility
of the final form

**11 parameters for TMD PDF
+ 2 for NP evolution + 14 for FF
= 27 free parameters**

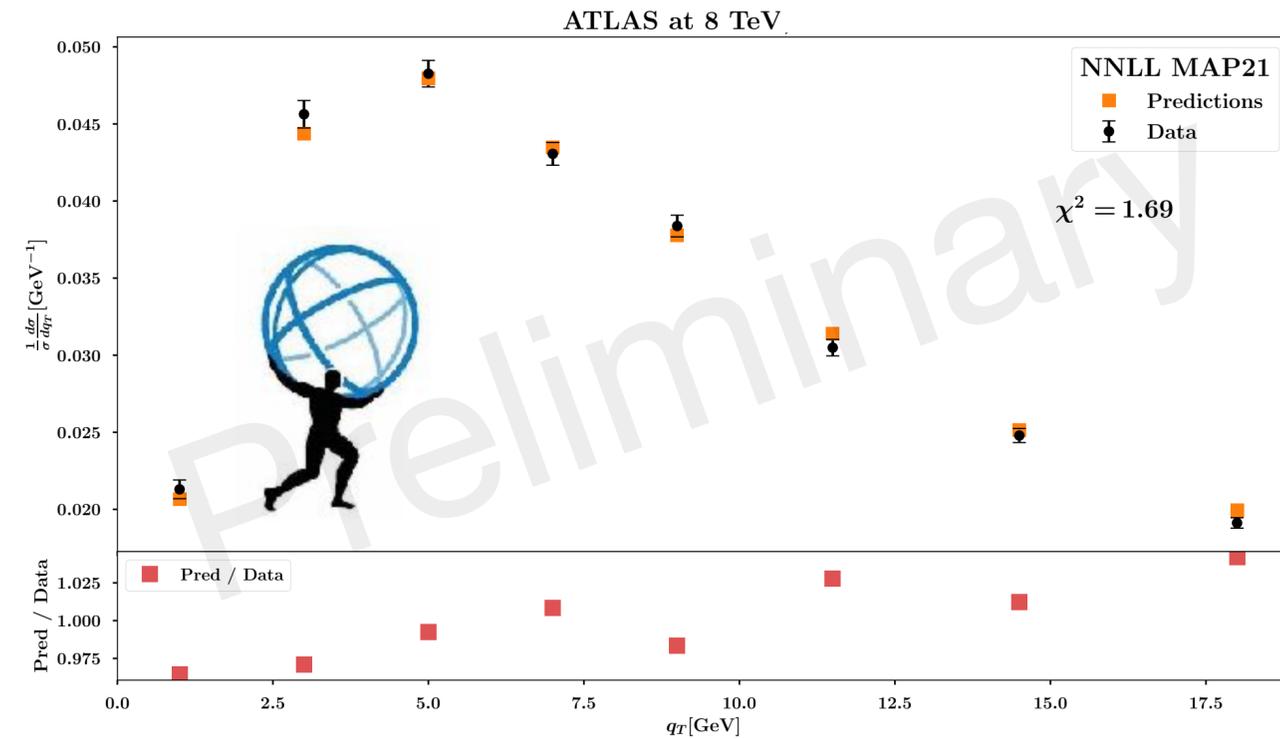
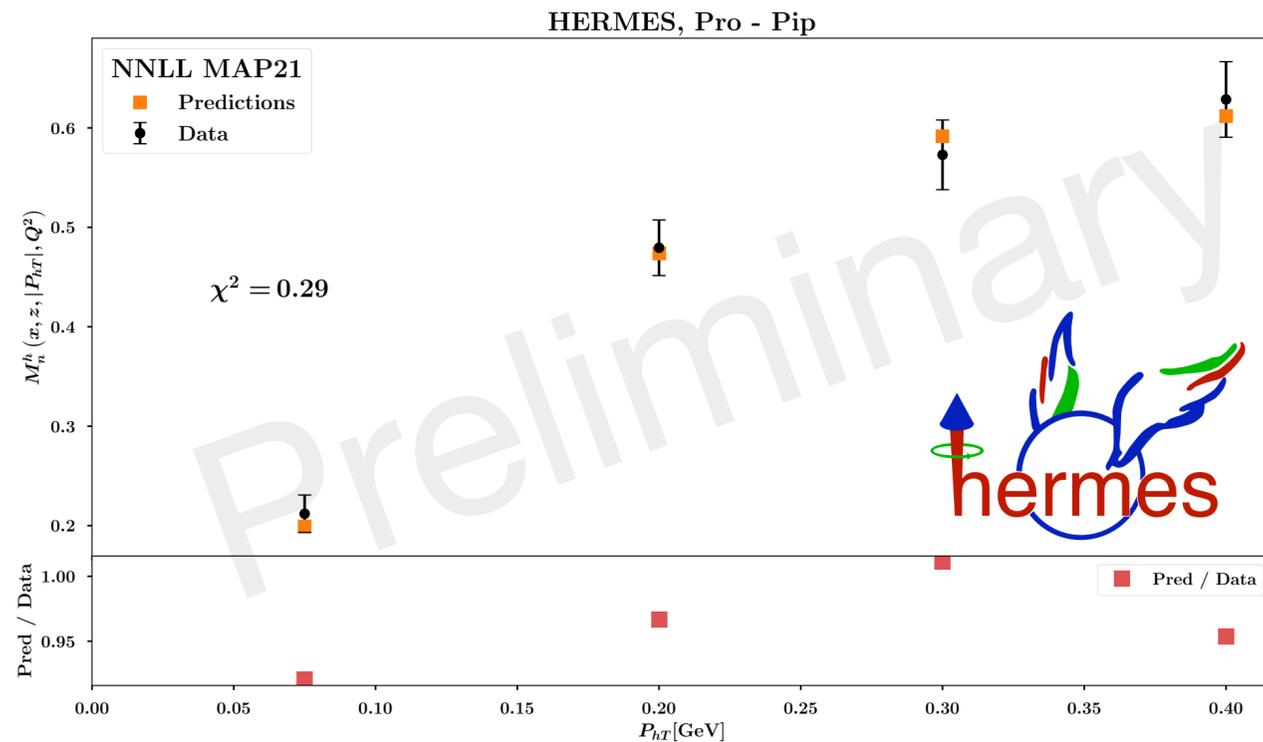
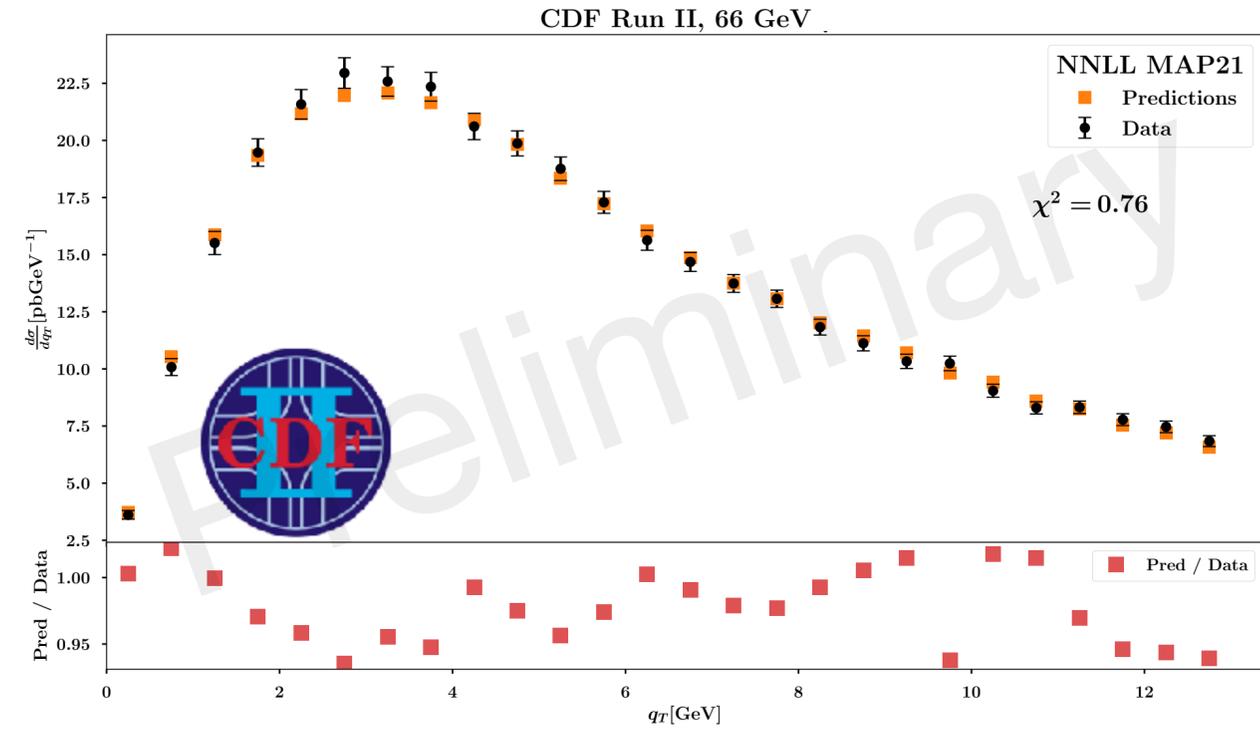
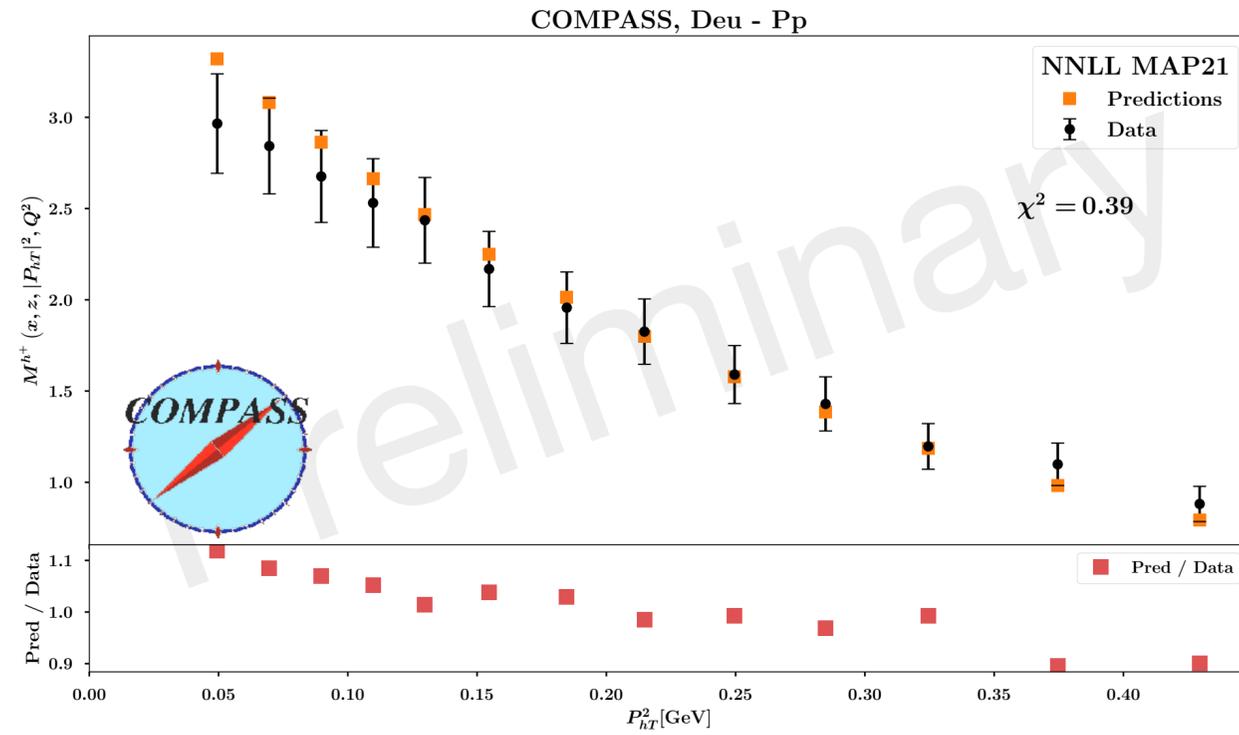
N²LL: EXAMPLE OF GOOD BINS

