The integrated angular bispectrum of the CMB

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Based on JCAP 06 (2020) 035, 2004.03574, *G. Jung, F. Oppizzi, A. Ravenni and M. Liguo*ri

Non-Gaussianity in the CMB

- **Bispectrum** $B_{\ell_1\ell_2\ell_3}$ $\leftrightarrow$ Three-point correlation function in harmonic space
- Main observable to search for non-Gaussianity (NG) in CMB anisotropies
- Many different shapes of NG, having different origins!

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Different origins of NG:

- **Primordial** (e.g. local): amount and shape of NG depend on the model of inflation
- **Late-time** (e.g. ISW-lensing): correlations between late-time effects modifying the CMB signal
- **Foregrounds** (e.g. dust): some foregrounds (galactic and extra-galactic) emit in the same frequency range as the CMB

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Many bispectra (local, ISW-lensing, dust) **peaks in the squeezed limit** ($\ell_3 \ll \ell_2 \simeq \ell_3$)
Correlations between large-scale and small-scale fluctuations

⇒ Simpler estimator: **integrated bispectrum**

Integrated bispectrum

Position-dependent approach
A physical approach to study NG by measuring how the power spectrum varies over the sky, initially introduced in *Komatsu et al.* (1403.3411) for large-scale structure studies

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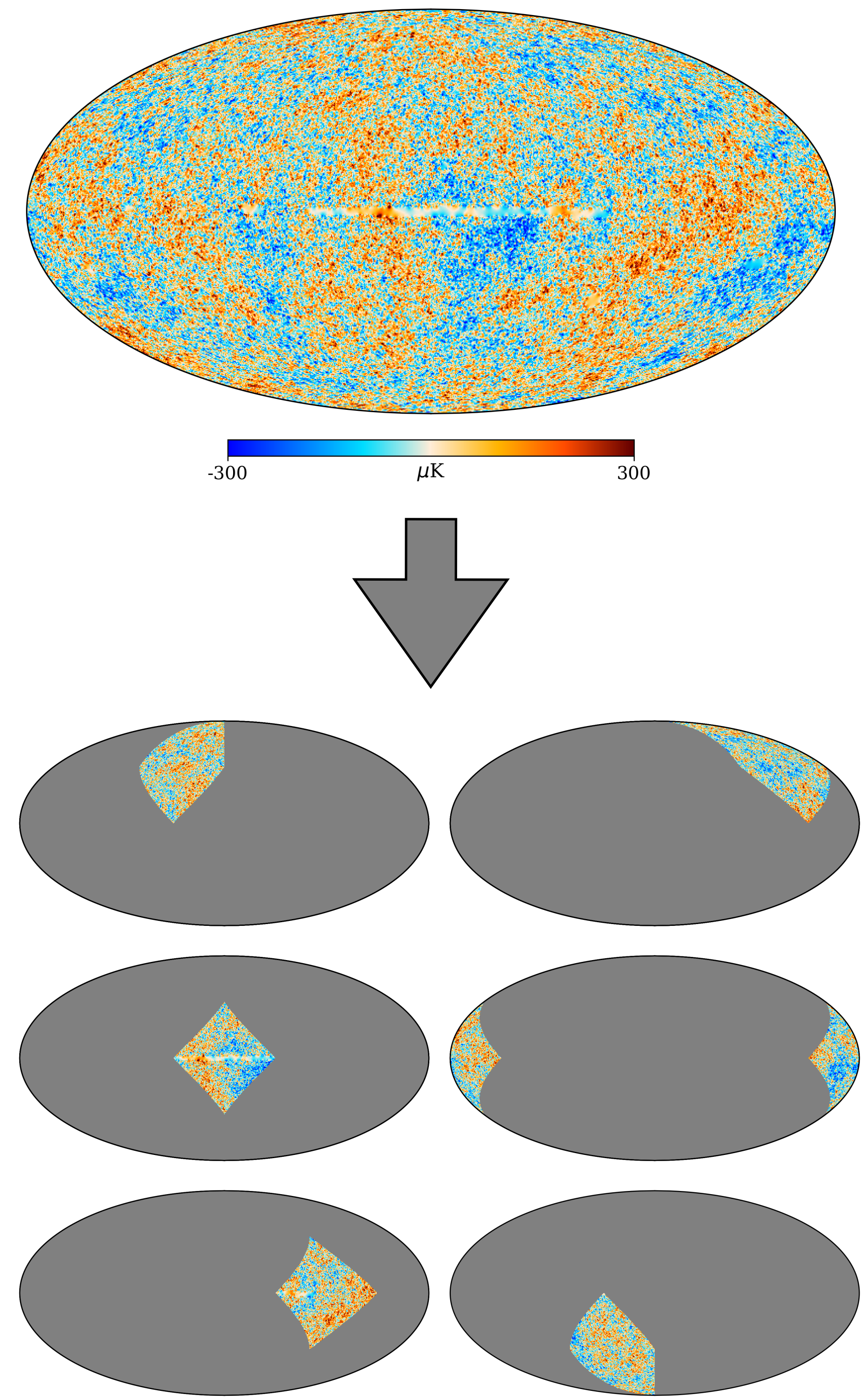


Method

1. Divide the sky into patches of equal size
2. In each patch, measure basic quantities:
   - Power spectrum (⇒ small-scale fluctuations)
   - Average value (⇒ large-scale mode)
3. Determine their patch-by-patch correlation
4. Average over the full sky

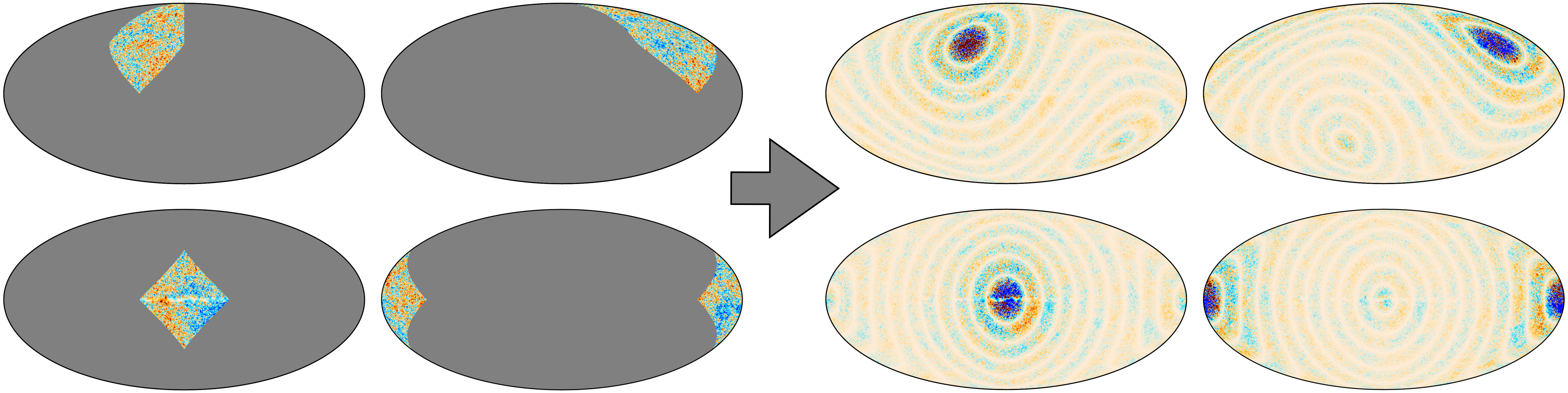
Integrated bispectrum: $\mathrm{I}B_\ell \equiv \frac{1}{N_{\mathrm{patch}}} \sum_{\mathrm{patch}} \bar{M}^{\mathrm{patch}} C_\ell^{\mathrm{patch}}$

