Nuclear transfer reactions with the Lagrange-mesh method

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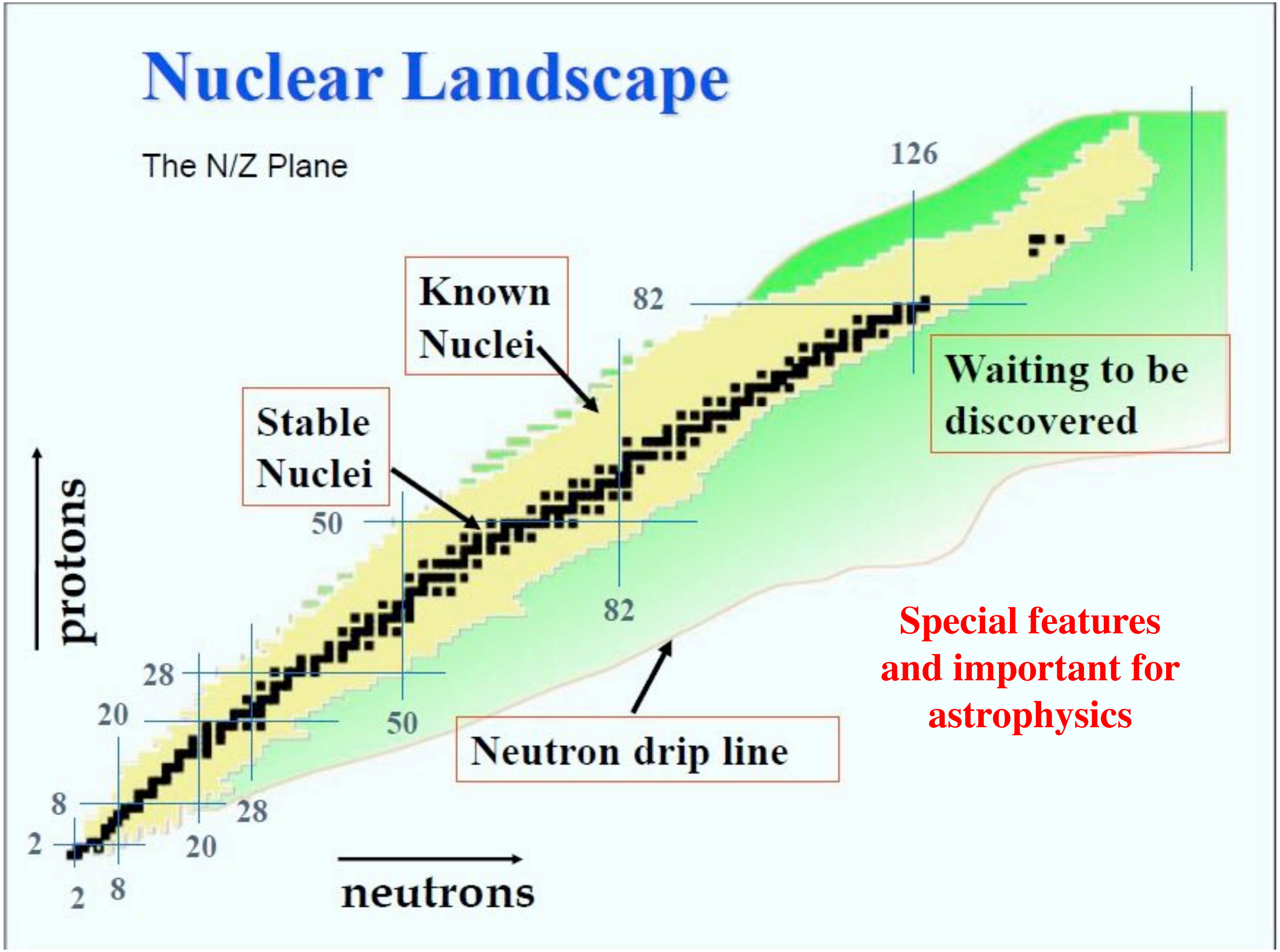
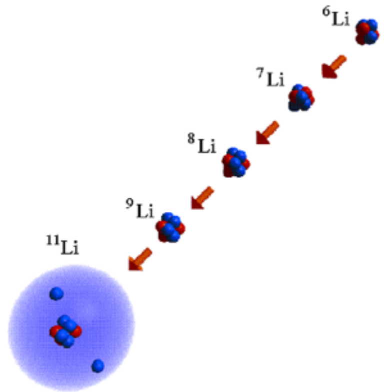
INFN Pisa, 11/11/2021