Global $\chi^2 < 1.1$



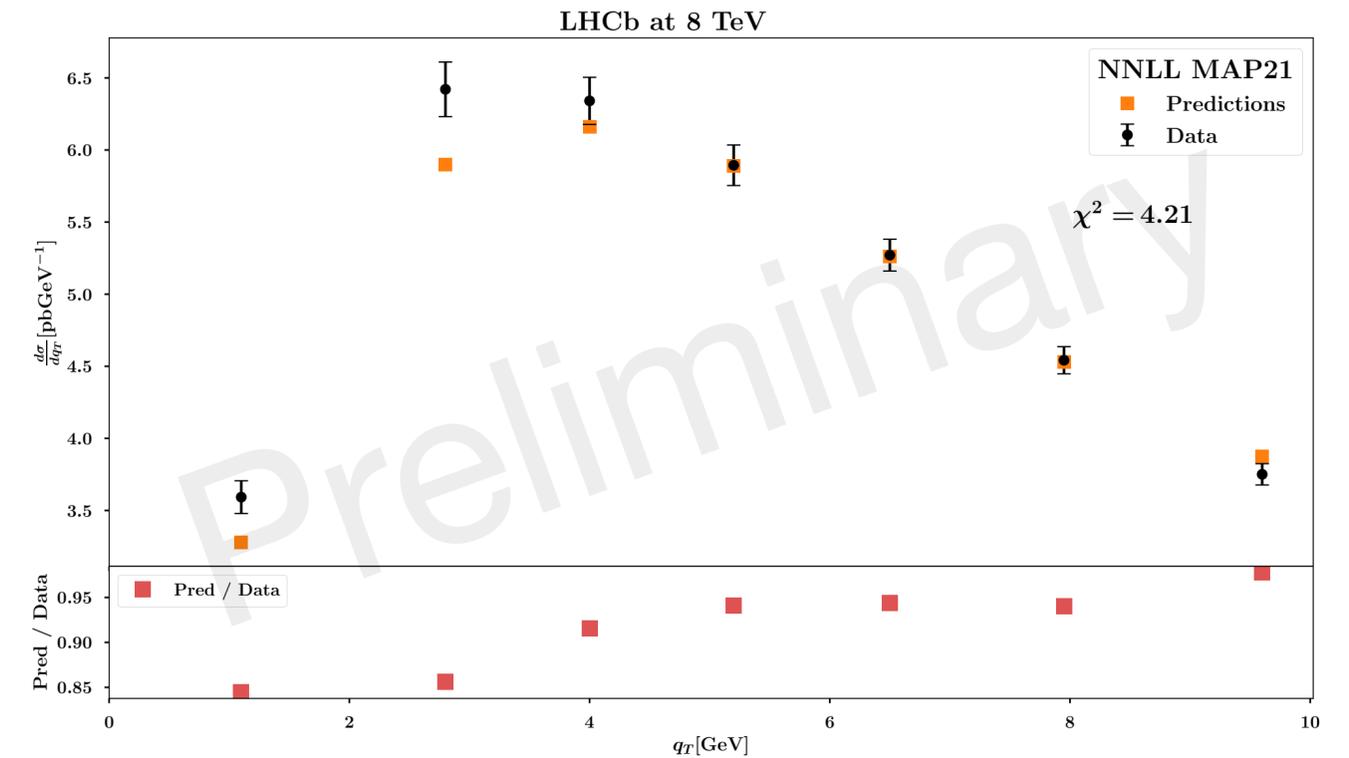
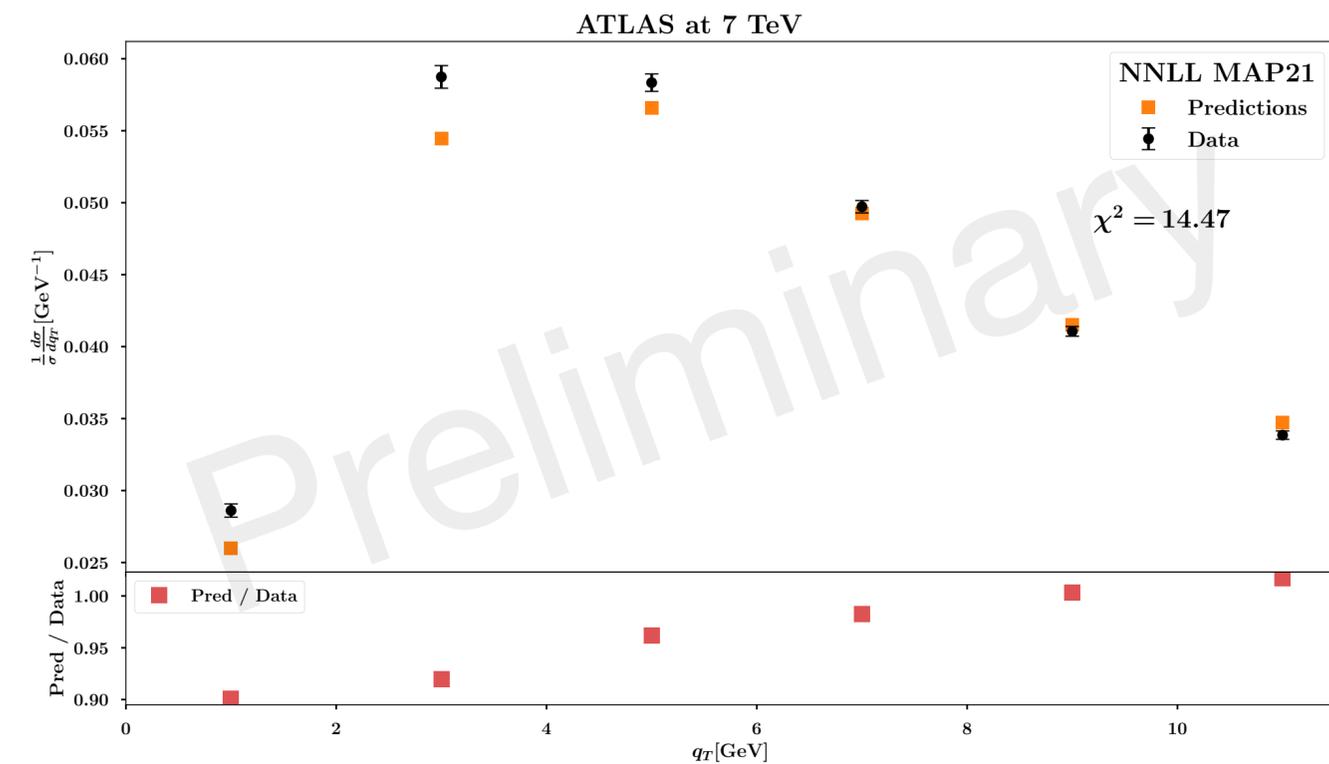
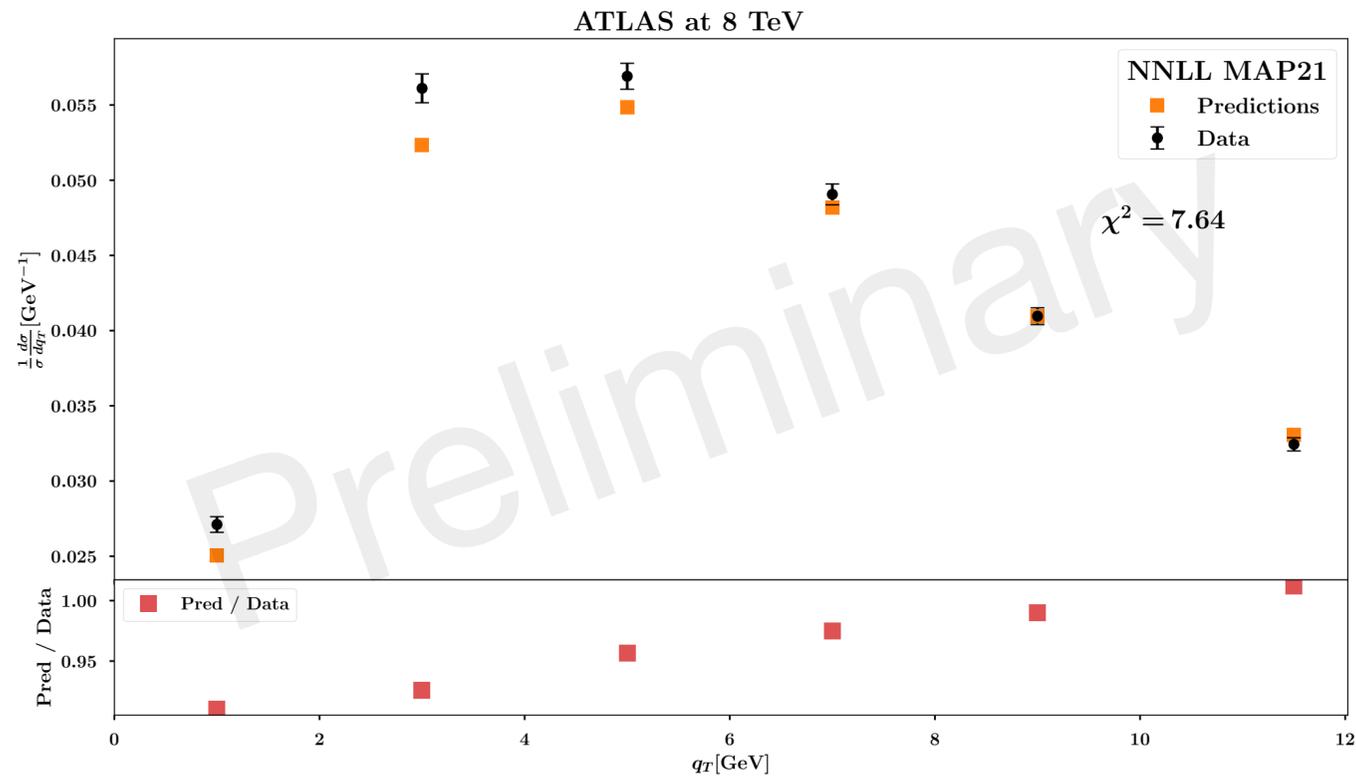
N²LL: EXAMPLE OF GOOD BINS

Global $\chi^2 < 1.1$



N²LL: EXAMPLE OF BAD BINS

Global $\chi^2 < 1.1$



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CONCLUSIONS

- 📌 DY data can NOT be described at NLL, but only at higher orders
- 📌 SIDIS data can be described very well at NLL, but require normalization prefactors at NLL' or higher
- 📌 The identification of the region of applicability of the TMD formalism is still an open issue
- 📌 Good global χ^2 can be reached at N²LL, but some LHC data remain hard to describe

BACKUP SLIDES

LOGARITHMIC ACCURACY

Sudakov form factor

Matching coefficient

LL $\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right)$

$$\tilde{C}^0$$

NLL $\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$

$$\tilde{C}^0$$

NLL' $\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$

$$\left(\tilde{C}^0 + \alpha_S \tilde{C}^1 \right)$$

the difference between the two is NNLL: $\alpha_S^n \ln^{2n-2} \left(\frac{Q^2}{\mu_b^2} \right)$

NON-MIXED TERMS IN COLLINEAR SIDIS CROSS SECTION

$$\begin{aligned} \frac{d\sigma^h}{dx dQ^2 dz} \Big|_{O(\alpha_s^1)} &= \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_s}{\pi} \left\{ \left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right. \\ &\quad \left. + \frac{1-y}{1+(1-y)^2} \left[D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right\}, \end{aligned}$$

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$$C_1^{qq} = \frac{C_F}{2} \left\{ -8\delta(1-x)\delta(1-z) \right. \\ \left. + \delta(1-x) \left[P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z) \right] \right. \\ \left. + \delta(1-z) \left[P_{qq}(x) \ln \frac{Q^2}{\mu^2} + L_1(x) - L_2(x) + (1-x) \right] \right. \\ \left. + 2 \frac{1}{(1-x)_+} \frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz) \right\},$$

