Perturbation Theory of Non-Perturbative QCD

A massive expansion from first principles

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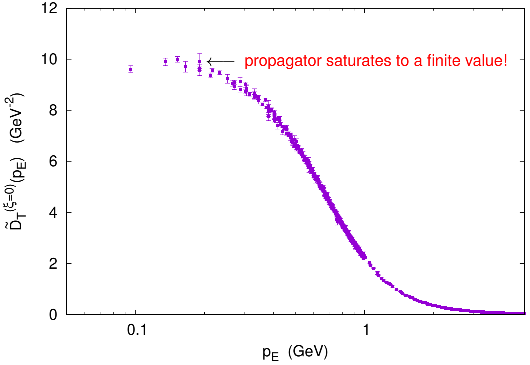
OUTLINE & REFERENCES

- Screened massive expansion in a general covariant gauge
  F. Siringo, NPB 907 (2016); F. Siringo, G.C., PRD 98 (2018);
  F. Siringo, PRD 99+100 (2019)
- RG analysis of the strong interactions
  G.C., F. Siringo, PRD 102 (2020)
- Dynamical mass generation in the quark sector
  G.C., D. Rizzo, M. Battello, F. Siringo, PRD 104 (2021)
- ... and more
  G.C., F. Siringo, PRD (2016-2021)

The aim is to formulate a predictive, self-contained, optimized perturbation theory for low-energy QCD from first principles

Motivation 1: dynamical mass generation (DMG)

Relatively recent results (2007-2009) from lattice calculations:
due to the strong interactions (no Higgs mechanism!)
at low energies the gluons acquire a mass!

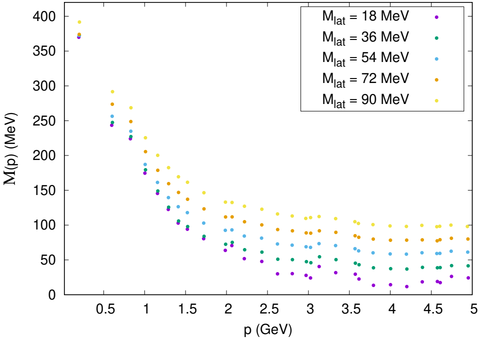


Duarte et al., Phys. Rev. D 94 (2016)

Pure YMT. No. of lattice sites up to 128^4, volumes up to $\sim (27\ \mathrm{fm})^4$: huge lattices!

Motivation 1: dynamical mass generation (DMG)

Something similar happens to the quarks: violation of the (approximate) chiral symmetry – due to the strong interactions
at low energies the quark mass is enhanced



Kamleh et al., Phys. Rev. D 71 (2005)
Quenched QCD (on a smaller lattice) - similar results for the unquenched theory

Motivation 1: dynamical mass generation (DMG)

Dynamical mass generation is of particular interest:

From a phenomenological standpoint

- since massive d.o.f. modify the IR behavior of the strong interactions – or perhaps, should we say, they allow us to describe it more faithfully?

Motivation 1: dynamical mass generation (DMG)

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From a theoretical standpoint

- since it cannot be described at any finite order in standard perturbation theory – for the gluons, gauge invariance forbids it; for the quarks, the corrections are too small
- it is thus a "non-perturbative" effect of the strong interactions

Motivation 2: Landau pole in the QCD coupling constant

Well known: ordinary QCD-PT breaks down in the IR

- From the standpoint of ordinary PT, the other side of the coin of UV-asymptotic freedom is the strong coupling regime in the IR: the running coupling $\alpha_s(p)$ blows up at $p \sim \Lambda_{\text{QCD}}$, making ordinary PT useless
- Nonetheless, the IR breakdown is **not** the main reason why DMG cannot be described in ordinary PT: as discussed, there are other constraints (e.g. gauge-invariance)
- On the other hand, it prevents us from doing "simple" and improvable perturbative calculations in the low-energy regime of QCD

Choice of an expansion point for QCD

Is the ordinary (massless) expansion point of QCD inadequate for describing the IR phenomenology?

- Can a change of expansion point account for gluon DMG?
- Does the change lead to a viable perturbation theory in the IR (i.e., no Landau pole, sufficiently small coupling, etc.)?
- What results do we get if the gluons are treated as massive already at tree-level?
- Can DMG for the quarks (i.e., mass enhancement) be accounted for by similar methods?

