

RECENT ADVANCES IN FINITE TEMPERATURE EQUATIONS OF STATE AND APPLICATIONS

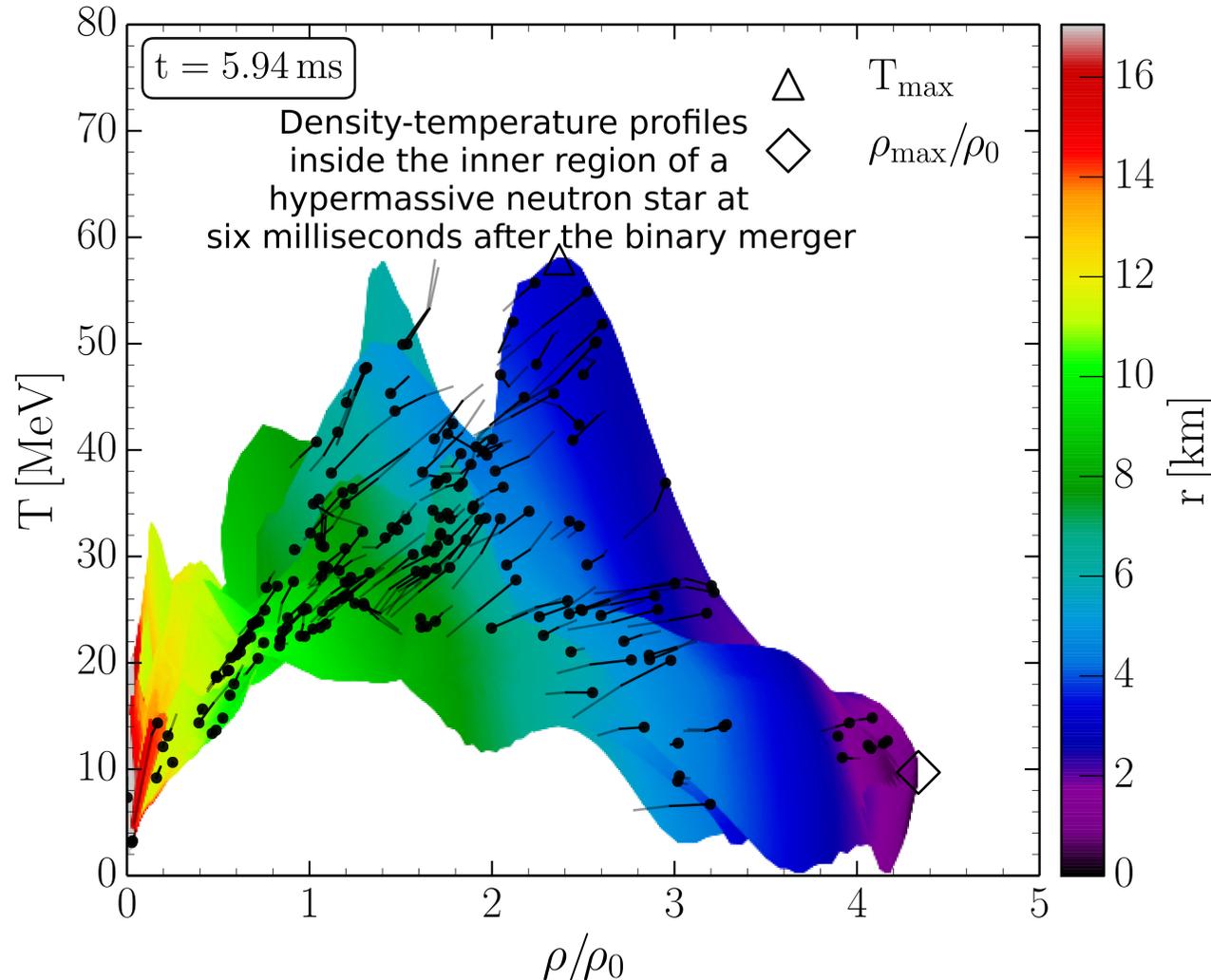
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based on : PRC **100**,054335 (2019)
PRD **102**,043006 (2020)
PRD **103**,083012 (2021)

Collaborators : A. Figura, J. B. Wei, J. J. Lu, Z.H. Li, A. Raduta, H.-J. Schulze