Measures the modulation of small-scale fluctuations by a large-scale mode

⇒ Probes the **squeezed limit of the bispectrum**

Integrated angular bispectrum

Theoretical predictions of the integrated bispectrum are possible using azimuthally symmetric patches



Center — Legendre polynomials

Patch: $W(\hat{\Omega}, \hat{\Omega}_c) = \sum_{\ell} w_{\ell} \frac{2\ell + 1}{4\pi} P_{\ell}(\hat{\Omega} \,.\, \hat{\Omega}_c)$

Here $w_{\ell} = 1$ for $\ell \leq 10$ and 0 otherwise

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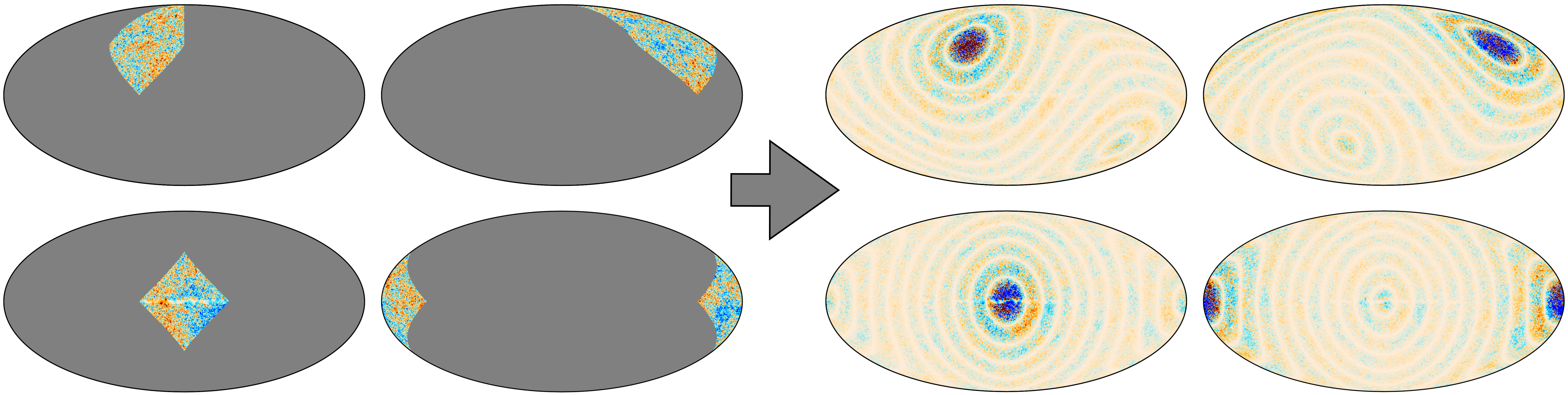

Product of Wigner symbols

$$\mathrm{I}B_\ell = \sum_{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5} B_{\ell_1\ell_2\ell_3} w_{\ell_3} w_{\ell_4} w_{\ell_5} \mathscr{F}_{\ell\ell_1\ell_2\ell_3\ell_4\ell_5}$$

Squeezed configurations (ℓ_3 small)

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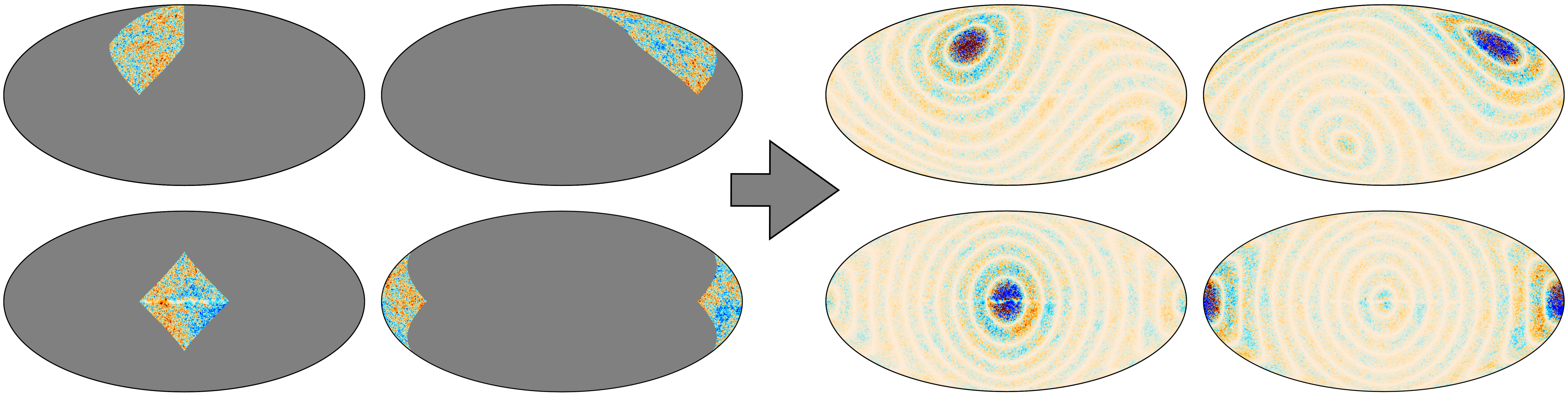

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Squeezed configurations (ℓ_3 small)

In the weak non-Gaussianity regime, similar expression for the covariance:

$$\mathrm{I}C_{\ell\ell'} = \sum_{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell'_4\ell'_5} C_{\ell_1}C_{\ell_2}C_{\ell_3} w_{\ell_3} w_{\ell_4} w_{\ell_5} w_{\ell'_4} w_{\ell'_5} \mathscr{F}_{\ell\ell_1\ell_2\ell_3\ell_4\ell_5} \times \left[w_{\ell_3} (\mathscr{F}_{\ell_1\ell_2\ell_3\ell'_4\ell'_5} + \mathscr{F}_{\ell_2\ell_1\ell_3\ell'_4\ell'_5}) + \ldots \right]$$