Nuclear structure from reaction



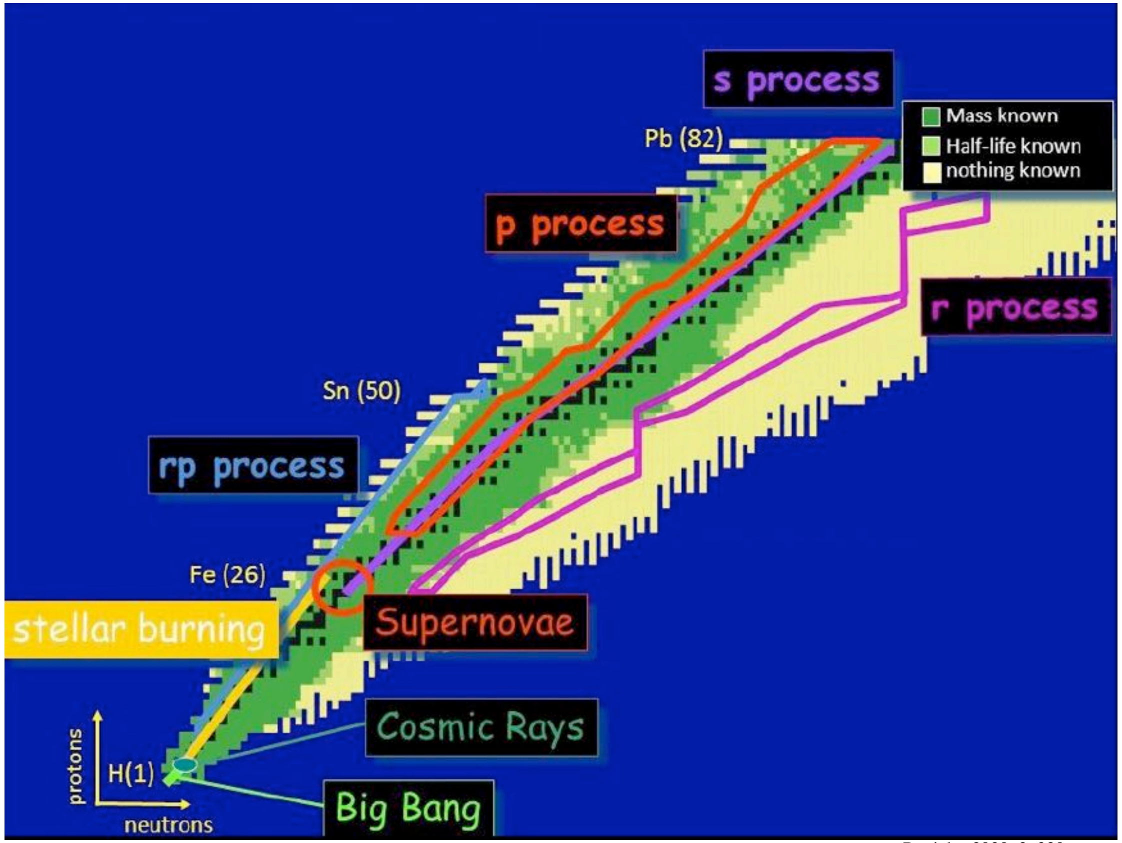


Special features and important for astrophysics

Nuclear Astrophysics

Formation of elements

- ❑BBN, Stellar nucleosynthesis, novae, supernovae
- ❑ Various processes



Particles 2020, 3, 320

To study various processes

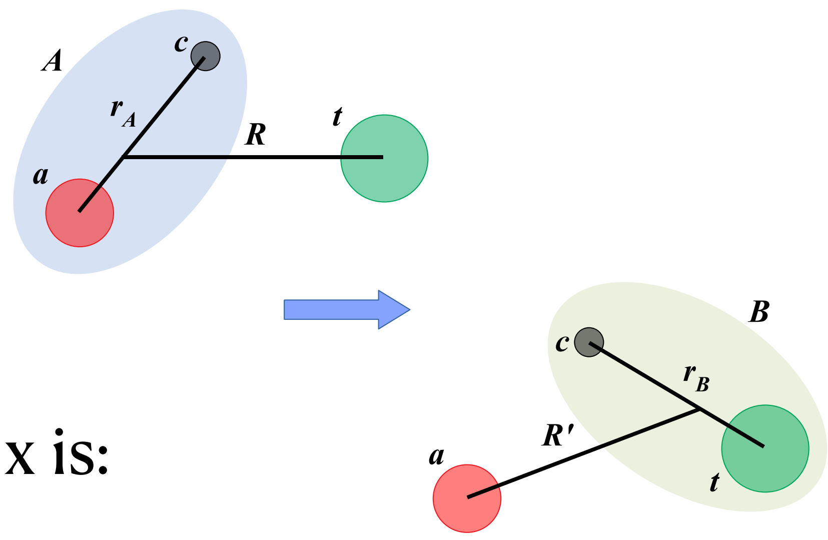
- ❑Nuclear reaction rates at small energies are needed.

For p + p, E_c= 550 keV

At T = 0.01x10^9 (av. Stellar temp.) E = 0.86 keV

Nuclear transfer reactions

$$A(=a+c)+t \rightarrow B(=c+t)+a$$



Different approaches:
DWBA, ADWA, CDCC, Faddeev method

In the DWBA formalism the scattering matrix is:

$$U^{J\pi}_{\alpha\beta} = -\frac{i\sqrt{SF_A SF_B}}{\hbar}\langle \chi^{J\pi}_{L_B}(\mathbf{R}')\Phi^{I_B}_{\ell_B}(\mathbf{r}_B)\,|\,\mathscr{V}\,|\,\chi^{J\pi}_{L_A}(\mathbf{R})\Phi^{I_A}_{\ell_A}(\mathbf{r}_A)\rangle$$

$$\mathscr{V}_{post} = V_{ac} + U_{at} - U_{aB}$$

$$\mathscr{V}_{prior} = V_{ct} + U_{at} - U_{At}$$

Remnant terms

Can be neglected ??

e.g. in (d, p) reaction post form p-A_t & p-(A_t+1) appear nearly same for a heavy target

Sources of uncertainties!

Which information?

