Credits: Geoff Chen

Accurate CMB covariance matrices for the SPT3G likelihood

Etienne Camphuis, with Silvia Galli, Karim Benabed, Eric Hivon and the SPT collaboration

[EC, Galli, Benabed, Hivon, Lilley] https://arxiv.org/abs/2204.13721

European Research Council
Established by the European Commission

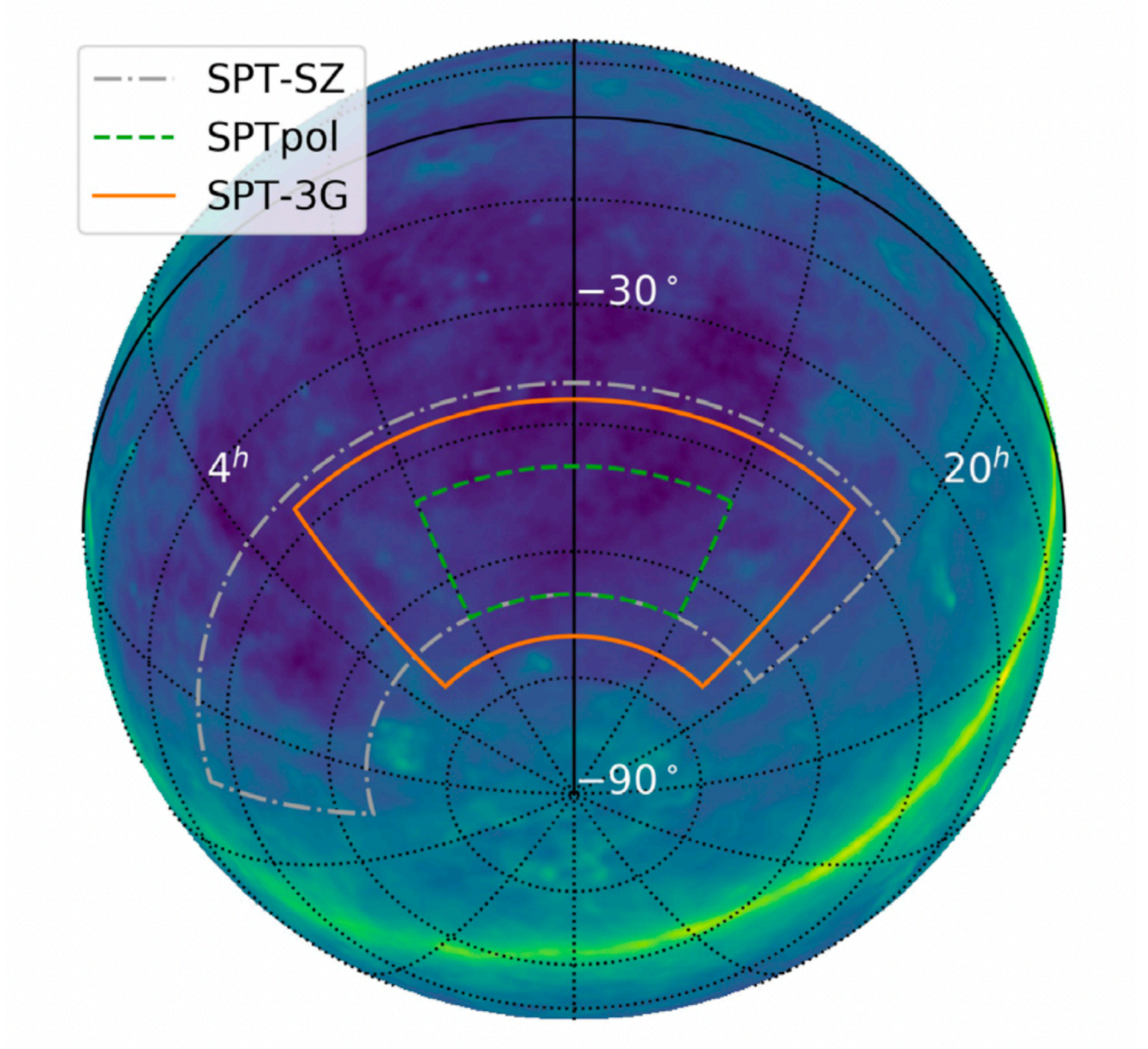
Etienne Camphuis - May 26th, 2022
From Planck to the future of CMB @ Ferrara

NEU CosmoS

SPT3G

- 10-meter diameter telescope
- Located at the South Pole
- 4% of the sky (with the **winter field**!)
- Also summer field (additional ~8%, see F. Guidi's talk this afternoon)
- 3 frequencies 90, 150, 220 GHz
- Beam: 1.2 arcmin (*Planck* is 5 arcmin)
- Final noise levels of 2.2μKarcmin in T (*Planck* is ~40μKarcmin)