Neutron Star Mergers: Probing the EoS of Hot, Dense Matter by Gravitational Waves

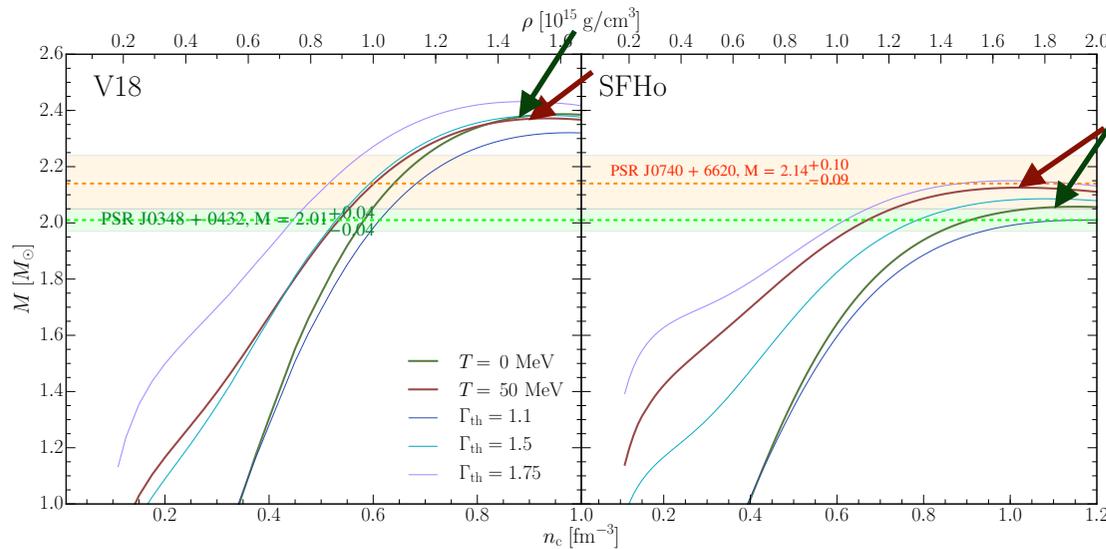
Hanauske, Steinheimer et. al, Particles **2019**, 2, 44-56



LS220-M135 simulation.

The color-coding indicates the radial position r of the corresponding fluid element inside the HMNS. The open triangle/diamond marks the maximum value of the temperature/density.

Rather different predictions for thermal effects on stellar stability



EoS	T(MeV)	M(Mo)	R(km)
V18	0	2.387	10.86
	50	2.372	11.40
SFHo	0	2.058	10.30
	50	2.126	11.81

A. Figura et al., *Hybrid equation of state approach in BNS merger simulations*. PRD **102**, 043006 (2020)



V18 Microscopic EoS : decreasing stability with temperature T

SFHo Phenomenological EoS : increasing stability with temperature T

SELECTING A BUNCH OF EOS @ T=0

for which an extension to finite T does exist ! Not many ...