Comparing the measured angular distribution with the theoretical calculations one can have:

- ℓ (orbital angular momentum), J
- Spectroscopic factor (SF)
- Or asymptotic normalisation coefficient (ANC)
- width (Γ) in case final state is a resonance
- ANC/SF also give information about radiative width

Indirect tool in Nuclear astrophysics:

$$\Gamma \rightarrow \sigma^{Res} \qquad \sigma^{DC} = \sum_{l_i l_f} SF(l_f)\, \sigma^{DC}_{l_i l_f} \qquad \sigma \longrightarrow \text{reaction rate}$$

For e.g. ^{13}C(^{3}He, d)^{14}N $\longrightarrow$ ANCs for ^{14}N states $\longrightarrow$ ^{13}C(p, γ)^{14}N

PRC ***62****, 024320 (2000).*

Similarly (^{6}Li, d) and (^{7}Li, t) were used for (α, γ) and (α, n). *J. Phys. G: Nucl. Part. Phys. 43 (2016) 043001*

Need of efficient numerical techniques!!

R-matrix and Lagrange mesh methods

Descouvemont, Baye, Rep. Prog. Phys. **73** (2010) 036301 D. Baye, Phy. Rep. 565 (2015) 1.

R-matrix method

Internal | External

$$\chi^{int}(r) = \sum_{j=1}^{N} c_j \phi_j(r) \qquad \chi^{ext}(r) = C_l\,[I_l(kr) - U_l\,O_l(kr)]$$

o a $r \longrightarrow$

To make Hamiltonian as Hermitian over (0, a), use Bloch operator

$$\mathscr{L} = \frac{\hbar^2}{2\mu}\,\delta(r-a)\frac{d}{dr} \qquad (H + \mathscr{L} - E)\chi^{int} = \mathscr{L}\chi^{int} = \mathscr{L}\chi^{ext}$$

- Expansion in square-integrable basis is now possible.
- Beyond making H+£ Hermitian, the Bloch operator enforces the continuity of the derivative of the wave function.

$\boxed{\text{Channel radius is not a fitting parameter}}$

R-matrix and Lagrange mesh methods

D. Baye, Phy. Rep. 565 (2015) 1.

In the internal region the wave function is expanded over the Lagrange basis

- They are orthonormal basis, vanishes at all but one mesh point.
- Gauss quadrature (GQ) associated with the mesh. N = basis size

Lagrange Condition

$$\phi_i(ax_j) = \frac{1}{\sqrt{a\lambda_j}}\delta_{ij}$$

Weight of the Gauss quadrature approximation associated

$$\int_a^b g(x)dx \approx \sum_{i=1}^{N} \lambda_i g(x_i)$$

Choice of Lagrange functions depend upon the interval

Matrix elements of the overlap $\langle \phi_i | \phi_j \rangle = \delta_{ij}$

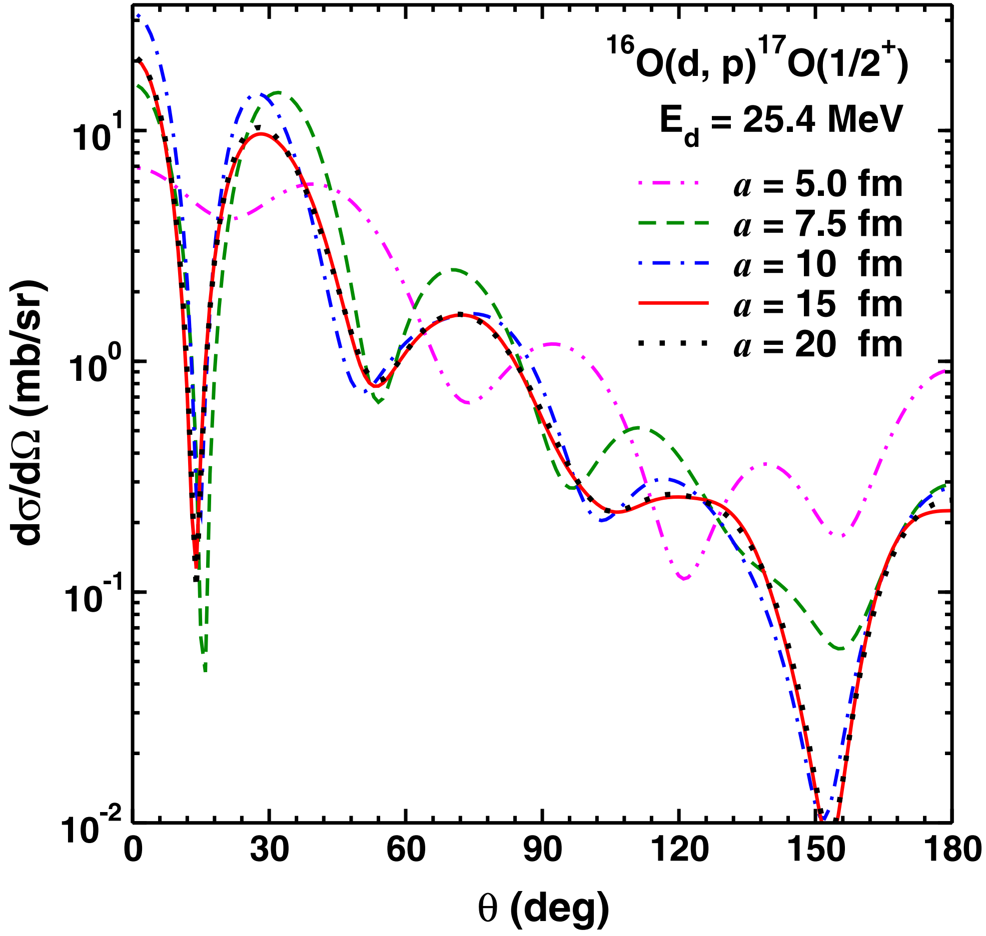
and of the potential $\langle \phi_i | V | \phi_j \rangle = V(ax_i)\delta_{ij}$