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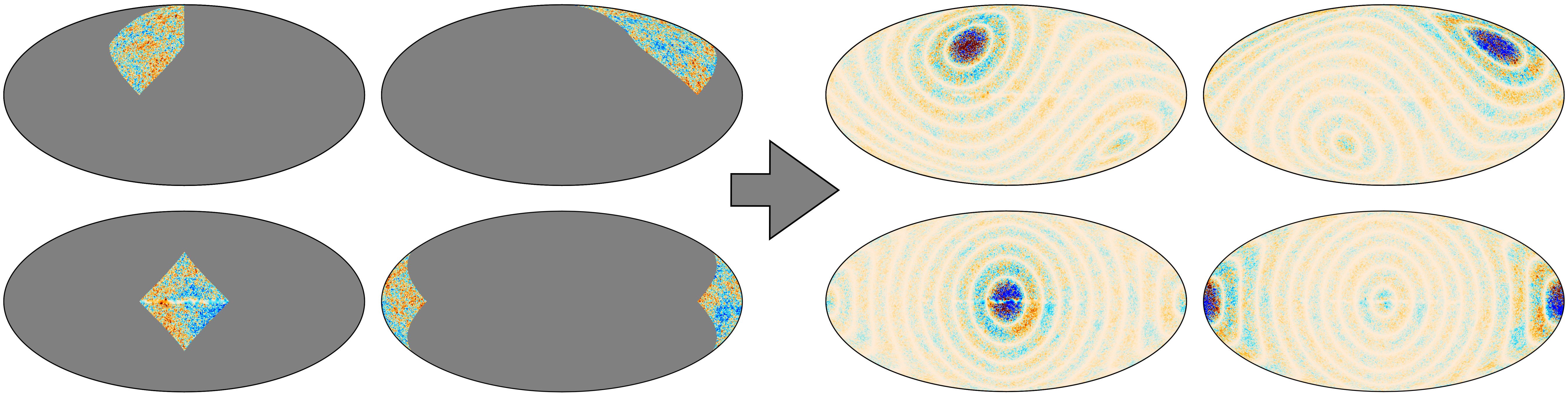

Product of Wigner symbols

$$IB_\ell = \sum_{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5} B_{\ell_1\ell_2\ell_3} w_{\ell_3} w_{\ell_4} w_{\ell_5} \mathcal{F}_{\ell\ell_1\ell_2\ell_3\ell_4\ell_5}$$

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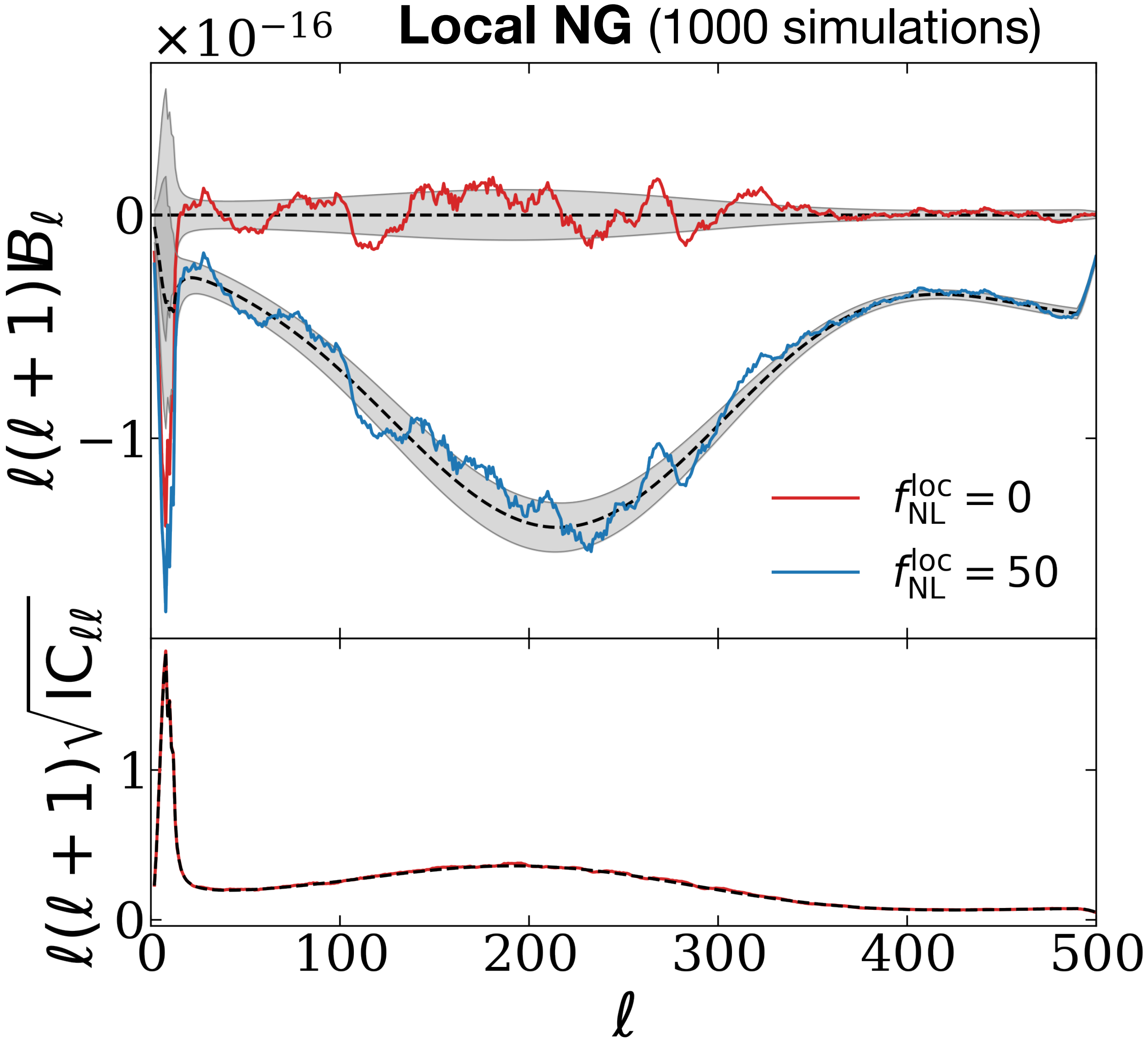
Integrated angular bispectrum:

⇒ Simple observable of NG (squeezed limit) for any 2D cosmological field

⇒ Easy to estimate from data (only requires to measure power spectra)