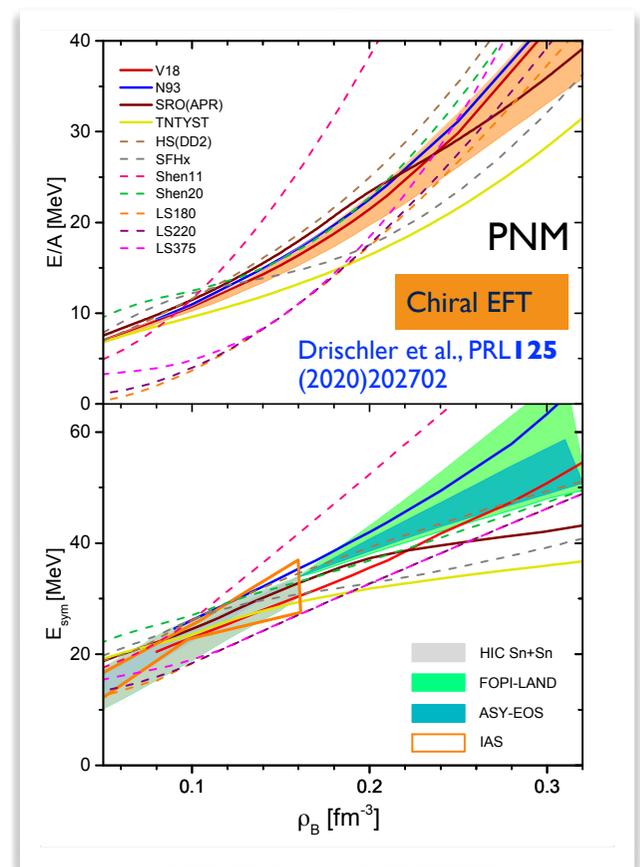
Microscopic EoS

- BHF with Argonne V18 or Nijmegen 93 2NF and microscopic 3NF consistent with 2NF (V18, N93)
Burgio&Schulze, A&A **518**, A17 (2010)
- Variational with Argonne V18 with phenomenological 3NF of Urbana UIX type (SRO(APR), TNTYST)
Schneider, Constantinou, Muccioli, Prakash, PRC **100**, 025803 (2019)
Togashi, Nakazato, Takehara, NP A**961**, 78 (2017)

Phenomenological EoS

- RMF model with TM1 2NF (Shen I I, Shen20)
Shen, Ji, Hu, Sumiyoshi, Astroph. J. **891**, 148 (2020)
- RMF model with new parametrization fitted to NS radius determination (SFHx)
Steiner, Hempel, Fischer, Astroph. J. **774**, 17 (2013)
- RMF model with excluded volume effects HS(DD2)
Hempel, Schaffner-Bielich NP A**837**, 210 (2010)
- LDM with Skyrme-type 2NF (LS180, LS220, LS375)
Lattimer & Swesty NP A**535**, 331 (1991)

Check w.r.t. available Nuclear Physics constraints @T=0

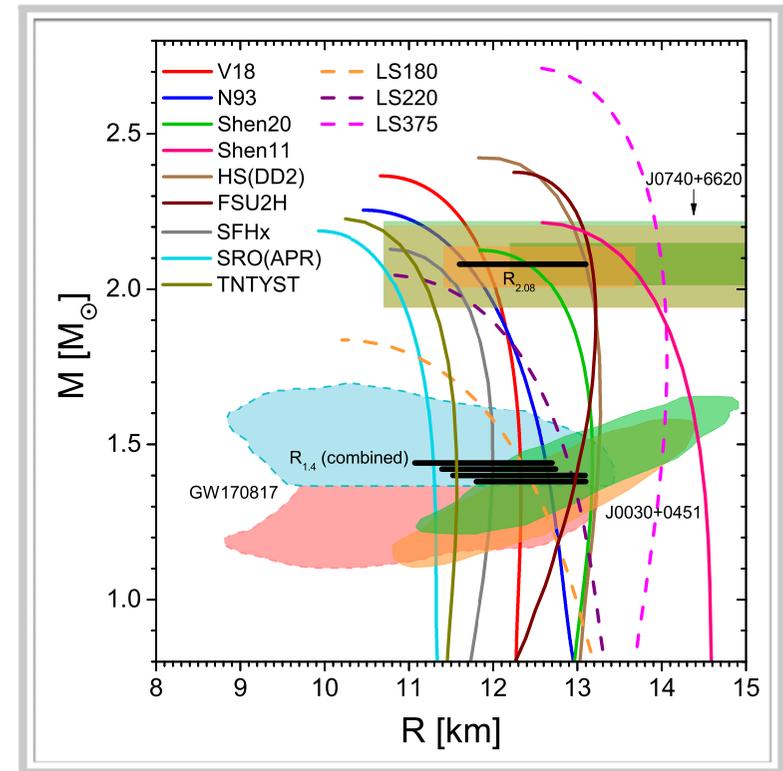
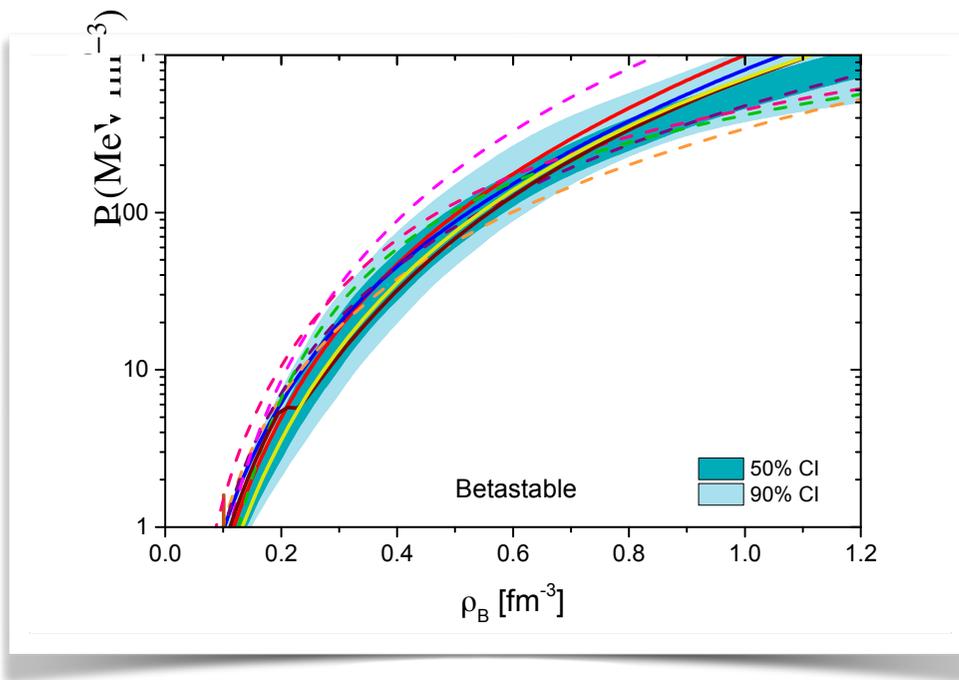