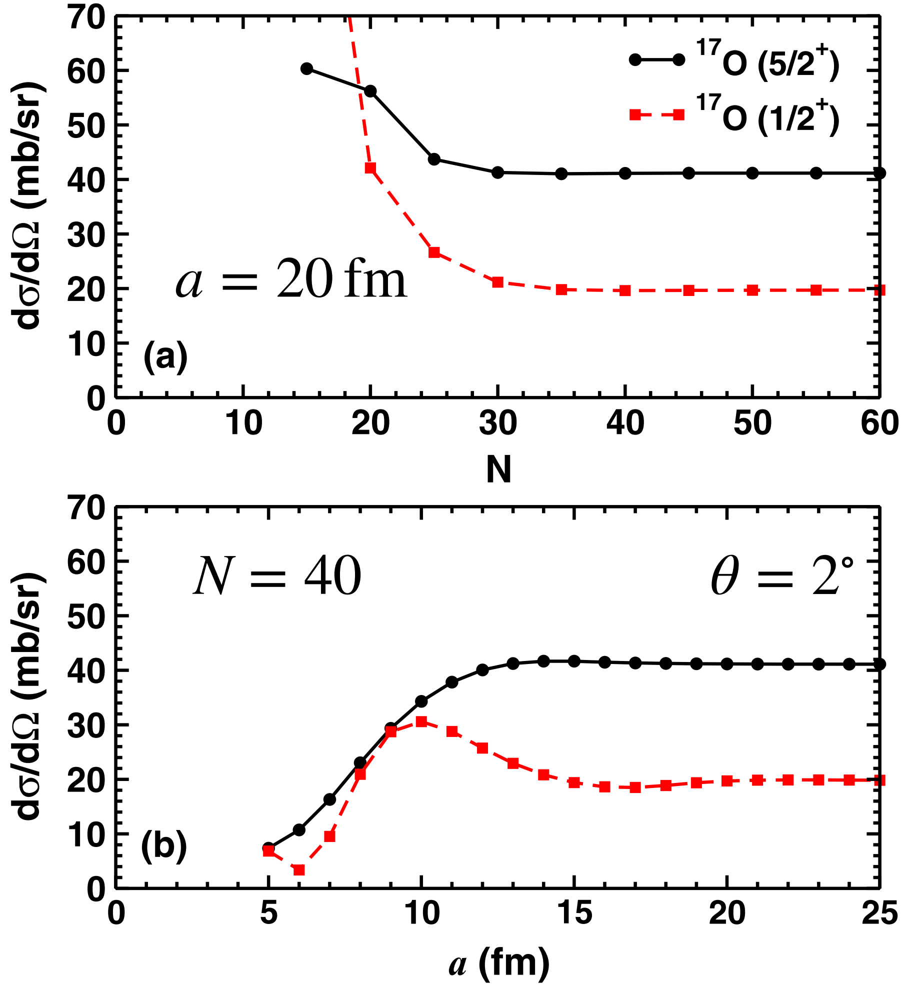
Large channel radius need large number of basis !!