Standard BRST invariant SU(N) YM Lagrangian:

$$S = S_0 + S_I = \left[S_0 + \frac{1}{2} \int A_\mu \, \delta\Gamma^{\mu\nu} \, A_\nu \right] + \left[S_I - \frac{1}{2} \int A_\mu \, \delta\Gamma^{\mu\nu} \, A_\nu \right]$$

$$\begin{cases} \delta\Gamma^{\mu\nu} = i \left[\Delta_m^{-1\,\mu\nu} - \Delta_0^{-1\,\mu\nu} \right] = m^2 \, t^{\mu\nu}(p) \quad \text{(2-point vertex)} \\ \\ \Delta_m^{\mu\nu}(p) = \dfrac{-i}{p^2 - m^2} \, t^{\mu\nu}(p) + \dfrac{-i\xi}{p^2} \, \ell^{\mu\nu}(p) \quad \text{(free propagator)} \end{cases}$$

P.T. with the new vertex set

$$\mathcal{L}_3 = -g f_{abc} (\partial_\mu A_{a\nu}) A_b^\mu A_c^\nu , \qquad \mathcal{L}_4 = -\frac{1}{4} g^2 f_{abc} f_{ade} A_{b\mu} A_{c\nu} A_d^\mu A_e^\nu$$

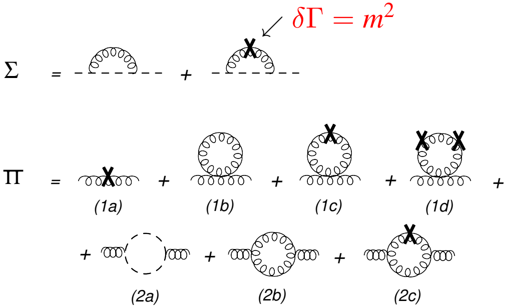
$$\mathcal{L}_{gh} = -g f_{abc} (\partial_\mu \omega_a^\star) \omega_b A_c^\mu , \qquad \mathcal{L}_m = -\frac{1}{2} \delta_{ab} \delta\Gamma_{\mu\nu} A_a^\mu A_b^\nu$$

Screened massive expansion of pure YMT

F.S. Nucl. Phys. B 907 (2016); F.S.+G.C. Phys. Rev. D 98 (2018); and more

Non-trivial mechanism for dynamical mass generation:

$$\Delta_T(p) = \frac{-i}{(p^2 - m^2) - \Pi^T} = \frac{-i}{(p^2 - m^2) - (-m^2 + \Pi^T_{Loops})} = \frac{-i}{p^2 - \Pi^T_{Loops}}$$



- The pole shift cancels at tree level
- All spurious diverging mass terms cancel without ren. c.t.'s
- Standard UV behavior

Standard UV behavior $\Longrightarrow \Pi^{finite} \sim -\frac{Ng^2}{(4\pi)^2} p^2 \left(\frac{13}{6} - \frac{\xi}{2}\right) \log \frac{p^2}{\mu^2}$

Screened massive expansion of pure YMT

F.S. Nucl. Phys. B 907 (2016); F.S.+G.C. Phys. Rev. D 98 (2018); and more

Setting $s = p^2/m^2 \leftarrow$ the scale m cannot be fixed by theory!

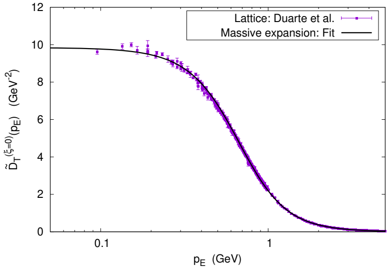
$$\Pi^T_{Loops} = -\frac{3Ng^2}{(4\pi)^2}\, p^2\, \left[F(s) + \xi\, F_\xi(s)\right] + \Pi^{diverg.} + \Pi^{c.t.}$$

After subtraction (field-strength renormalization):

$$\boxed{\Delta(p) = \frac{Z}{p^2\left[F(s) + \xi\, F_\xi(s) + F_0\right]}} \qquad \mu \Leftrightarrow F_0$$

- Results depend on $\mu/m \rightarrow F_0$
- Propagator saturates at $p = 0$ $\longrightarrow$ gluon DMG!