Compare with observations

Composition : beta-stable and charge neutral matter with nucleons and leptons

GW170817 → Abbott et al., PRL119 (2017) 161101

For a given density value, the value of the pressure has a 50% chance to lie within the darker blue area and a 90% chance to lie within the lighter blue band.

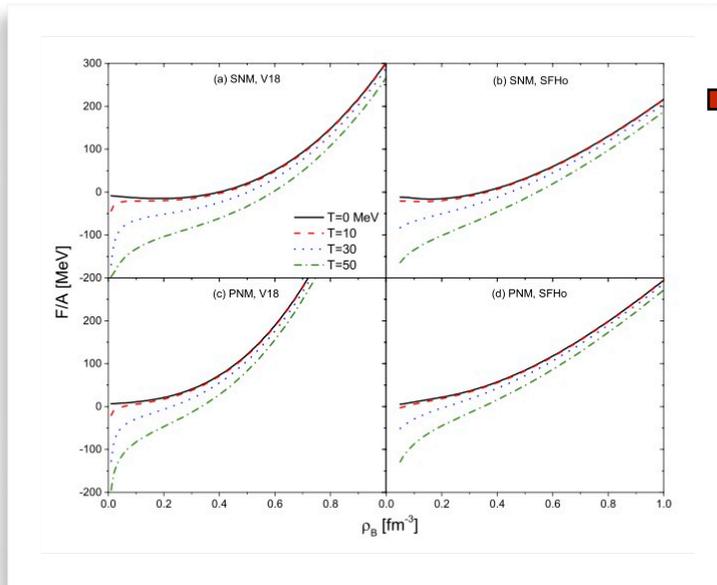


Mass-radius relations obtained with different EOSs. The mass of the most heavy pulsar PSR J0740+6620 observed until now is also shown, together with the constraints from the GW170817 event and the mass-radius constraints on the pulsars J0030+0451 and J0740+6620 of the NICER mission. The black bars indicate the limits on $R_{2.08}$ and $R_{1.4}$ obtained in combined data analyses.

Most of the EoS are compatible with the data, except LS180 and LS375.