SPT3G patch



Sky path overlaid over thermal dust emission

Kavli Institute
for Cosmological Physics
at The University of Chicago

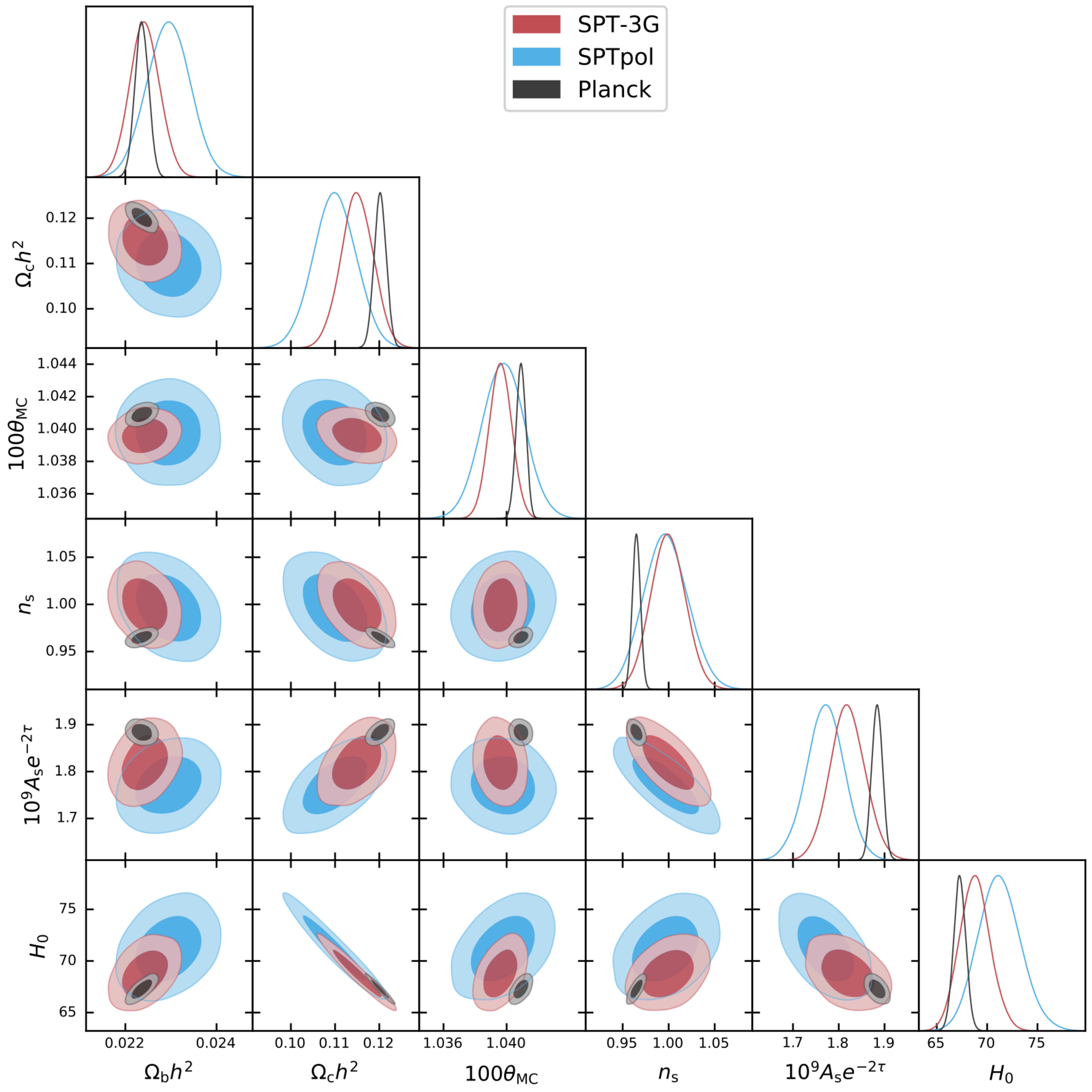
SPT3G - winter field

Cosmological parameters

- 5 years (2019-2023) of SPT3G observations => **very high quality data.** My goal is to perform the cosmological analysis of the first two years.
- Final constraints on cosmological parameters **comparable to Planck**

- More details this afternoon by F. Guidi
- [Dutcher et al., 2021] data + ΛCDM
- [Balkenhol et al., 2021] ΛCDM extensions

Constraining cosmological parameters with SPT3G 2018



[Dutcher et al., 2021]

Plan

A. Context

B. Exact covariances, at last !

C. Approximations, old and new

D. Accuracy of approximations

Plan

A. Context

Accurate covariance matrices

Core component of the likelihood

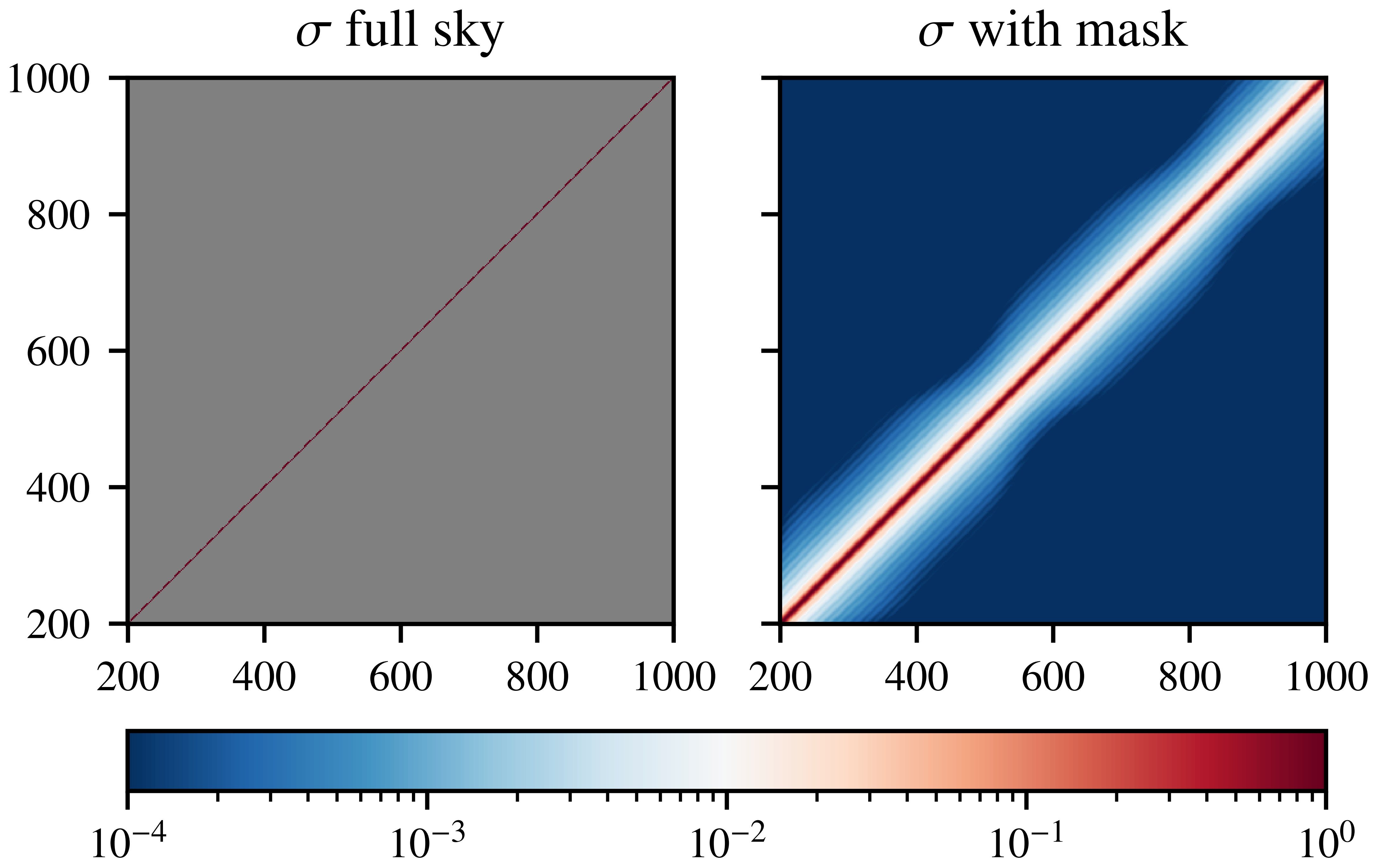
- Previous data release: simulations + empirical estimators, which requires computing resources and regularization [Balkenhol et al. 2021]
- Next data release: (semi-)**analytical computation,** precision and no need for regularization [EC et al. 2022] https://arxiv.org/abs/2204.13721. **Curved-sky analysis**
- Ingredients: mask (introduces coupling) W and theoretical spectrum $C_\ell^{\rm th}$