Best fit at $\xi = 0$: $\begin{cases} m = 0.654 \text{ GeV} \\ F_0 = -0.887 \end{cases}$



- Via the exact BRST symmetry and the Nielsen identities, the poles p_0 are gauge-invariant (i.e. ξ-independent)

$$\text{N.I.} \rightarrow \boxed{\frac{\partial}{\partial \xi}\frac{1}{\Delta(p)} = G^T(p)\left[\frac{1}{\Delta(p)}\right]^2} \qquad G \sim \langle T\left[D^\mu \omega_a A_a^\nu \omega_b^\star B_b\right]\rangle$$

$$\frac{1}{\Delta\left(p_0(\xi)\right)} = 0; \qquad \frac{\mathrm{d}}{\mathrm{d}\xi}\frac{1}{\Delta\left(p_0(\xi)\right)} = 0 \qquad \Longrightarrow \qquad \boxed{\frac{\mathrm{d}}{\mathrm{d}\xi}\, p_0(\xi) = 0}$$

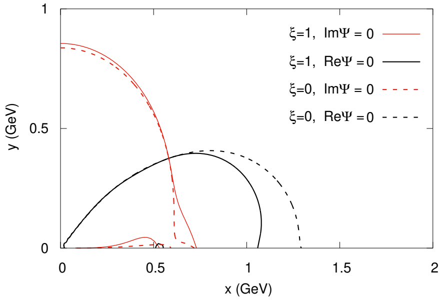
- In the S.M.E. this is not automatic at finite order, but can be enforced by tuning $m^2(\xi)$ and $F_0(\xi)$.

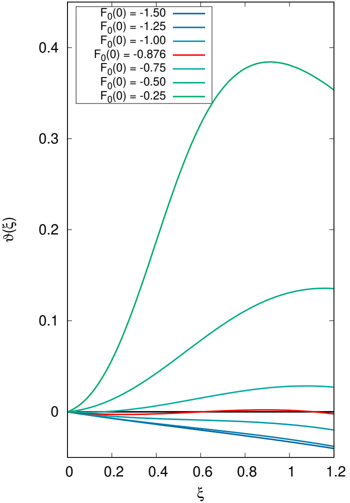
- We noticed that with the fitted F_0 the **phase** of the residue also is almost ξ-independent $\Longrightarrow$

reverse the reasoning and determine F_0 by **fixing** the phase

Optimized S.M.E. of pure YMT

Optimization by ξ-independence of principal part – F.S.+G.C. Phys. Rev. D 98 (2018)





$$F_0(0) = -0.876, \quad m_0 = m(0) = 0.656 \text{ GeV}, \quad Z(0) = 2.684$$

$$|\theta(\xi)| < 2.76 \cdot 10^{-3}, \qquad 0 < \xi < 1.2$$

$$F_0(\xi) \approx -0.8759 - 0.01260\xi + 0.009536\xi^2 + 0.009012\xi^3$$

$$m^2(\xi)/m_0^2 \approx 1 - 0.39997\xi + 0.064141\xi^2$$

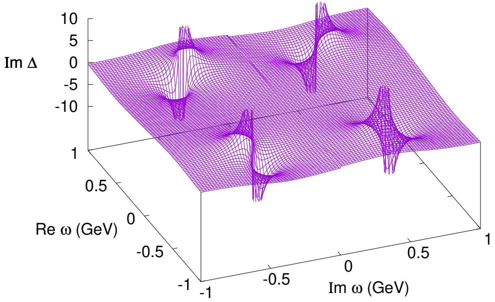
$$z_0/m_0 = 0.8857 + 0.5718\, i, \quad t_R = \mathrm{Im}\, R(0)/\, \mathrm{Re}\, R(0) = 3.132$$

$$M = 0.581 \text{ GeV}, \gamma = 0.375 \text{ GeV} \quad \text{(invariant pole)}$$

Complex-conjugate poles and confinement

F.S.+G.C. Phys. Rev. D 98 (2018)

In the long wavelength limit $p^2 = \omega^2 - \mathbf{k}^2 \to \omega^2$ the poles are at $\omega = \pm(M \pm i\gamma)$ where $M = \mathbf{0.581}$ **GeV** and $\gamma = \mathbf{0.375}$ **GeV**.