Test cases

Shubhchintak, Descouvemont, Phys. Rev. C ***100****, 034611 (2019).*

Convergence of cross sections with channel radius and Number of basis

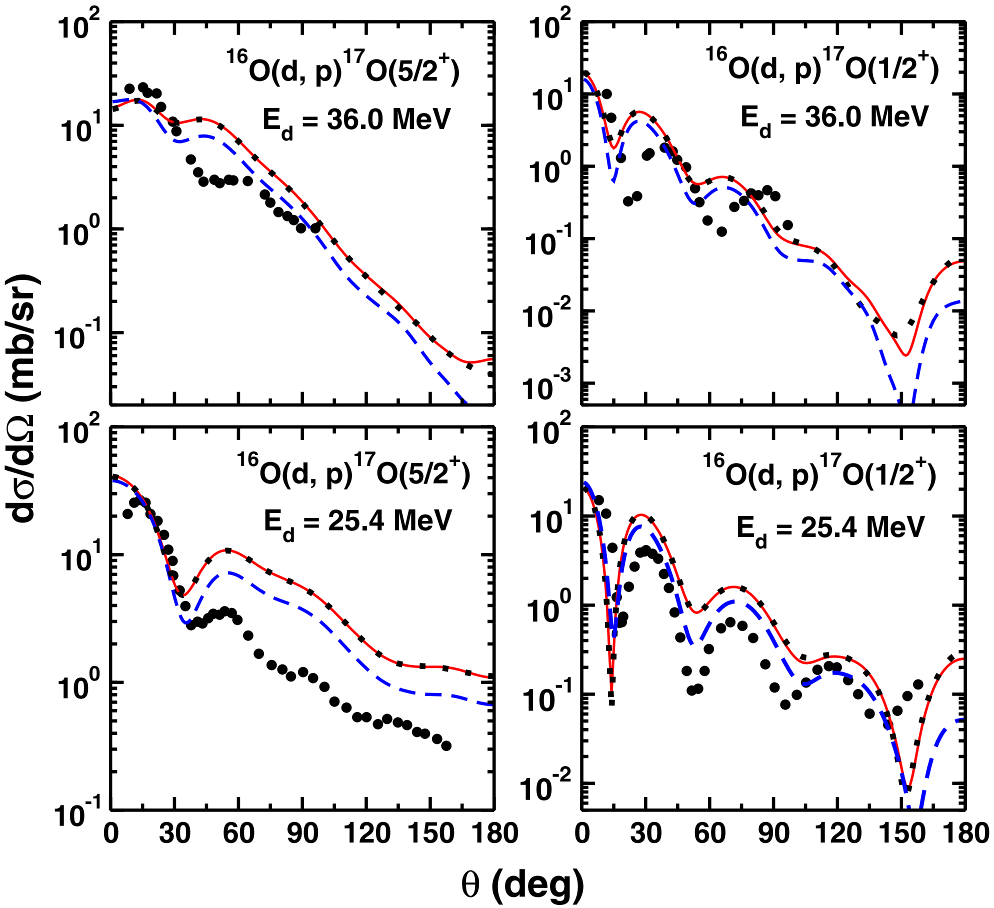




N ~ 40 is sufficient, much lesser than other methods

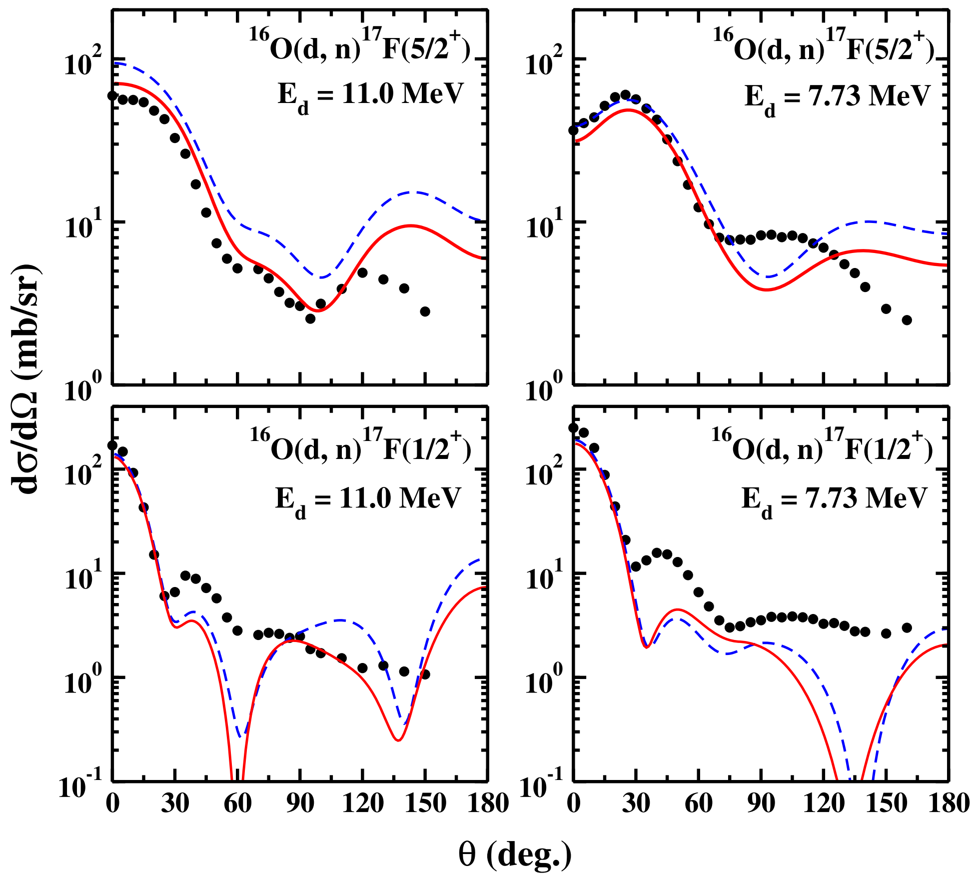
Test cases (nucleon transfer)

***Shubhchintak**, Descouvemont, Phys. Rev. C **100**, 034611 (2019).*



Dashed lines: With remnant terms
Solid lines: Without remnant terms
Dotted lines: FRESCO
Exp. data and potentials: *NPA 218, 249 (1974)*

***Shubhchintak**, Eur. Phys. J. A **57**, 32 (2021).*



Dashed lines: Without remnant terms
Solid lines: With remnant terms
Exp. data and potentials: *NPA 127, 567 (1969)*
SF with Remnant: For g.s. change by 20-30% for e.s. change by < 10%