Power spectrum gaussian likelihood :

$$-\ln \mathscr{L}(\hat{C}\,|\,\Lambda\mathrm{CDM})$$

$$\propto \frac{1}{2}(\hat{C} - C^{\rm th})^T \boxed{\Sigma^{-1}} (\hat{C} - C^{\rm th}))$$

Unbinned correlation matrices full sky vs masked sky



Accurate covariance matrices

PolSpice and pseudo-power spectrum

- For this analysis we will use `PolSpice` estimator $\hat{C}_\ell$ [Szapudi et al. 2001][Chon et al. 2004]
- It is built on the pseudo-power spectrum $\tilde{C}_\ell$, the power spectrum of the masked maps $(\hat{C}_\ell^{\mathrm{TT}} = \sum_{\ell'} {}_0G_{\ell\ell'}\tilde{C}_{\ell'}^{\mathrm{TT}})$
- **Our goal is to compute analytically the covariance matrix of the pseudo-power spectrum**

- This work can thus be extended to any estimator built on pseudo-power spectrum

Power spectrum gaussian likelihood :

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Accurate covariance matrices

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Built on the pseudo-power spectrum = power spectrum of the masked maps $\tilde{C}_\ell$

$$\tilde{\Sigma}_{\ell\ell'} = \mathrm{cov}(\tilde{C}_\ell, \tilde{C}_{\ell'})$$

Formalism

Covariance matrix of the pseudo-power spectrum

$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1} \, \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W] \; C^{\mathrm{th}}_{\ell_2}$$

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Pure geometric coupling - MASTER matrix

Well known [Hivon et al. 2002]

Scales as $\mathcal{O}(\ell^3_{\mathrm{max}})$ (or even $\mathcal{O}(\ell^2_{\mathrm{max}})$ using [Louis et al. 2020])

Formalism

Covariance matrix of the pseudo-power spectrum

Theoretical power spectrum from model

Can include beam, transfer function, noise, pixel window function.

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Covariance coupling kernel

Scales as $\mathcal{O}(\ell^6_{\mathrm{max}})$ and $\ell_{\mathrm{max}} \sim 4000$

Always approximated in the literature

UNTIL NOW!

Pure geometric coupling - MASTER matrix

Well known [Hivon et al. 2002]

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Plan

A. Context

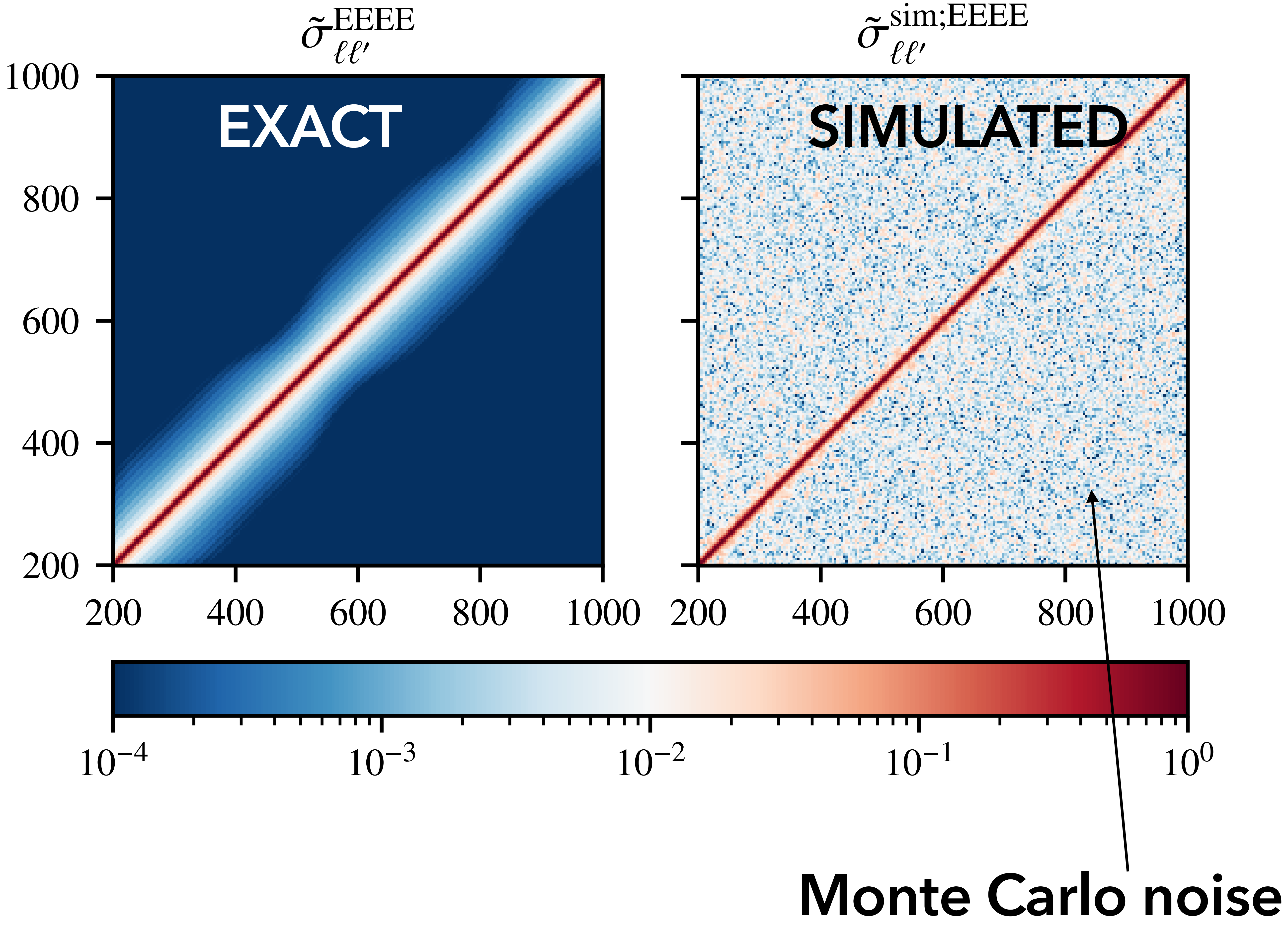
B. Exact covariances, at last !

