Polarization angle requirements for CMB B-mode experiments. Application to the LiteBIRD satellite

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On behalf of the LiteBIRD collaboration

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Preliminaries

- Next generation of CMB polarization experiments will be limited by a combination of astrophysical and instrumental systematics
- Galactic and extragalactic foregrounds are orders of magnitude above instrumental sensitivities
- Strong requirements on key instrumental quantities must be imposed
- Novel instrumental calibration strategies are needed to acomplish those requirements
- The polarization angle is a key quantity for CMB polarization experiments (r parameter, birrefringence, …)
- This presentation is based on LiteBIRD coll. JCAP04(2022)029
- The proposed work is focused on the estimation of requirements, while the establishment of a methodology to meet them is out of the scope of this paper (see LiteBIRD coll. JCAP01(2022)039 , de la Hoz et al. JCAP 03 (2022) 032).

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Preliminaries

Given an experiment with n frequency channels, the CMB polarization signal is estimated as a (linear) combination of the form:

$$\begin{pmatrix} \hat{Q} \\ \hat{U} \end{pmatrix}(p) = \sum_{\nu=1}^{n} w_\nu \begin{pmatrix} Q_\nu \\ U_\nu \end{pmatrix}(p) \quad , \qquad \sum_{\nu=1}^{n} w_\nu = 1$$

Estimated CMB at position p

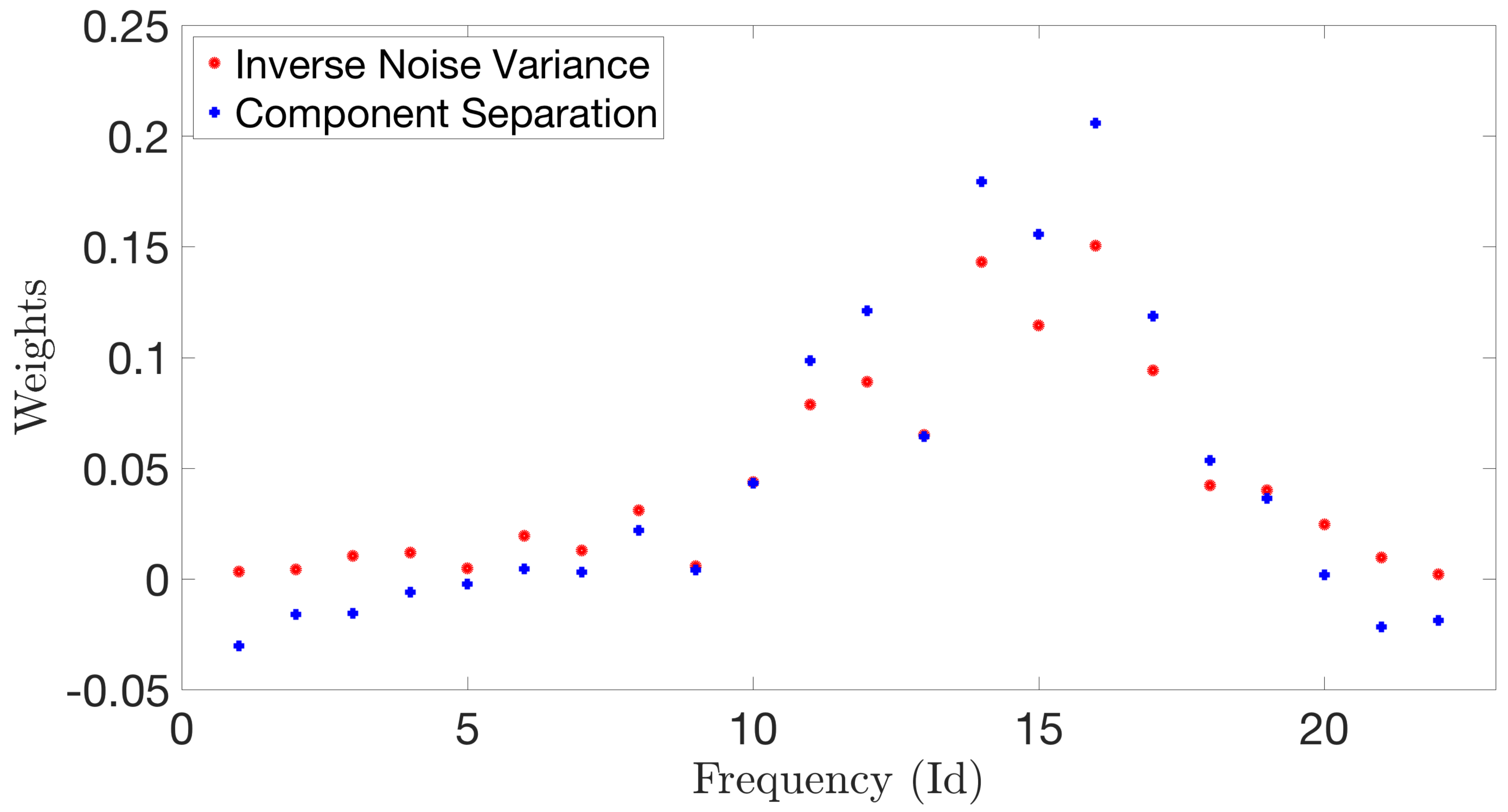
Weight at frequency ν

Data at frequency ν and position p

or equivalently for the spherical harmonic coefficients:

$$\begin{pmatrix} \hat{e}_{\ell m} \\ \hat{b}_{\ell m} \end{pmatrix} = \sum_{\nu=1}^{n} w_\nu \begin{pmatrix} e_{\ell m}^{\nu} \\ b_{\ell m}^{\nu} \end{pmatrix}$$

This linear combination is typical of the ILC method (e.g. Fernández-Cobos et al. 2016). On the other hand, a parametric method as FGBuster (Errard & Stompor 2018), used as the baseline for LiteBIRD, is expected to provide inverse noise weighting.



Preliminaries

- The rotation of the polarization axes by an angle α transforms the intrinsic polarization Stokes parameters (Q, U) in the rotated ones (Q^{rot}, U^{rot}):

$$\begin{pmatrix} Q^{rot} \\ U^{rot} \end{pmatrix}(p) = \begin{pmatrix} \cos(2\alpha) & -\sin(2\alpha) \\ \sin(2\alpha) & \cos(2\alpha) \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}(p)$$

or equivalently for the polarization modes $e_{\ell m}$ and $b_{\ell m}$ (assuming a uniform rotation over the sky):

$$e_{\ell m}^{rot} = \cos(2\alpha)\, e_{\ell m} - \sin(2\alpha)\, b_{\ell m}$$

$$b_{\ell m}^{rot} = \sin(2\alpha)\, e_{\ell m} + \cos(2\alpha)\, b_{\ell m}$$

- The CMB polarization signal estimated from the combination of the n frequency channels ν rotated by angles α_ν becomes:

$$\hat{e}_{\ell m} = e_{\ell m} \sum_{\nu=1}^{n} w_\nu \cos(2\alpha_\nu) - b_{\ell m} \sum_{\nu=1}^{n} w_\nu \sin(2\alpha_\nu)$$

$$\hat{b}_{\ell m} = e_{\ell m} \sum_{\nu=1}^{n} w_\nu \sin(2\alpha_\nu) + b_{\ell m} \sum_{\nu=1}^{n} w_\nu \cos(2\alpha_\nu)$$

and the corresponding change in the $\hat{B}_\ell$ power spectrum would be (assuming a null primordial B_ℓ):

$$\hat{B}_\ell = E_\ell \left(\sum_{\nu=1}^{n} w_\nu \sin(2\alpha_\nu) \right)^2 + B_\ell \left(\sum_{\nu=1}^{n} w_\nu \cos(2\alpha_\nu) \right)^2$$