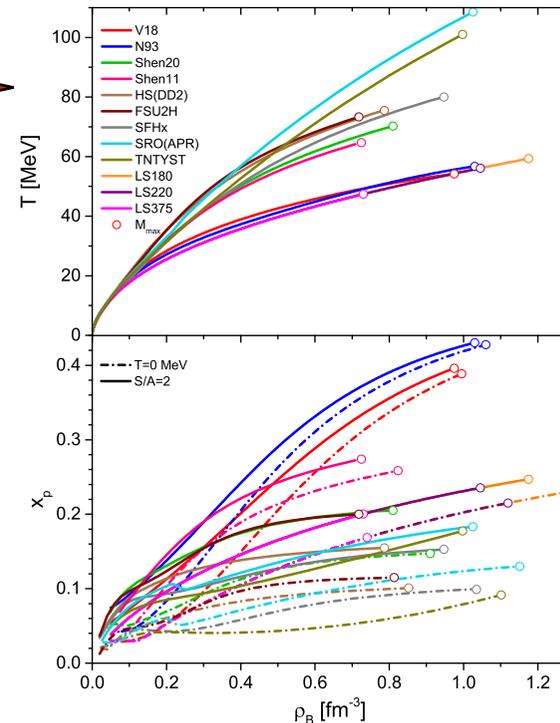
Models Extension at finite Temperature

Isothermal curves for BHF and SFHo



Free energy per particle for symmetric and pure neutron matter vs. nucleon density for different T values. The typical Van Der Waals behaviour is evident.

Iisentropic $S/A=2$

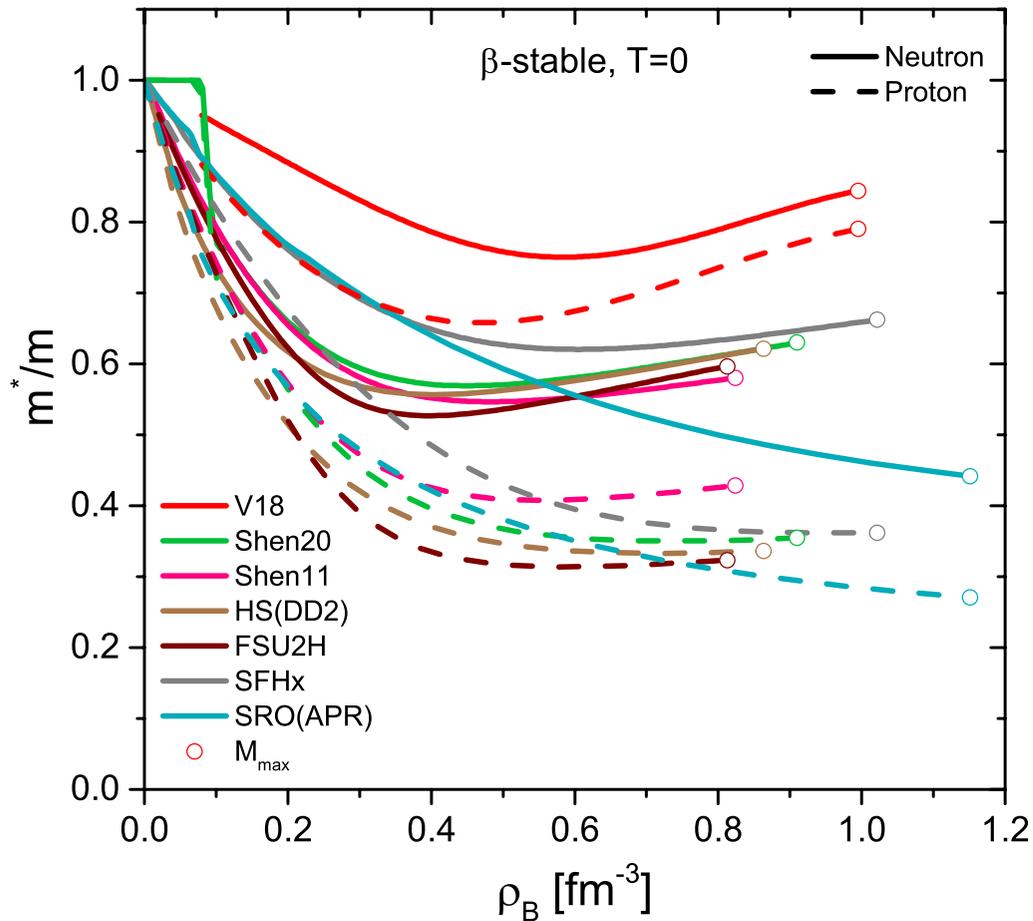


- T decreases monotonically from the core to the outer layers. Large dispersion at high densities.
- Highest temperatures at large density reached by the two variational EOSs, SRO(APR) and TNTYST.
- Lowest temperatures obtained for the two BHF EOSs and the three LS EOSs.
- Intermediate values for RMF models.