⇒ Easy to compute exactly analytically for any given bispectrum template

CMB integrated bispectrum



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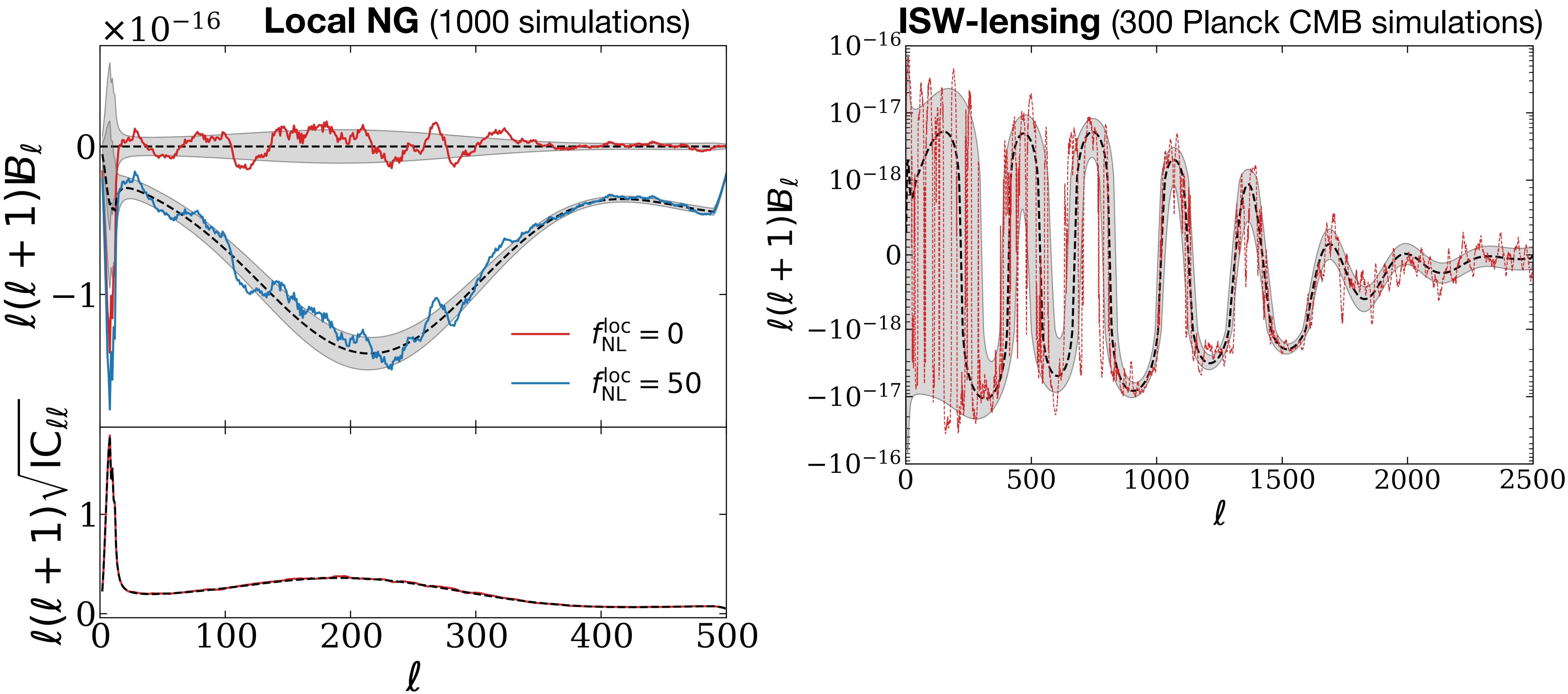

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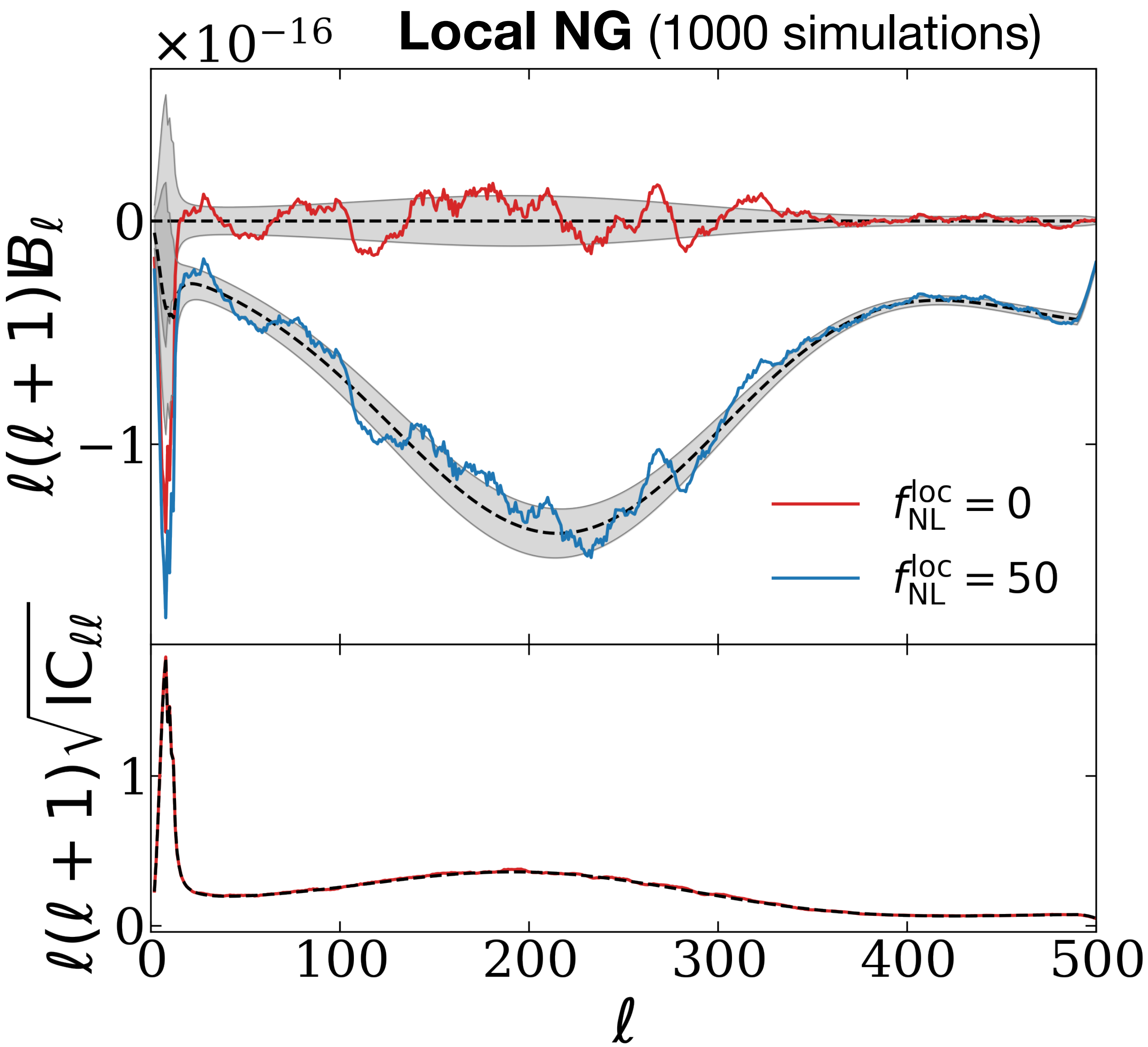
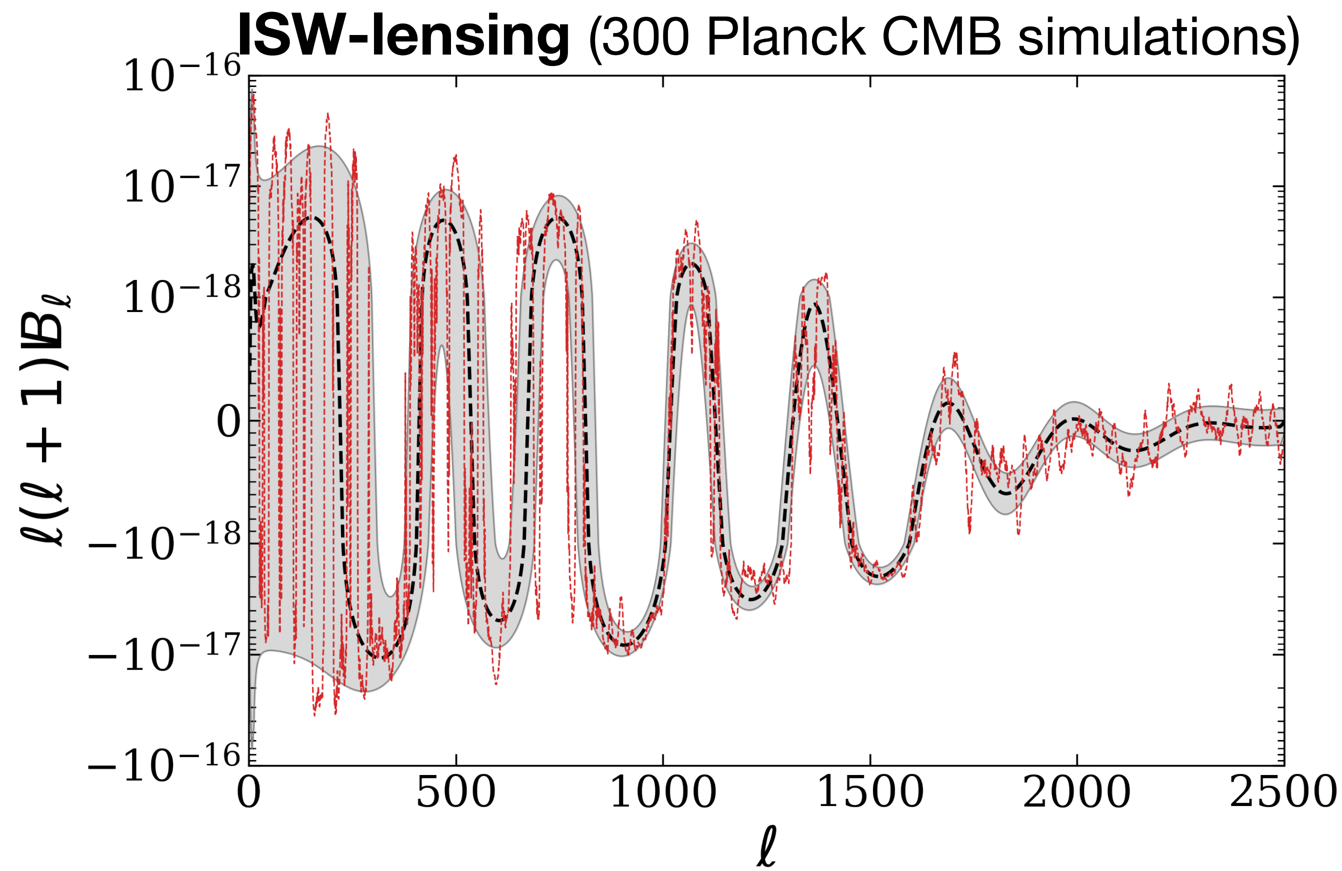