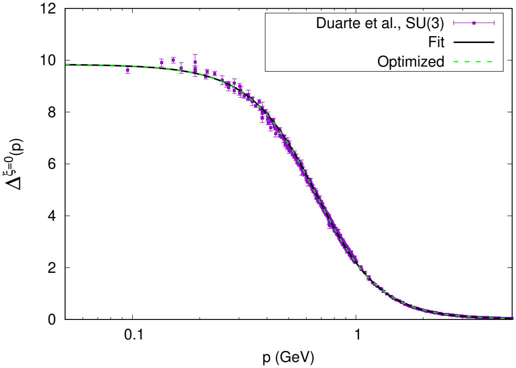


No violation of unitarity and causality (Stingl, 1996):

short-lived quasigluons with lifetime $\tau = 1/\gamma$ are canceled from the asymptotic states

$$\boxed{\Delta(p) = \frac{Z}{p^2\left[F(s) + \xi\, F_\xi(s) + F_0(\xi)\right]}}$$

$$\xi = 0,\ \text{opt.}: \begin{cases} F_0(0) = -0.876 \\ m(0) = 656\ \text{MeV} \end{cases}$$



RG-improvement of the screened massive expansion

G.C.+F.S. Phys. Rev. D 102 (2020)

The Renormalization Group (RG) equations can be used to extend the validity of the perturbative results over a wide range of energies (resummation of large logarithms).

- A strong running coupling $\alpha_s(\mu)$ can be defined like in standard PT – use the lattice-friendly Taylor scheme

$$\alpha_s(\mu) = \alpha_s(\mu_0)\,\frac{Z_A(\mu)\,Z_c^2(\mu)}{Z_A(\mu_0)\,Z_c^2(\mu_0)} \qquad \mu\frac{d\alpha_s}{d\mu}(\mu) = \beta(\mu/m, \alpha_s(\mu/m))$$

- In the Taylor-scheme S.M.E., $\beta = \beta(\mu/m, \alpha_s(\mu/m))$: the beta function depends explicitly on the renormalization scale!

$$\beta = -\frac{3N\alpha_s^2}{4\pi}\,\frac{\mu^2}{m^2}\,H'(\mu^2/m^2) \qquad H(s) = F(s) + 2G(s)$$

from ghost prop. $\nearrow$

RG-improvement of the screened massive expansion

G.C.+F.S. Phys. Rev. D 102 (2020)

What happens to the strong running coupling when it is computed in the screened massive expansion?

It does not have a Landau pole in the IR!

The function $H(s)$ is not monotonic – in the UV, $H'(s) > 0$ (usual behavior); in the IR, $H'(s) < 0$: the β function **changes sign!**

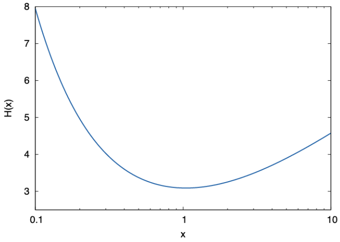
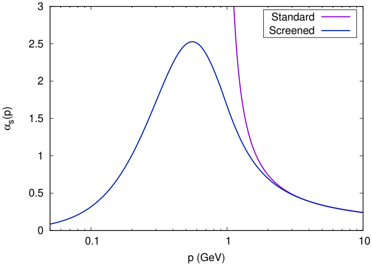


Figure 2. Function $H(x)$. The minimum $H(x_0) \approx 3.090$ is found at $x_0 \approx 1.044$.



The gluon mass **screens** the IR from the divergences

RG-improvement of the screened massive expansion

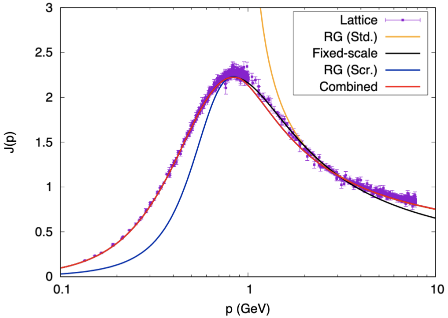
G.C.+F.S. Phys. Rev. D 102 (2020)

Upsides of the method

- Finite (!) and moderately small running coupling $\alpha_s(p)$
- Good agreement @1L with the lattice propagators at momenta $p \gtrsim m$

Downsides of the method

- $\alpha_s(p)$ not small enough for a 1L truncation for $p \lesssim m$
- Need to match with the fixed-scale scheme



Is it possible to apply the same method in the **quark sector** to describe IR-DMG for the quarks in full QCD?

Recall that the light quarks ($M_q \sim$ few MeV in the UV) acquire an IR mass $M_q \sim 300 - 400$ MeV due to the strong interactions.