Methodology

Gaussian likelihood approximation for the B_ℓ spectrum:

$$\delta_r = \left[\frac{\sum_{\ell=2}^{\ell_{max}} \Delta B_\ell B_\ell^{fid}}{Var(B_\ell)}\right]\left[\frac{\sum_{\ell=2}^{\ell_{max}} \left(B_\ell^{fid}\right)^2}{Var(B_\ell)}\right]^{-1}$$

- B_ℓ^{fid} is the B-mode spectrum corresponding to the fiducial ΛCDM model for $r = 1$.
- ΔB_ℓ is the biased B-mode spectrum after subtracting the known contributions to the estimated signal $\hat{B}_\ell$: the fiducial spectra for primordial B-mode and lensing and the effective noise. $\hat{B}_\ell$ is given by:

$$\hat{B}_\ell = \left(r\, B_\ell^{fid} + L_\ell + R_\ell^B\right)\Sigma_{cos} + \left(E_\ell + R_\ell^E\right)\Sigma_{sin} + N_\ell^{eff}$$

- E_ℓ, L_ℓ, R_ℓ^B, R_ℓ^E and N_ℓ^{eff} are the fiducial E-mode and lensing, residual foregrounds and effective noise spectra resulting from the linear combination of the frequency channels w_ν:

$$N_\ell^{eff} = \sum_{\nu=1}^{n} N_\ell^\nu w_\nu^2 (b_\ell^\nu)^{-2}$$

- Σ_{cos} and Σ_{sin} terms account for the impact of the polarization angle offsets of each frequency channel (see below).

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Methodology

- ΔB_l is given by the subtraction of the known contributions to the estimated $\hat{B}_\ell$:

$$\Delta B_\ell = \left(r\, B_\ell^{fid} + L_\ell + R_\ell^B\right)\left(\sum_{cos} - 1\right) + \left(E_\ell + R_\ell^E\right)\sum_{sin}$$

Obviously these contributions can be removed at the power spectrum level but not from the cosmic variance (here we do not attemp to do delensing at map level):

$$Var(B_\ell) = \frac{2}{f_{sky}(2\ell+1)}\hat{B}_\ell^2$$

with f_{sky} accounting for the sampling variance.

- $\sum_{cos}$ and $\sum_{sin}$ terms account for the impact of the polarization angle offsets at each frequency channel

$$\sum_{cos} = \left(\sum_{\nu=1}^{n} \cos(2\alpha_\nu)\, w_\nu\right)^2 \qquad \sum_{sin} = \left(\sum_{\nu=1}^{n} \sin(2\alpha_\nu)\, w_\nu\right)^2$$

where n is the number of channels and α_ν is the polarization angle offset of channel ν.

In the limit of very small angle offsets: $\sum_{cos}$=1 and $\sum_{sin}$=0, and therefore ΔB_ℓ=0 and also $\delta_r = 0$ as one would expect.

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Methodology

- Typical instrumental offsets are expected to be at the degree level at most, then it is worth considering the small angle approximation. In this case the previous expression for $\sum_{cos}$ and $\sum_{sin}$ take the following form up to first order:

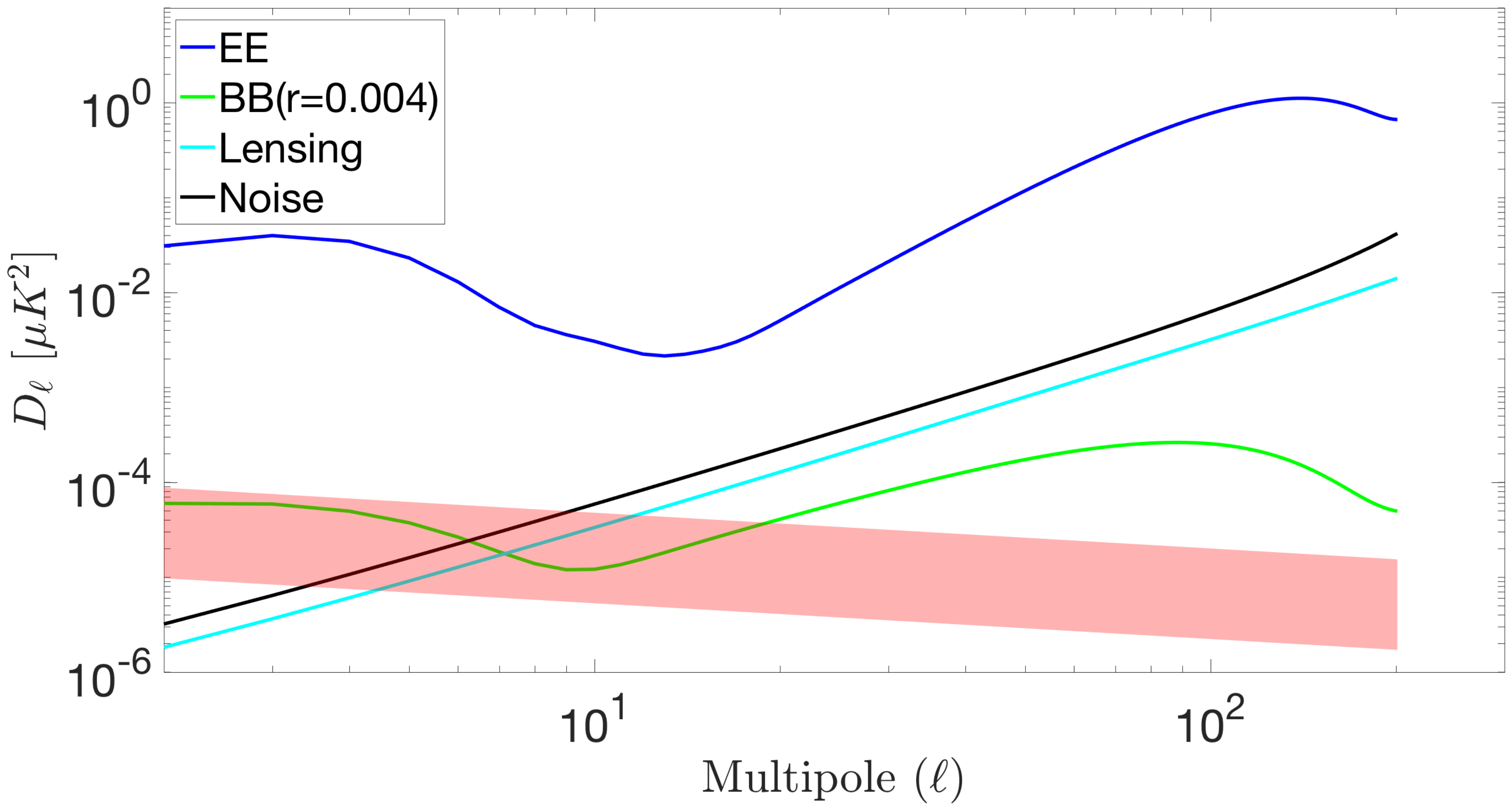
$$\Sigma_{cos} \approx 1 - 4\sum_{\nu=1}^{n} \alpha_\nu^2 w_\nu^2$$

$$\Sigma_{sin} \approx 4\left(\sum_{\nu=1}^{n} \alpha_\nu w_\nu\right)^2$$

and the bias in the r parameter, δ_r, is given by (also considering that $r\, B_\ell^{fid} + L_\ell \ll E_\ell$):

$$\delta_r \approx 4A\left(\sum_{\nu=1}^{n} \alpha_\nu w_\nu\right)^2 \quad , \quad A = \left[\sum_{\ell=2}^{\ell_{max}} \frac{(E_\ell + R_\ell^E) B_\ell^{fid}}{Var(B_\ell)}\right]\left[\sum_{\ell=2}^{\ell_{max}} \frac{\left(B_\ell^{fid}\right)^2}{Var(B_\ell)}\right]^{-1}$$

