Single frequency CMB B-mode inference with realistic foregrounds from a single training image

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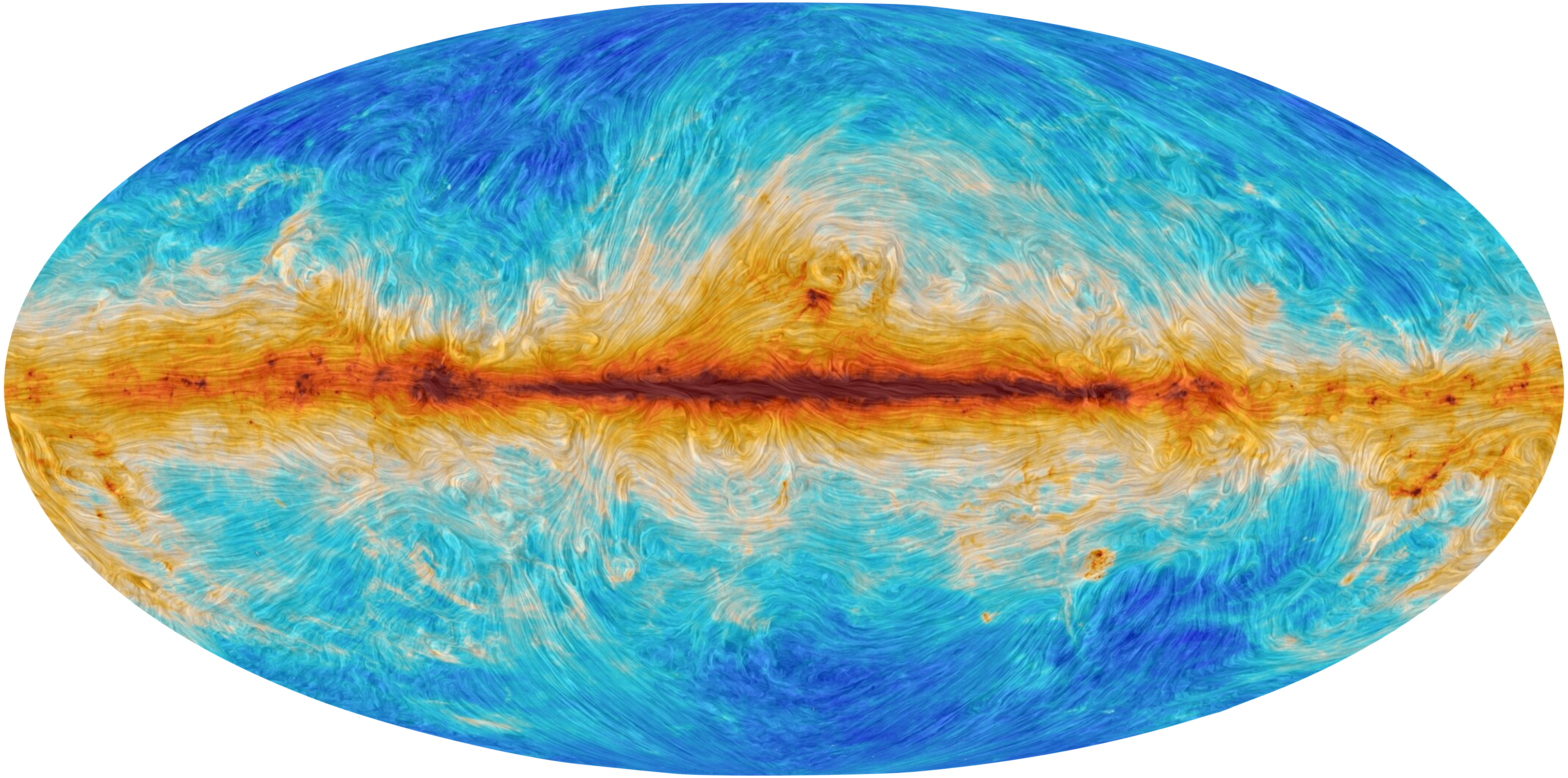
From Planck to the future of the CMB, May 2022

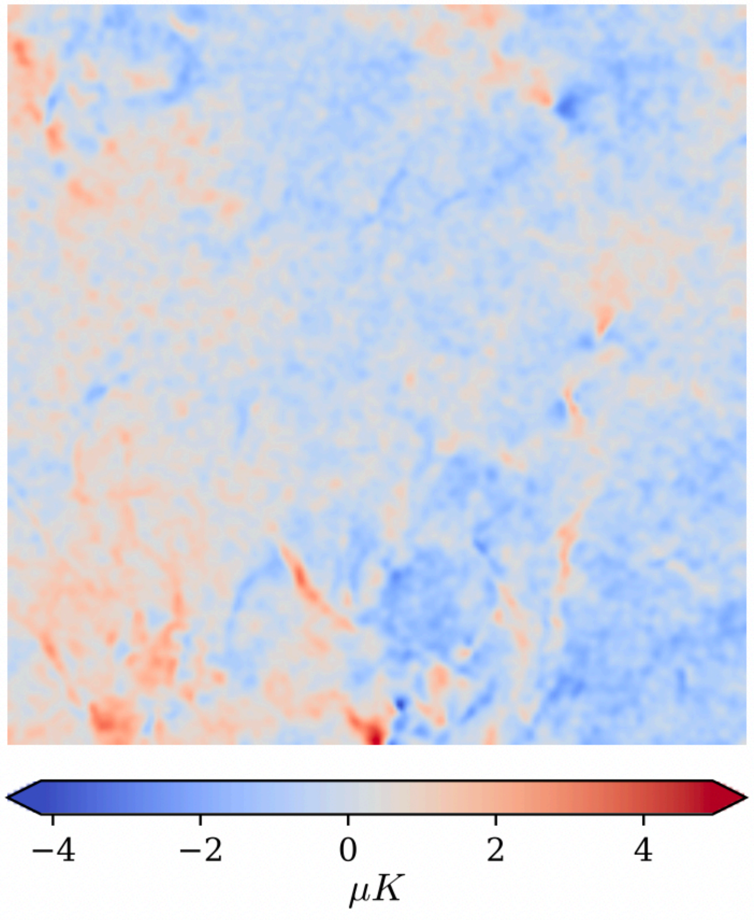
Search for "B-modes" signal of cosmic inflation