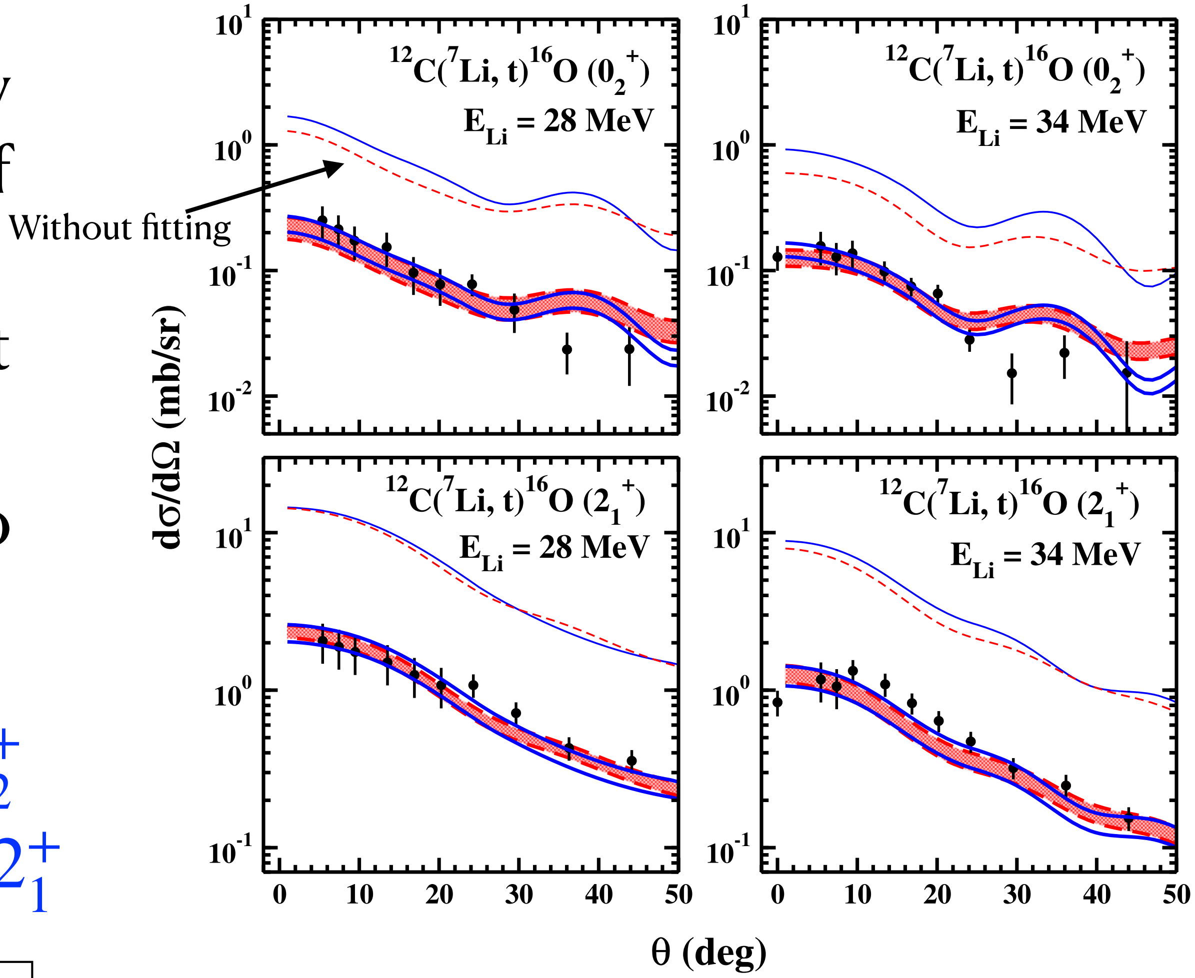
$^{12}C(^{7}Li, t)^{16}O$

- Case of α transfer
- Has been used for many indirect measurements of $^{12}C(\alpha, \gamma)^{16}O$
- $^{12}C(\alpha, \gamma)^{16}O$ is an important astrophysical reaction.
- Cross sections below 300 keV has uncertainties.

$$\text{Exp. SFs} = 0.13^{+0.07}_{-0.06} \text{ for } 0_2^+$$
$$= 0.15 \pm 0.05 \text{ for } 2_1^+$$

SFs with Remnant: For 2_1^+ increase by 6 - 14 %, for 0_2^+ state increased by 30 - 50 %

Shubhchintak, *Eur. Phys. J. A* ***57****, 32 (2021).*



Data and potentials: *Oulebsir, Hammache et al. PRC* ***85****, 035804 (2012).*
C + t potential: D. Y. Pang et al. PRC ***91****, 024611 (2015).*