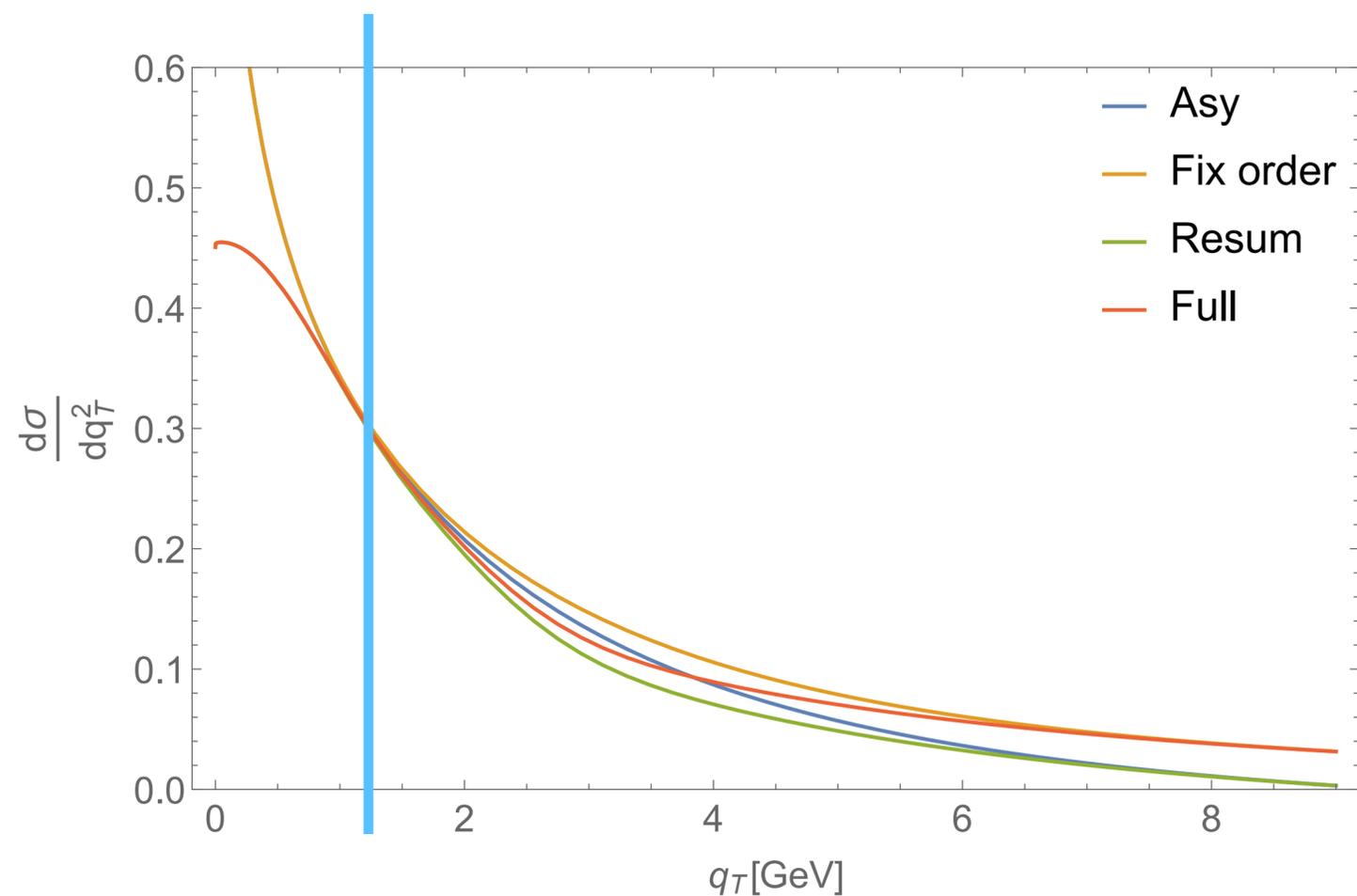
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SOURCE OF W TERM SUPPRESSION – IDEAL SITUATION – LARGE Q

Ideal situation at high Q

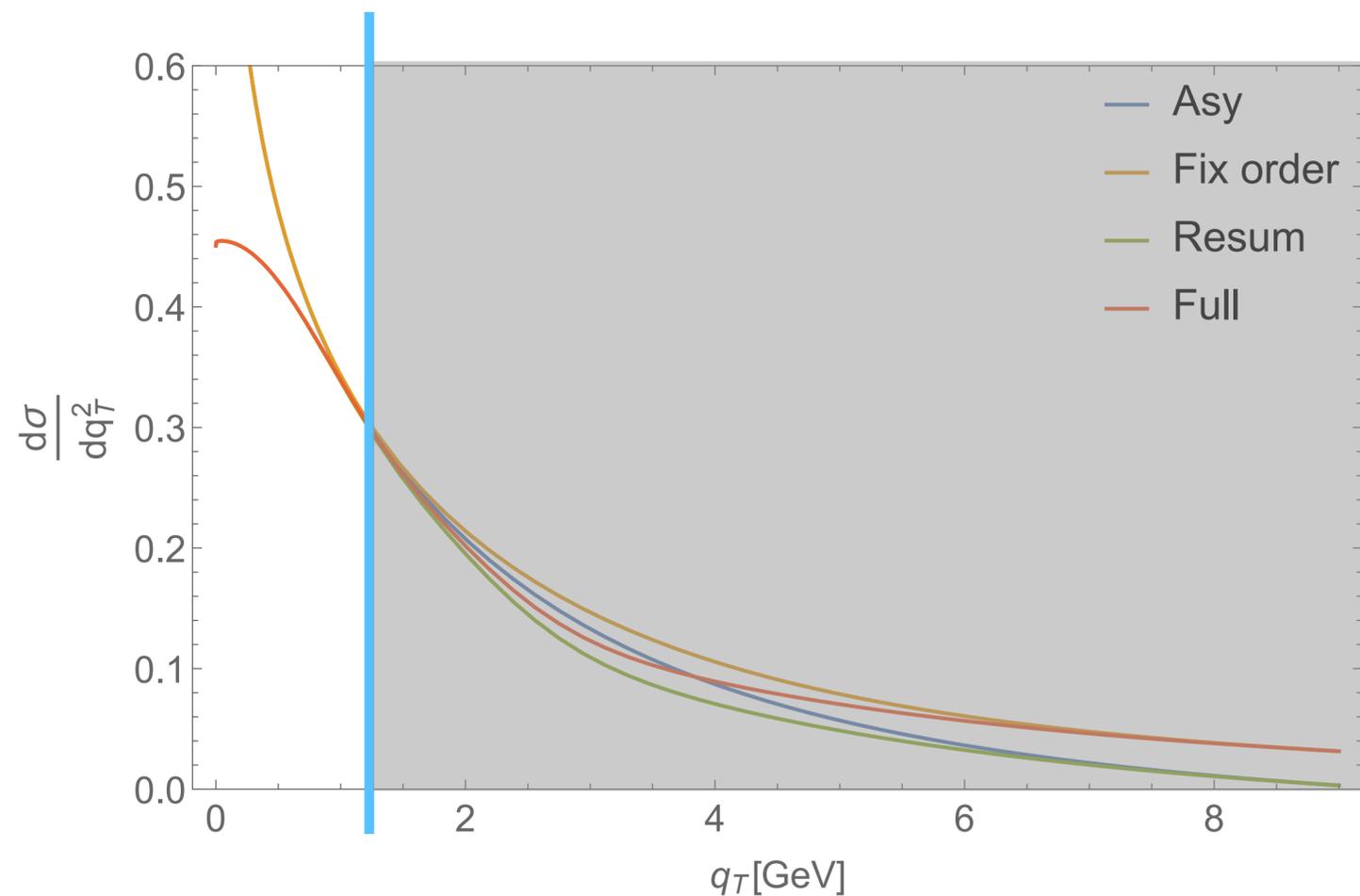


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order

SOURCE OF W TERM SUPPRESSION – IDEAL SITUATION – LARGE Q

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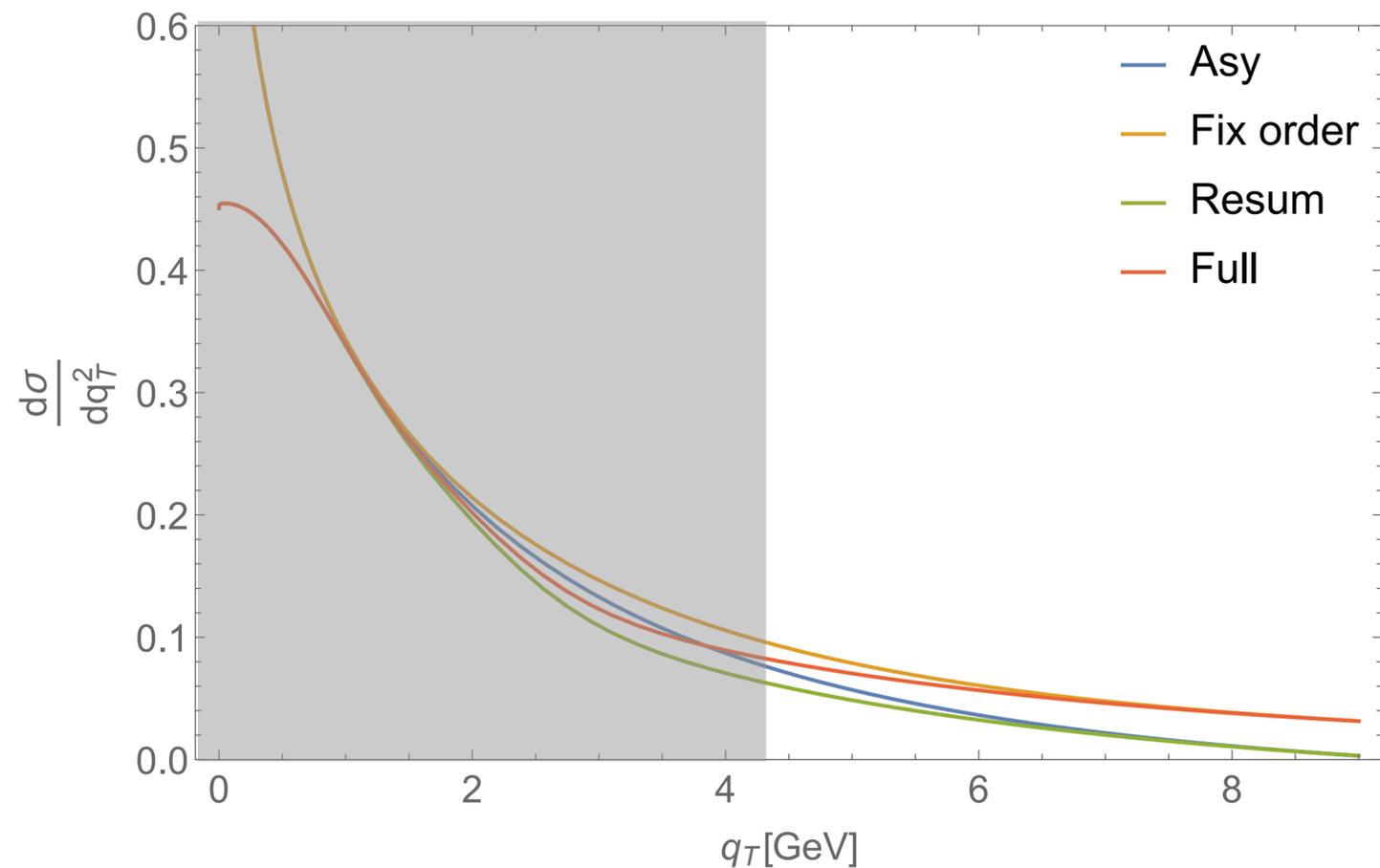


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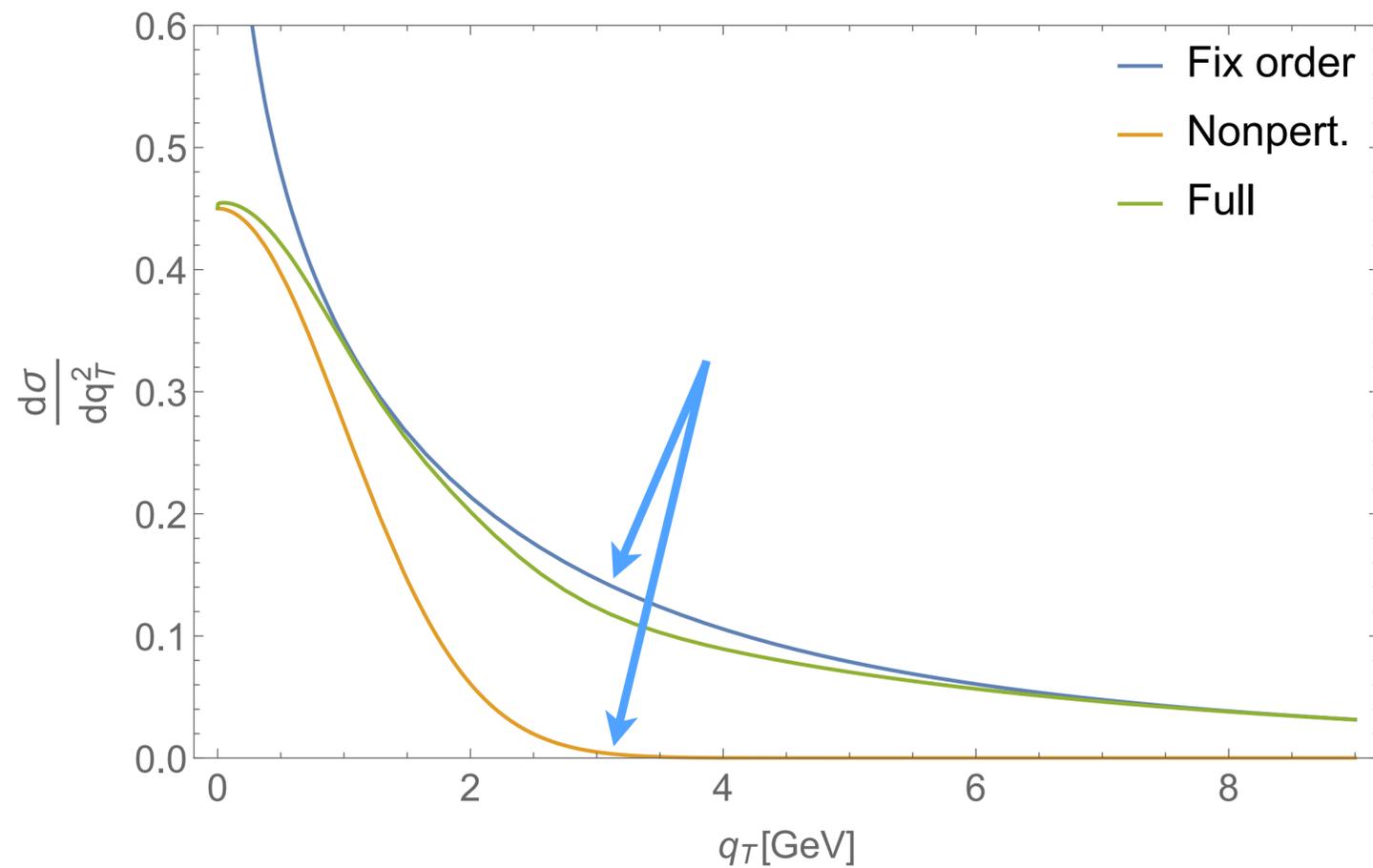