image credit: ESA

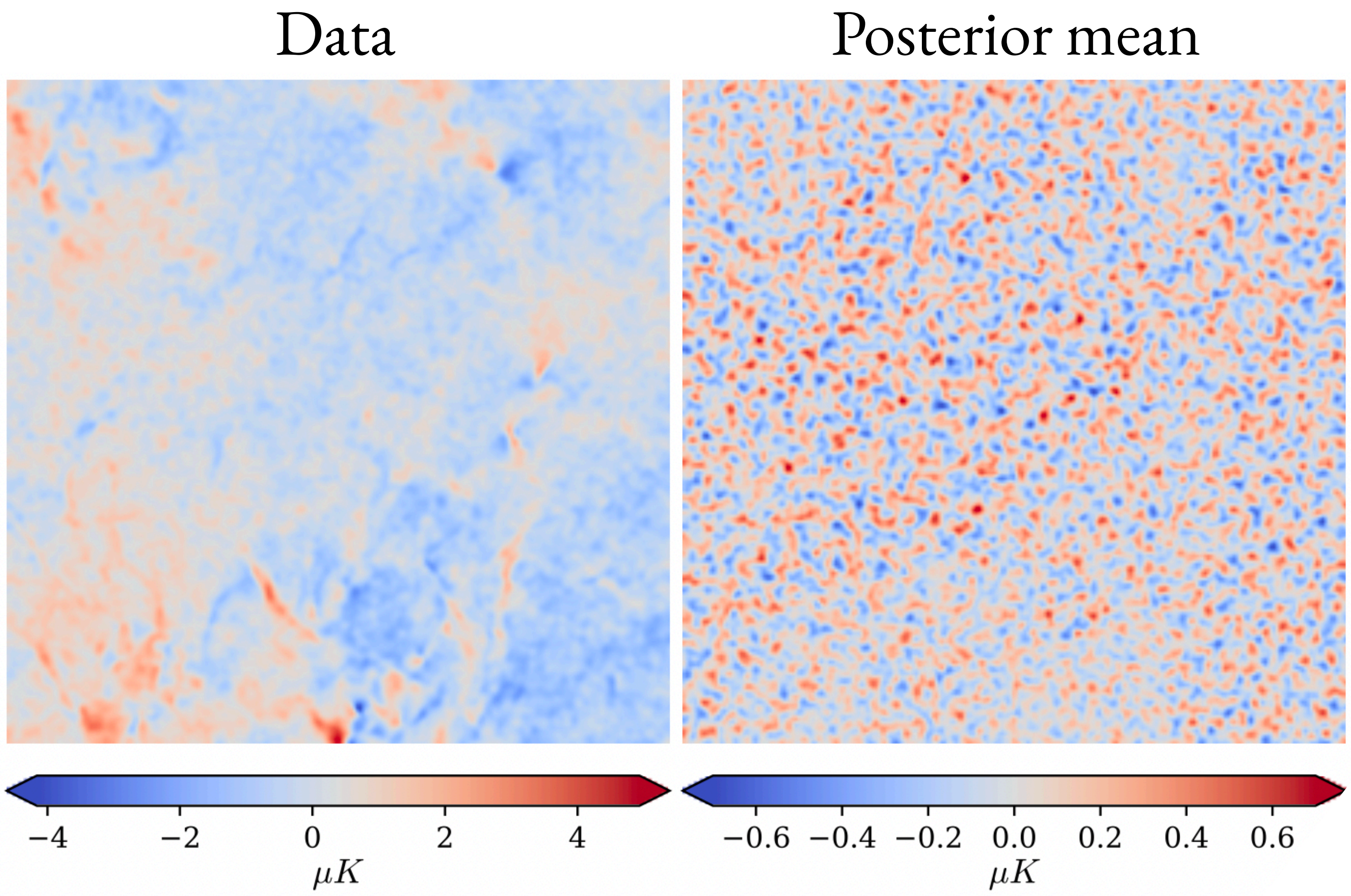


B-mode inference

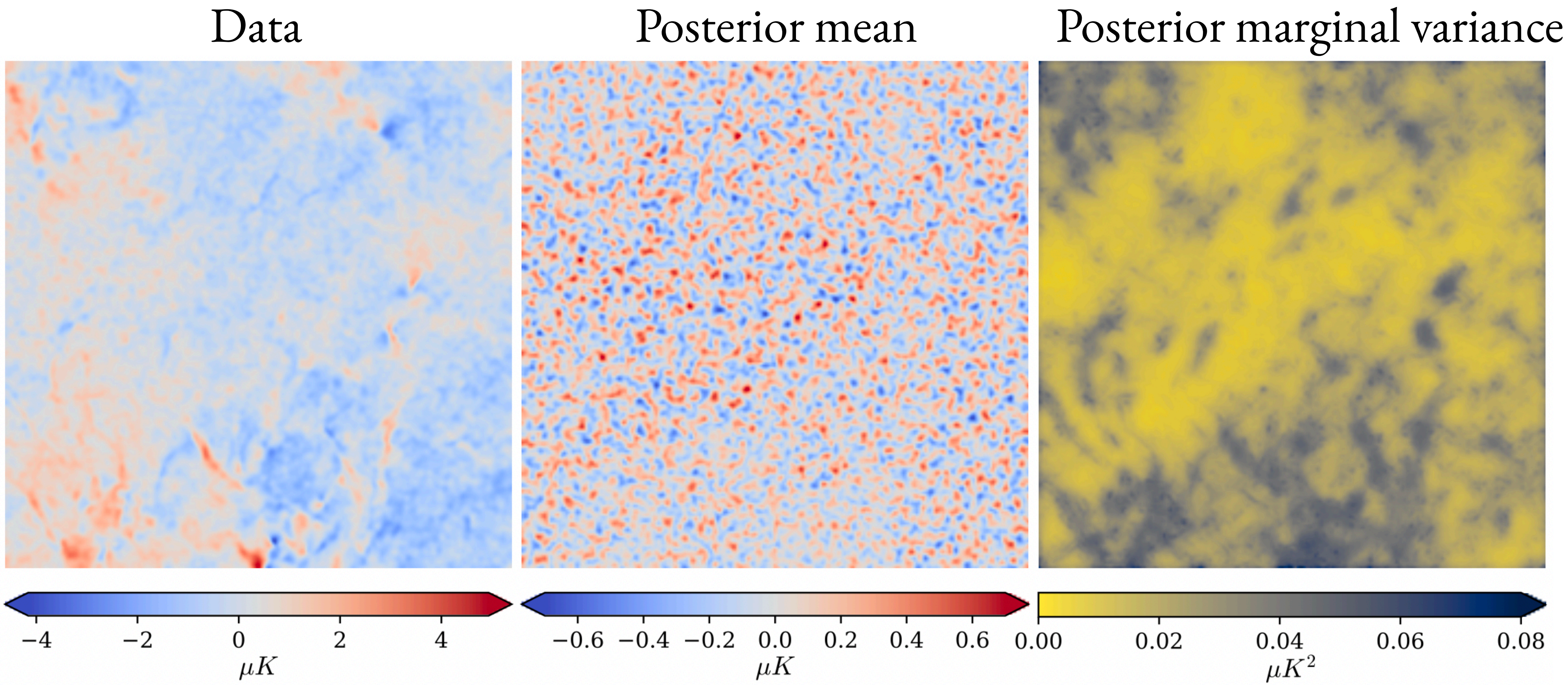
Data



B-mode inference



B-mode inference



Outline

1. Likelihood-free inference
2. Realistic foreground synthesis
3. Posterior validation
4. Neural foreground generator

1. Likelihood-free inference
(in high-dimension)

Why "likelihood-free"?

Observed summary statistic d_o & unknown parameters θ

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Observed summary statistic d_o & unknown parameters θ

$$p(\theta \mid d_o) \propto p(d_o \mid \theta)\ p(\theta)$$

LIKELIHOOD

PRIOR

Density-estimation likelihood-free

$$p(d \mid \theta)\ ?$$

Density-estimation likelihood-free

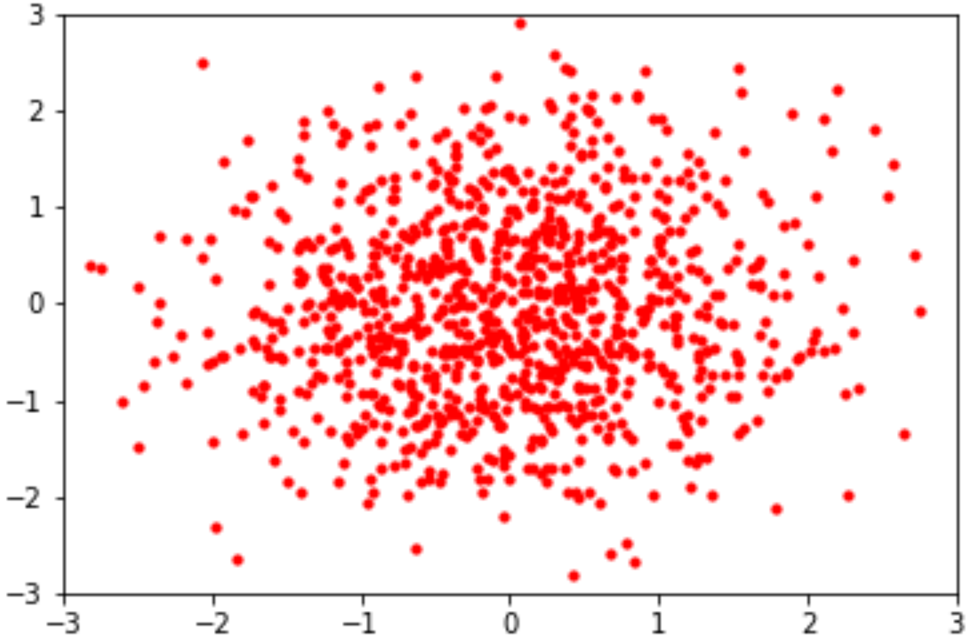
$$p(d\,|\,\theta)\ ?$$

Draw d_i from the distribution $p(d\,|\,\theta_i)$ by running a simulation:

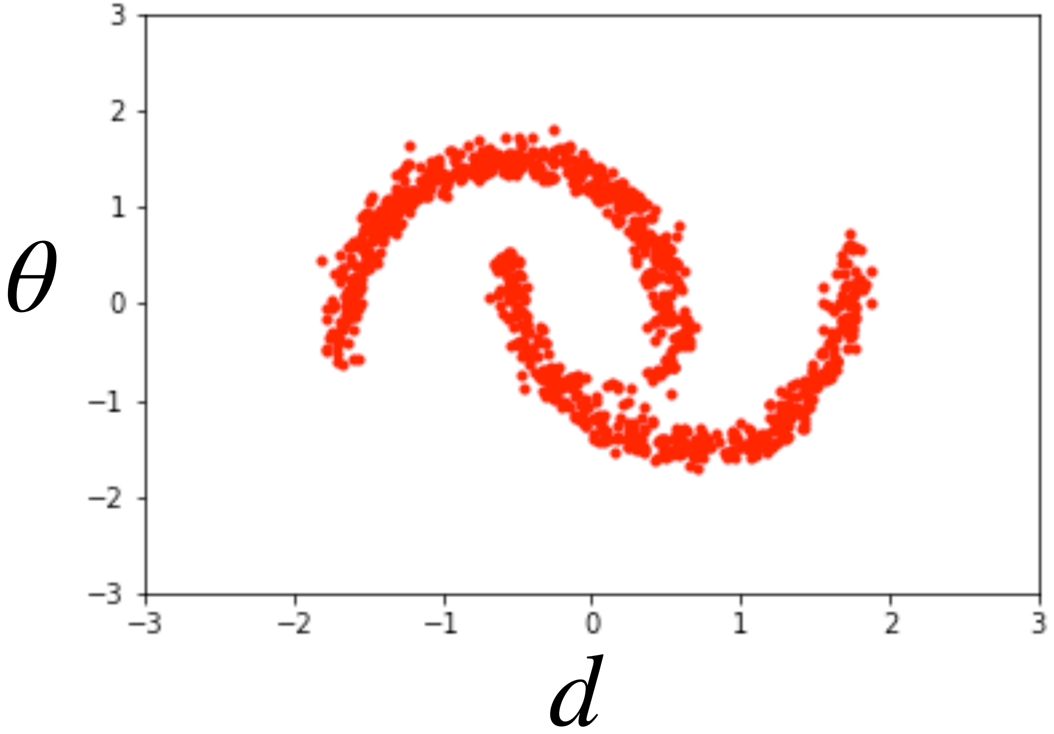
$$\{d_i, \theta_i\}$$

Estimate density from simulations: $p(d\,|\,\theta)$

Image credit: Eric Jang



Normalising Flow



Simulated data realizations

Moment Networks

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1. Hierarchy of networks
2. Estimate marginal posterior moments

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$$J_0 = \int ||\boldsymbol{s} - \mathcal{F}(\boldsymbol{d})||^2 p(\boldsymbol{d}, \boldsymbol{s}) \, \mathrm{d}\boldsymbol{d} \, \mathrm{d}\boldsymbol{s}$$