Exact covariance

- **I implemented for the first time an exact computation, with a x1000 speedup** (`Healpix` based algorithm)
- **This code allows to compute any rank of covariance at any multipole**
- Scales as $\mathcal{O}(n_{\rm side}^5) = \mathcal{O}(\ell_{\rm max}^5)$ instead of $\mathcal{O}(\ell_{\rm max}^6)$
- Full computation up to $\ell = 1000$
- 300h CPU time for a slice at $\ell = 1000$

$$\sigma_{\ell\ell'} = \frac{\Sigma_{\ell\ell'}}{\sqrt{\Sigma_{\ell\ell}\Sigma_{\ell'\ell'}}}$$

Unbinned correlation matrices exact vs simulations



$N_{\rm sim} = 10\ 000$

Plan

A. Context

B. Exact covariances, at last !

C. Approximations, old and new

Approximations

It is not realistic to run the exact computation for all multipoles => we use approximations that work for every multipole !

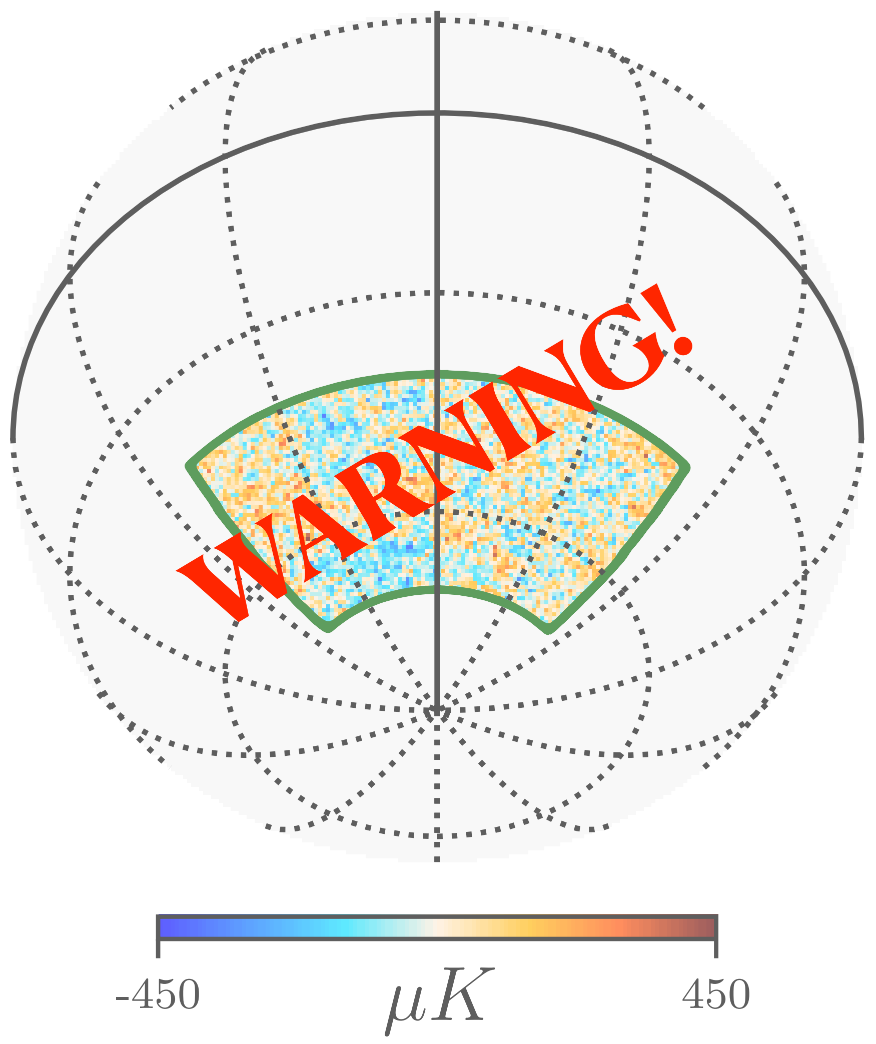
- [Efstathiou 2004]+[Challinor&Chon 2004] **NKA** - *Planck* and others => $\mathcal{O}(\ell^3_{\max})$
- [Friedrich et al. 2021] **FRI** => $\mathcal{O}(\ell^3_{\max})$ - *DESY3*
- [Nicola et al. 2021] **INKA** => $\mathcal{O}(\ell^3_{\max})$

- [EC et al. 2022] **ACC** - obtained with our knowledge from the exact computation

Scales as $\mathcal{O}(d_{\max} n^4_{\mathrm{side}}) \gg \mathcal{O}(\ell^3_{\max})$ (~100h of CPU-time vs few minutes) but it has to be computed only once per mask

Approximations are expected to be less precise on small survey area

No source masking !!