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order
- From a certain value of q_T the total cross section follows the Fixed Order term

SOURCE OF W TERM SUPPRESSION – IDEAL SITUATION – LARGE Q

Ideal situation at high Q

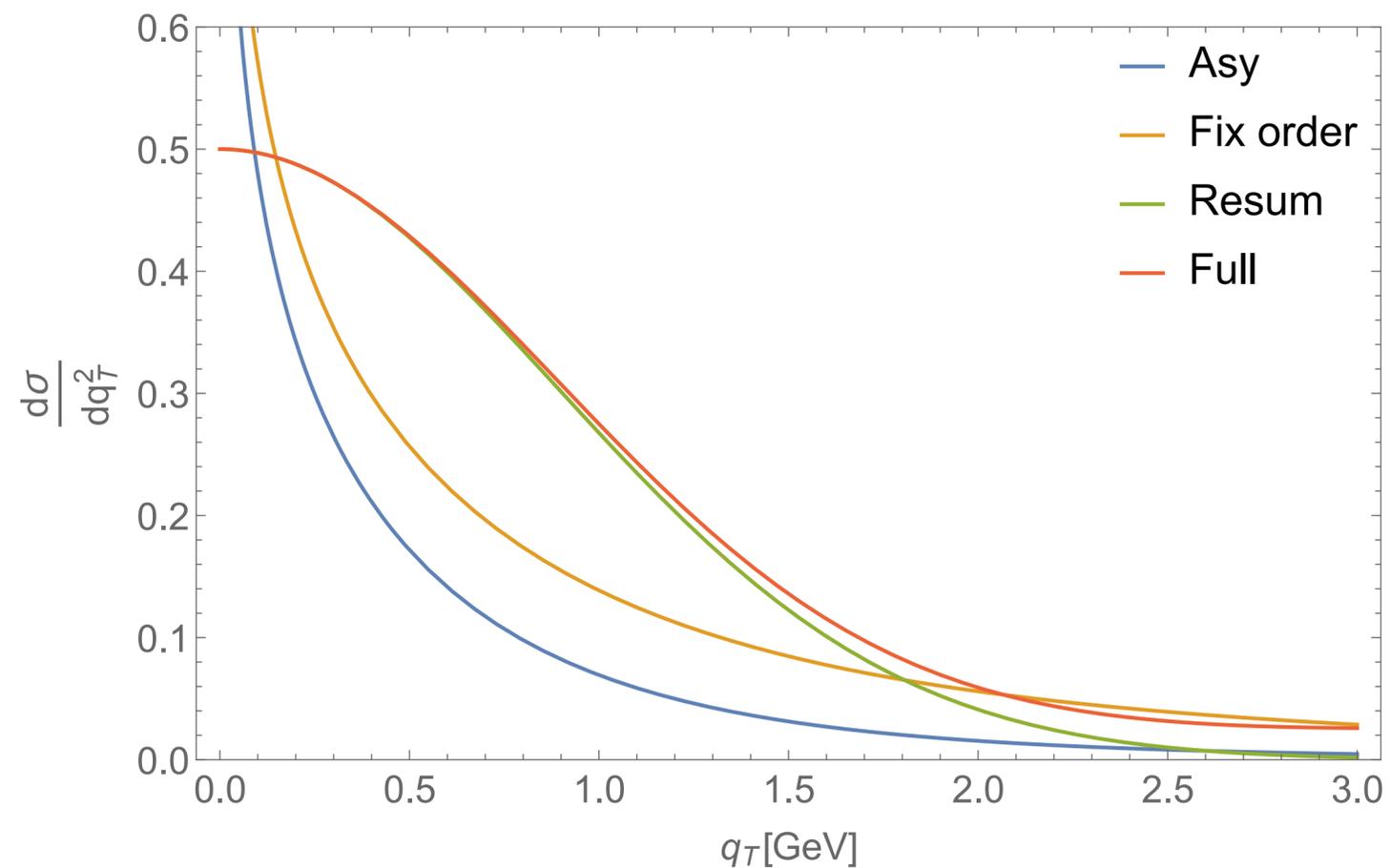


Standard approach

- Collinear result is mostly given by the integral of the Fixed Order
- The Non-Perturbative term is only a small correction

SOURCE OF W TERM SUPPRESSION – IDEAL SITUATION – LOW Q

Ideal situation at low Q

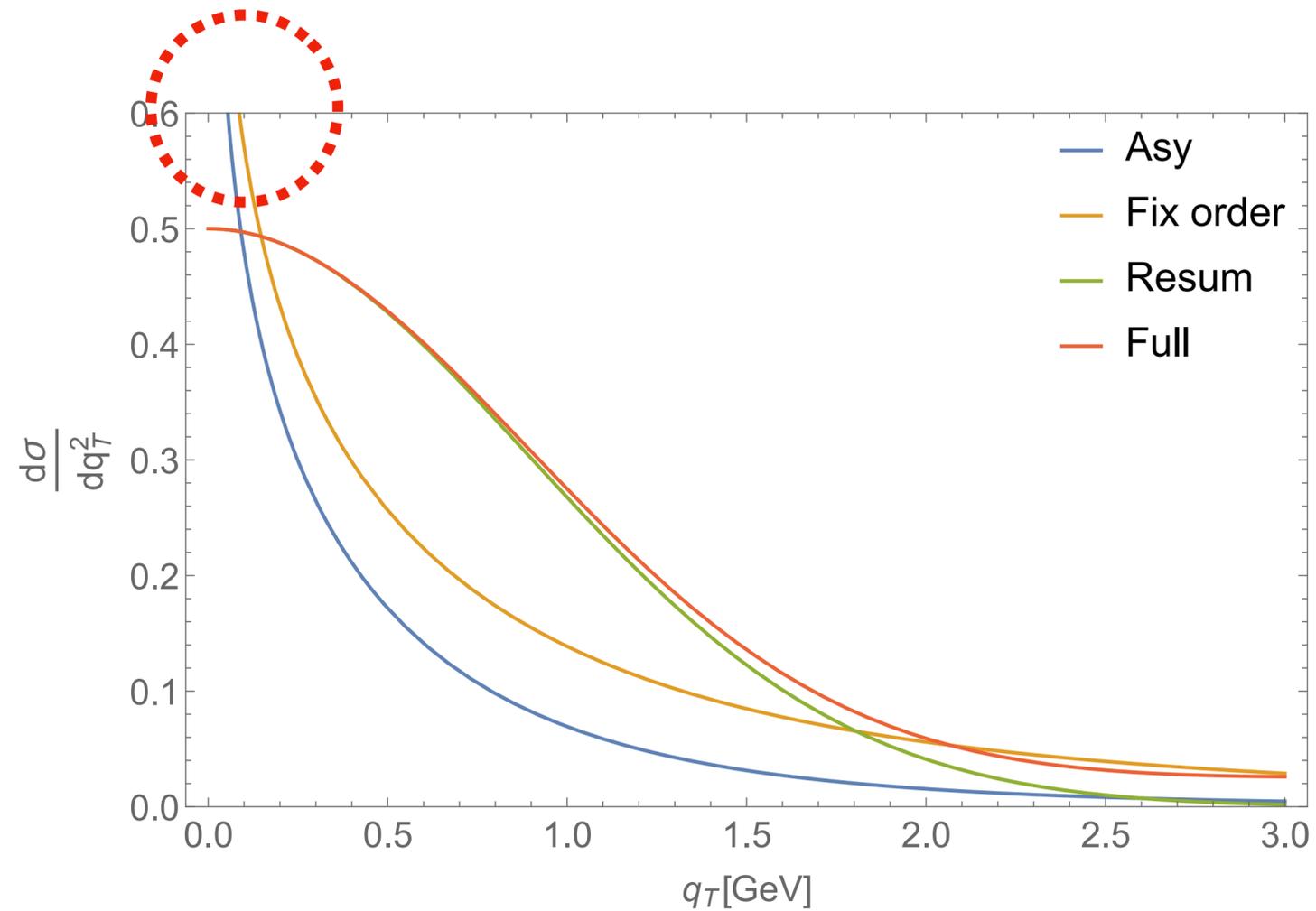


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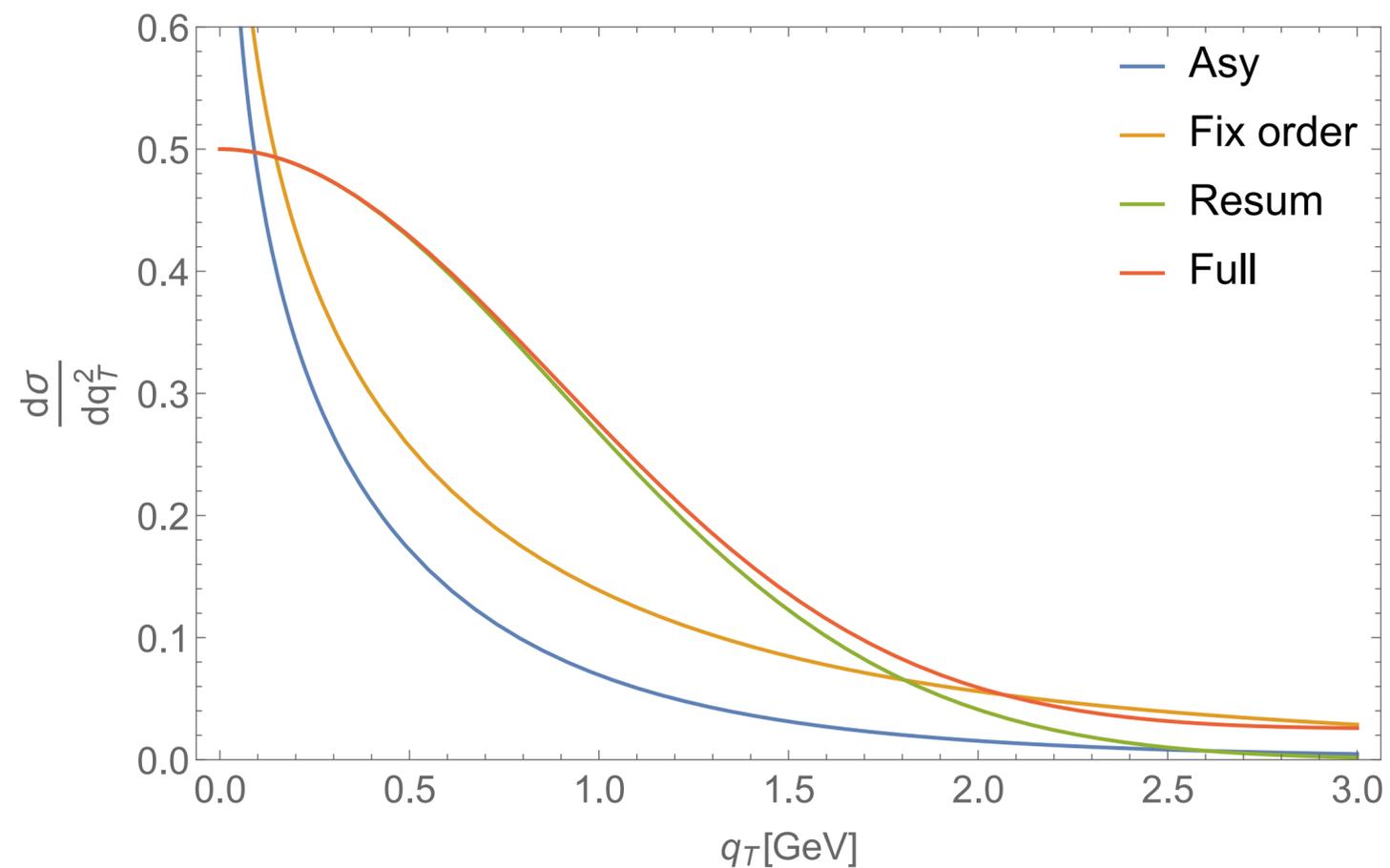