Moment Networks

NJ & Wandelt 2011.05991

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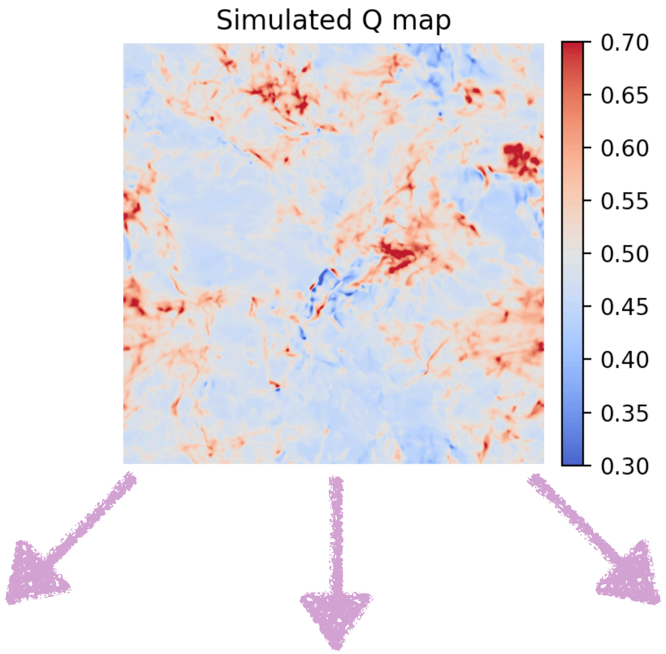
$$J_0 = \int ||\boldsymbol{s} - \mathcal{F}(\boldsymbol{d})||^2 p(\boldsymbol{d}, \boldsymbol{s})\ \mathrm{d}\boldsymbol{d}\ \mathrm{d}\boldsymbol{s}$$

$$J_1 = \int ||(\boldsymbol{s} - \mathcal{F}_{\mathrm{fixed}}(\boldsymbol{d}))^2 - \mathcal{G}(\boldsymbol{d})||^2 p(\boldsymbol{d}, \boldsymbol{s})\ \mathrm{d}\boldsymbol{d}\ \mathrm{d}\boldsymbol{s}$$

2. Forward model &
B-mode inference

Generative model for data

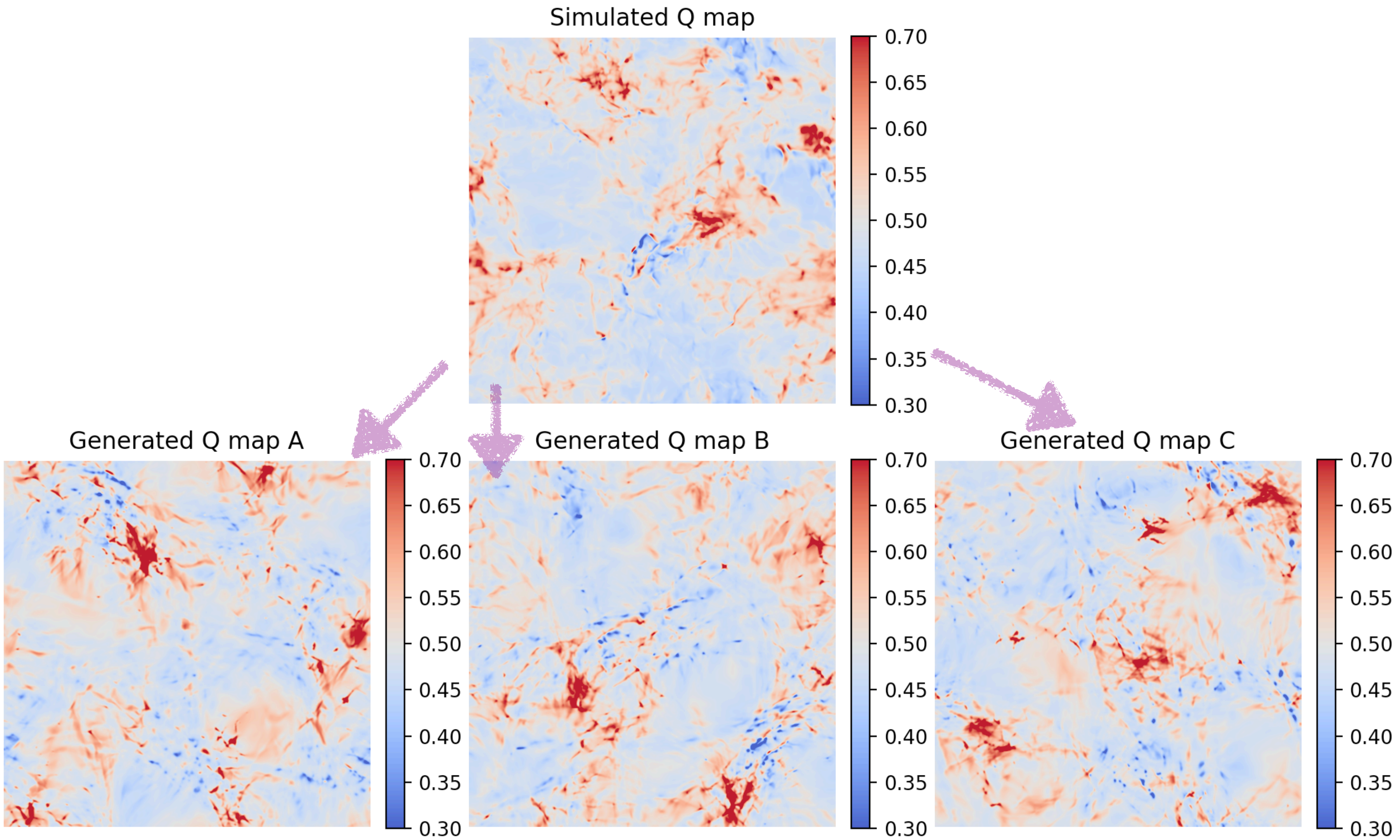


Wavelet Phase Harmonics

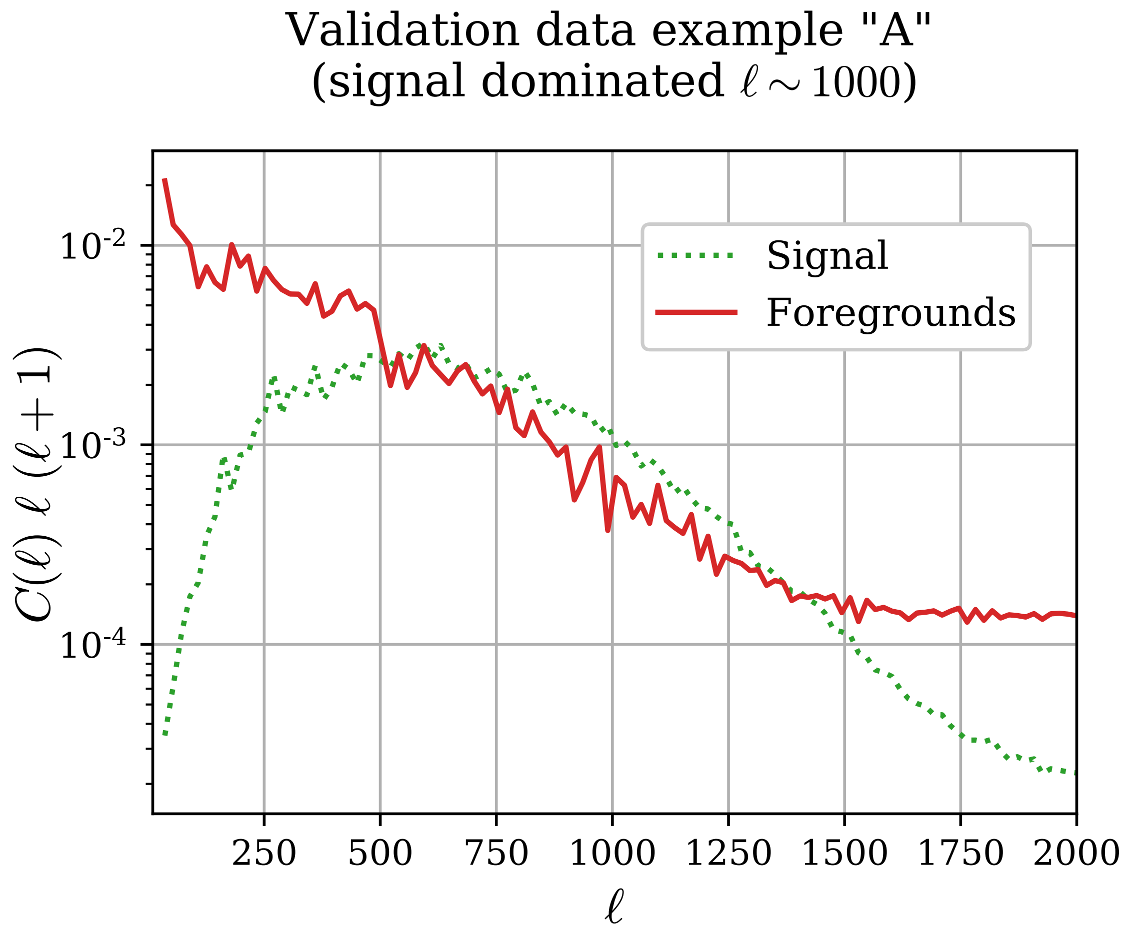
$$\phi(f)_{p_1=0,p_2=1} = \mathrm{Cov}\Big(\psi_{j_1,l_1} \circledast f(\overrightarrow{x}) \ , \ ||\psi_{j_2,l_2} \circledast f(\overrightarrow{x} + \vec{\tau})||_1\Big)$$

(see Allys++ 2006.06298)