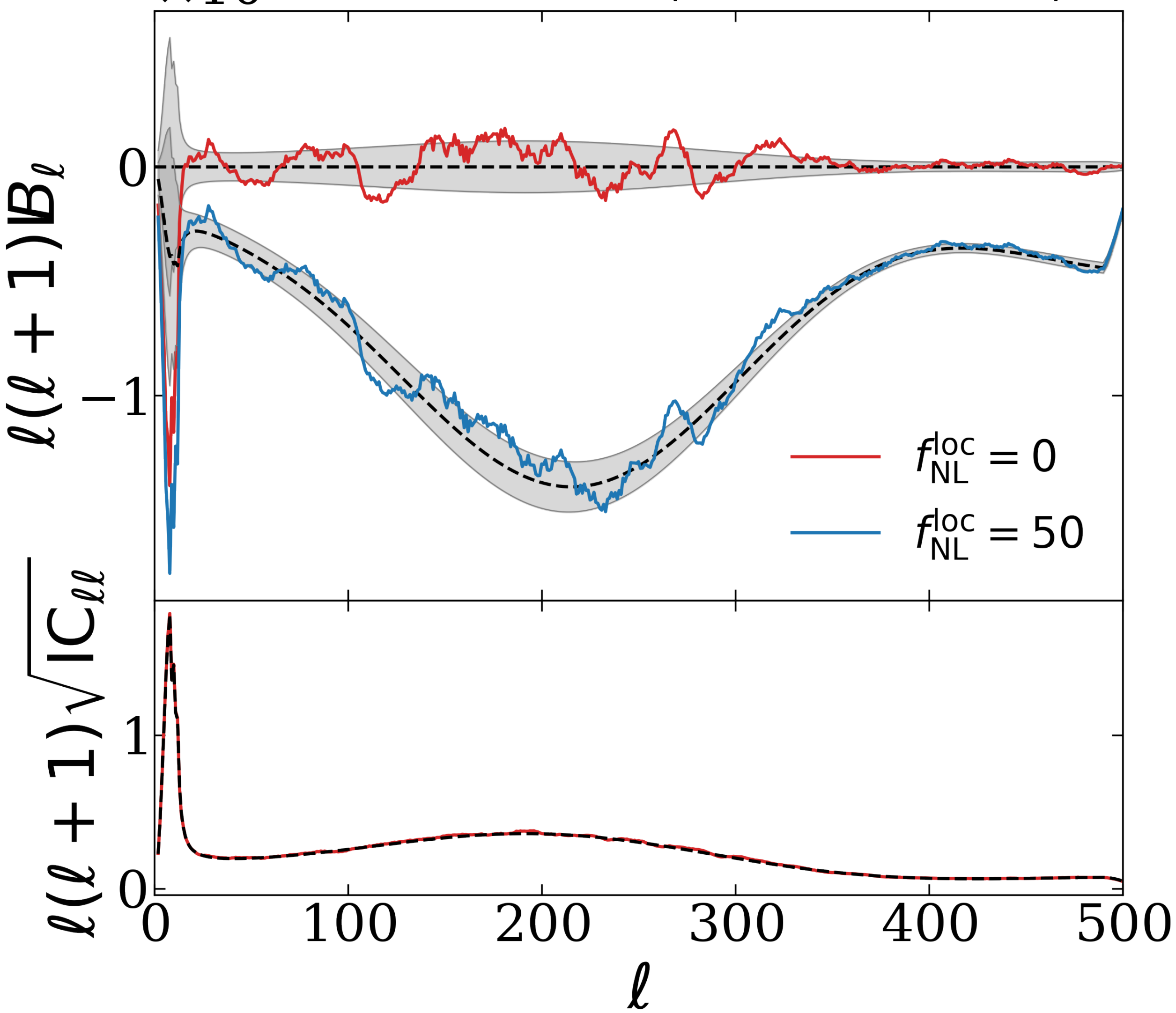
$f_{\rm NL}$**-estimator:** $$\hat{f}_{\rm NL} = \frac{\sum_{\ell\ell'} {\rm I}B_\ell^{\rm obs}\,{\rm I}C^{-1}_{\ell\ell'}\,{\rm I}B_{\ell'}^{\rm th}}{\sum_{\ell\ell'} {\rm I}B_\ell^{\rm th}\,{\rm I}C^{-1}_{\ell\ell'}\,{\rm I}B_{\ell'}^{\rm th}},$$