Dashed lines: With remnant terms
Solid lines: Without remnant terms

Post-Prior Equivalence in DWBA

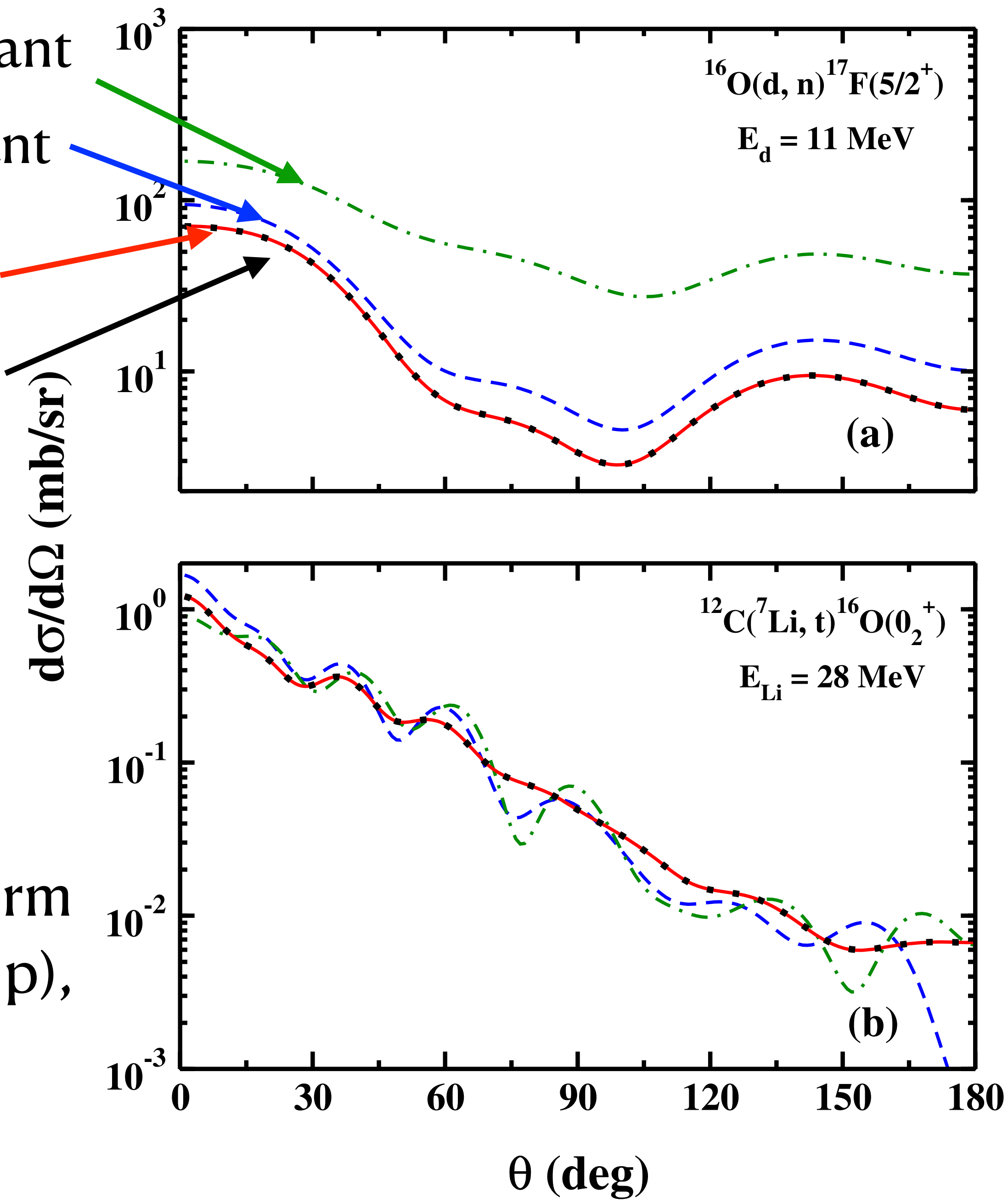
Shubhchintak, *Eur. Phys. J. A* ***57****, 32 (2021).*

Prior-form without remnant

Post-form without remnant

Post-form with remnant

Prior-form with remnant



$$\mathscr{V}_{post} = V_{ac} + \boxed{U_{at} - U_{aB}}$$

$$\mathscr{V}_{prior} = V_{ct} + \boxed{U_{at} - U_{At}}$$

Also justify the use of post form without remnant for the (d, p), (d, n) reactions.

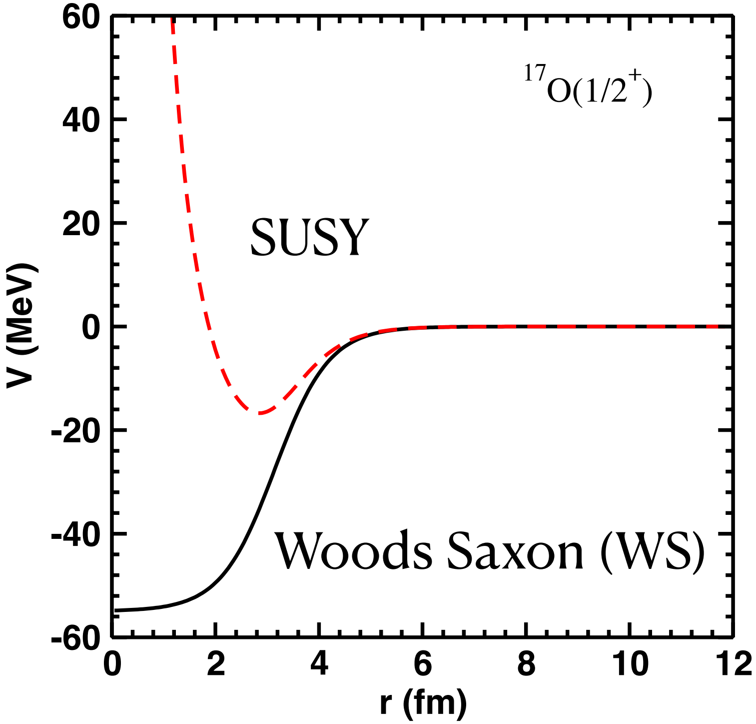
Sensitivity to the bound state wave functions