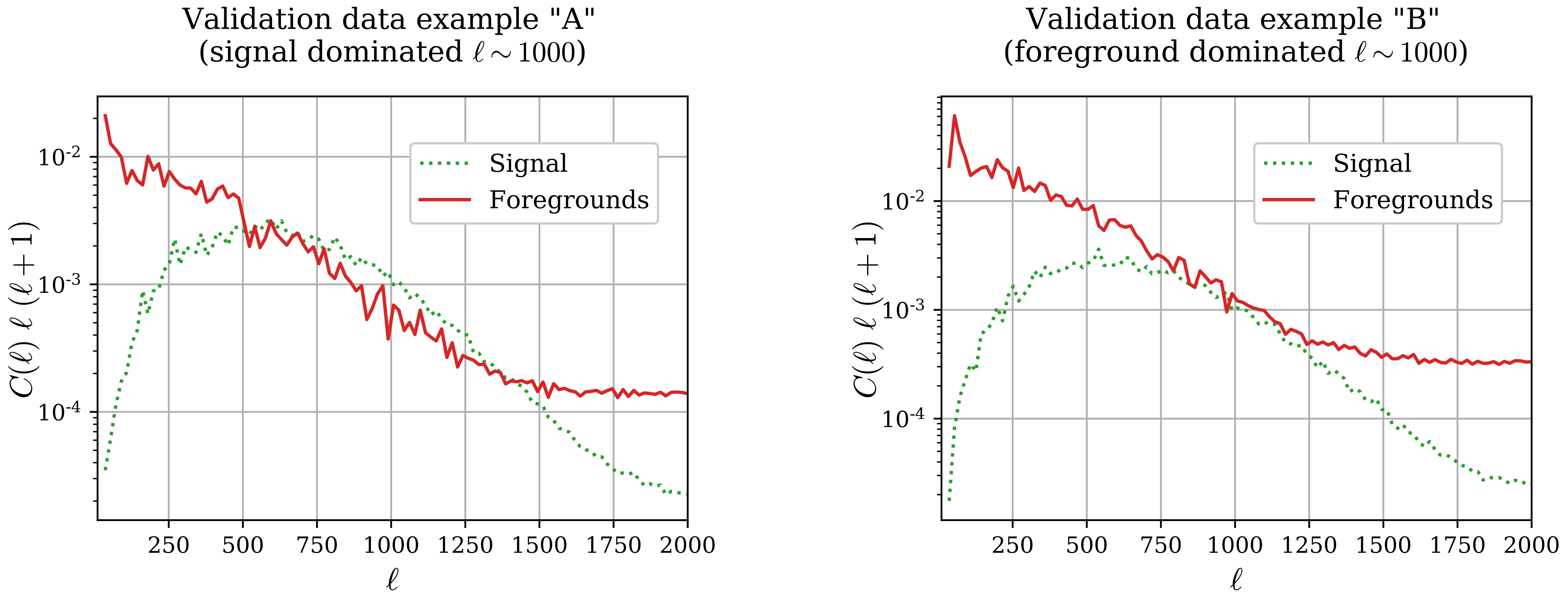
Generative for foregrounds



Nuisance parameters from prior



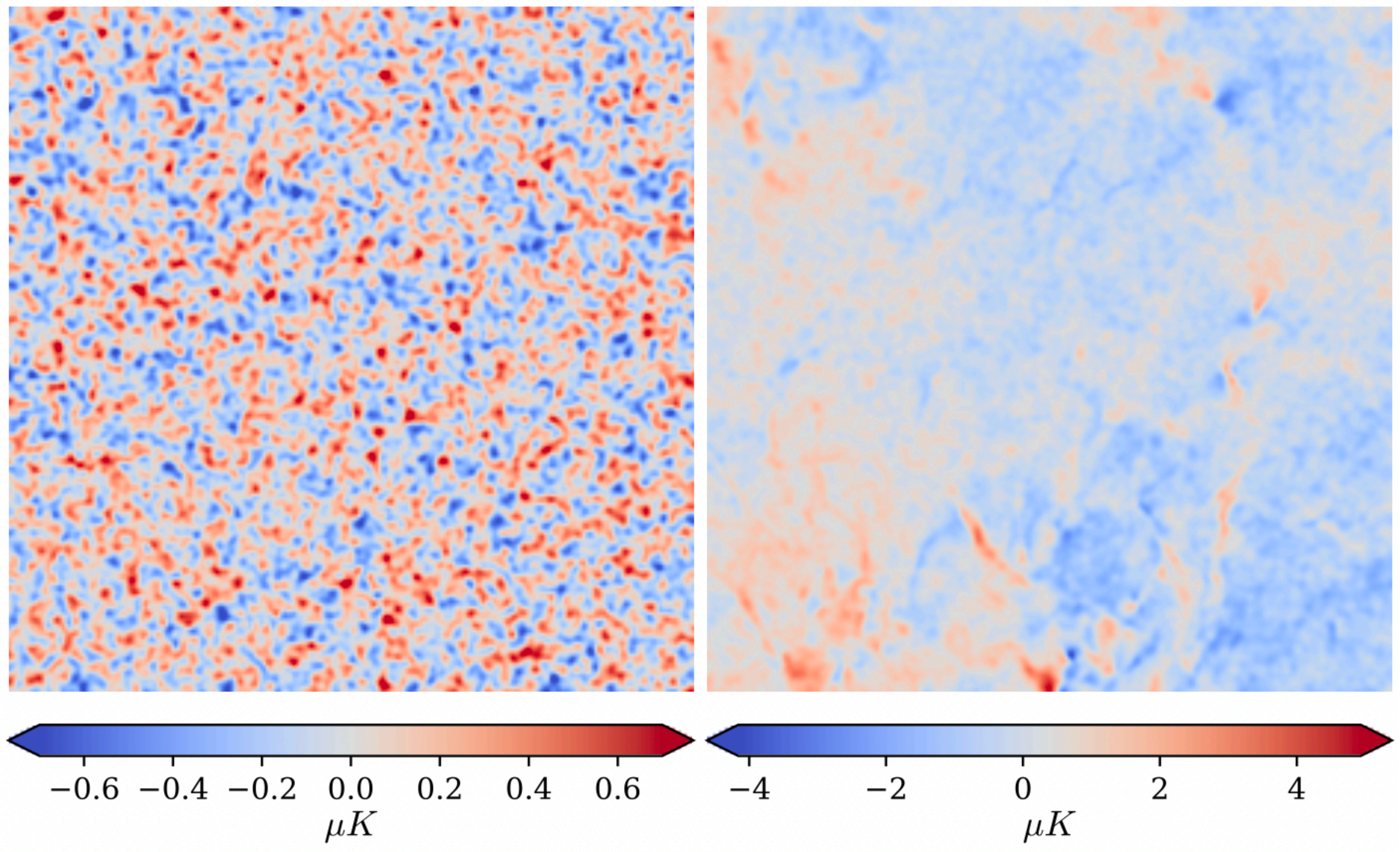
Nuisance parameters from prior



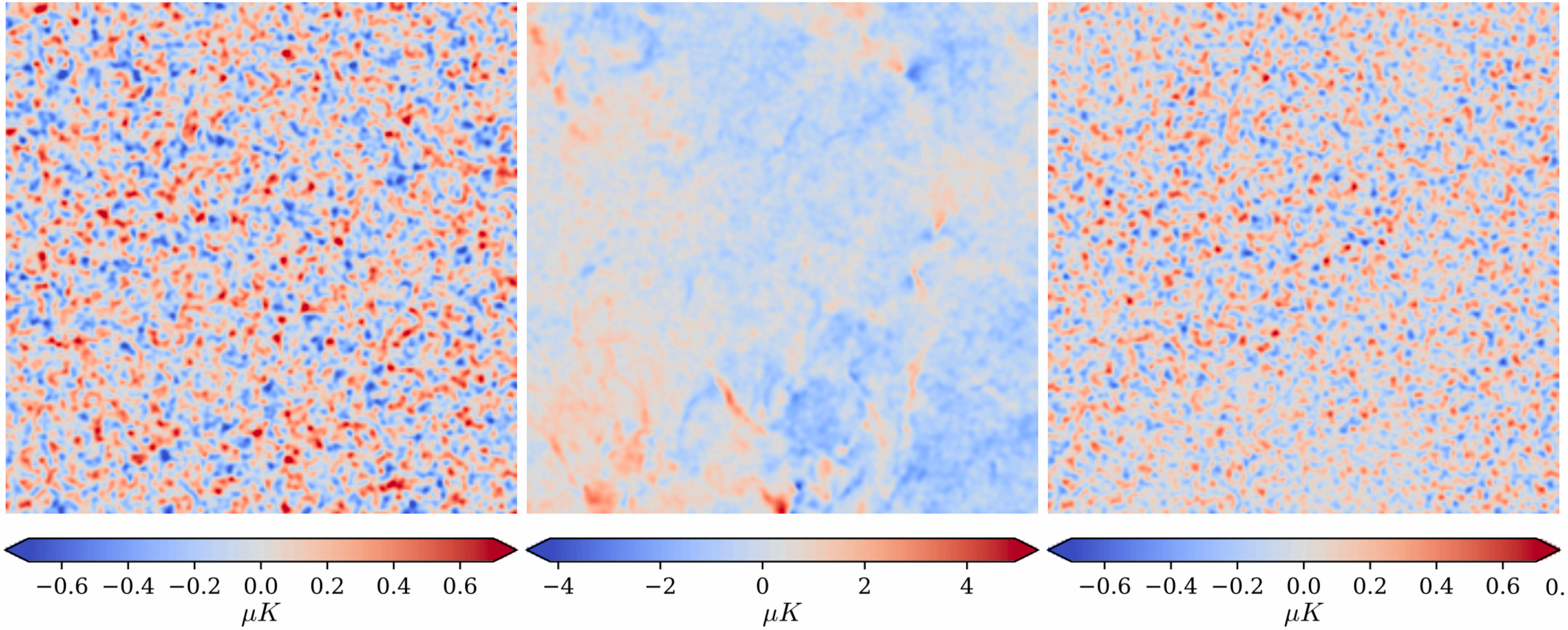
We now have:

- CMB simulation ✔
- Foreground "synthesiser" ✔
- Inference tool (Moment Network) ✔