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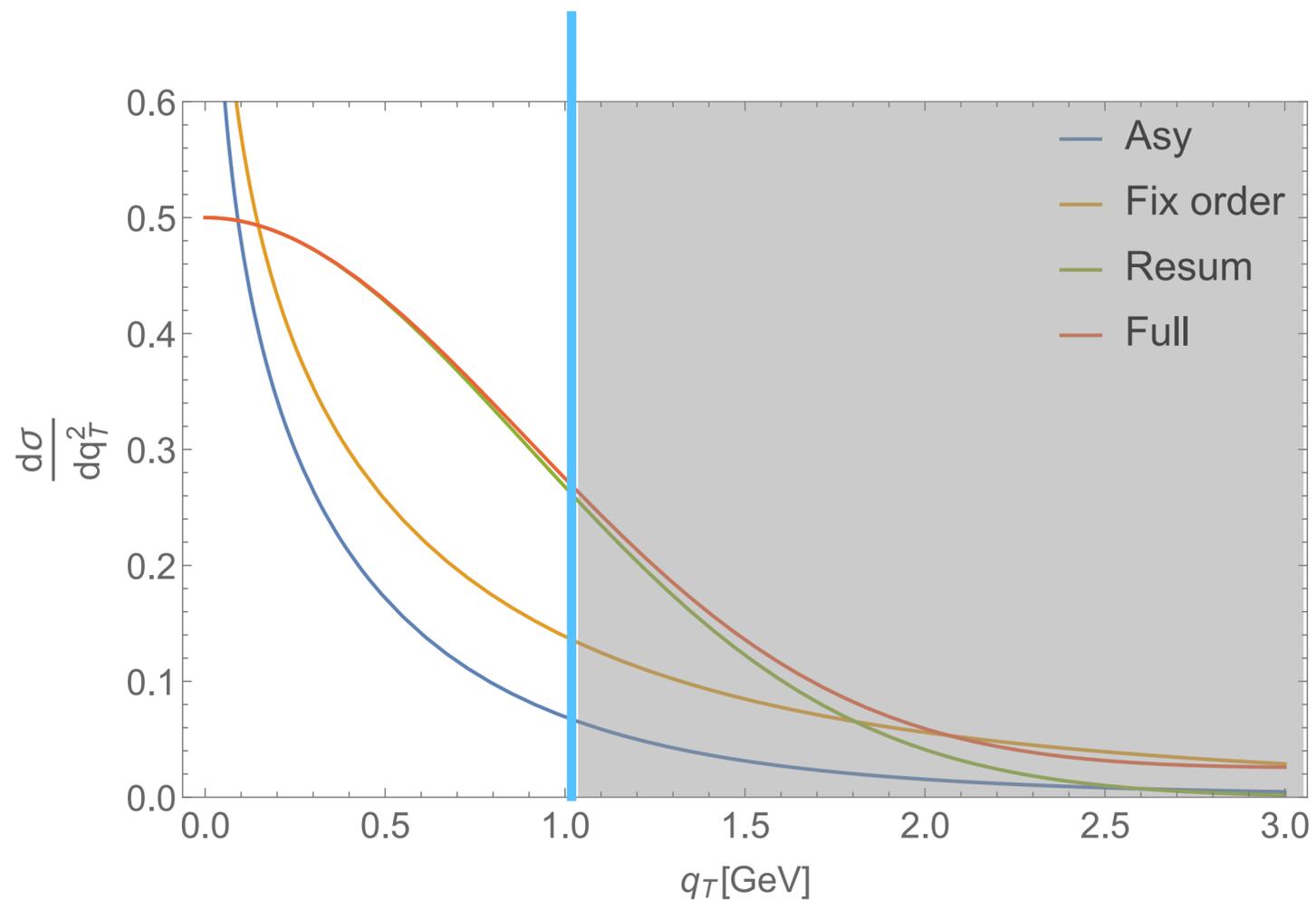


Non-Perturbative approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order OR the Non-Perturbative contributions dominates

SOURCE OF W TERM SUPPRESSION – IDEAL SITUATION – LOW Q

Ideal situation at low Q



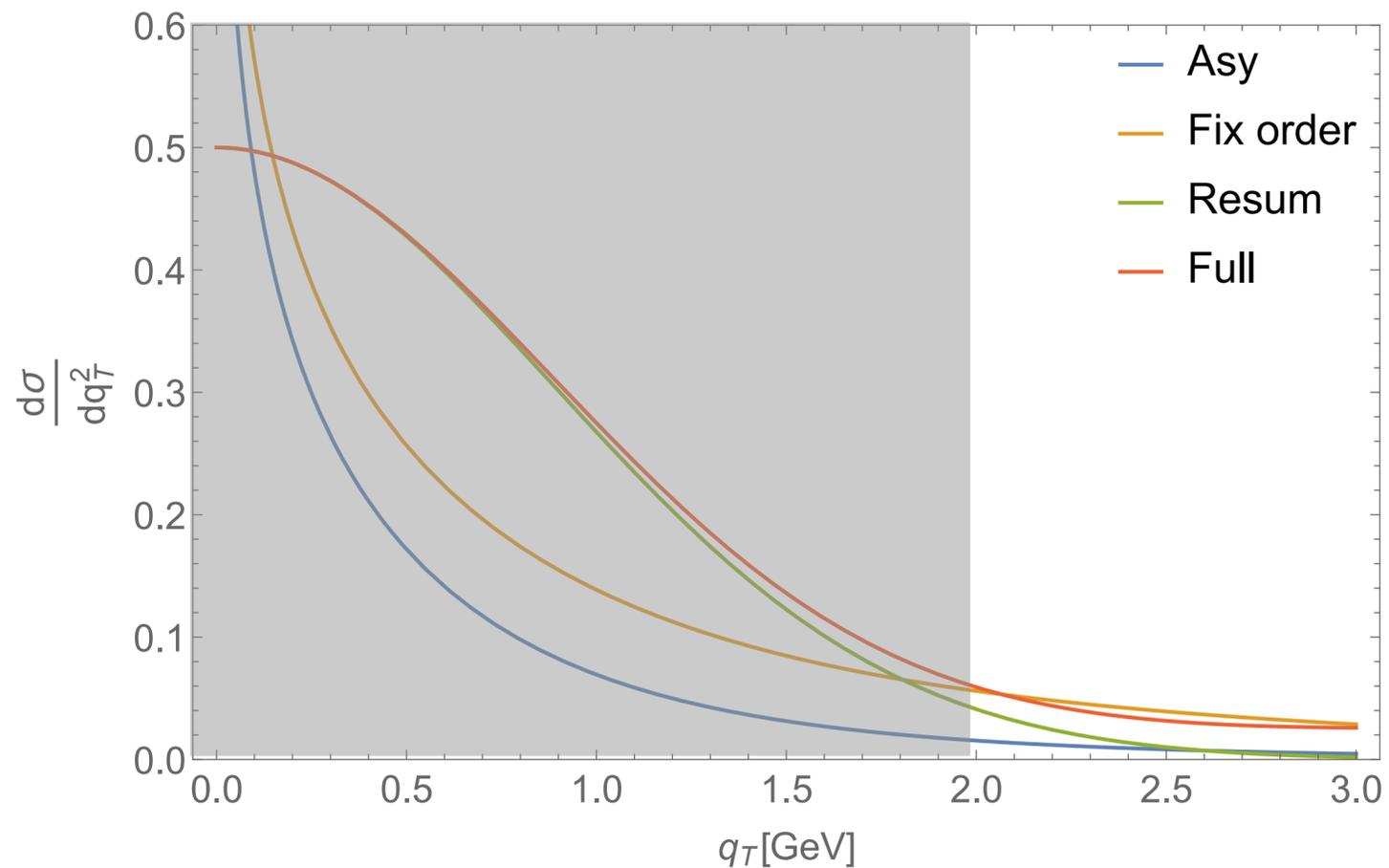
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→ TMD Region

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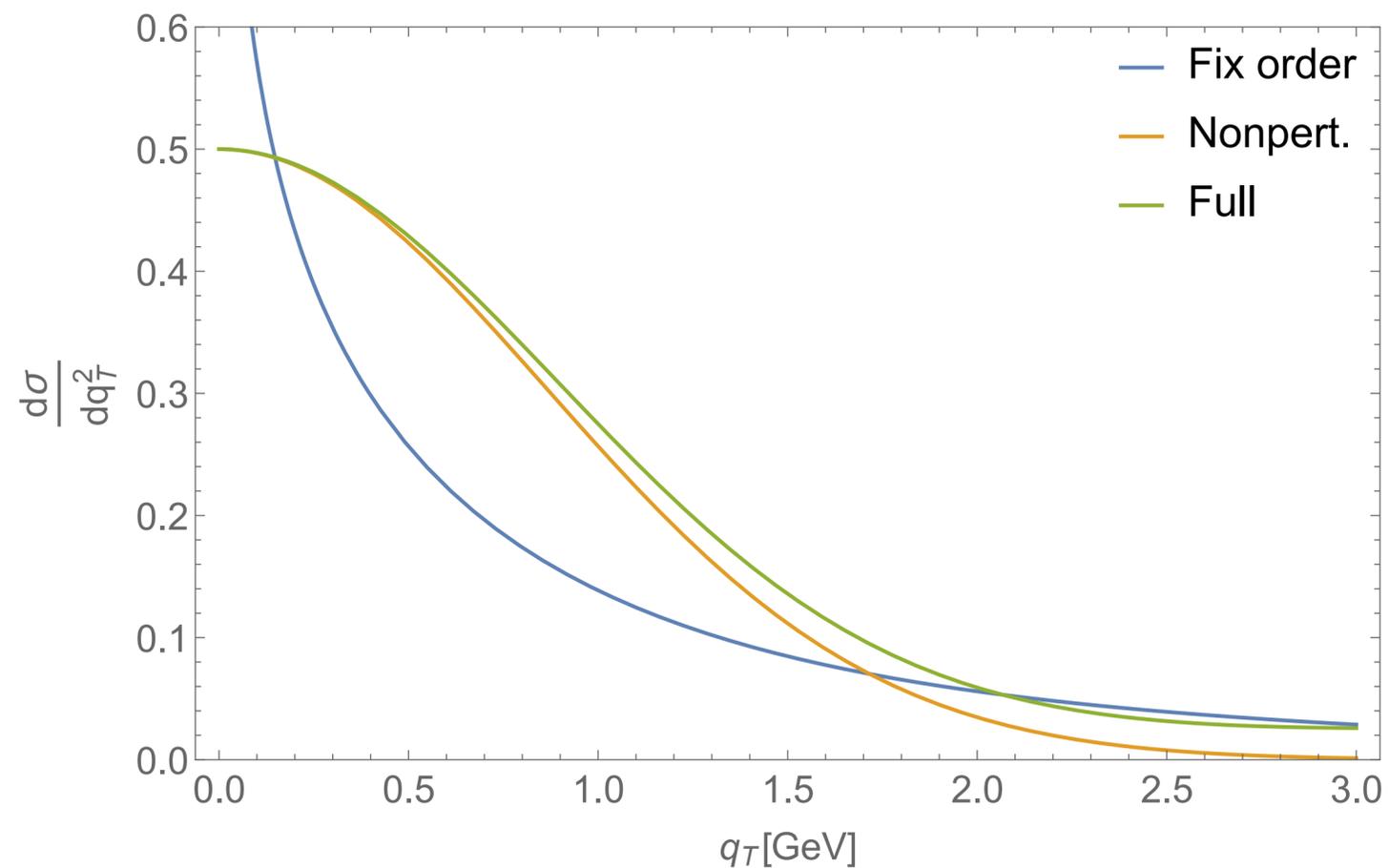