This is a general expression that only depends on the polarization angle mismatch per channel, α_ν, and the weight that each channel has to build the final CMB map, w_ν.



Correlations among detectors and bias in r

Correlation of the polarization angle offsets across different detectors may come from several systems of the experiment, such as the focal plane, optical components or the platform. Such instrumental systematic effects affect different sets of detectors at the same time, but in general at different levels.

Correlations between offsets α_{ν_1} and α_{ν_2}, corresponding to frequency elements ν_1 and ν_2, can be characterized by the matrix $\boldsymbol{C}$:

$$\langle\alpha_{\nu_1}\alpha_{\nu_2}\rangle \equiv C_{\nu_1\nu_2} = \rho_{\nu_1\nu_2}\sigma_{\nu_1}\sigma_{\nu_2}$$

where $\rho_{\nu_1\nu_2}$ is the correlation coefficient. Considering the expected value of δ_r:

$$\langle\delta_r\rangle \approx 4A \sum_{\nu_1,\nu_2=1}^{n} C_{\nu_1\nu_2} w_{\nu_1} w_{\nu_2}$$

Many different combinations of the n uncertainties α_ν lead to the same $\langle\delta_r\rangle$. A natural assumption is that all the terms in the sum of the expression for δ_r add evenly, i.e., $\sigma_\nu = c/w_\nu/^{-1}$, with c a constant. Under this assumption, the requirements on σ_ν can be derived unambiguously:

$$\langle\delta_r\rangle \approx 4c^2 A \sum_{\nu_1,\nu_2=1}^{n} \rho_{\nu_1\nu_2}$$

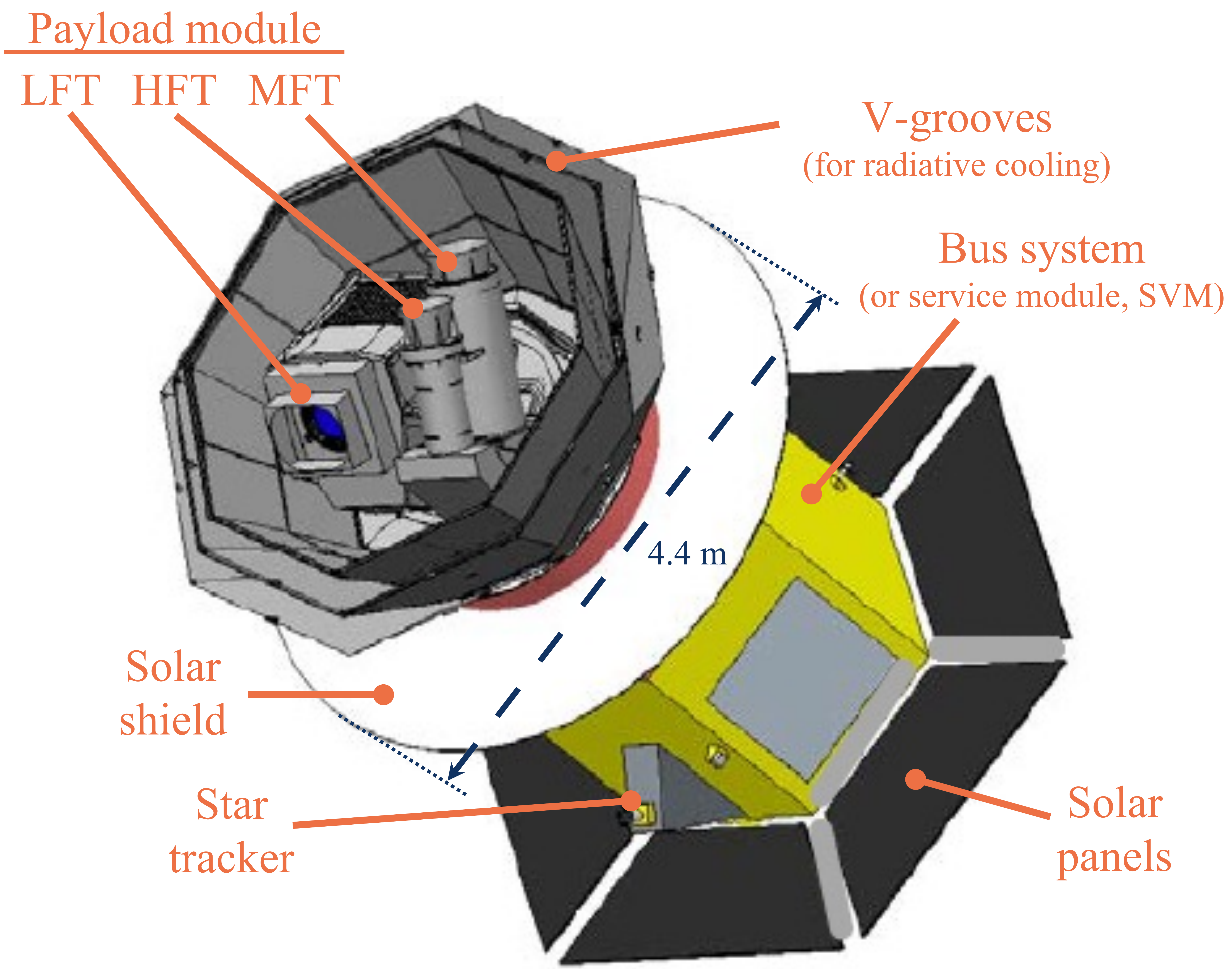
Two extreme cases:

- $\langle\delta_r\rangle \approx 4n^2c^2A$ ($\rho_{\nu_1\nu_2}$=1), fully correlated → strongest requirements
- $\langle\delta_r\rangle \approx 4nc^2A$ ($\rho_{\nu_1\nu_2}$=0), uncorrelated → weakest requirements

LiteBIRD spacecraft overview

- **3 telescopes** are used to provide the **40-402 GHz** frequency coverage
  1. **LFT** (low frequency telescope)
  2. **MFT** (middle frequency telescope)
  3. **HFT** (high frequency telescope)
- Multi-chroic transition-edge sensor (TES) **bolometer arrays** cooled to **100 mK**
- Polarization modulation unit (PMU) in each telescope with **rotating half-wave plate** (HWP), for $1/f$ noise and systematics reduction
- Optics cooled to **5 K**