Finite T/S increases the proton fraction due to the increased lepton fraction as a result of Fermi distributions at finite temperature because of the charge-neutrality condition.

T vs. Effective mass

Landau effective mass m_L^*



Quasi-particle approximation for the entropy density

$$s = -2 \sum_k \left(n(k) \ln n(k) + [1 - n(k)] \ln[1 - n(k)] \right)$$

$$n(k) = \{ \exp([e(k) - \tilde{\mu}]/T) + 1 \}^{-1}$$

$$e(k) = k^2/2m_L^*$$

Quindi un valore ridotto della massa di Landau implica un aumento della stiffness di $e(k)$ e, per un fissato valore di T , una diminuzione della densità di entropia.

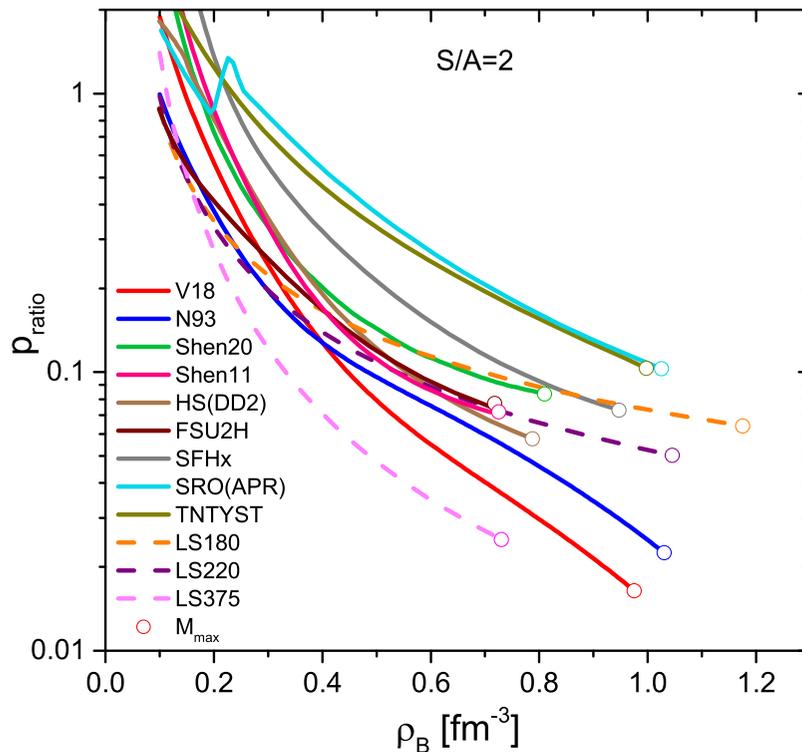
Larger effective mass (LS, BHF) implies smaller T for fixed S/A

Thermal effects on the EoS : the thermal pressure

Crucial for the stability of the star against collapse !

$$p_{th}(\rho, T) = p(\rho, x_T, T) - p(\rho, x_0, 0)$$

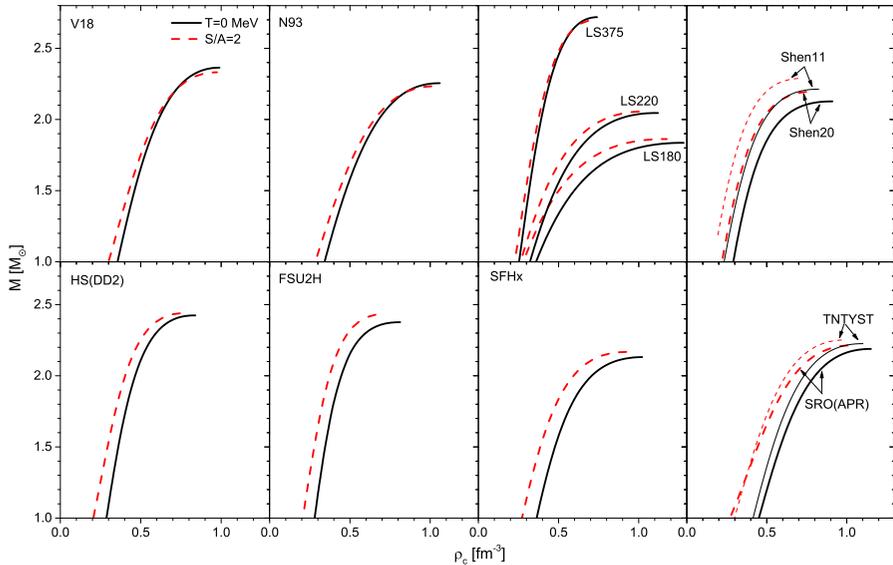
$$p_{ratio} = \frac{p_{th}}{p_0}(\rho, T) = \frac{p(\rho, x_T, T) - p(\rho, x_0, 0)}{p(\rho, x_0, 0)}$$