CMB temperature anisotropies

Approximations

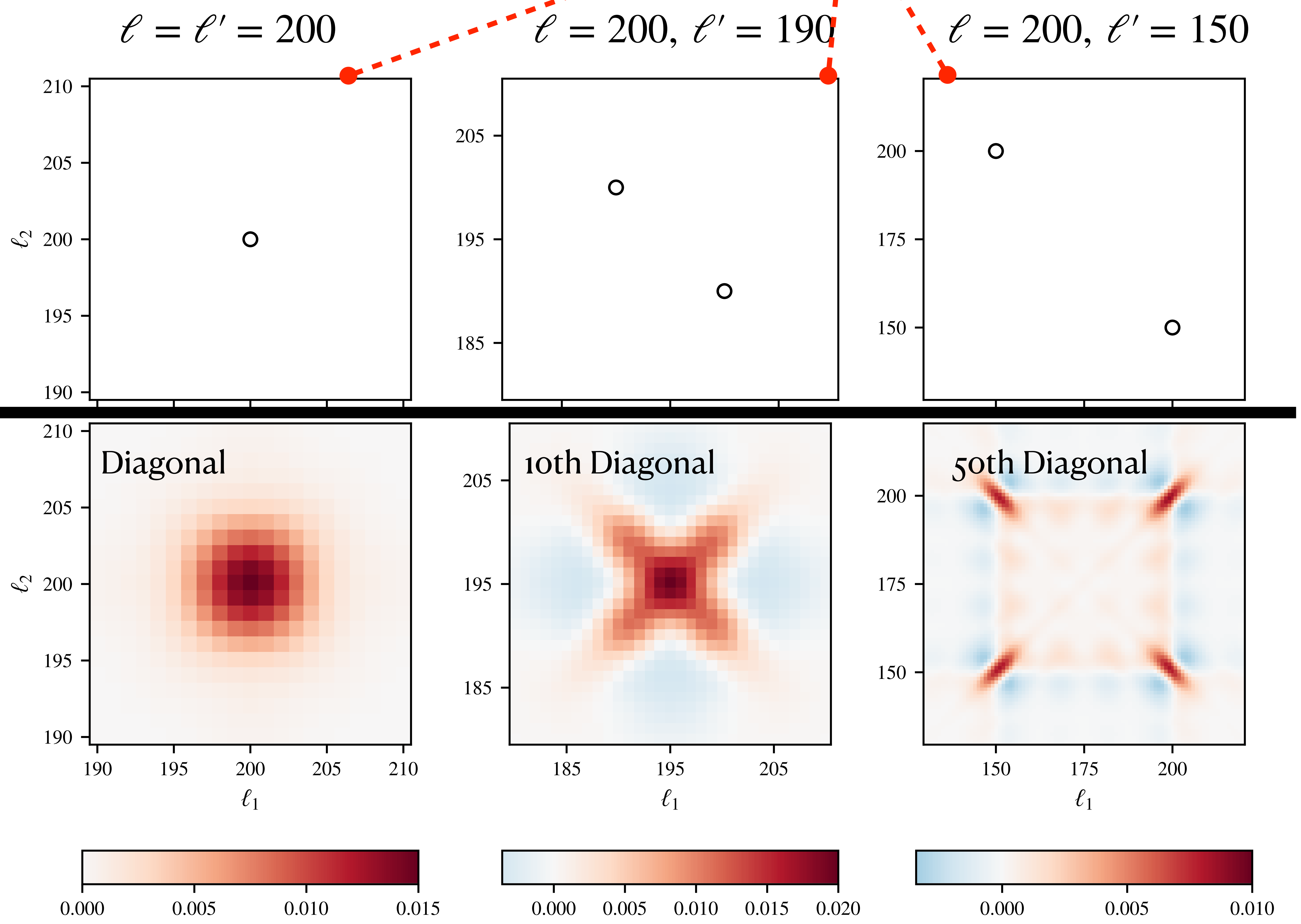
$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\rm th}_{\ell_1}\, \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W]\, C^{\rm th}_{\ell_2}$$

Comparing the covariance coupling kernels

Approximations:

- NKA (Planck) (o)

Exact



Approximations

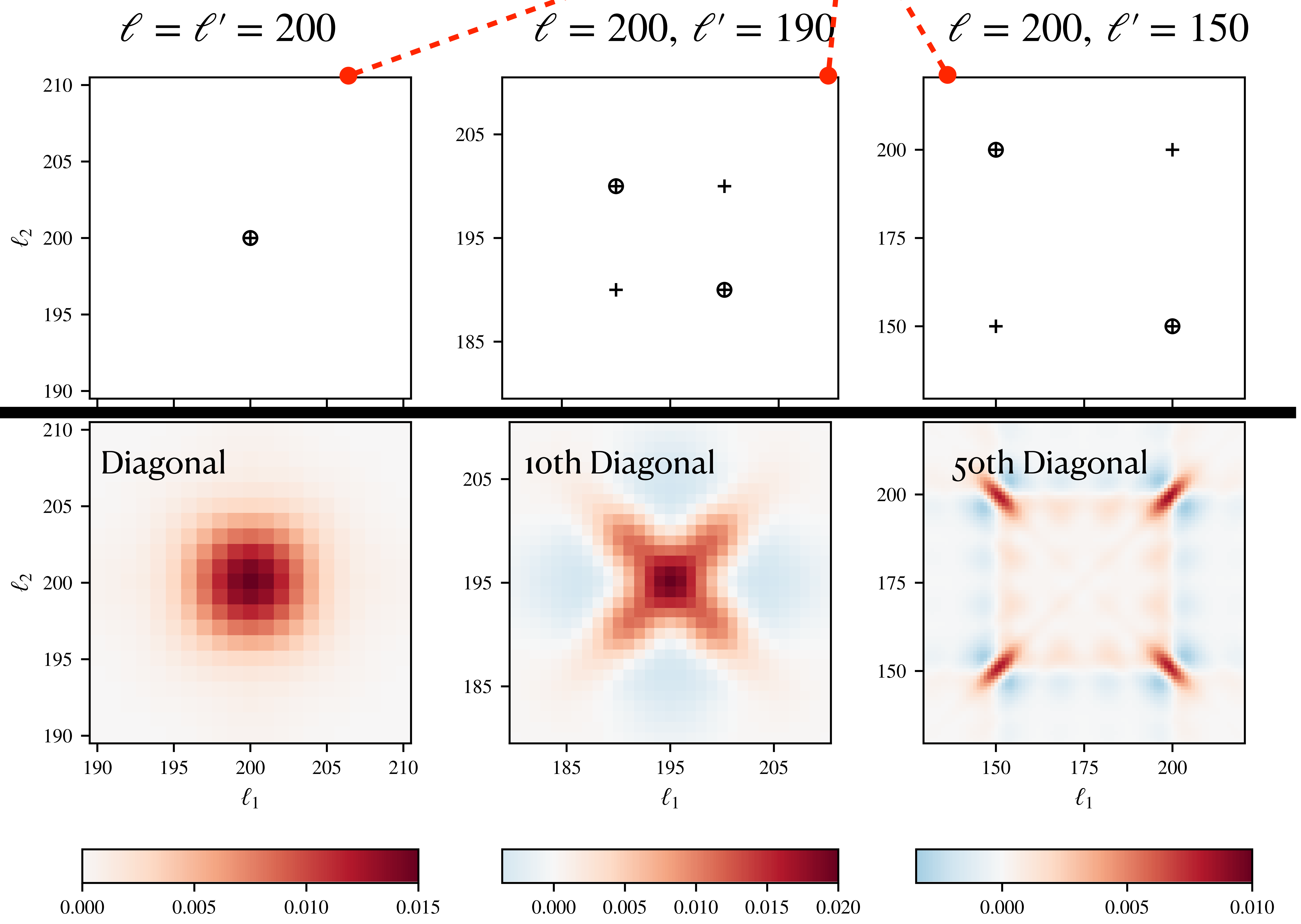
$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1} \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W] \, C^{\mathrm{th}}_{\ell_2}$$