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SOURCE OF W TERM SUPPRESSION – IDEAL SITUATION – LOW Q

Ideal situation at low Q



Non-Perturbative approach

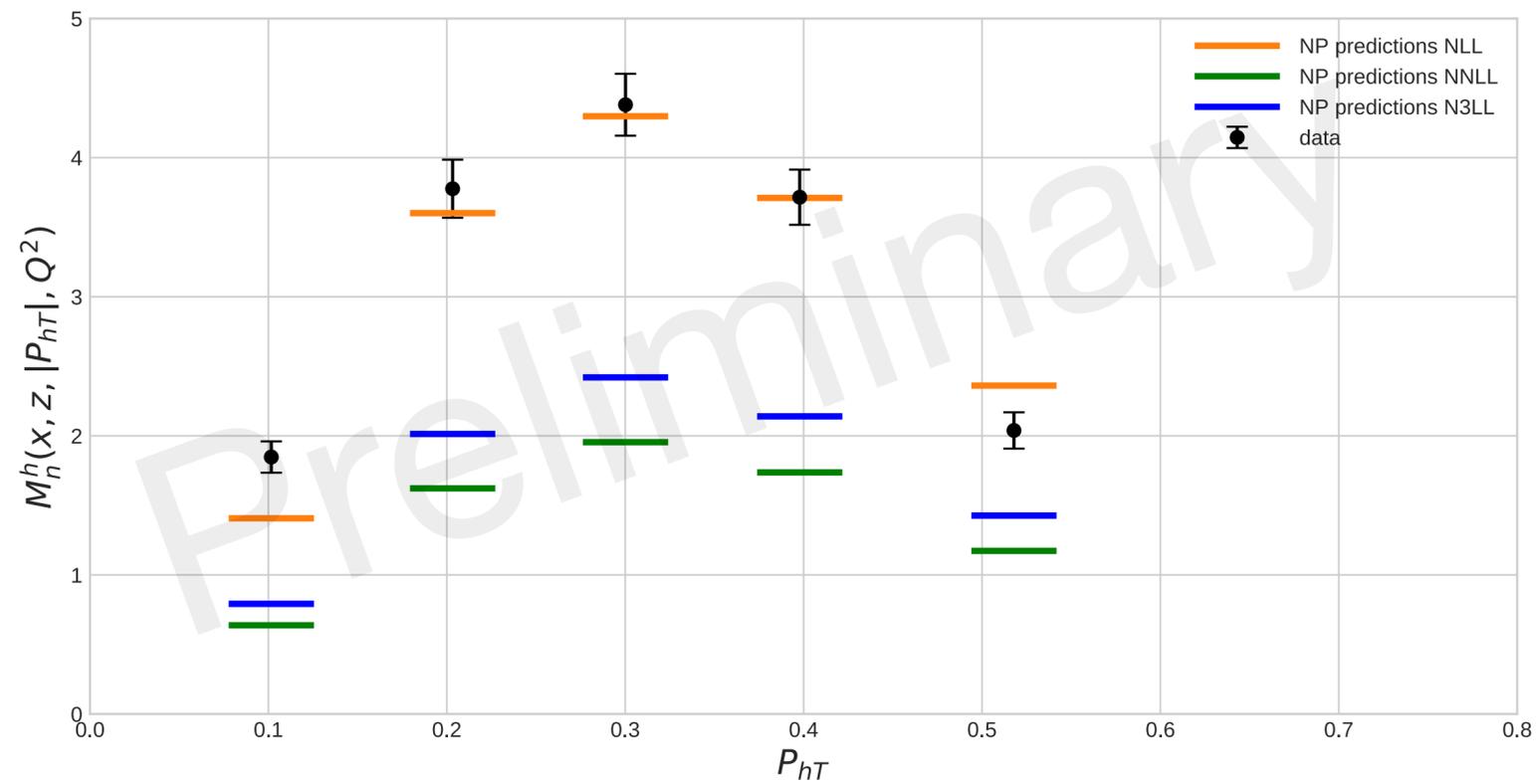
- Collinear result is no more mostly given by the integral of the Fixed Order
- The Non-Perturbative term is not only a small correction, but is even larger than the Fixed Order contribution