- Mass: 2.6 t
- Power: 3.0 kW
- Data: 17.9 Gb/day

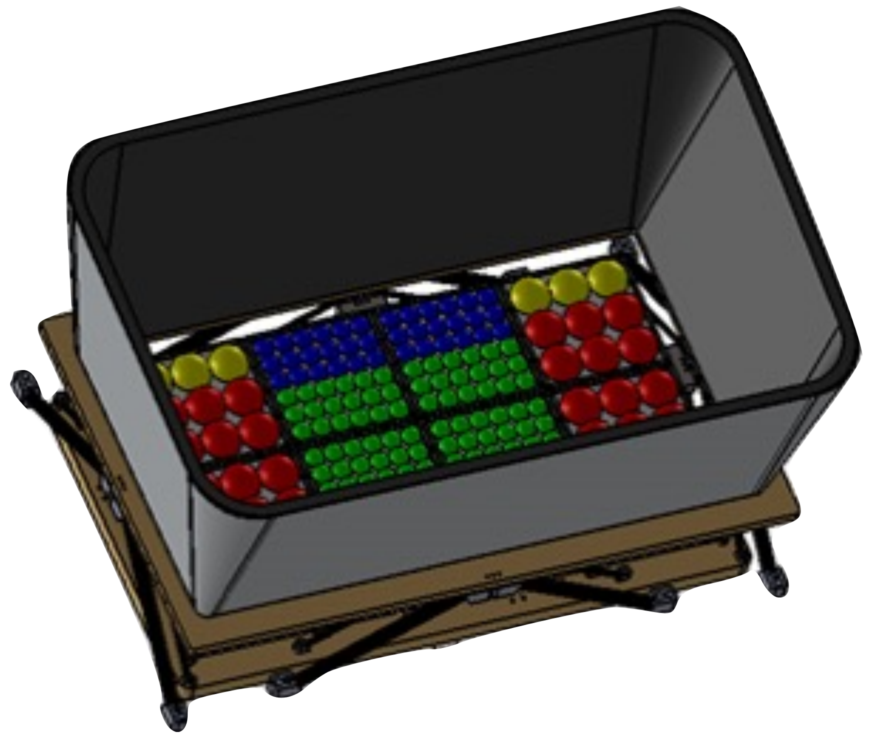


Focal plane configuration

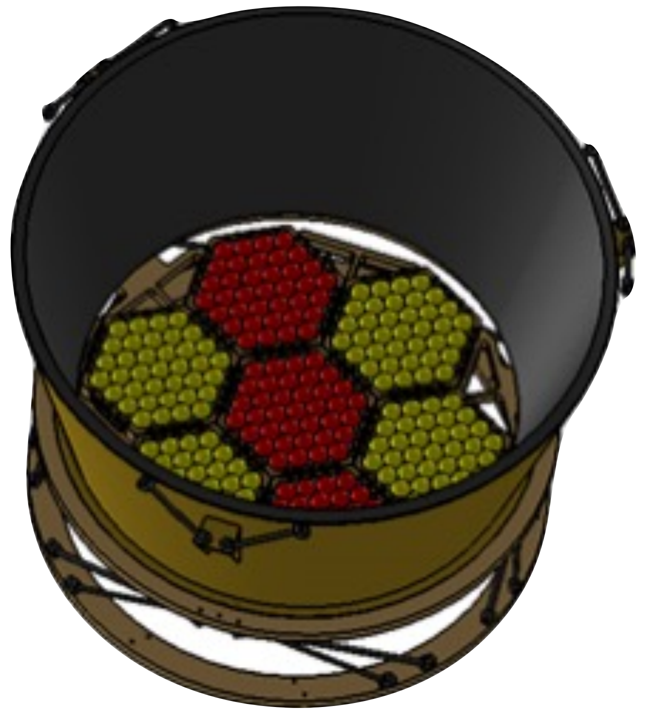
- Transition-Edge Sensor (TES) arrays
- Multichroic detectors
- Number of sensors: 4508
- 15 bands including overlap between instruments