- Shift the quark Lagrangian so that the quarks propagate with an **enhanced mass** M

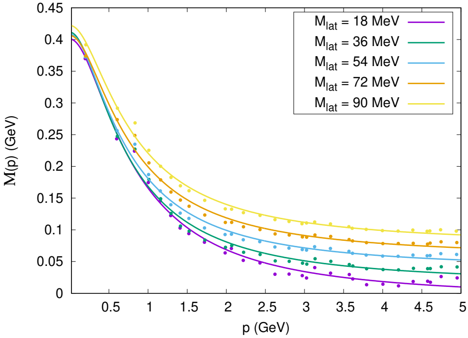
$$\begin{aligned}\mathcal{L}_q &= \overline{\psi}_B (i\not{D}_B - M_B)\psi_B = \\ &= \overline{\psi}(i\not{\partial} - M)\psi + (M - M_B)\overline{\psi}\psi + g\,\overline{\psi}\not{A}\psi + \mathcal{L}_{\text{c.t.}}\end{aligned}$$

- Do not treat M as if it were $M = M_B\,[1 + O(\alpha_s)]$

$$S_M(p) = \frac{i}{\not{p} - M} \qquad\qquad \delta\Gamma_q = i(M - M_B)$$

Unlike in the pure YMT setting, here the S.M.E. is not optimized $\Longrightarrow$ we must rely on the lattice to fit the parameters

(The only parameter we don't fit directly is the gluon m^2)



$M^\star_{\text{lat}}$	$M^\star$	$M^\star_B$
18	318.1	10
36	330.8	21.5
54	320.0	55.2
72	330.7	79.9
90	336.9	106.1

* (MeV)

Dynamical mass generation in the quark sector

G.C.+D.R.+M.B.+F.S. Phys. Rev. D 104 (2021)

In the quark sector also we find **complex-conjugate poles** $\Longrightarrow$ **quark confinement**

M_{lat} (MeV)	p_0 (MeV)
18	$\pm 373.7 \pm 202.3i$
36	$\pm 388.0 \pm 194.2i$
54	$\pm 390.7 \pm 185.6i$
72	$\pm 407.7 \pm 174.9i$
90	$\pm 424.4 \pm 177.3i$

(At variance with a previous result)

SUMMARY

- **The screened massive expansion works well in the IR**
  Excellent agreement with lattice with few free parameters

- **Optimization by gauge invariance**
  Self-contained optimization + predictivity
  Complex conjugate poles $\rightarrow$ gluon confinement

- **Readily extended to quarks**
  DMG in the quark sector
  C.c. poles $\rightarrow$ quark confinement

SUMMARY

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THANK YOU!

BACKUP SLIDES

$$\Delta_E(t_E) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}p_4}{2\pi}\, e^{ip_4 t_E}\, \Delta(\vec{p}=0, p_4) \quad (t_E = \text{Euclidean time})$$

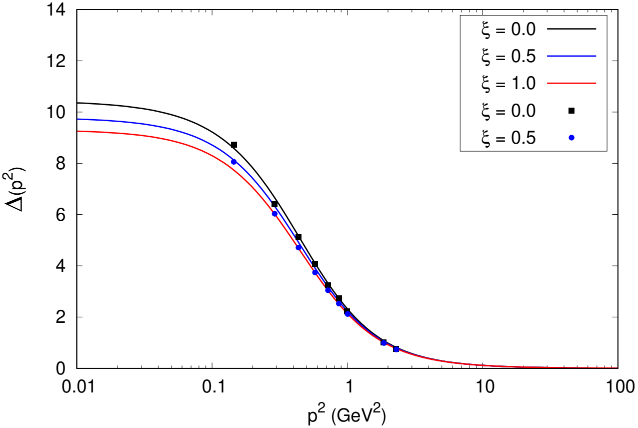
$$\begin{cases} \Delta_E(p) = \frac{R}{p^2+z_0^2} + \frac{R^\star}{p^2+z_0^{\star\,2}} \quad \text{where} \quad \boxed{z_0 = M + i\gamma} \\ \\ \Delta_E(t_E) = \left[\frac{|R|}{\sqrt{M^2+\gamma^2}}\right] e^{-M|t_E|} \cos\left(\gamma|t_E| - \phi\right), \quad \phi = \mathrm{Arg}[R] - \tan^{-1}\frac{\gamma}{M} \end{cases}$$

$$\Delta_M(t) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}p_0}{2\pi}\, e^{ip_0 t}\, \Delta_M(p_0, \vec{p}=0) \quad (t = \text{real time})$$

$$\begin{cases} \Delta_M(p) = \frac{R}{-p^2+z_0^2} + \frac{R^\star}{-p^2+z_0^{\star\,2}} \\ \\ \Delta_M(t) = \left[\frac{|R|}{\sqrt{M^2+\gamma^2}}\right] e^{-\gamma|t|} \sin\left(M|t| + \phi\right) \end{cases}$$