SOURCE OF W TERM SUPPRESSION

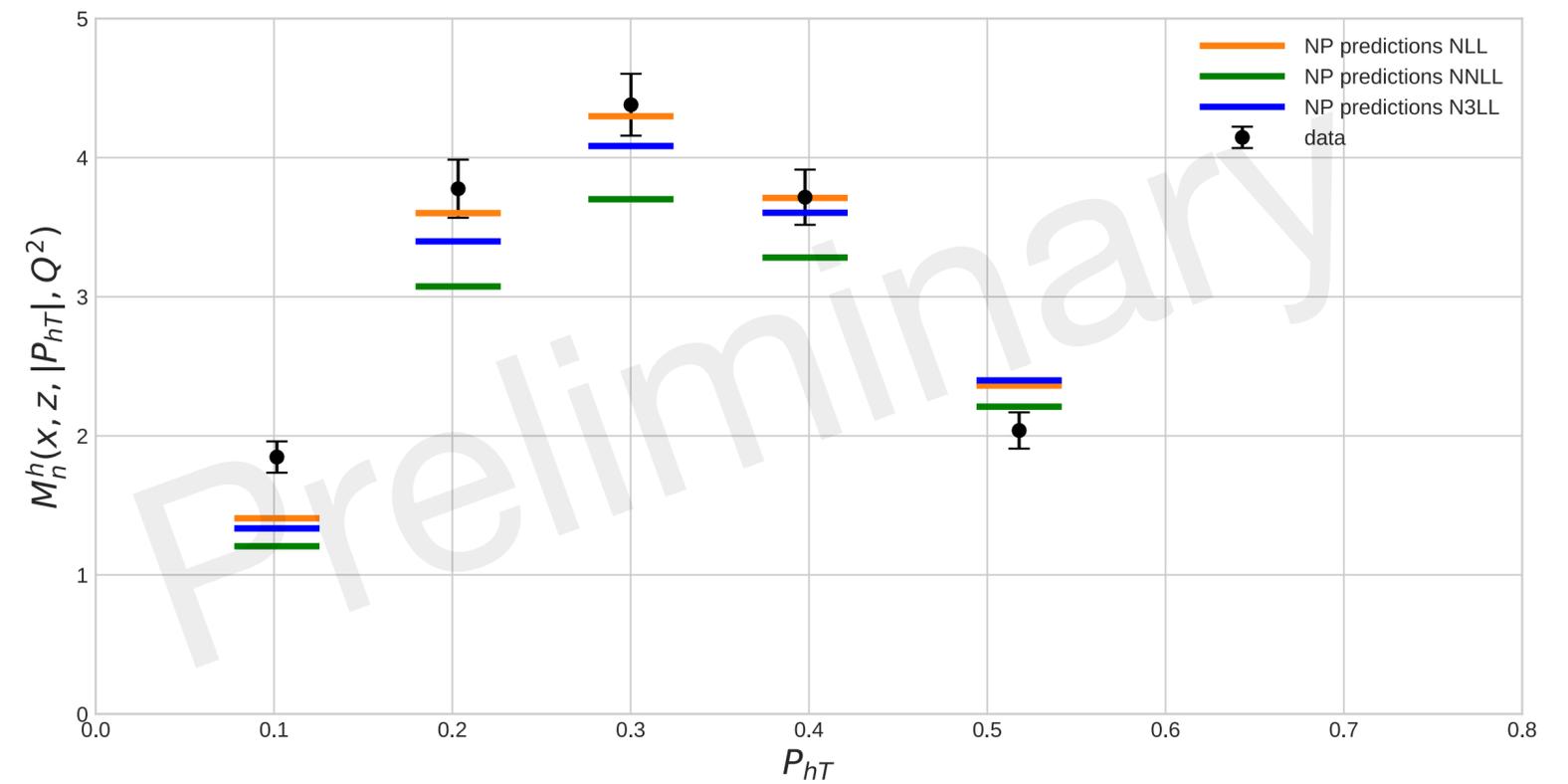
Present situation at low Q

HERMES multiplicity

Full Hard Factor



Hard Factor = 1

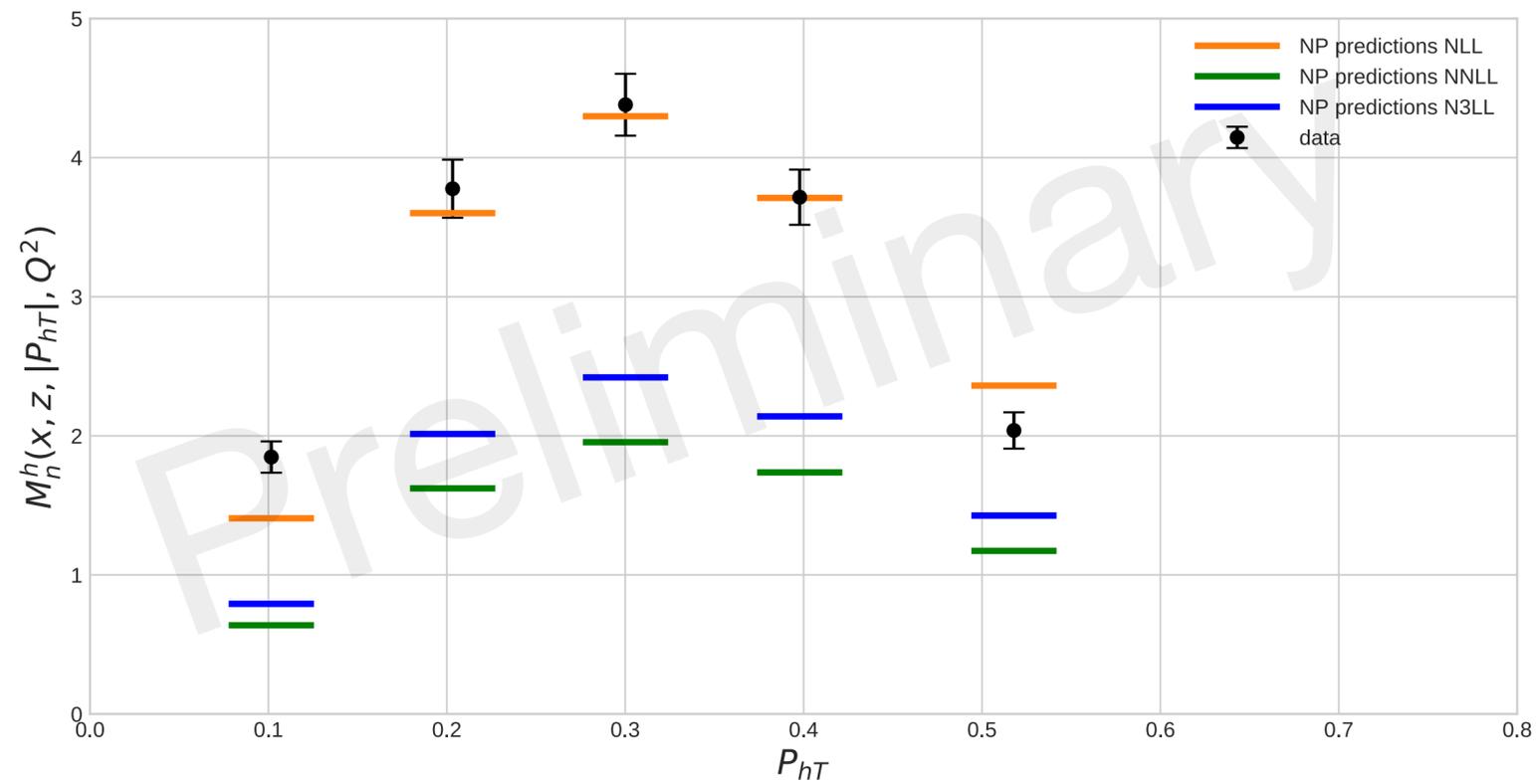


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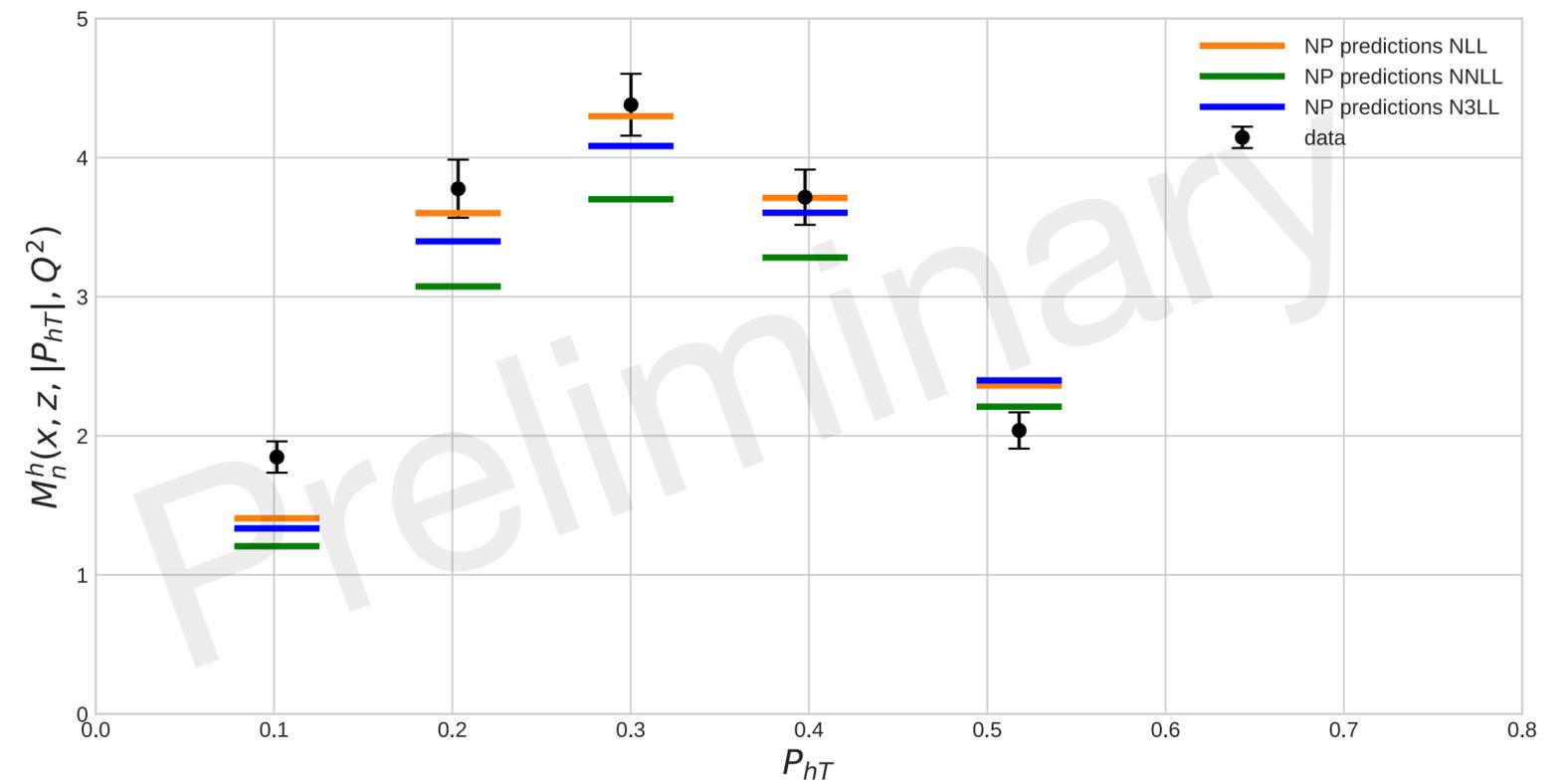
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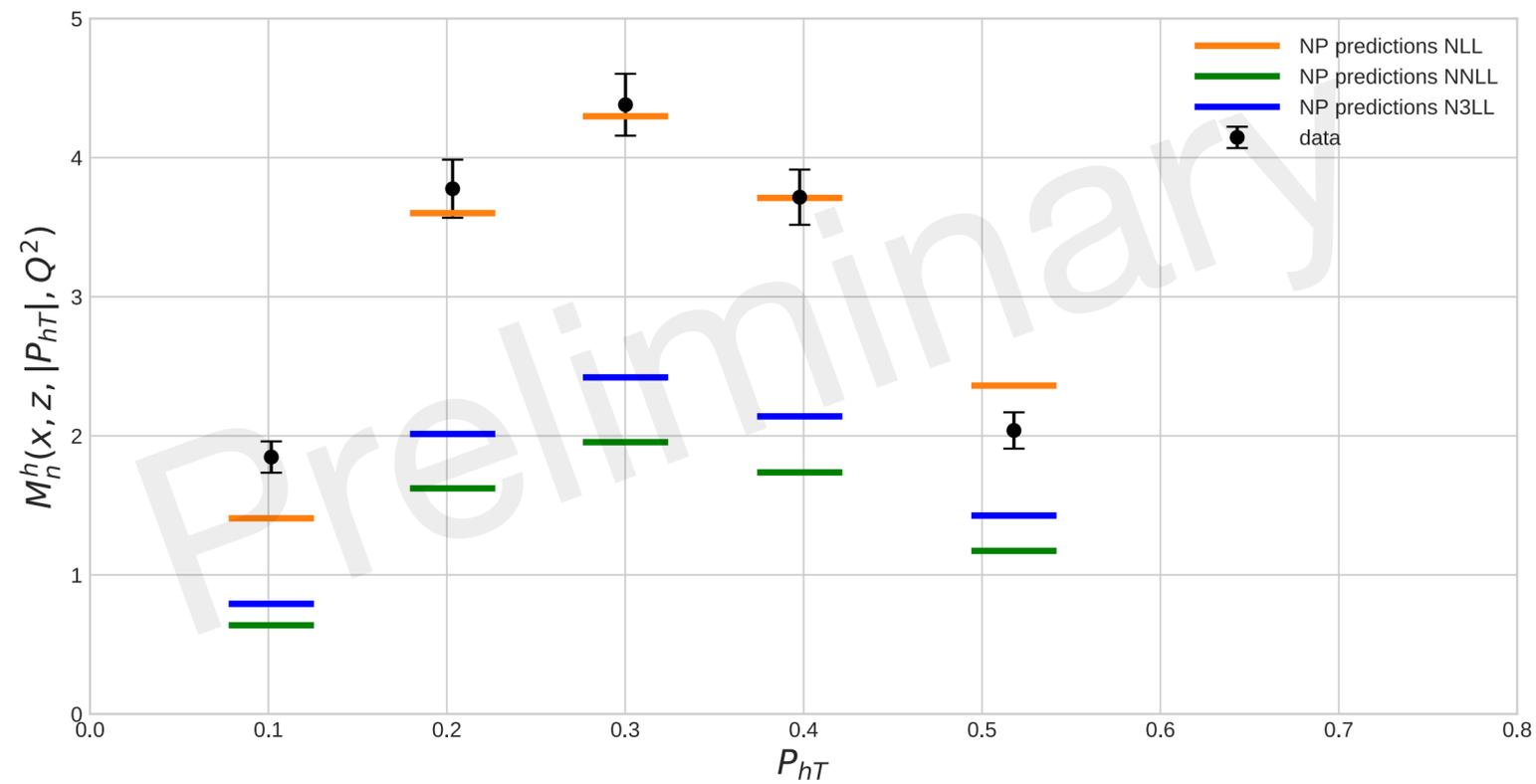


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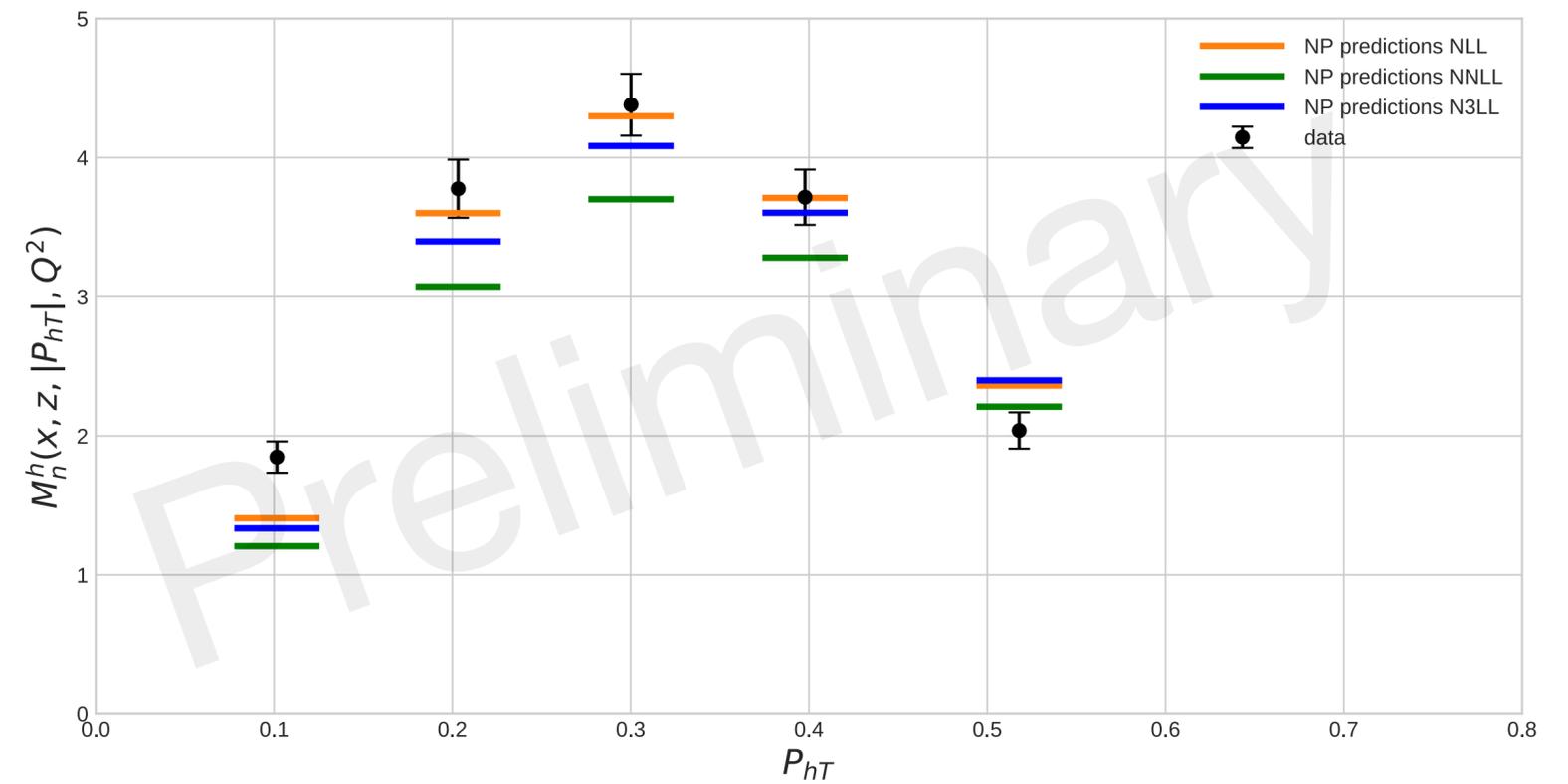
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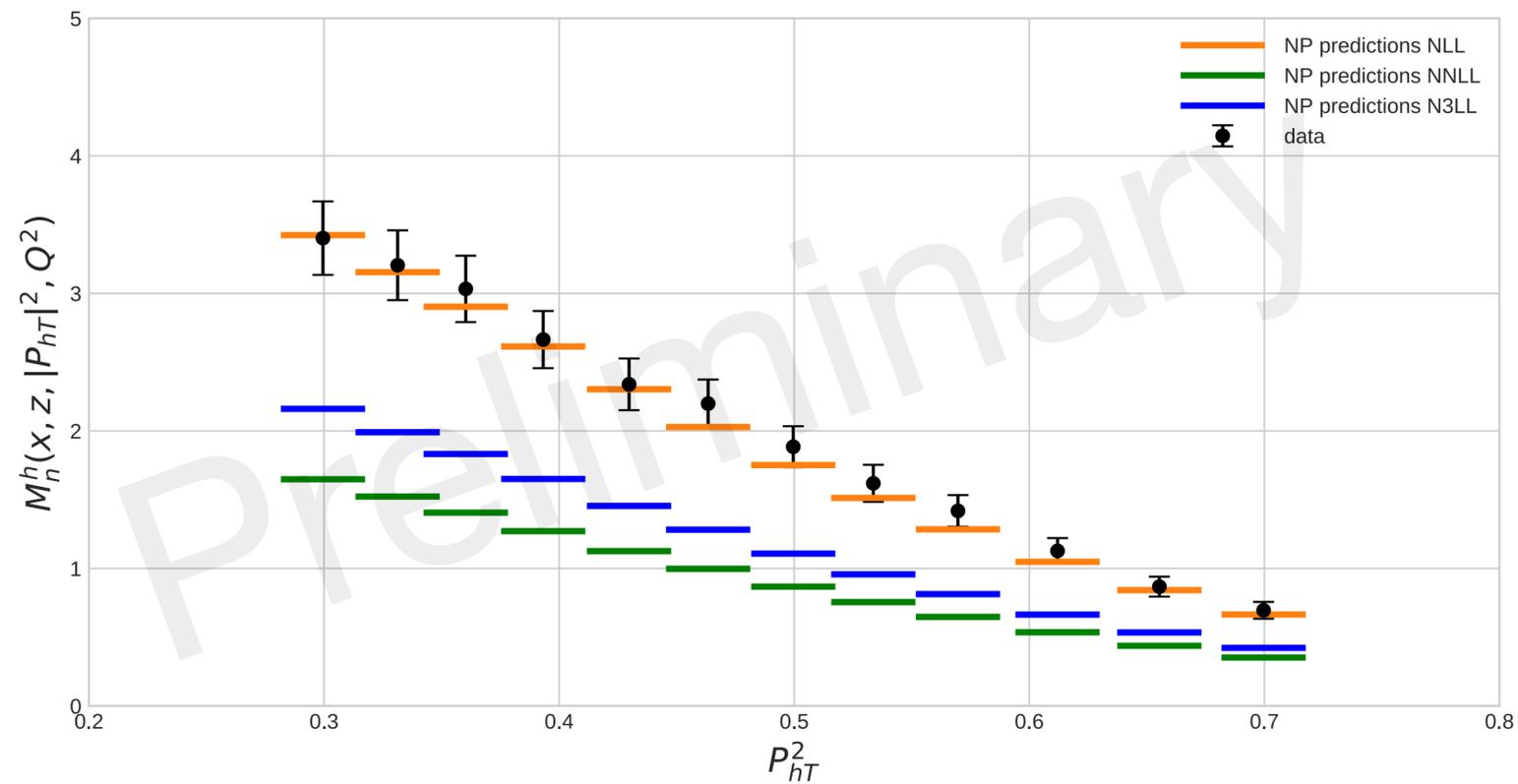