Rule of thumb:
1000 detectors in space = 100 000 detectors on ground

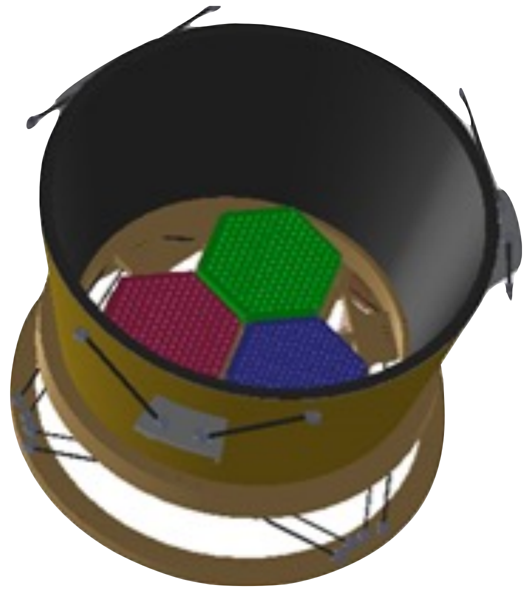
LFT



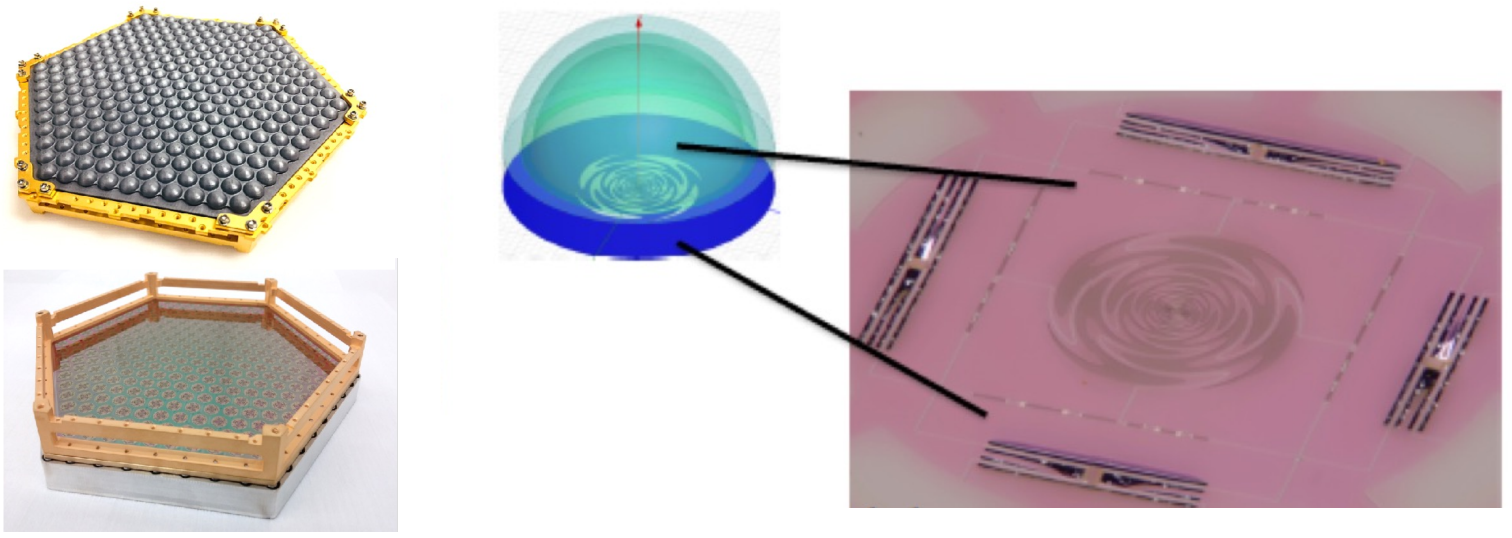
MFT



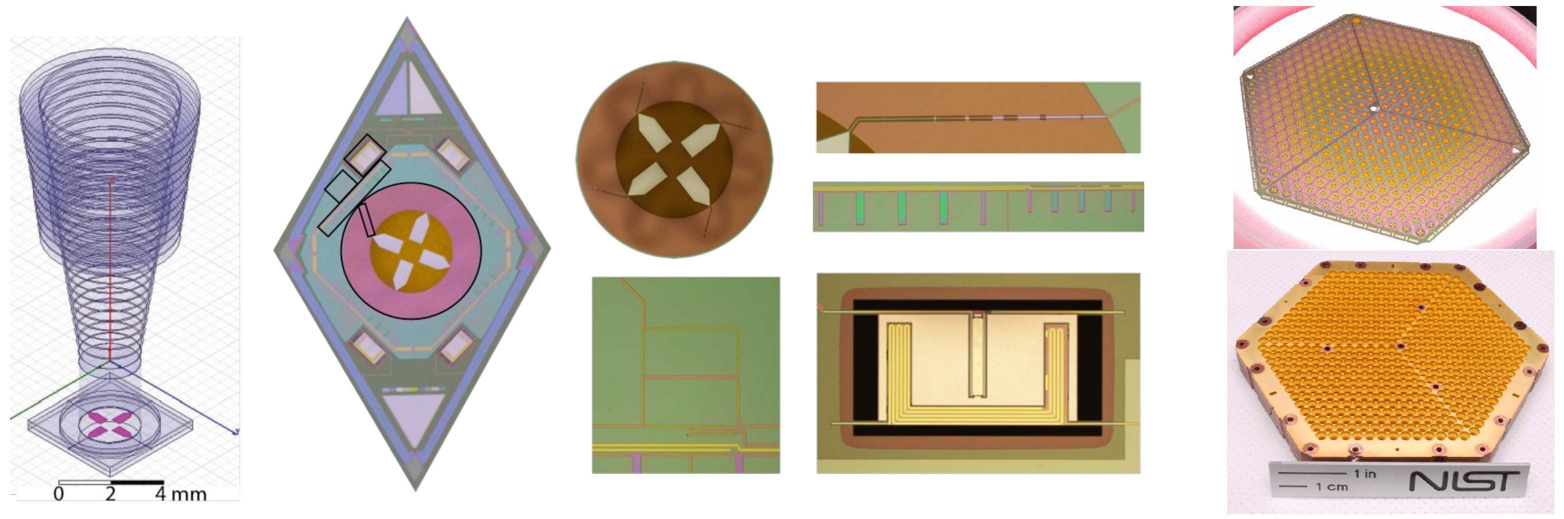
HFT



Lensed coupled detectors
Lenslets



Horn coupled detectors
Platelets



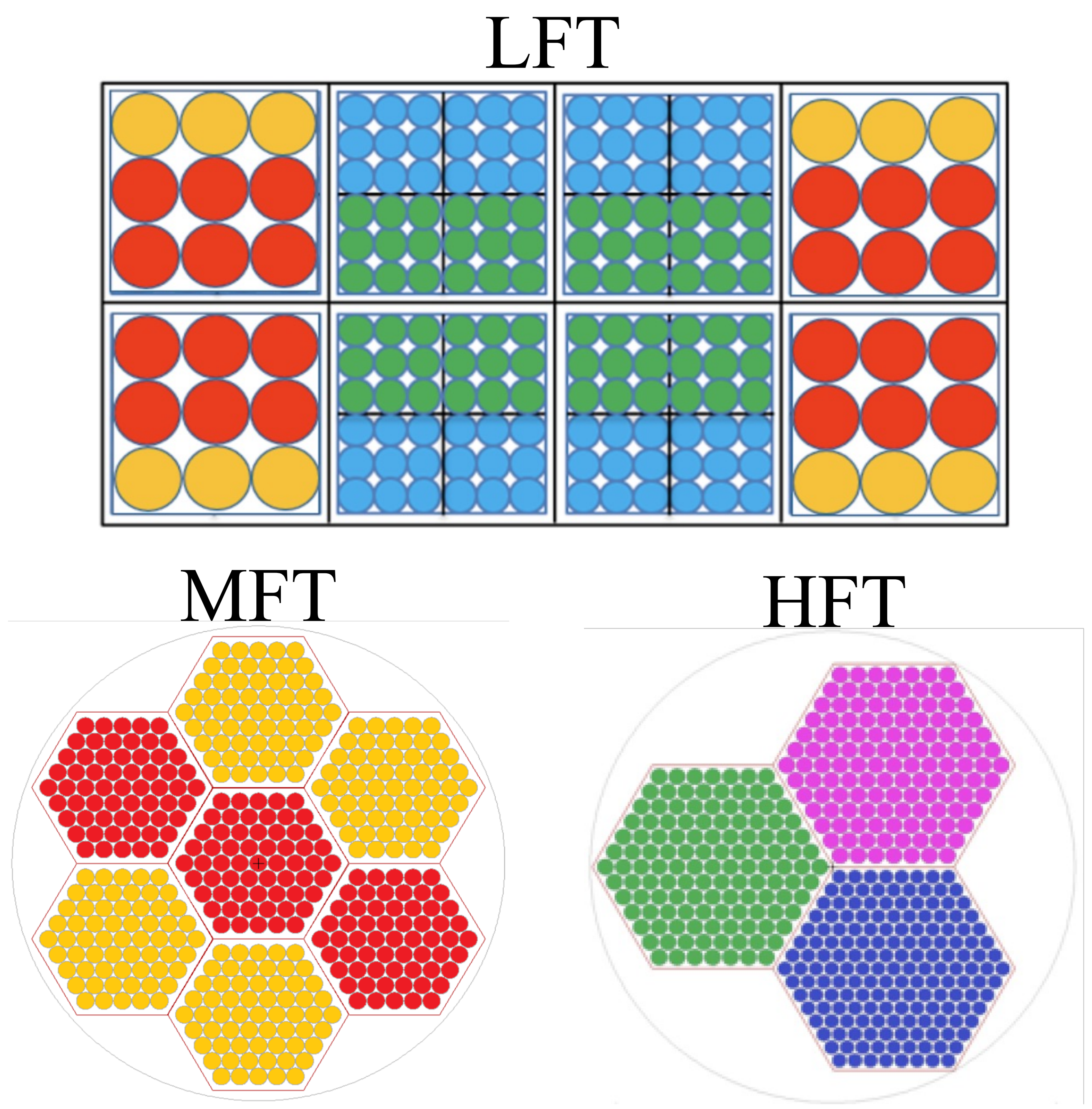
Westbrook+ SPIE 2020

MFT (2.5:1)
89GHz — 225 GHz
2075 detectors
366 Trichroic TES
588 Dichroic TES
Bands: 100, 119, 140, 166, 195