Gluon propagator in general R_ξ gauges

F.S.+G.C. PRD 98 (2018)

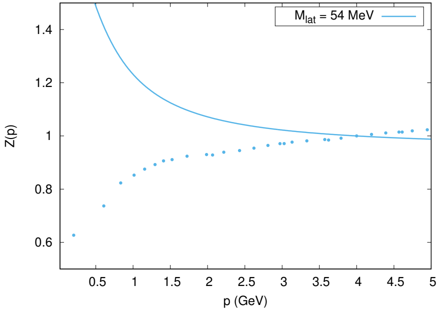


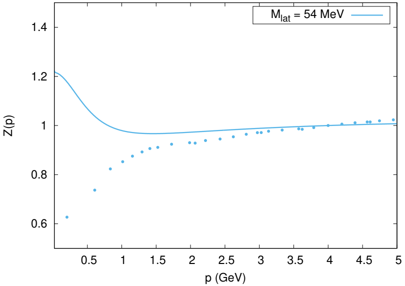
Gluon propagator $\Delta(p)$ in various linear covariant (R_ξ) gauges

Dynamical mass generation in the quark sector

G.C.+D.R.+M.B.+F.S. Phys. Rev. D 104 (2021)

The S.M.E. LO approx. of the quark Z-function is not good (left)



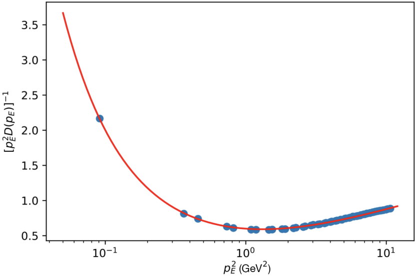


On the right, we used a resummed gluon propagator – takes into account the gluon c.c. poles, higher orders (partially), etc.

We expect a NLO calculation to solve the mismatch (e.g. CF model)

Optimized S.M.E. vs. Lattice data in the Landau gauge – w/ L. Leone (unpublished)

$$\boxed{\Delta(p) = \frac{Z}{p^2\left[F(s) + F_q(s, M_q) + \xi\, F_\xi(s) + F_0(\xi)\right]}}$$



$M_q = 386$ MeV (quark IR mass)

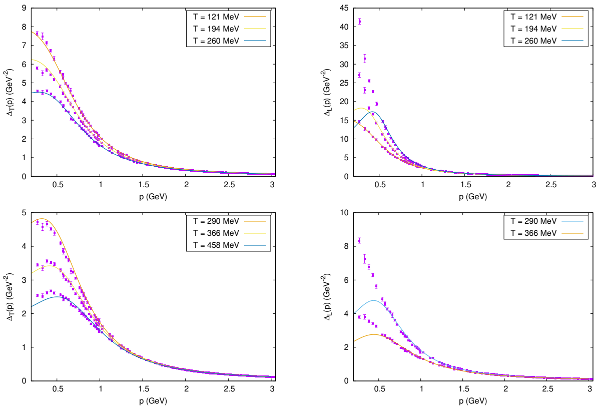
$\xi = 0$, opt.: $F_0(0) = -0.432$
$|\theta(\xi)| < 0.015$

$\boldsymbol{M = 0.887}$ **GeV**,
$\boldsymbol{\gamma = 0.385}$ **GeV**

Analogous results in full QCD, with a **heavier gluon** (expected based on other approaches)

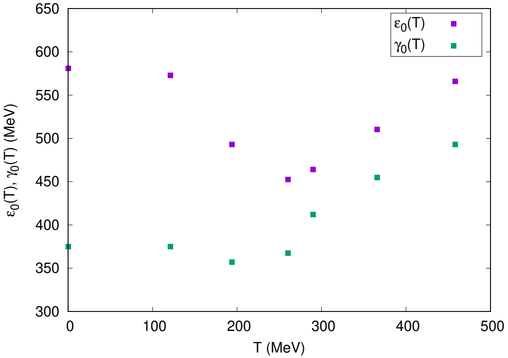
Are the gluon poles stable? Probe the theory at **finite temperature.**

- Compute the gluon propagator at $T \neq 0$ (using the S.M.E.)
- Fix the T-dependent free parameters using the lattice
- Obtain the dispersion relations for the quasi-gluons



Quasi-gluon dispersion relations at $T \neq 0$
F.S.+G.C. Phys. Rev. D 103 (2021)

- Compute the poles as a function of the temperature



A phase transition is distinguishable: **deconfinement**
($T \approx 270$ MeV for pure YMT)