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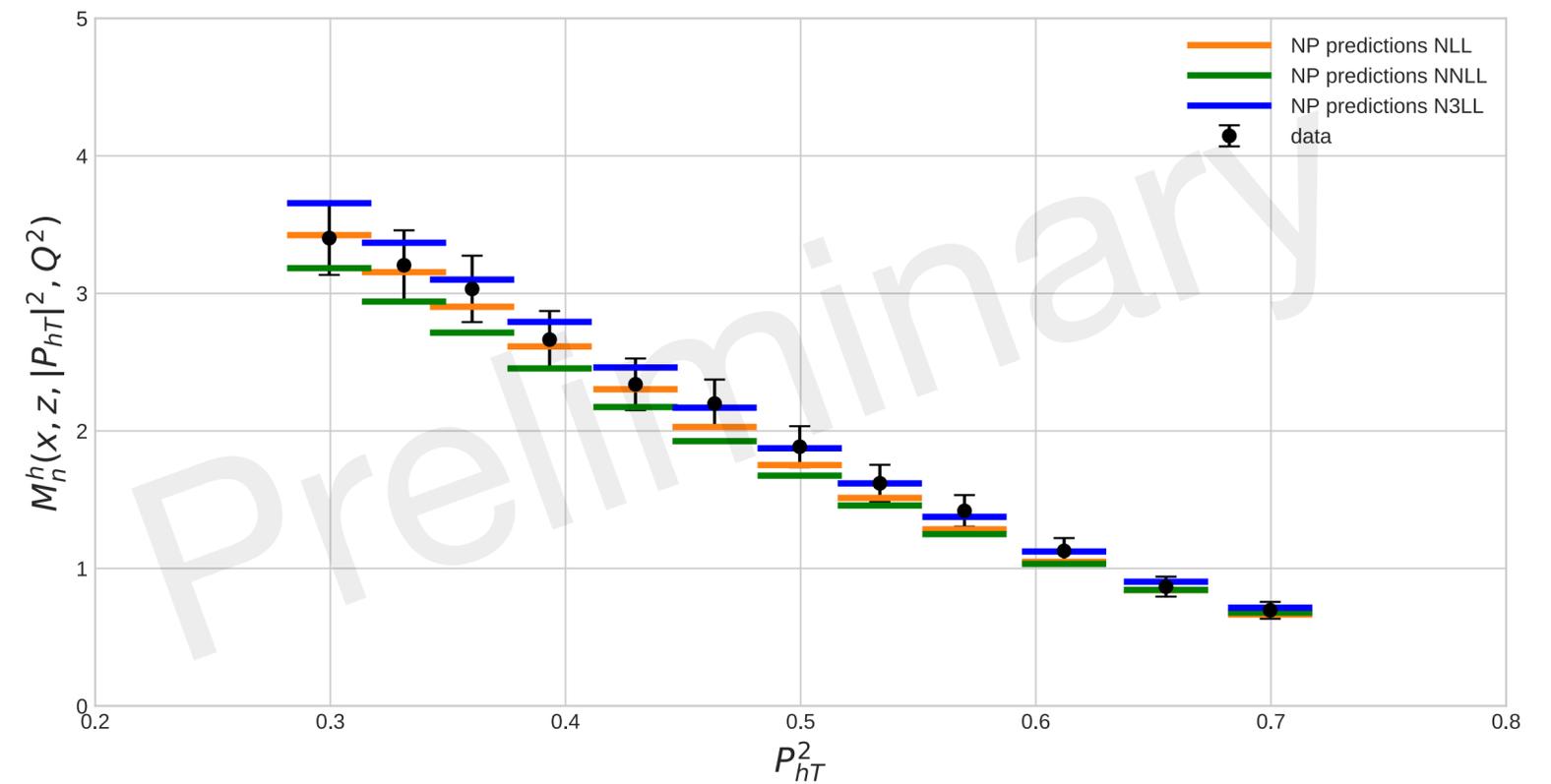
Present situation at low Q

COMPASS multiplicity

Full Hard Factor



Hard Factor = 1

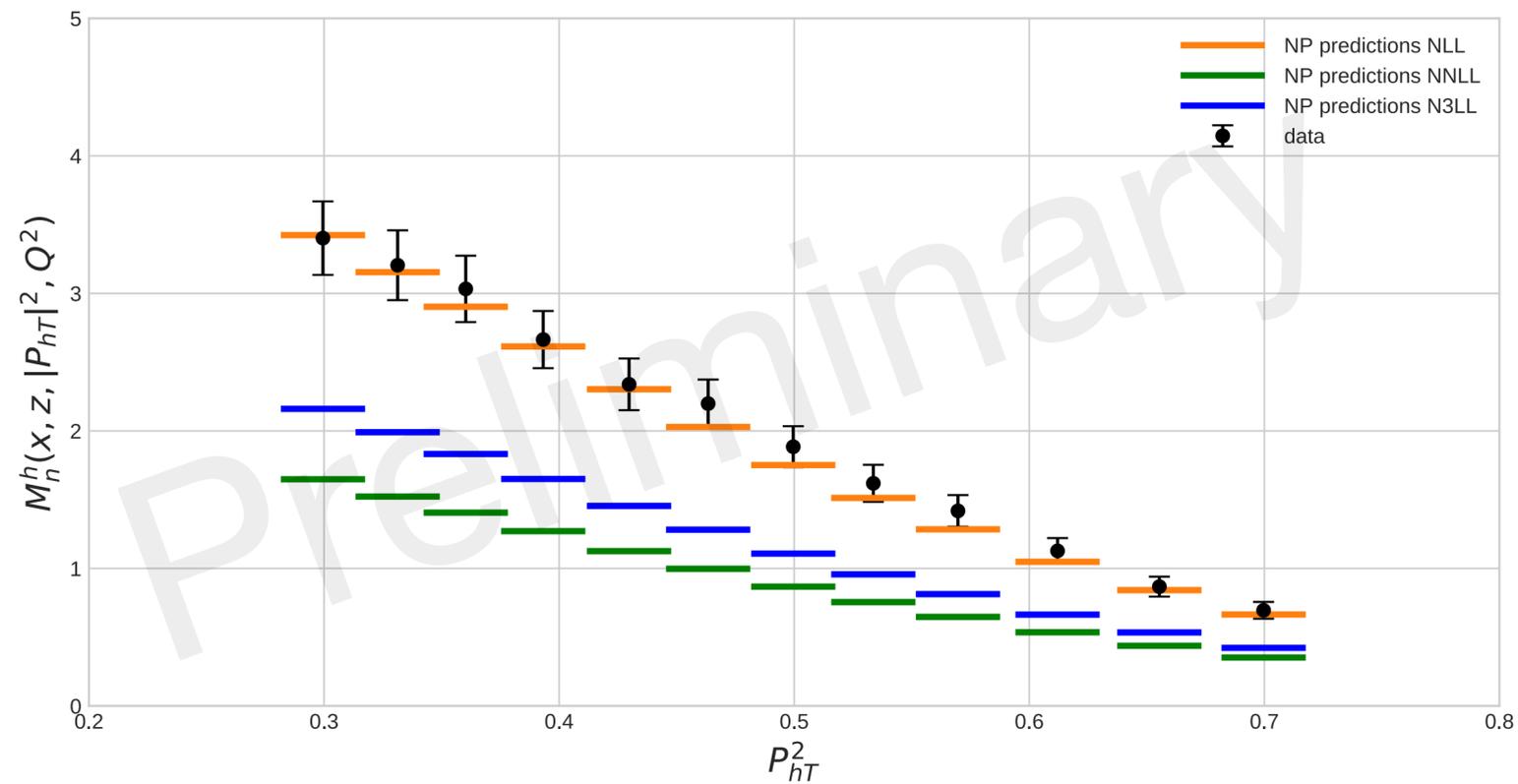


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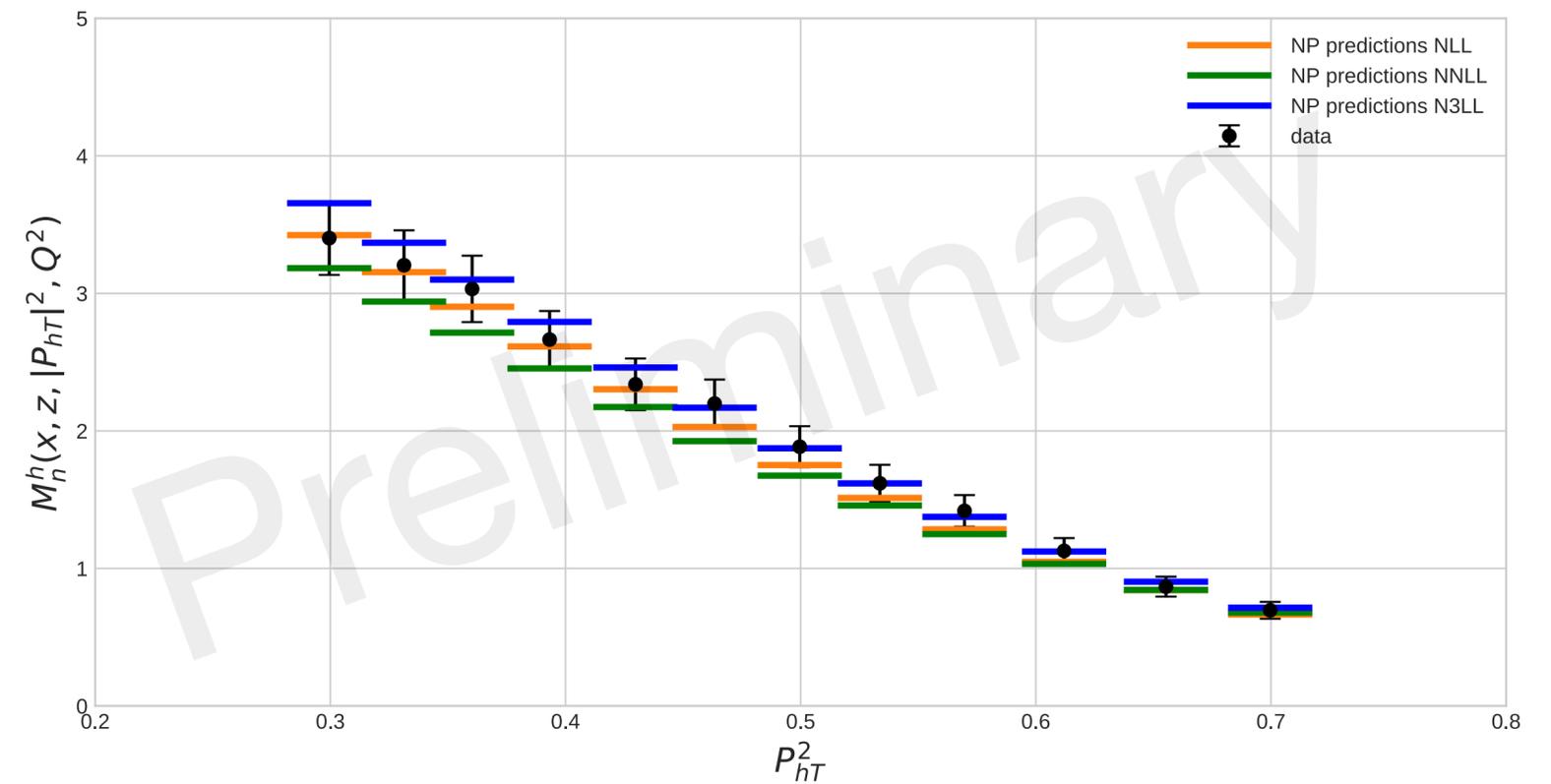
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