LFT (5.7:1)
34GHz — 161 GHz
Bands: 40, 50, 60, 68, 78, 89, 100, 119, 140; 68, 78, 89
1258 detectors
2 x (65 + 155) Trichroic TES

HFT (2.7:1)
166 GHz — 448 GHz
Bands: 195, 235, 280, 337, 402
1355 detectors
2 x 255 Dichroic TES
338 Monochromatic TES

LiteBIRD sensitivities



Total budget assigned to systematics is 1/3 of the overall sensitivity on *r*.
Maximum systematic error in δ_r induced by biased polarization angle of 1% of the total budget assigned to systematics:

$$\langle\delta_r\rangle = \frac{10^{-3}}{\sqrt{3}} \times 0.01 = 5.77\times10^{-6}$$

(see LiteBIRD coll. arXiv:2202.02773 for more details)

Element name	ID	Frequency [GHz]	FWHM [arcmin]	Pol. sensitivity [μK-arcmin]	Number of Bolometers
LFT_040GHz	1	40	70.5	37.42	48
LFT_050GHz	2	50	58.5	33.46	24
LFT_60GHz	3	60	51.1	21.31	48
LFT_68GHz_a	4	68	41.6	19.91	144
LFT_68GHz_b	5	68	47.1	31.77	24
LFT_78GHz_a	6	78	36.9	15.55	144
LFT_78GHz_b	7	78	43.8	19.13	48
LFT_89GHz_a	8	89	33.0	12.28	144
LFT_89GHz_b	9	89	41.5	28.77	24
LFT_100GHz	10	100	30.2	10.34	144
LFT_119GHz	11	119	26.3	7.69	144
LFT_140GHz	12	140	23.7	7.25	144
MFT_100GHz	13	100	37.8	8.48	366
MFT_119GHz	14	119	33.6	5.70	488
MFT_140GHz	15	140	30.8	6.38	366
MFT_166GHz	16	166	28.9	5.57	488
MFT_195GHz	17	195	28.0	7.05	366
HFT_195GHz	18	195	28.6	10.50	254
HFT_235GHz	19	235	24.7	10.79	254
HFT_280GHz	20	280	22.5	13.80	254
HFT_337GHz	21	337	20.9	21.95	254
HFT_402GHz	22	402	17.9	47.45	338

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Absolute angle requirements

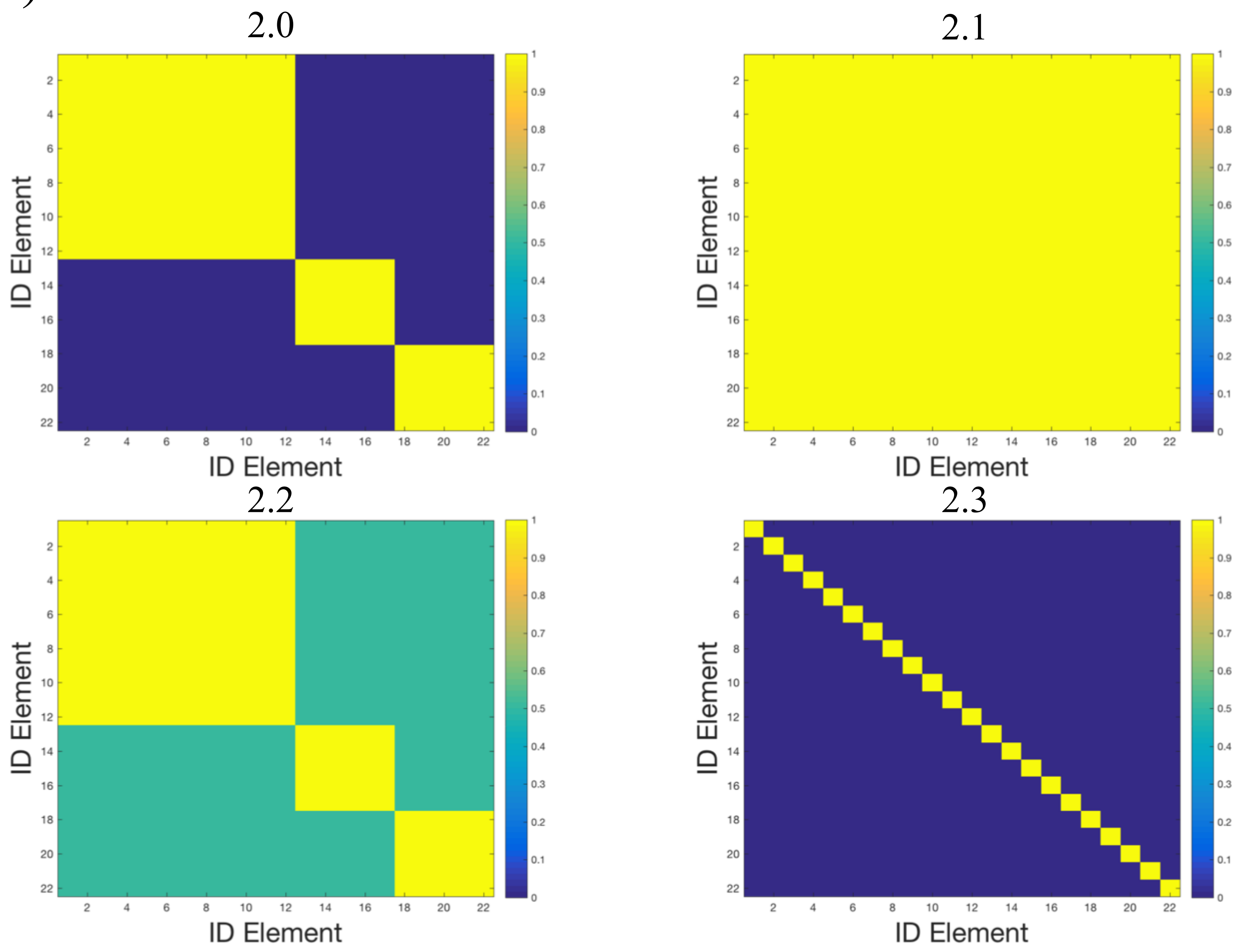
Four absolute angles: one global and one for each focal plane

- Case 1.0: no correlations.
- Case 1.1: the four offsets are fully correlated.
- Case 1.2: the global offset is uncorrelated with any of the focal plane ones, with the latter fully correlated.
- Case 1.3: the global offset is fully correlated with any of the focal plane ones, with the latter uncorrelated.

	Offset (arcmin)		
Label	Case 1.0	Case 1.1	Cases 1.2, 1.3
GLB	3.7	1.8	2.3
LFT	11.6	5.8	7.4
MFT	6.4	3.2	4.1
HFT	30.8	15.4	19.5

Relative angle requirements: frequency level

- Case 2.0: the offsets of all the n=22 elements are uncorrelated, except for those in the same telescope focal plane which are fully correlated.
- Case 2.1: the offsets of all the n elements are fully correlated (strongest constraints).
- Case 2.2: the offsets of all the n elements are partially correlated, in particular, we chose $\rho_{\nu_1\nu_2} = 0.5$ (for any $\nu_1 \neq \nu_2$), except those within the same telescope which are fully correlated.
- Case 2.3: the offsets of all the n elements are uncorrelated (weakest constraints).