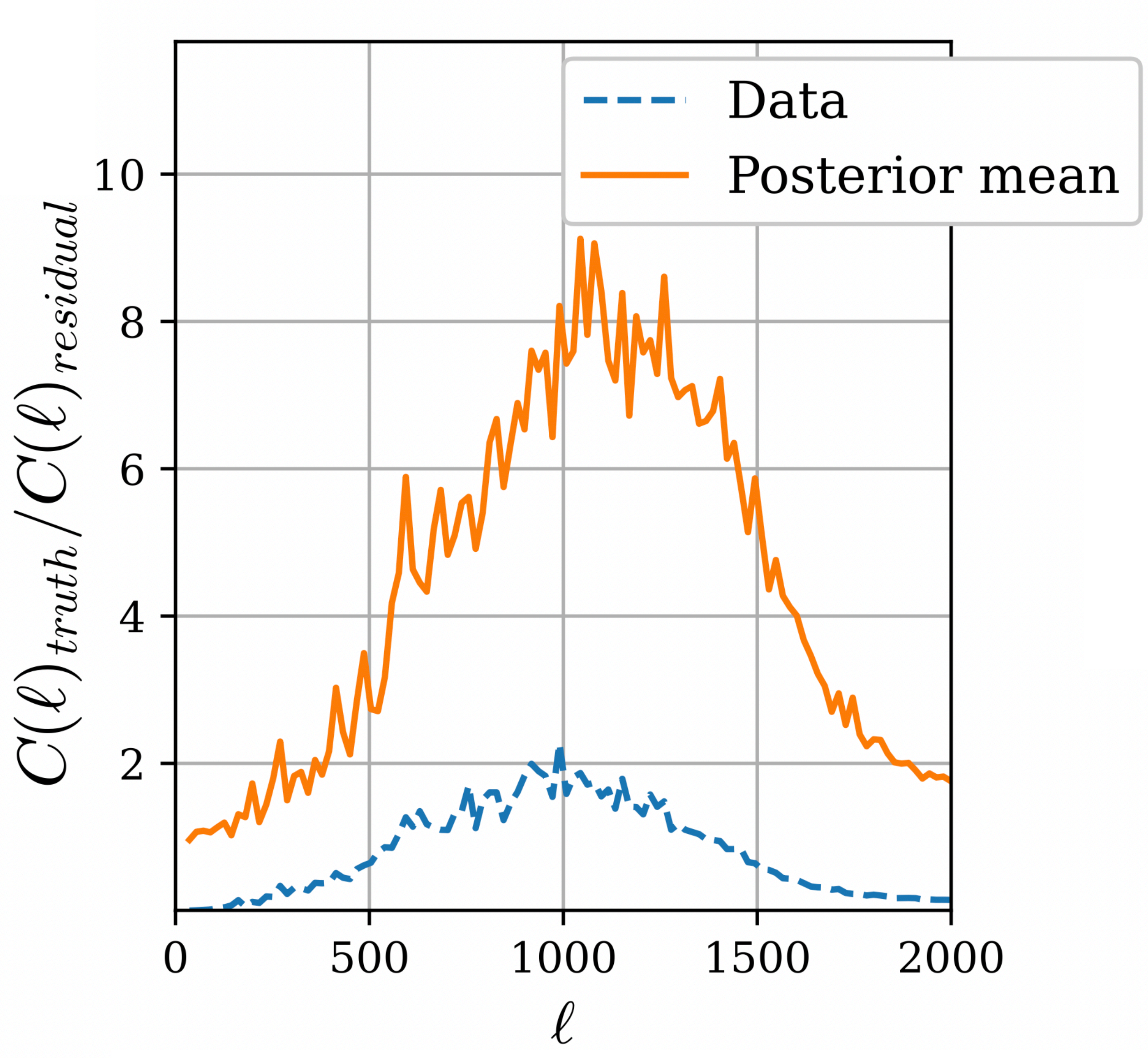
B-mode inference



B-mode inference



Recovered "signal-to-noise"

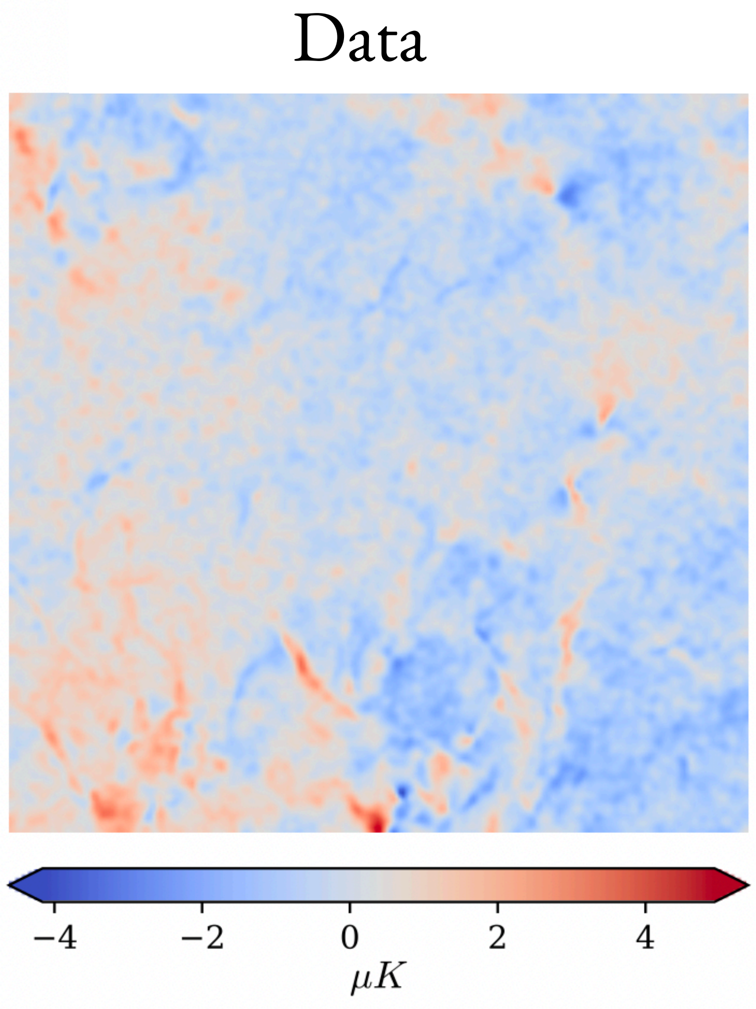


3. Posterior validation

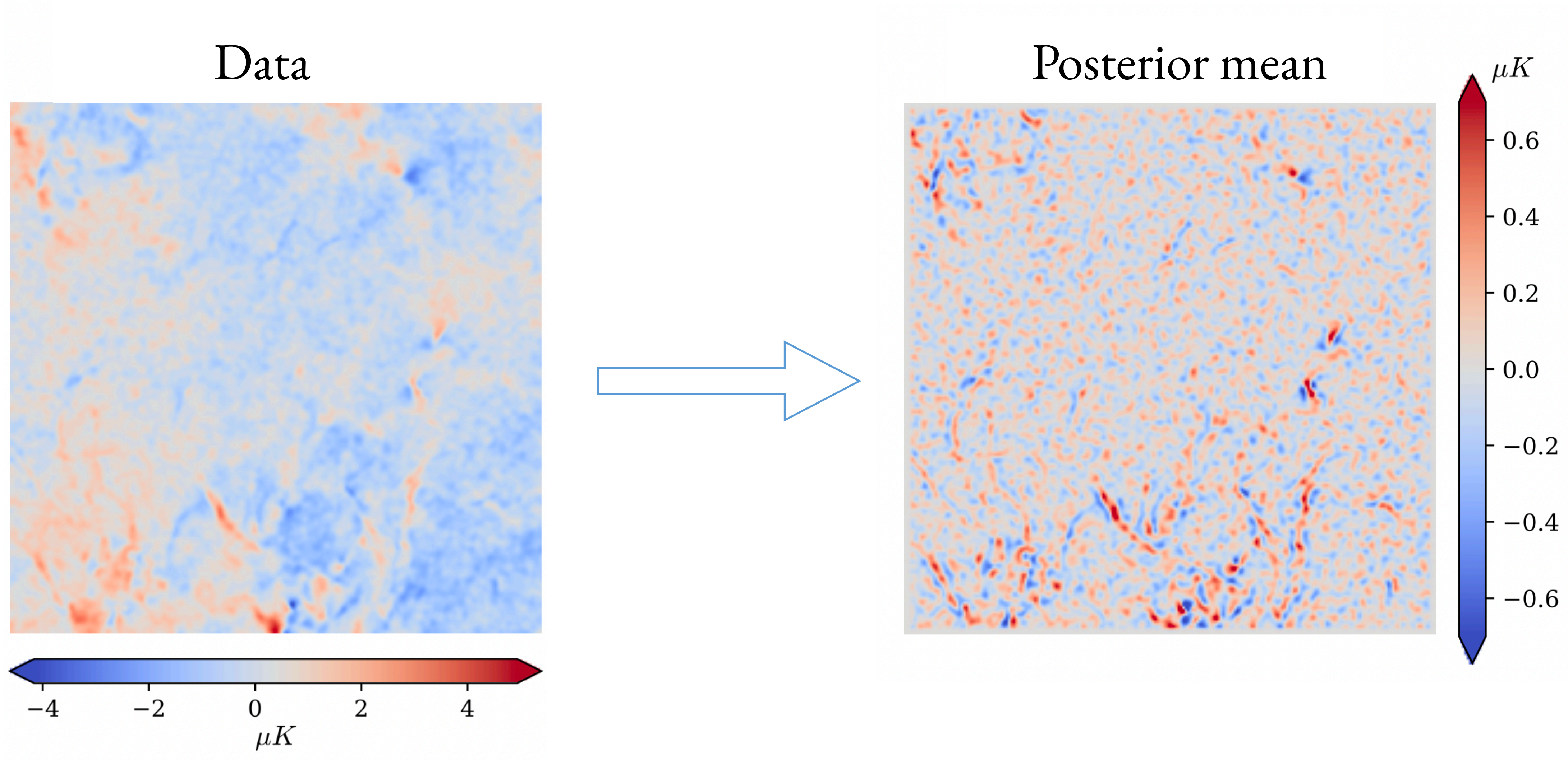
How can we test the posterior?