Comparing the covariance coupling kernels

<ins>Approximations:</ins>

- NKA (Planck) (o)
- FRI (+)

<ins>Exact</ins>



Approximations

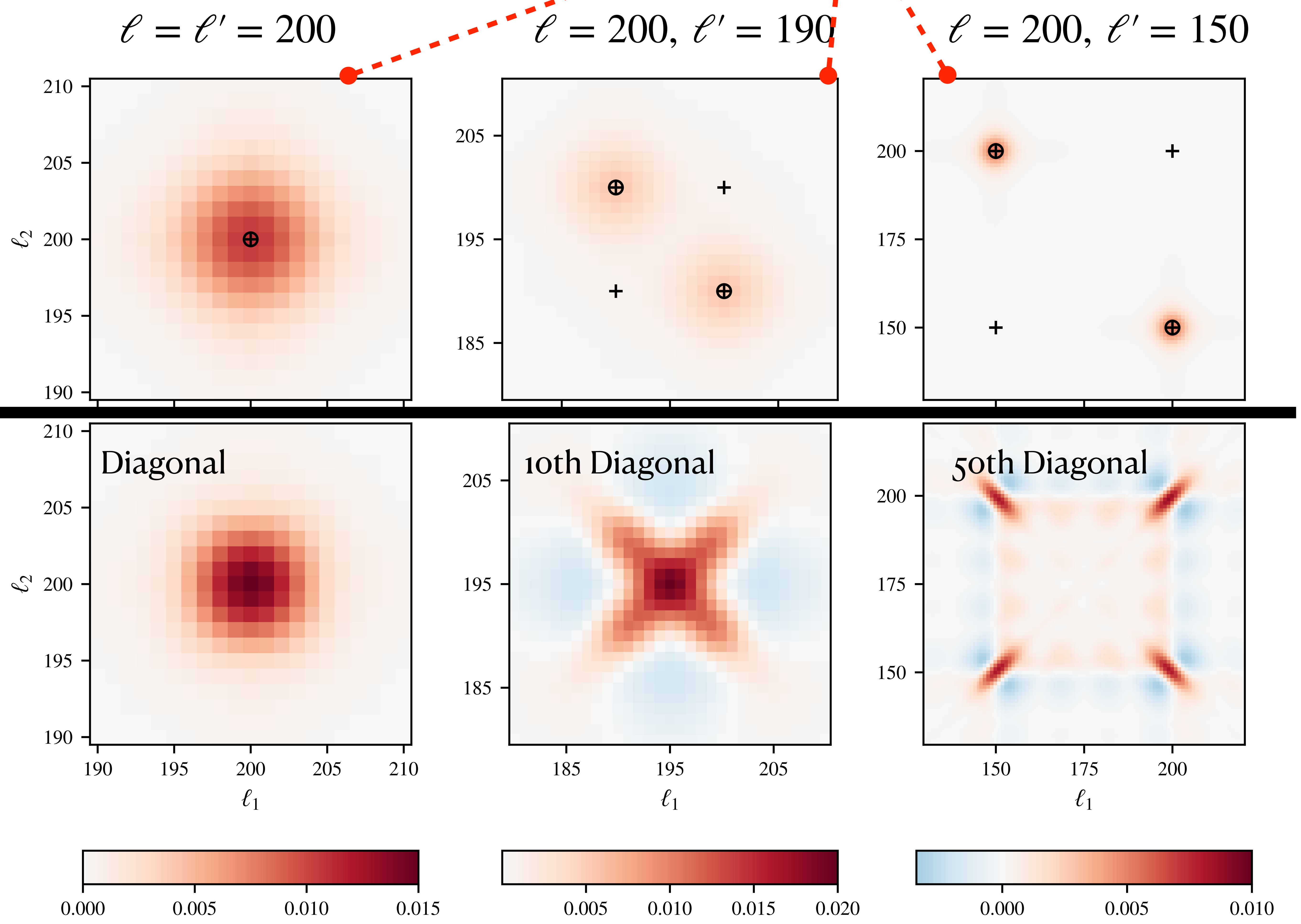
$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1} \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W] \, C^{\mathrm{th}}_{\ell_2}$$

Comparing the covariance coupling kernels

<u>Approximations:</u>

- NKA (Planck) (o)
- FRI (+)
- INKA (image)

<u>Exact</u>



Approximations

$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1} \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W] \, C^{\mathrm{th}}_{\ell_2}$$