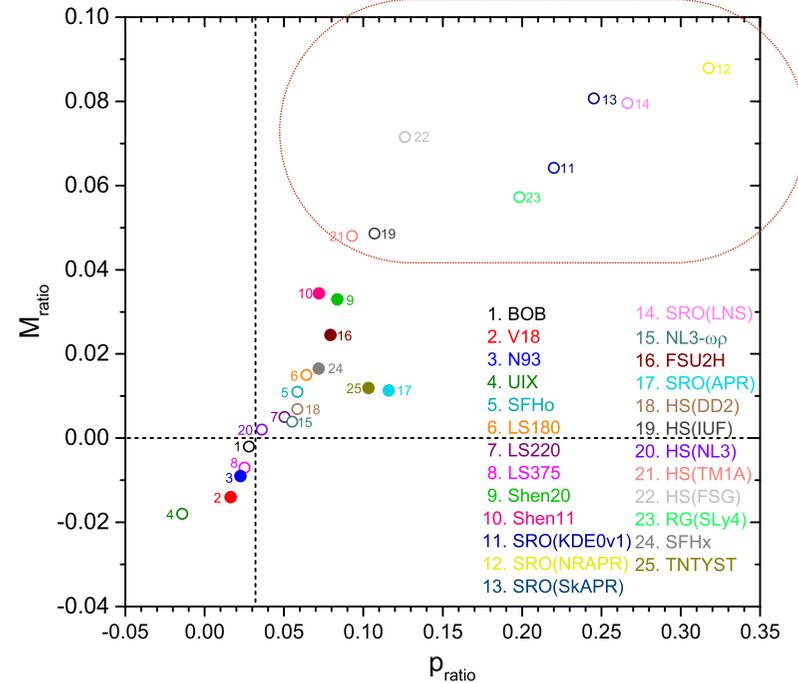
- p_{ratio} decreases with increasing density and reaches a few percent at the maximum-mass configurations.
- The V18, N93, and LS375 ratios are below 3 percent, while the others are up to 10 percent for the SRO(APR) and TNTYST.
- For LS EOSs the thermal pressure is identical for all three models, and the different ratios are caused solely by different Fermi pressures of cold matter, related to the different incompressibility values.

Mass-central density relation

The final EoS is a complicated interplay between the increased lepton thermal pressure, and the increased nucleonic thermal pressure which is limited by the increased symmetry.



$$M_{ratio} = \frac{M_{max}^{hot} - M_{max}^{cold}}{M_{max}^{cold}}$$



Thermal effects on the maximum mass are very small !
Change of the maximum masse is just a few percent,
and can be both positive or negative.

A clear correlation does exist for all of them!
For the subset of realistic EoS the relative
increase of M_{max} is limited to *less than 4 percent*.

Application to BNS merger

A. Figura et al., PRD **102**, 043006 (2020)
PhD Thesis 2021, Catania Univ.

Simulations performed in full general relativity
Mathematical and numerical setup as in
Papenfort, Gold & Rezzolla, PRD **98**, 104028 (2018)

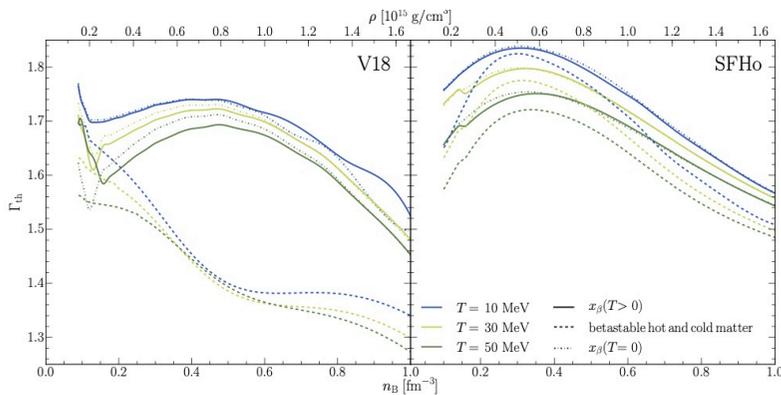
- a) FT approach : a fully temperature dependent EoS
- b) Hybrid EoS approach : a cold EoS **plus** a thermal contribution obeying the ideal-fluid EoS