What does the "wrong" answer look like…

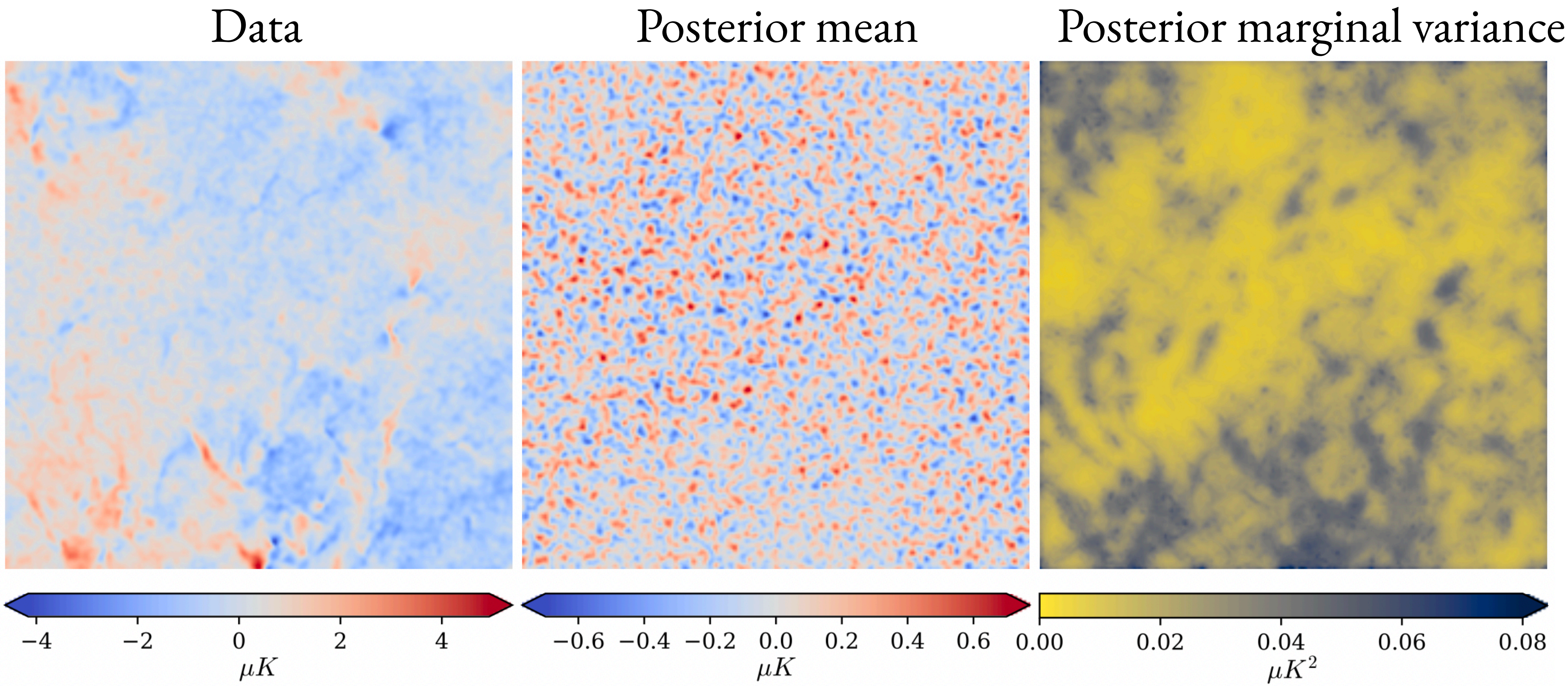
Naive Gaussian model:



Naive Gaussian model:



Validation of posterior



Rescaled residuals

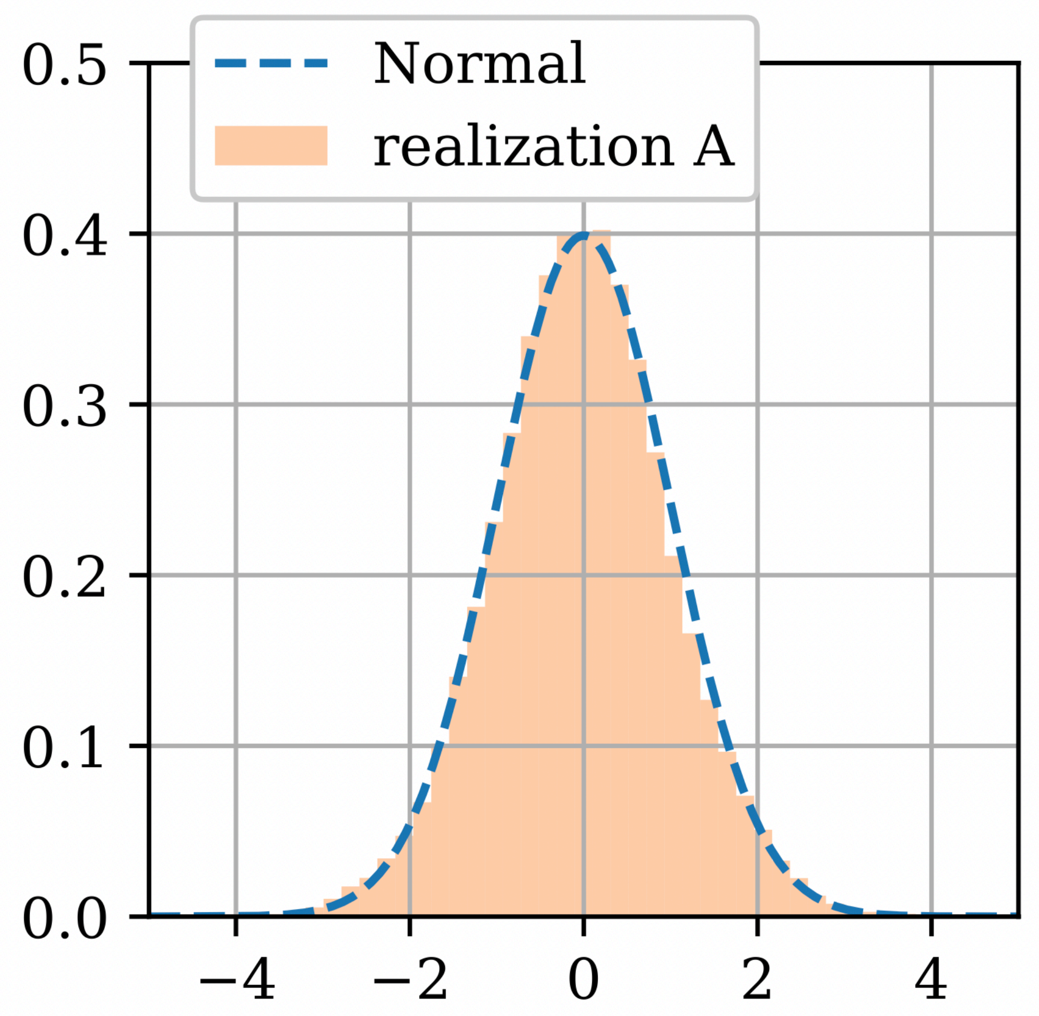
$$(\mu_B - s_B)/\sigma_B$$

POSTERIOR MEAN

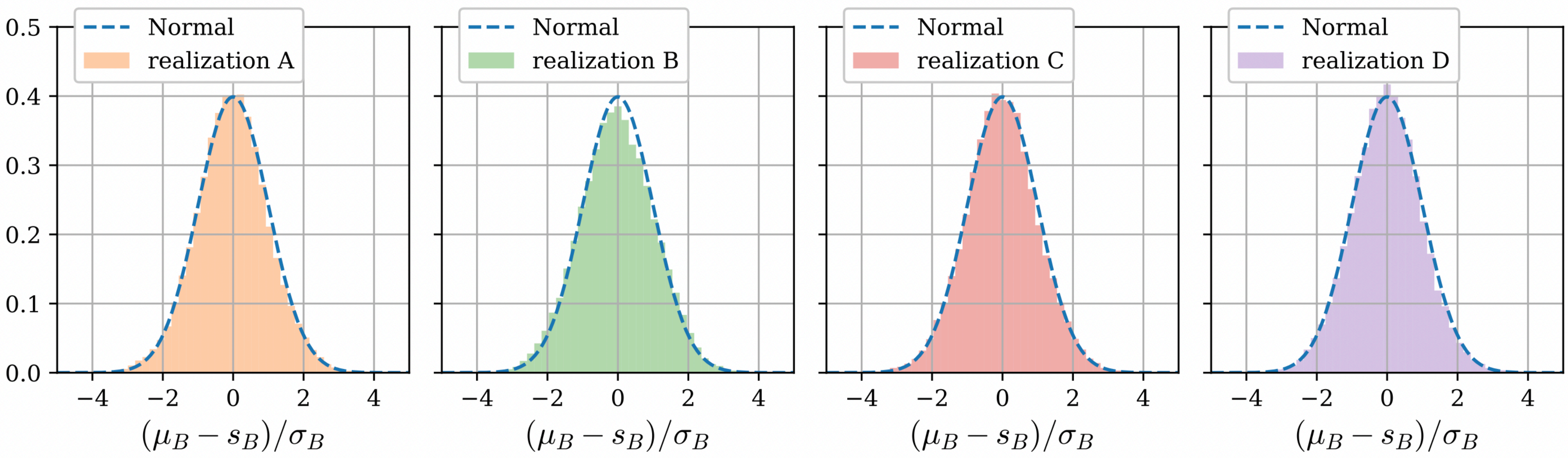
TRUE PIXEL VALUE

POSTERIOR VARIANCE

Rescaled residuals



Posterior estimates are excellent



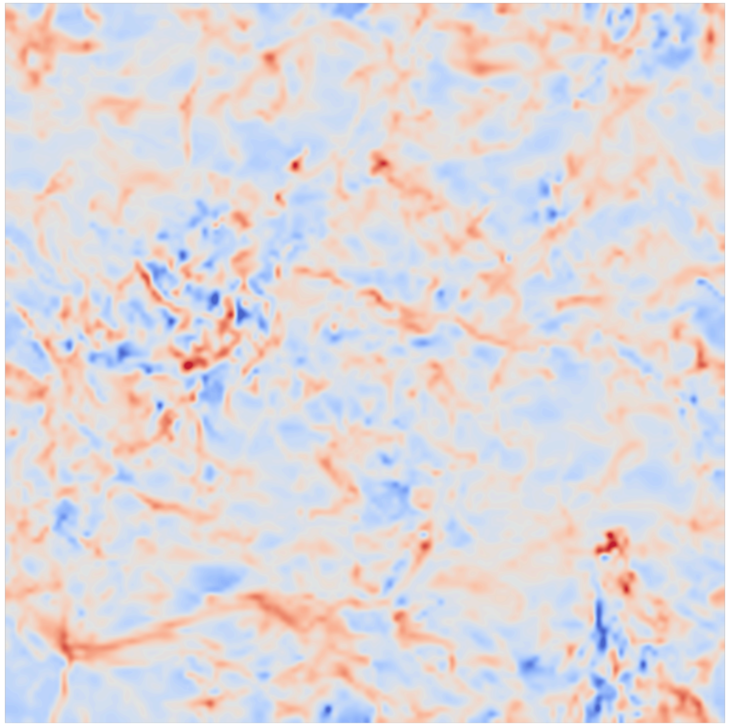
4. Neural foreground generator

(in progress!)