Element name	ID	σ_α (arcmin) Case 2.0	Case 2.1	Case 2.2	Case 2.3
LFT_040GHz	1	49.8	31.5	37.7	147.8
LFT_050GHz	2	39.8	25.2	30.1	118.2
LFT_060GHz	3	16.1	10.2	12.2	47.9
LFT_068GHz_a	4	1.09	8.9	10.7	41.8
LFT_068GHz_b	5	35.9	22.7	27.1	106.5
LFT_078GHz_a	6	8.6	5.4	6.5	25.5
LFT_078GHz_b	7	13.0	8.2	9.8	38.6
LFT_089GHz_a	8	5.4	3.4	4.1	15.9
LFT_089GHz_b	9	29.4	18.6	22.3	87.4
LFT_100GHz	10	3.8	2.4	2.9	11.3
LFT_119GHz	11	2.1	1.3	1.6	6.2
LFT_140GHz	12	1.8	1.2	1.4	5.6
MFT_100GHz	13	2.6	1.6	1.9	7.6
MFT_119GHz	14	1.2	0.7	0.9	3.4
MFT_140GHz	15	1.5	0.9	1.1	4.3
MFT_166GHz	16	1.1	0.7	0.8	3.3
MFT_195GHz	17	1.8	1.1	1.3	5.3
HFT_195GHz	18	3.9	2.5	3.0	11.6
HFT_235GHz	19	4.1	2.6	3.1	12.3
HFT_280GHz	20	6.8	4.3	5.1	20.1
HFT_337GHz	21	17.1	10.8	13.0	50.9
HFT_402GHz	22	80.0	50.7	60.5	237.6

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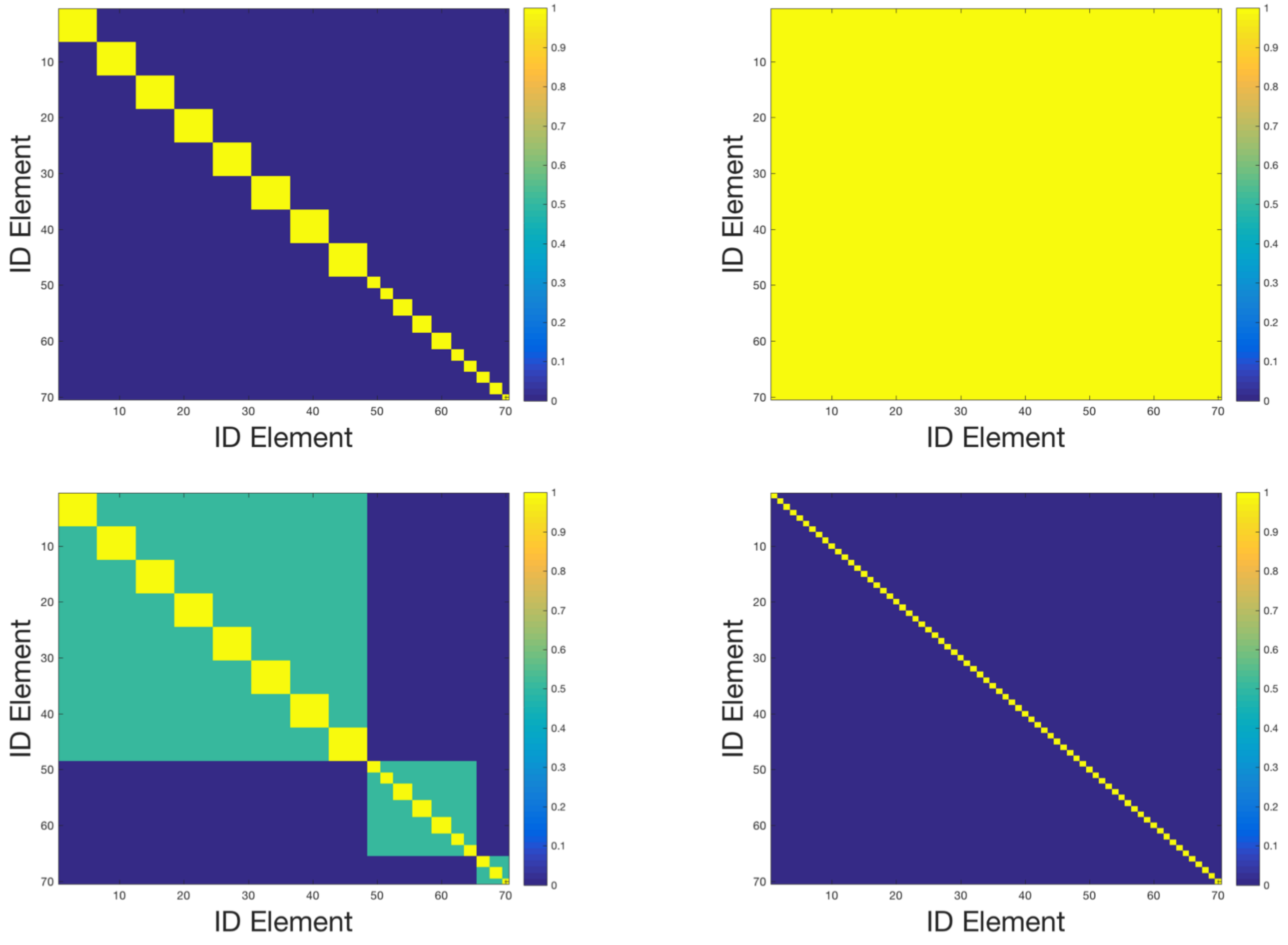
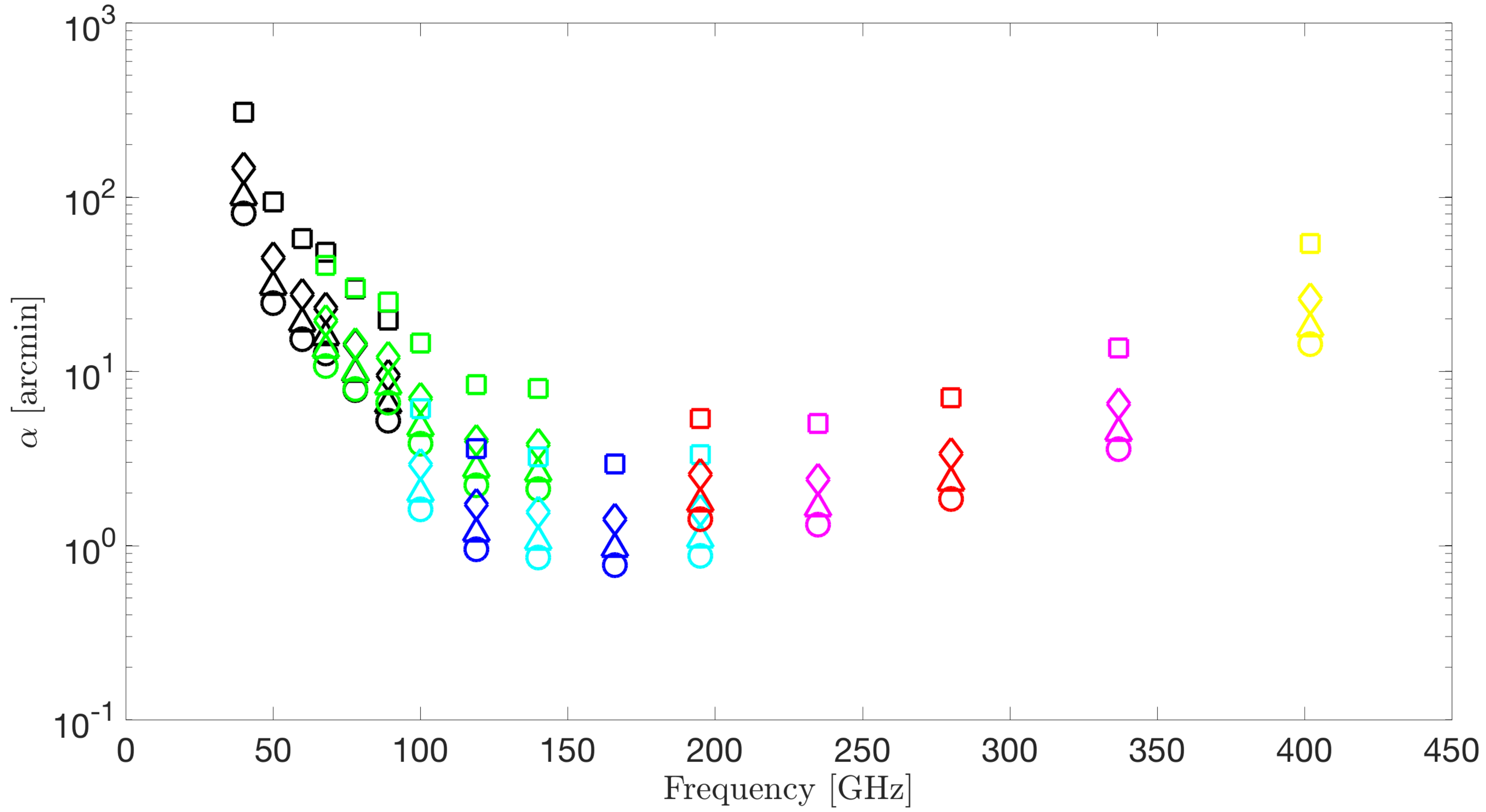
Relative angle requirements: wafer-frequency level