Planck temperature data:

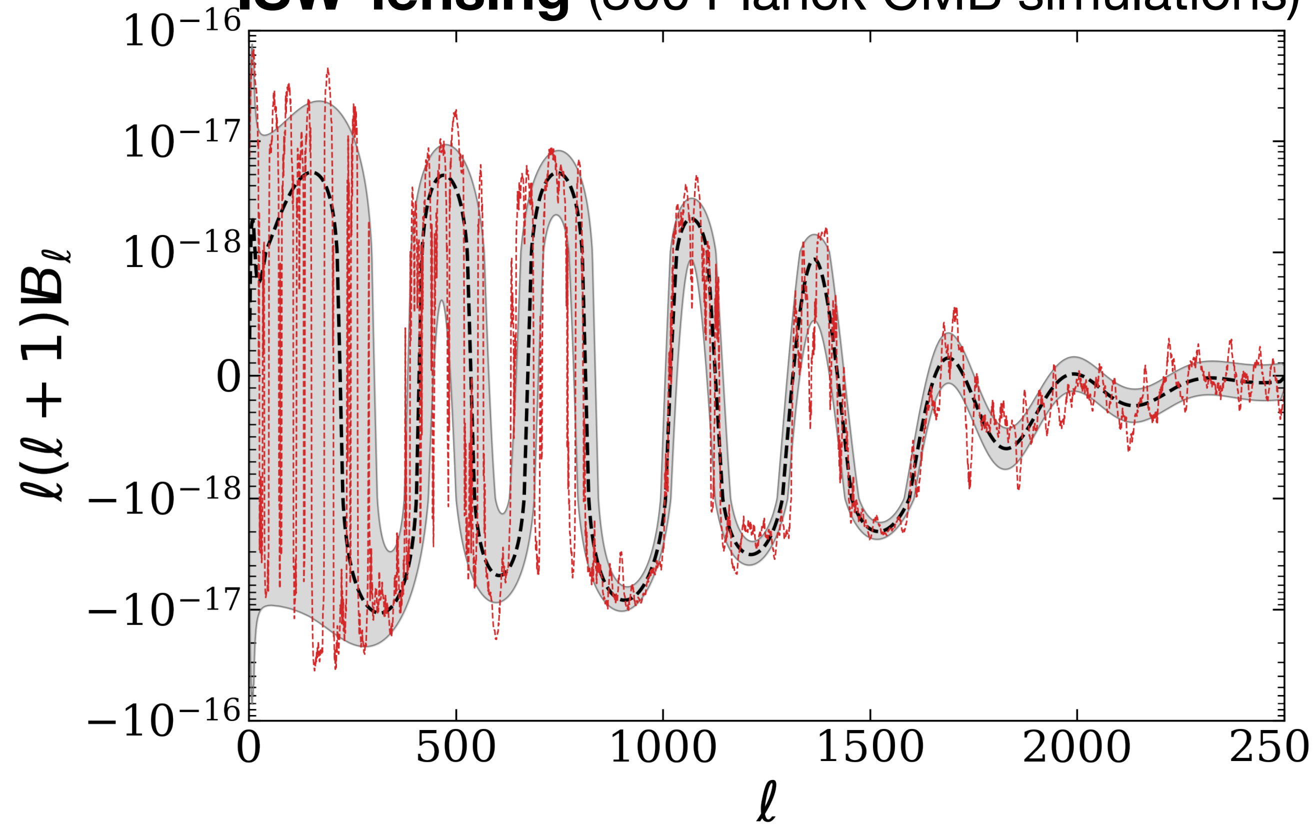
	IB-estimator	Planck 2018
Local	7.1 ± 7.5	6.7 ± 5.6
ISW-lensing	0.8 ± 1.0	0.7 ± 0.3
Local (ISW-lensing removed)	5.0 ± 8.5	-0.5 ± 5.6

CMB integrated bispectrum

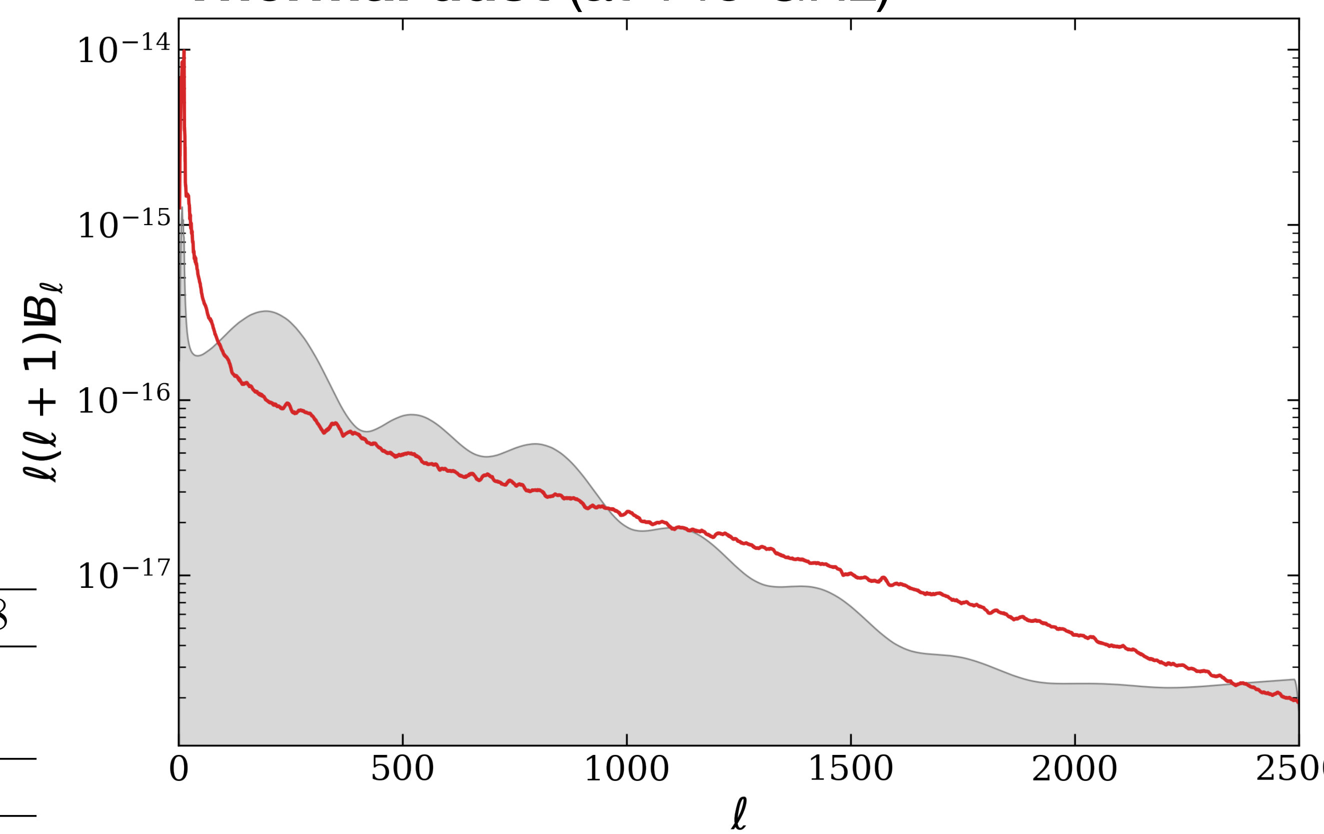
Local NG (1000 simulations)