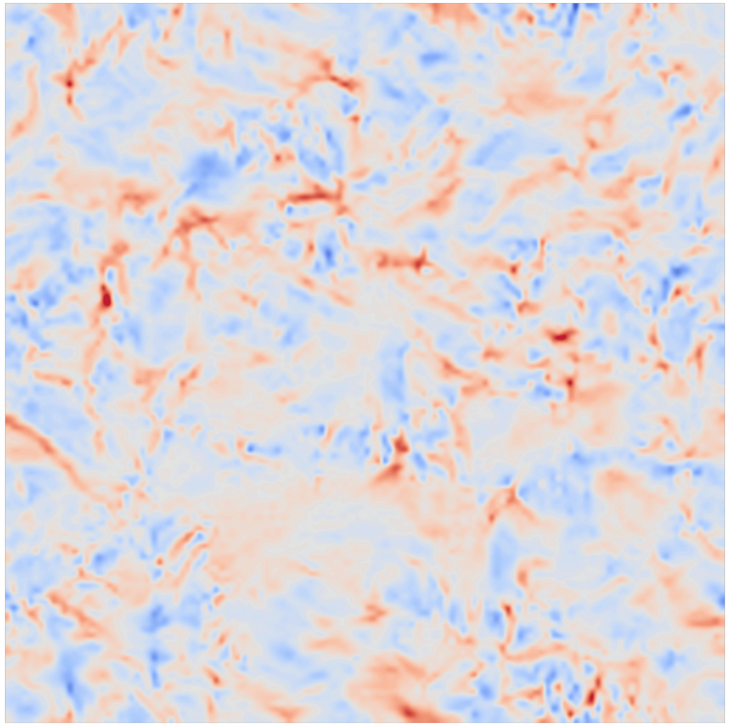
Standard synthesis procedure

1. Measure target wavelet phase coefficients: ϕ^*
2. Generate foreground image f with coefficients $\phi(f) = \phi^*$

Target (simulation)



Gradient descent synthesis



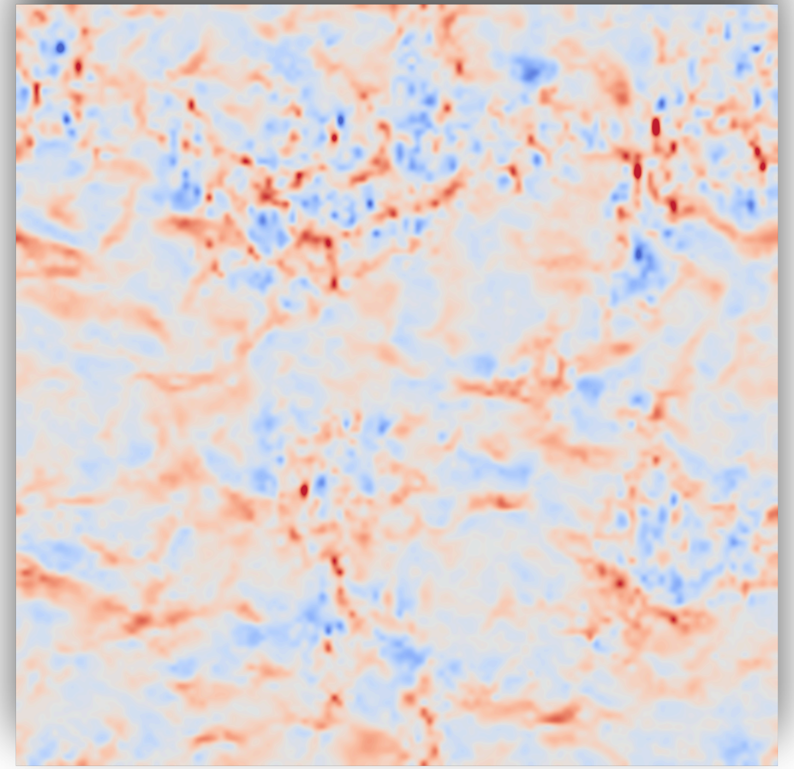
Neural foreground sampler

$$z_i \sim \mathcal{N}(0,\Sigma)$$

$$f_i = \mathrm{W}(z_i) \quad \text{such that} \quad \langle \phi(f_i) \rangle = \phi^*$$

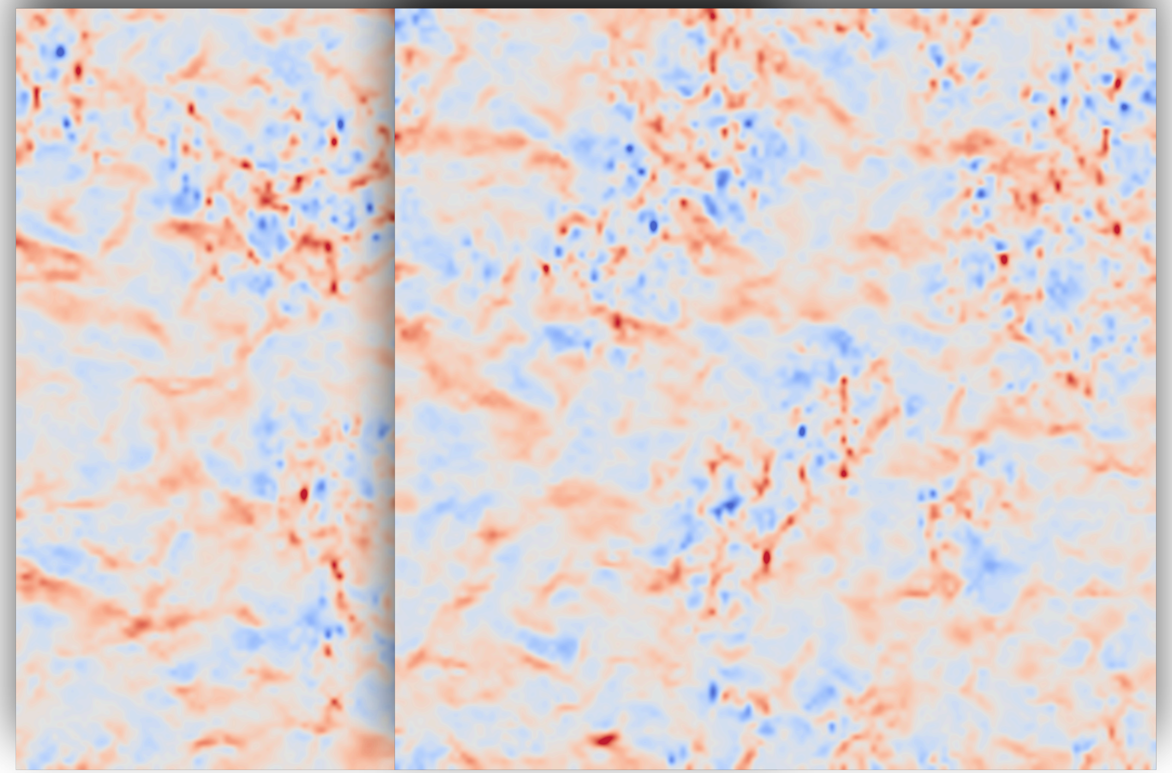
Neural foreground sampler

$$f_i \sim p(f \mid \phi^*)$$



Neural foreground sampler

$$f_i \sim p(f \,|\, \phi^*)$$



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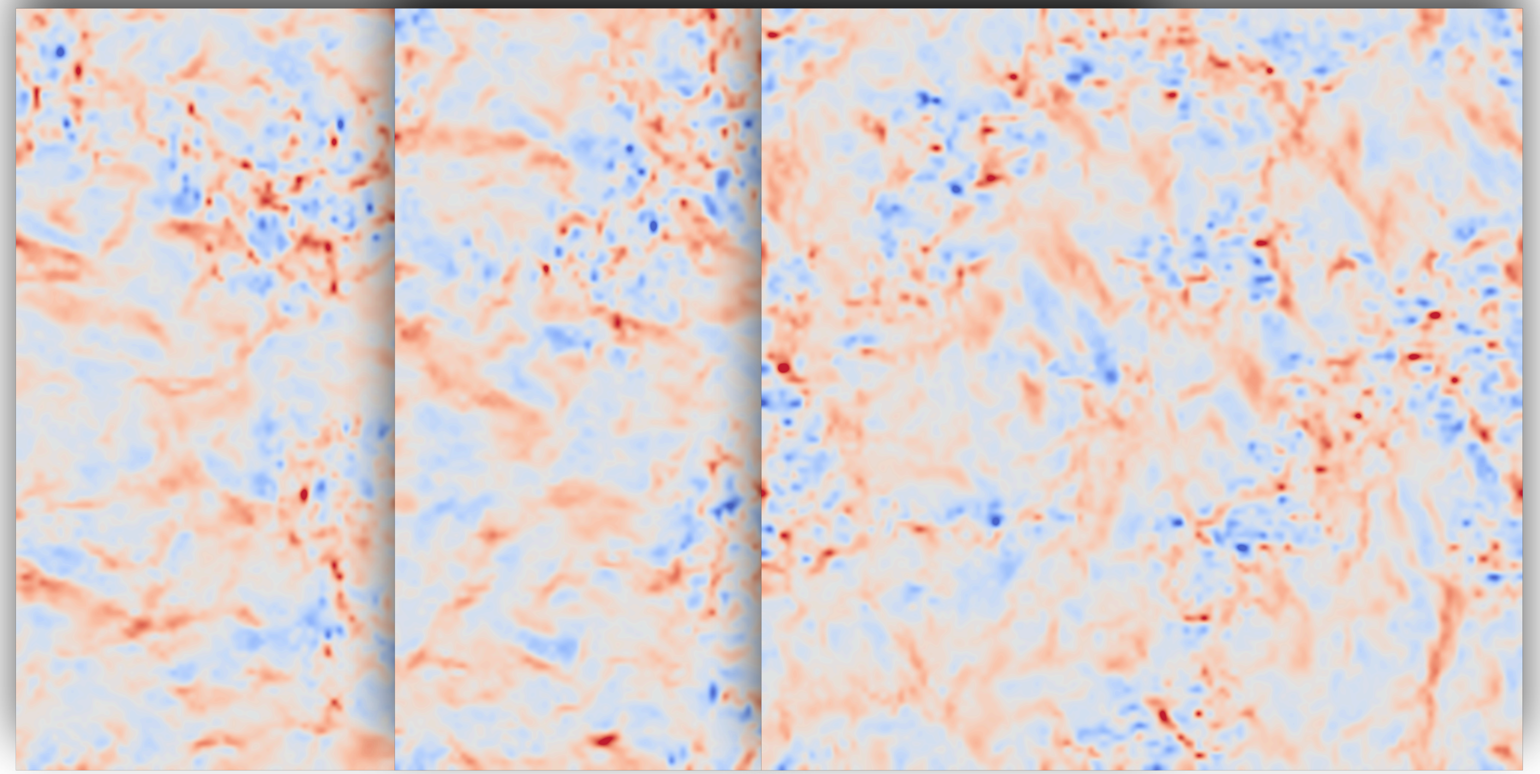