- Case 3.0 (square): the offsets of all the n=70 elements are uncorrelated, except for those in the same telescope focal plane, which are fully correlated.
- Case 3.1 (circle): the offsets of all the n elements are fully correlated (strongest constraints).
- Case 3.2 (diamond): the offsets of all the n elements are partially correlated, in particular, we chose $\rho_{\nu_1\nu_2} = 0.5$ (for any ν_1 and ν_2 in the same telescope), except those within the same element which are fully correlated.
- Case 3.3 (triangle): the offsets of all the n elements are uncorrelated (weakest constraints).



Cases:
- 3.0: □
- 3.1: ○
- 3.2: ◇
- 3.3: △

Wafers:
- LFT-type 1
- LFT-type 2
- MFT-type 1
- MFT-type 2
- HFT-type 1
- HFT-type 2
- HFT-type 3



Relative angle requirements: detector level

Element (Band) (Telescope + Freq.)	σ_α (arcmin) Case 4.0	Case 4.1	Case 4.2
LFT_040GHz	7.4	495.6	28.3
LFT_050GHz	3.0	198.1	11.3
LFT_060GHz	2.4	160.7	9.2
LFT_068GHz_a	6.3	420.9	24.0
LFT_068GHz_b	2.7	178.6	10.2
LFT_078GHz_a	3.8	256.7	14.7
LFT_078GHz_b	1.9	129.5	7.4
LFT_089GHz_a	2.4	160.2	9.1
LFT_089GHz_b	2.2	146.5	8.4
LFT_100GHz	1.7	113.5	6.5
LFT_119GHz	0.9	62.8	3.6
LFT_140GHz	0.8	55.8	3.2
MFT_100GHz	2.9	194.1	11.1
MFT_119GHz	1.7	116.9	6.7
MFT_140GHz	1.6	109.9	6.3
MFT_166GHz	1.7	111.5	6.4
MFT_195GHz	2.0	134.2	7.7
HFT_195GHz	3.1	206.5	11.8
HFT_235GHz	3.3	218.1	12.5
HFT_280GHz	5.3	356.7	20.4
HFT_337GHz	13.4	902.3	51.5
HFT_402GHz	83.6	5611.2	320.3

- Case 4.0: the offsets of all the detectors (n=4508) and, therefore, all the frequency elements are fully correlated.
- Case 4.1: the offsets of all the detectors and, therefore, all the frequency elements are uncorrelated.
- Case 4.2: the offsets of all the detectors of a given frequency element are fully correlated, but frequency elements are uncorrelated among them.

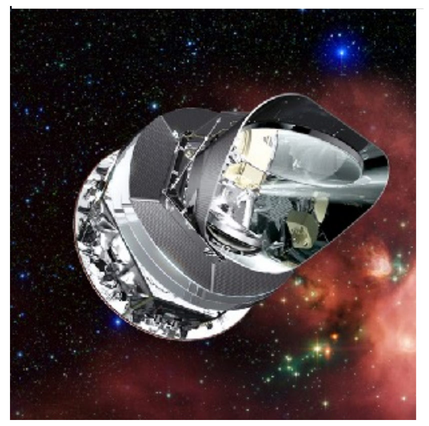
Conclusions

- A new methodology to establish requirements on the polarization angle accuracy of the CMB detectors is presented.
- The method assumes that the CMB solution can be obtained through a linear combination of the different sets of detectors that observed the microwave sky at different frequency elements. The coefficients are considered to be inversely proportional to the noise variance, providing a similar scheme to the one obtained with FGBuster (used as baseline for LiteBIRD).
- Assuming that the requirements on the polarization angle are small enough to work on the small angle limit, we are able to obtain analytical solutions relating the bias on the r parameter with the polarization angle uncertainties.
- At the global and telescope levels, requirements vary from a few arcminutes (full correlation) to a factor of 2 larger (no correlation).
- At the frequency element level, requirements are between slightly below 1 arcmin (full correlation) and several arcminutes (no correlation) for the most sensitivy frequencies around 150GHz.
- At the waver-frequency element level, requirements are again between slightly below 1 arcmin (full correlation) and several arcminutes (no correlation) for the most sensitivy frequencies around 150GHz.
- At the detector level, requirements are between slightly below 1 arcmin (full correlation) and several tens of arcminutes (no correlation) for the most sensitivy frequencies around 150GHz.
- These requirements appear to be achievable, as the first attempts made in LiteBIRD coll. 2022 and de la Hoz et al. 2022 seem to indicate.
- More specific analyses considering a detailed modellisation of the expected level of correlation for a given design of the instrument are needed.

LiteBIRD Joint Study Group

Over 300 researchers from **Japan**, **North America** and **Europe**

Team experience in CMB experiments, X-ray satellites and other large projects (ALMA, HEP experiments, …)





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IPMU Institute for the Physics and Mathematics of the Universe
iPNS 素粒子原子核研究所 Institute of Particle and Nuclear Studies
irap astrophysique & planétologie
JAXA
KEK
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KMI Kobayashi-Maskawa Institute Nagoya University
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