ISW-lensing (300 Planck CMB simulations)



Thermal dust (at 143 GHz)



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ISW-tSZ-tSZ

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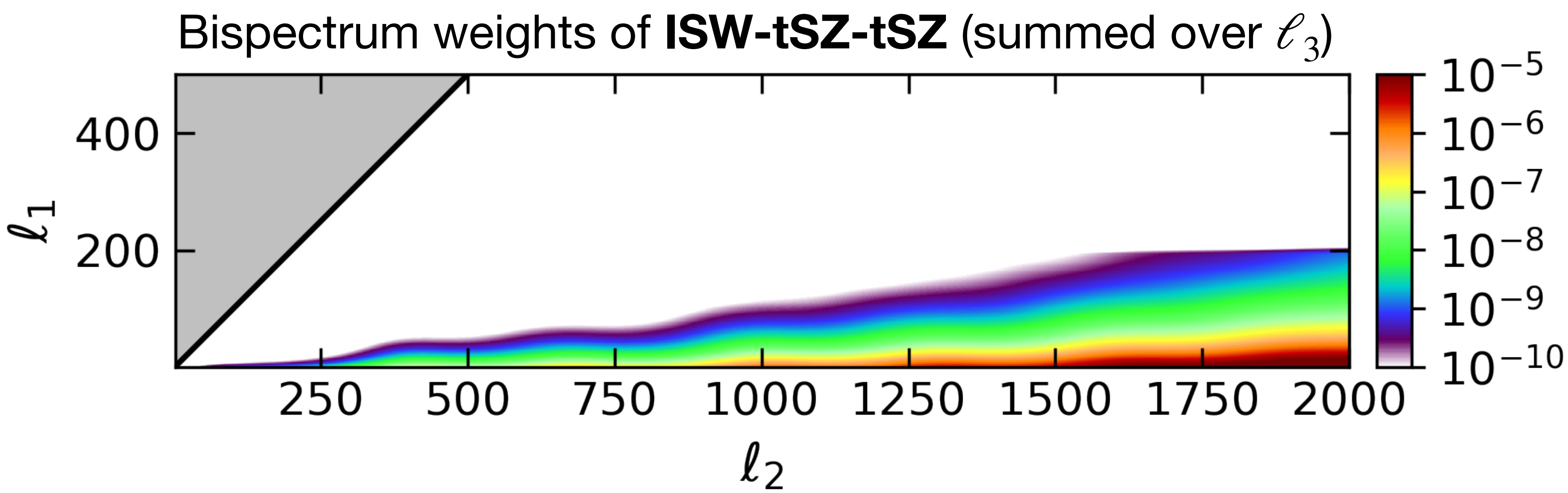

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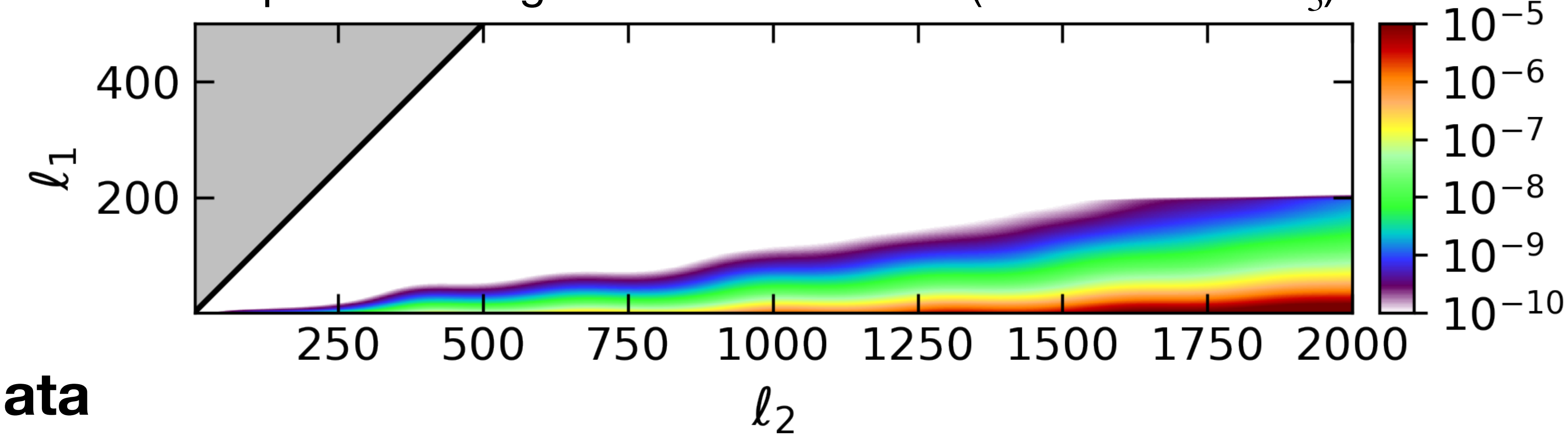
Bispectrum weights of **ISW-tSZ-tSZ** (summed over ℓ_3)



In Planck data

Using cleaned single-frequency maps (SEVEM)

	Local	ISW-lensing	ISW-tSZ-tSZ
100 GHz	11.8 ± 10.9	0.4 ± 1.9	-5.8 ± 8.0
143 GHz	10.3 ± 8.7	0.7 ± 1.2	-4.1 ± 7.5