$$p_{th}(\rho, T) = e_{th}(\Gamma_{th} - 1)$$

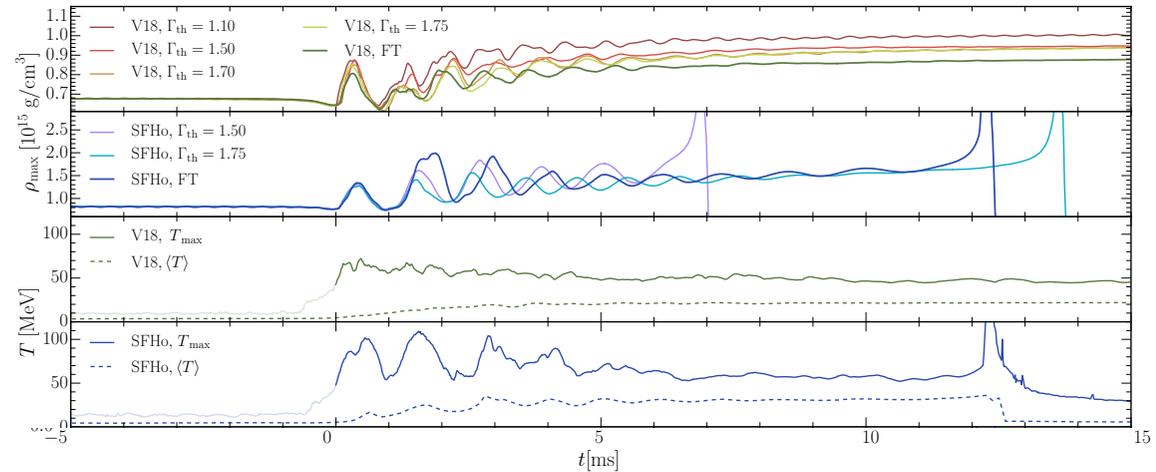
Usually $\Gamma_{th} = \text{const}$

Bauswein, Janka and Oechslin, PRD **82**, 084043 (2010)

Strong impact on the stability of the merger remnant
hence on its lifetime before collapsing to BH.



Simulation of equal-mass binaries with $M_G = 1.35 M_\odot$,
and initial separation 45 km.



Evolution of the maximum rest-mass density ρ_{\max}

Increasing Γ_{th} leads to a less dense remnant. FT EoS leads to a remnant with even smaller maximum rest-mass density than the hybrid-EoS case.
V18 produces a metastable HMNS up to the largest time. SFHo leads to a collapse into a BH, in a time which is dependent on Γ_{th} .

Maximum and density-weighted aver. temperature, T_{\max} and $\langle T \rangle$

Temperature fluctuations during the metastable phase before collapse, stronger for SFHo. In the post merger phase T_{\max} peak around 70 (110) MeV for V18 (SFHo).

Additional simulations with chiral NN interaction in Logoteta, Perego, Bombaci, arXiv:2012.03599, published in A&A

CONCLUSIONS

- ❖ We extended several EoS, compatible with observations, to finite T and focussed on beta-stable isentropic matter with $S/A=2$.
- ❖ BHF EoS with large proton fraction feature low ratios of thermal pressure to Fermi pressure, and predict no increase of stellar stability due to finite temperature. Matter with large proton fraction as a consequence of beta-stability.
- ❖ In variational approaches thermal effects are quite large due to small symmetry energy hence proton fraction, and effective masses.
- ❖ All EoS exhibit a clear correlation between the ratio of thermal and Fermi pressures at the NS maximum central density and the relative change of the maximum gravitational mass:
The mass increases only for EOSs with a pressure ratio larger than about 3%.

For the realistic EOSs, the relative change of the maximum gravitational mass varies between about -2% and +4%, depending on details of the EoS and its extension to finite temperature.