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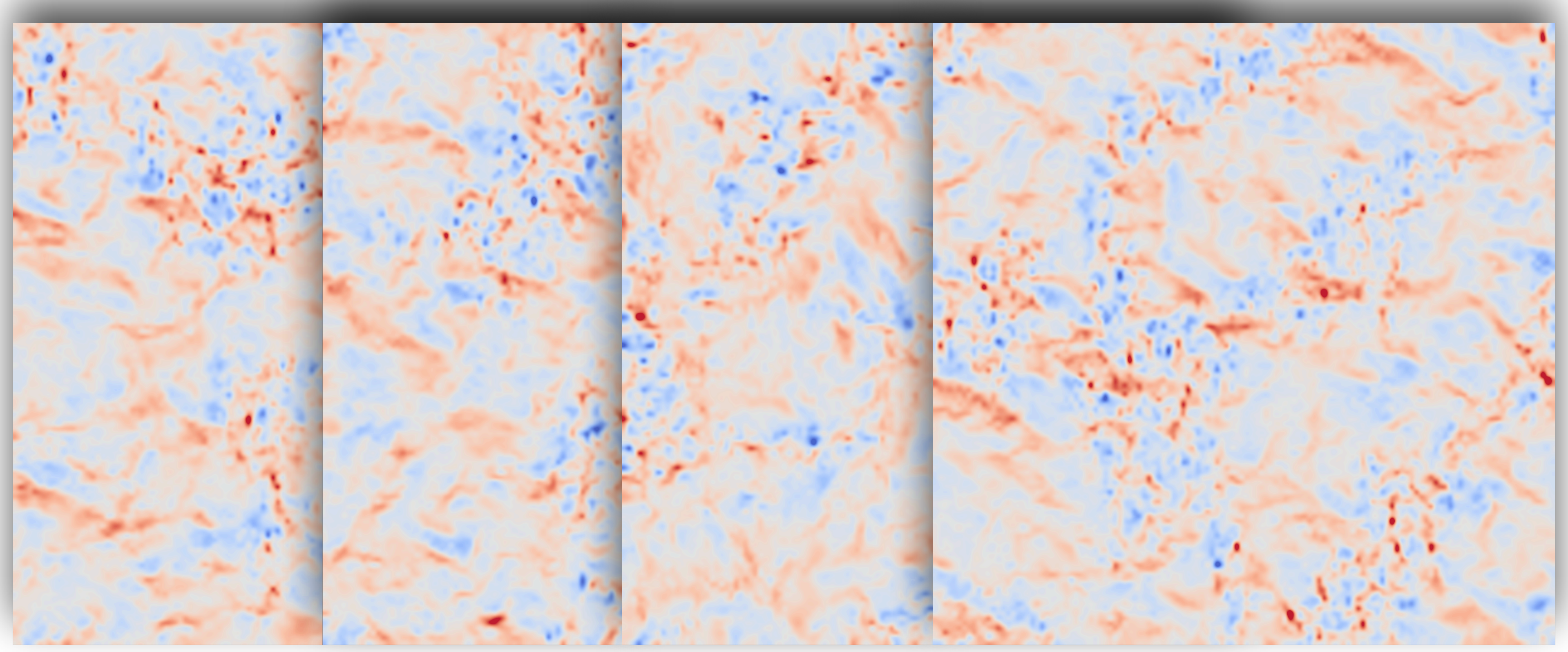

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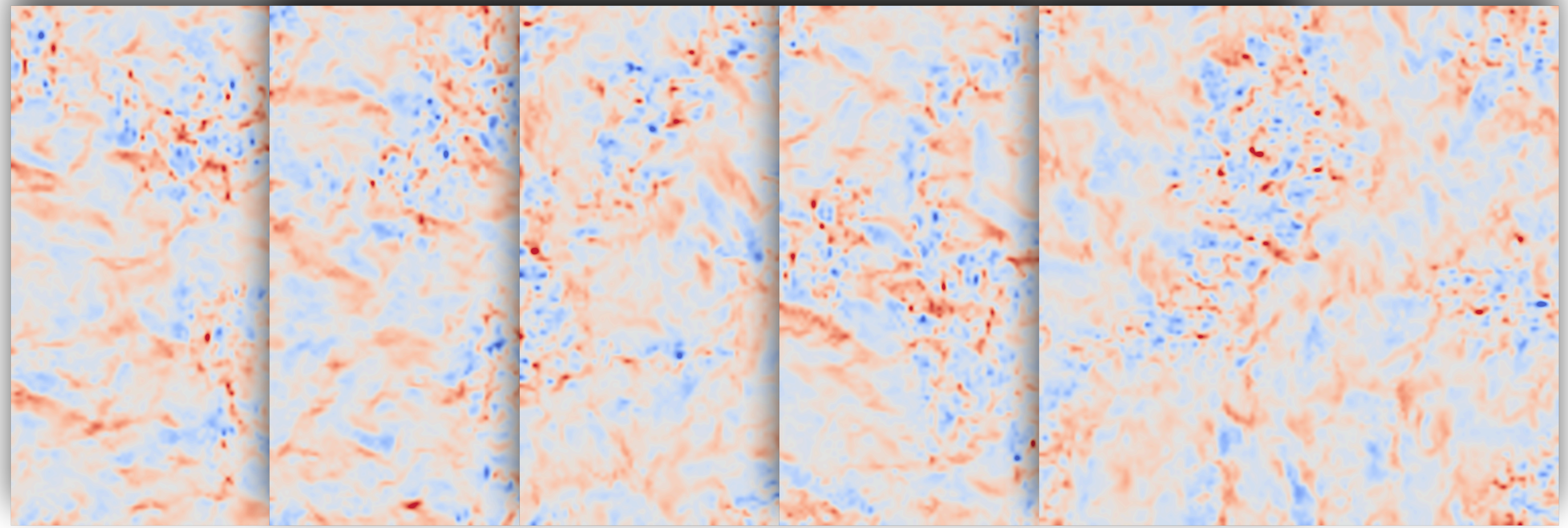
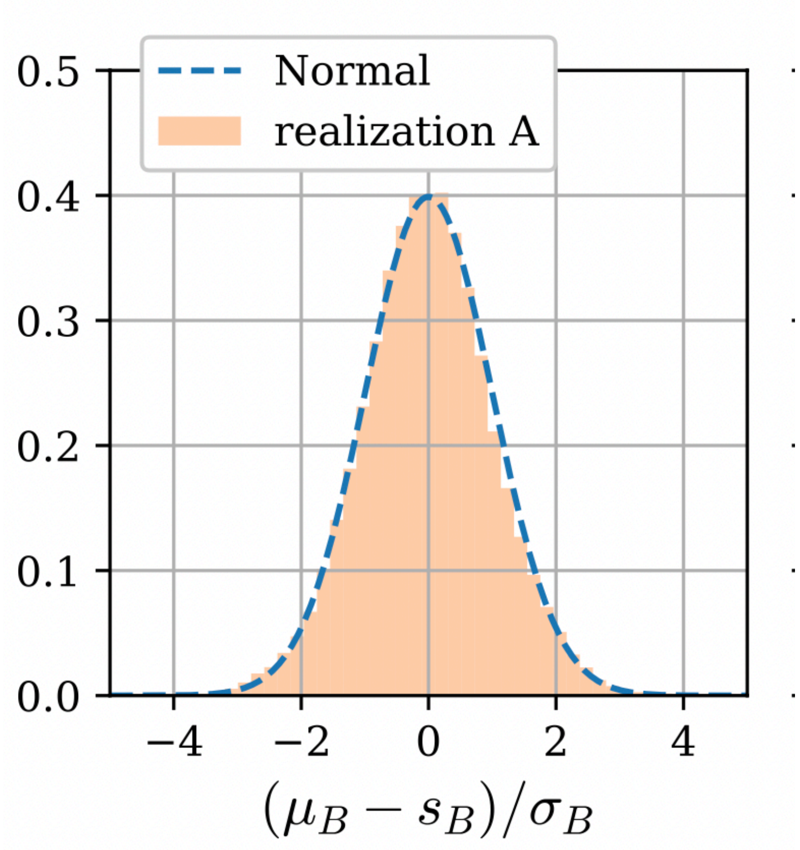
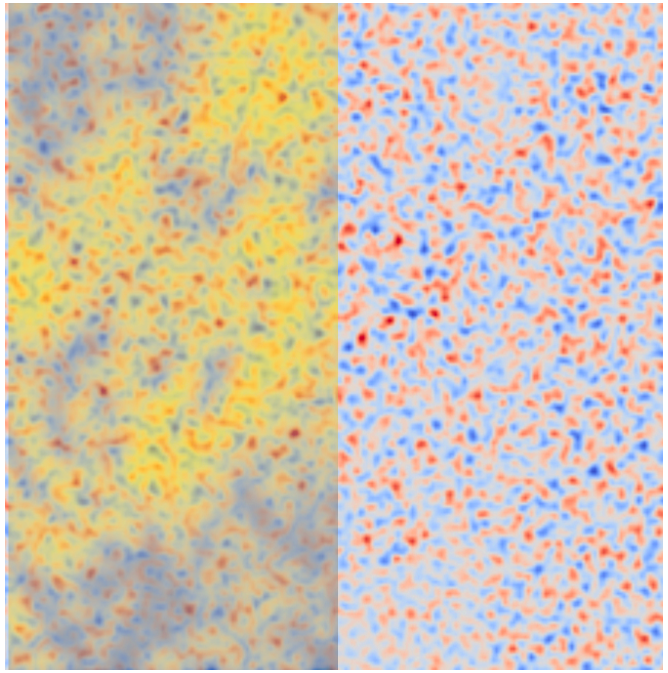




Grazie!



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