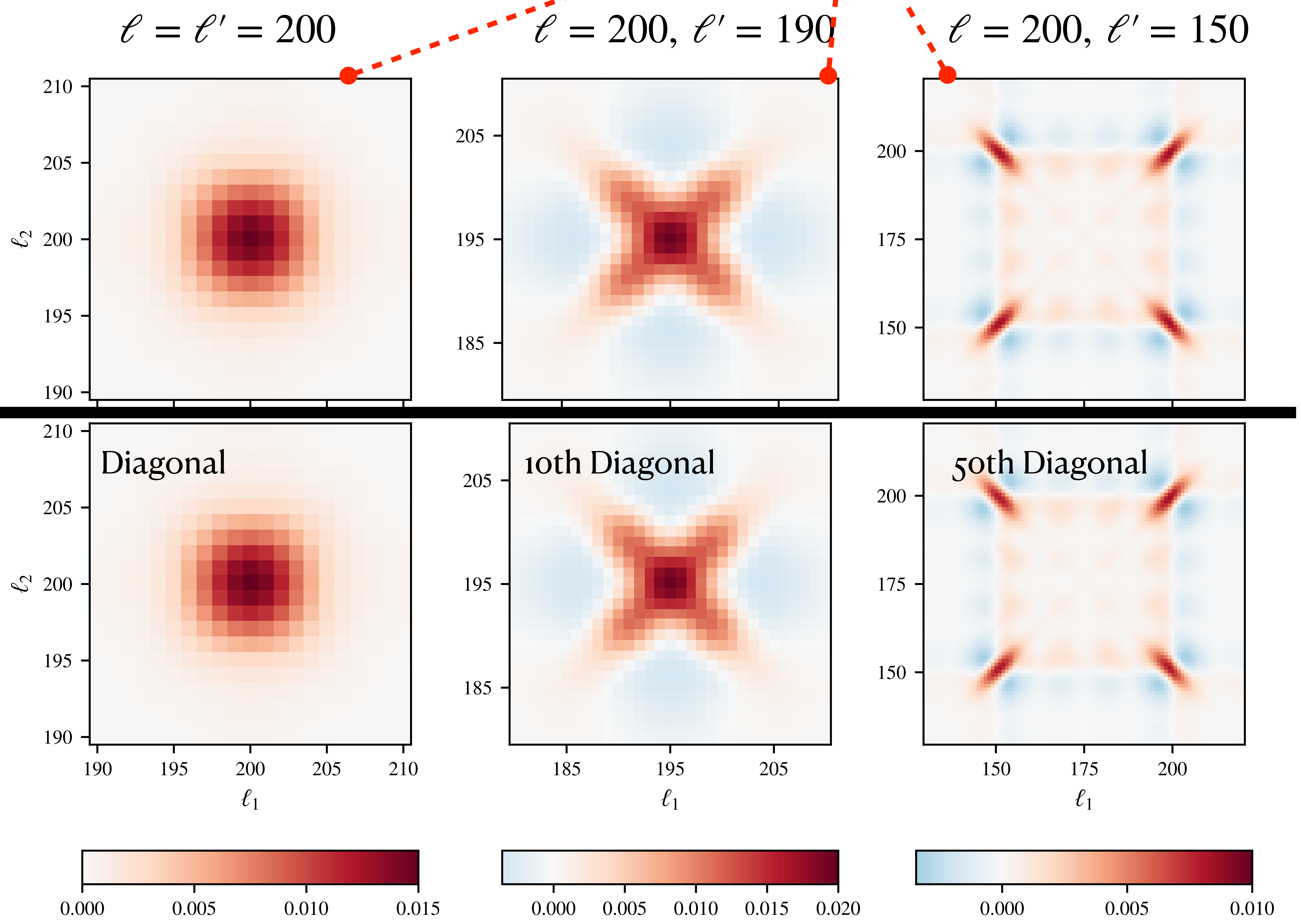
Comparing the covariance coupling kernels

Approximations:

- **ACC (this work)**

Using the same $\bar{\Theta}$ for identical multipole separation $|\ell - \ell'|$

Exact



Plan

A. Introducing covariance matrices

B. Exact covariances, at last !

C. Approximations, old and new

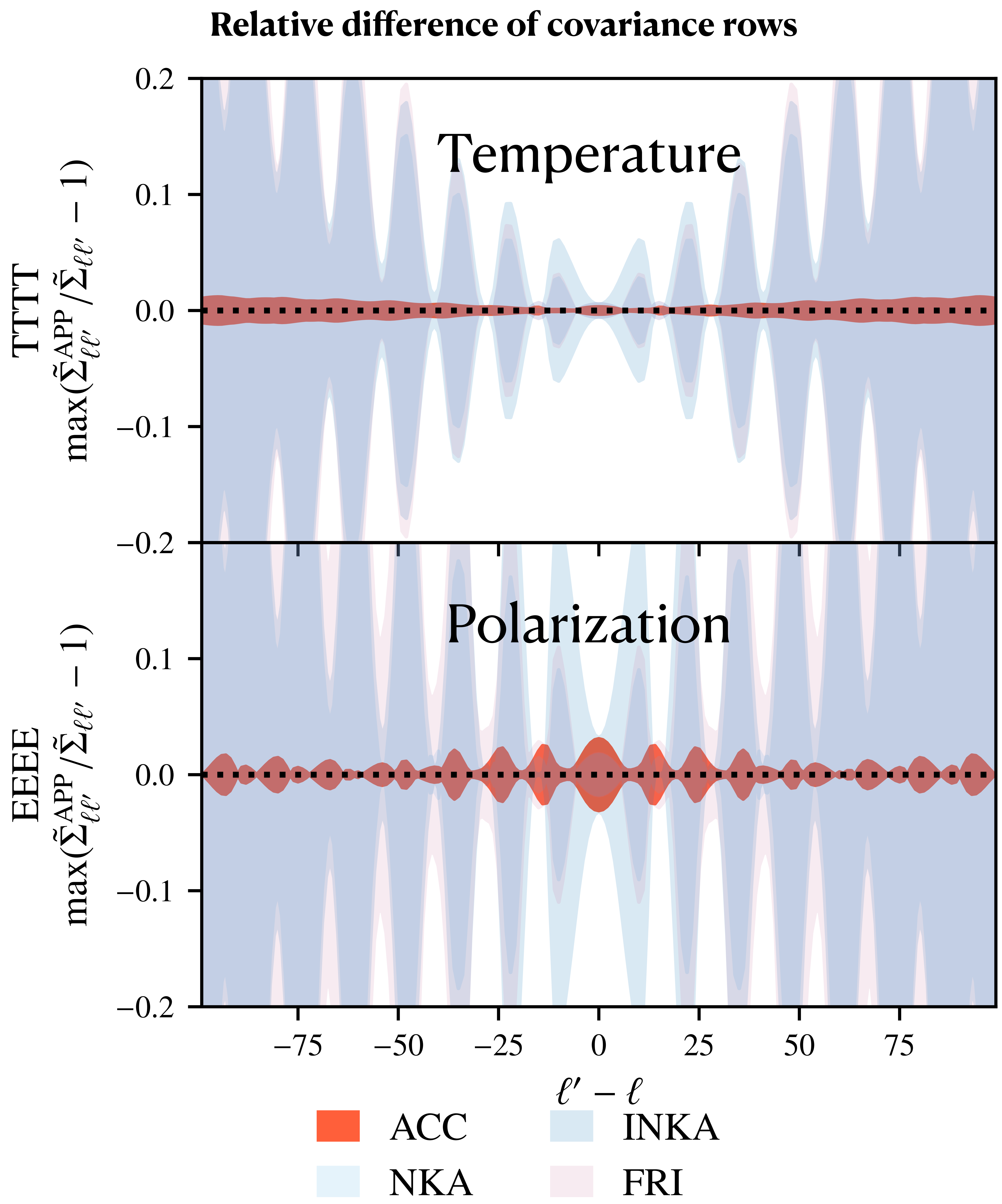
D. Accuracy of approximations

Results

Accuracy of approximations

- We look at the relative difference of rows of the covariance centered on the diagonal