⇒ ISW-tSZ-tSZ too small to have an impact on Planck results

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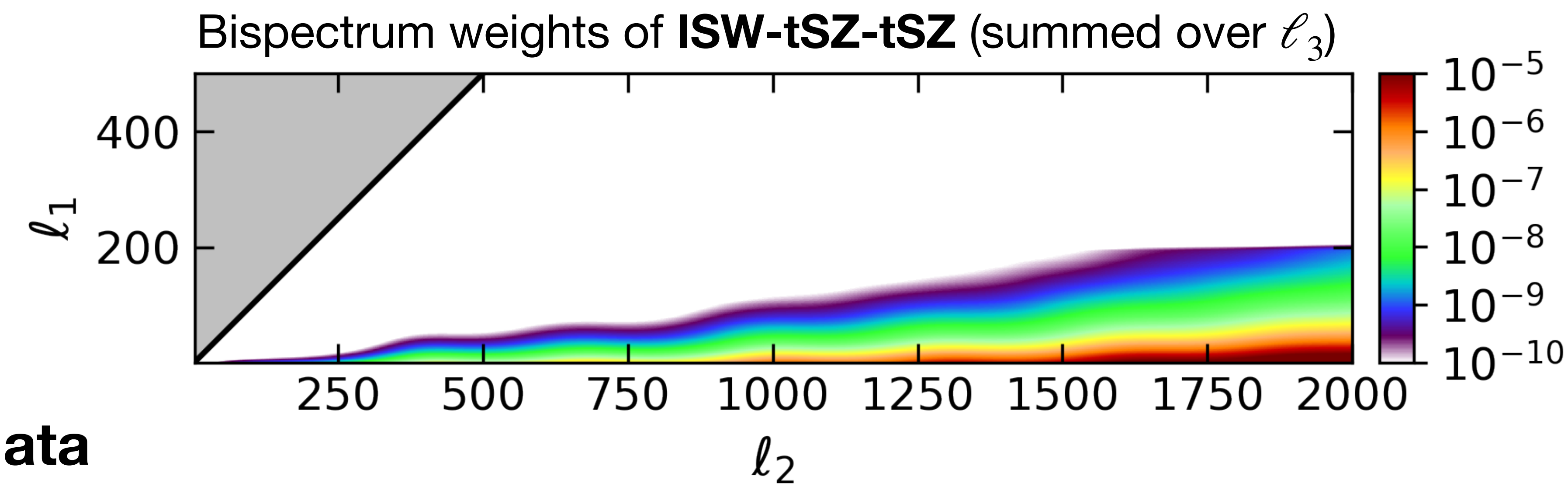

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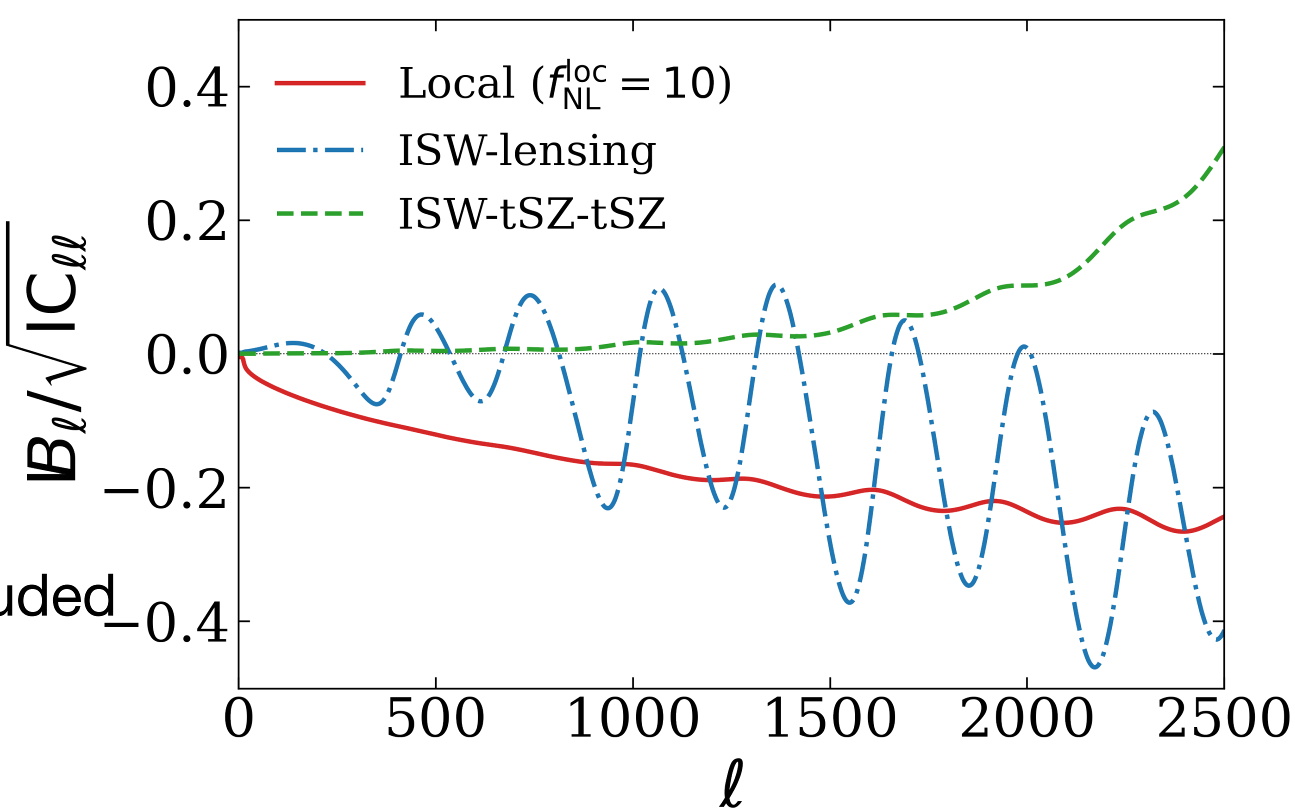

Future ($\ell_{\max} > 2000$)

- Large bispectrum when smaller scales are included
- Can bias primordial NG results
- Contains useful cosmological information

Conclusions

Integrated angular bispectrum: new pipeline for the study of squeezed NG

- **Simple**: only requires to estimate power spectra
- **Exact theoretical predictions** (integrated bispectrum + covariance)

Valid for any 2D cosmological field defined on the celestial sphere

- CMB:
  - Validation of the pipeline on simulations
  - Application to Planck data ($f_{\rm NL}^{\rm local}$, ISW-lensing, ISW-tSZ-tSZ)
- Further applications for **weak lensing**
  - JCAP 06 (2021) 055, 2102.05521, *G. Jung, T. Namikawa, M. Liguori, D. Munshi and A. Heavens*
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Thanks for your attention !