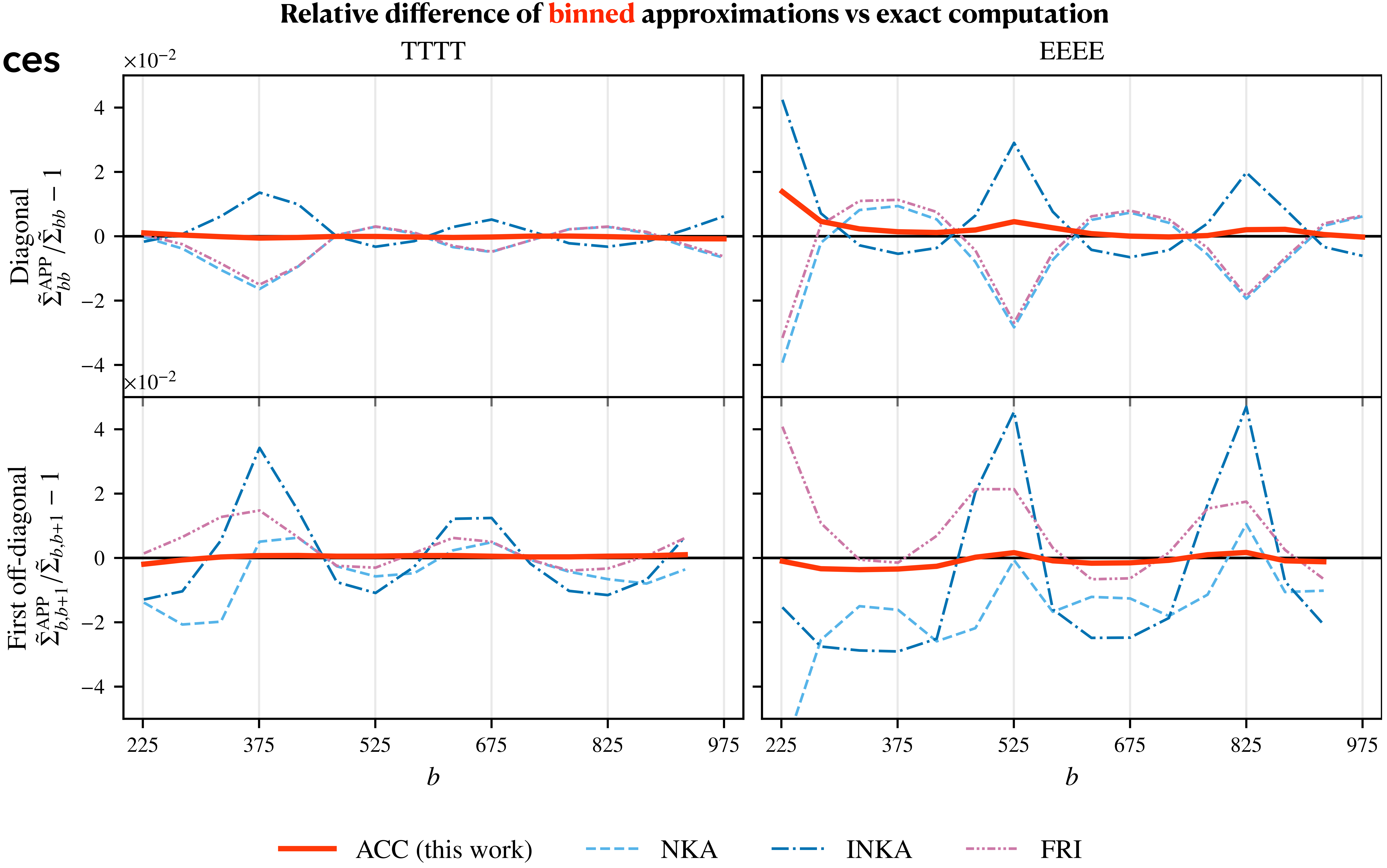
- In red **ACC**



Results

Binned covariances

- Looking at binned covariance ($\Delta\ell = 50$)
- Literature approximations work with precision up to 5%.
- ACC is more precise, percent level



Future developments

- This formalism can already 1D include instrumental effects. More work is needed for 2D transfer function, but exact computation helps a lot !

- Main problem: **point sources masking** but the problem already existed for other approximations. I am working on 3 solutions:
    - Analytical model by treating the mask as a stochastic process [Gratton, Challinor, Migliaccio, Hivon, Lilley, Camphuis *in prep*]
    - Gaussian constrained realization in the holes with **polcork** [Benoit-Levy et al. 2013] with K. Benabed
    - CarPool [Chartier et al. 2021][Chartier, Camphuis et al. *in prep*]

Summary

- We are now able to compute an exact covariance matrix
- We showed that current approximations work fine on small footprints
- We built a new one that works even better
- This work can be applied to other probes, masks, experiments.

- https://arxiv.org/abs/2204.13721 -submitted to A&A for more details.

Accurate CMB covariance matrices: exact calculation and approximations

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May 2, 2022

ABSTRACT

Accurate covariance matrices are required for a reliable estimation of cosmological parameters from pseudo-power spectrum estimators. In this work, we focus on the analytical calculation of covariance matrices. We consider the case of observations of the Cosmic Microwave Background in temperature and polarization on a small footprint such as in the SPT-3G experiment, which observes 4% of the sky. Power spectra evaluated on small footprints are expected to have large correlations between modes, and these need to be accurately modelled. We present, for the first time, an algorithm that allows an efficient (but computationally expensive) exact calculation of analytic covariance matrices. Using it as our reference, we test the accuracy of existing fast approximations of the covariance matrix. We find that, when the power spectrum is binned in wide bandpowers, current approaches are correct up to the 5% level on the SPT-3G small sky footprint. Furthermore, we propose a new approximation which improves over the previous ones reaching a precision of 1% in the wide bandpowers case and generally more than 4 times more accurate than current approaches. Finally, we derive the covariance matrices for mask-corrected power spectra estimated by the `PolSpice` code. In particular, we include, in the case of a small sky fraction, the effect of the apodization of the large scale modes. While we considered the specific case of the CMB, our results are applicable to any other cosmological probe which requires the calculation of pseudo-power spectrum covariance matrices.

Key words. cosmic background radiation – cosmology: observations – cosmological parameters – methods: data analysis

Please contact me if you have any question !

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