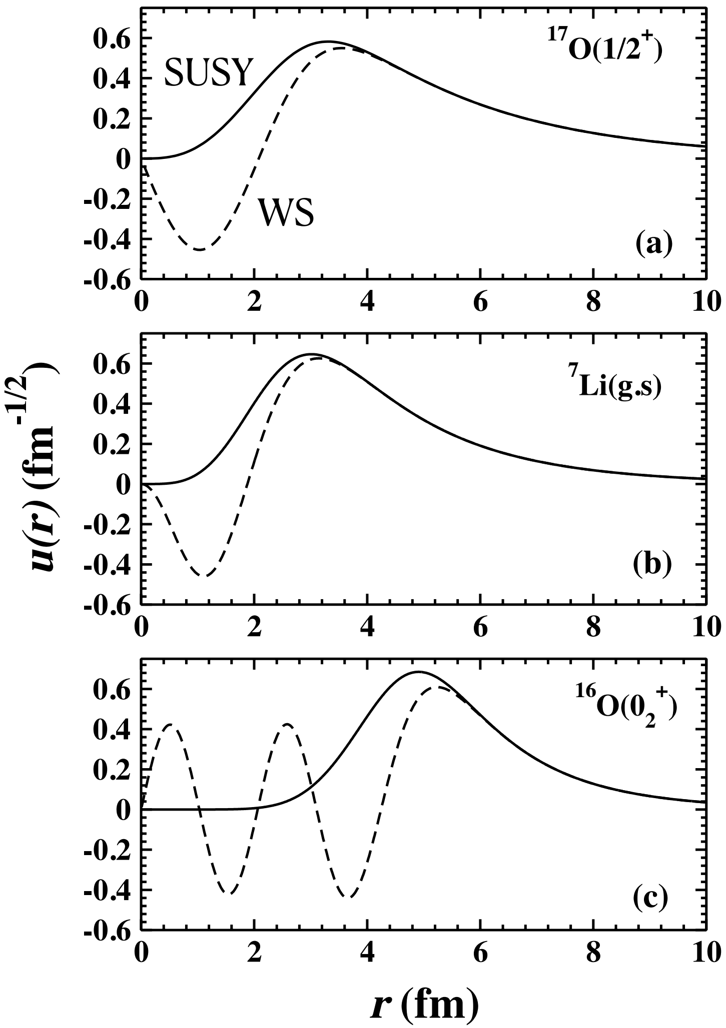
Supersymmetric (SUSY) transformation: *D. **Baye**, PRL **58**, 2738 (1987).*

$$H_0 = -\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + V_0^{\ell j}(r) \longrightarrow H_2 \quad \text{With} \quad V_2^{\ell j} = V_0^{\ell j} - \frac{\hbar^2}{\mu}\frac{d^2}{dr^2} log \int_0^r |u_0(s)|^2 ds$$

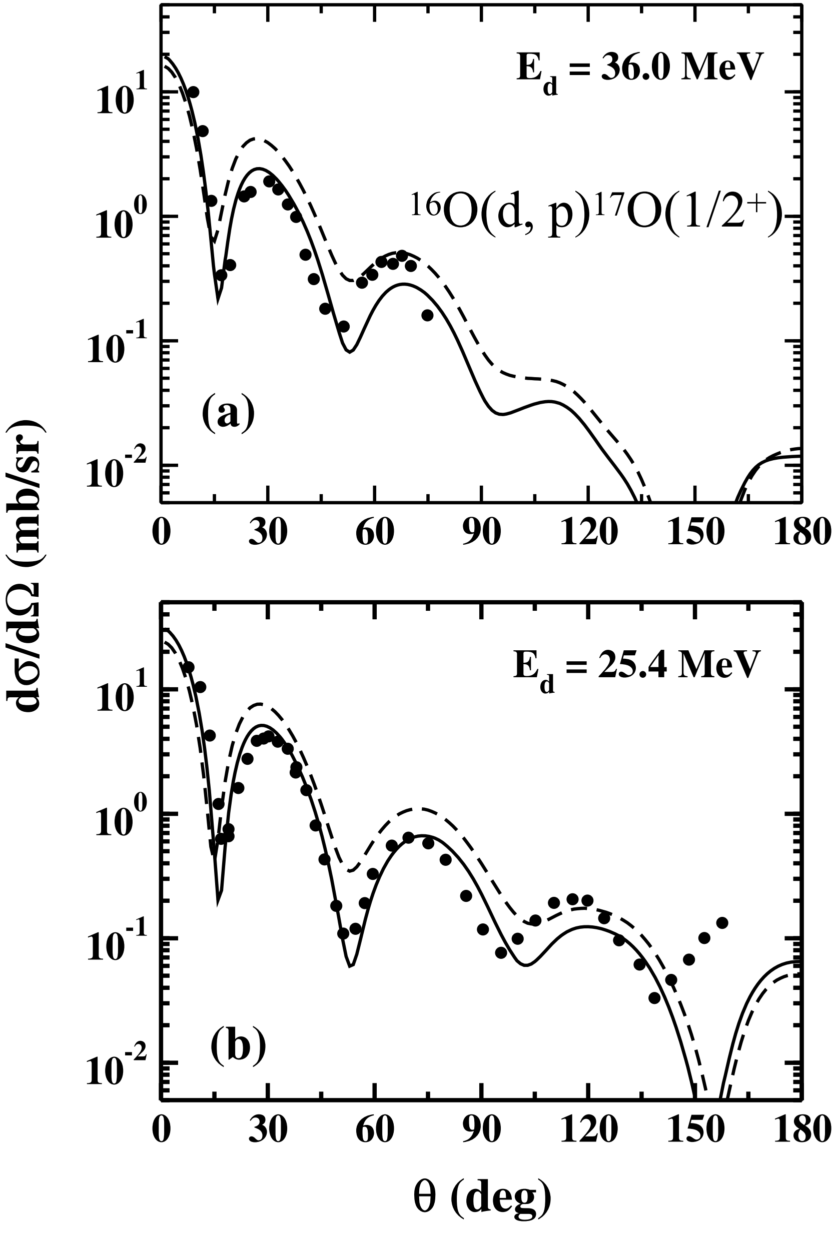
Number of Pauli-forbidden states (n) in the different systems considered here.

System	State	ℓ	n
$n+^{16}$O	$1/2^+$	0	1
$\alpha+t$	$3/2^-$	1	1
$\alpha+^{12}$C	0_2^+	0	4
$\alpha+^{12}$C	2_1^+	2	3





Sensitivity to the bound state wave functions



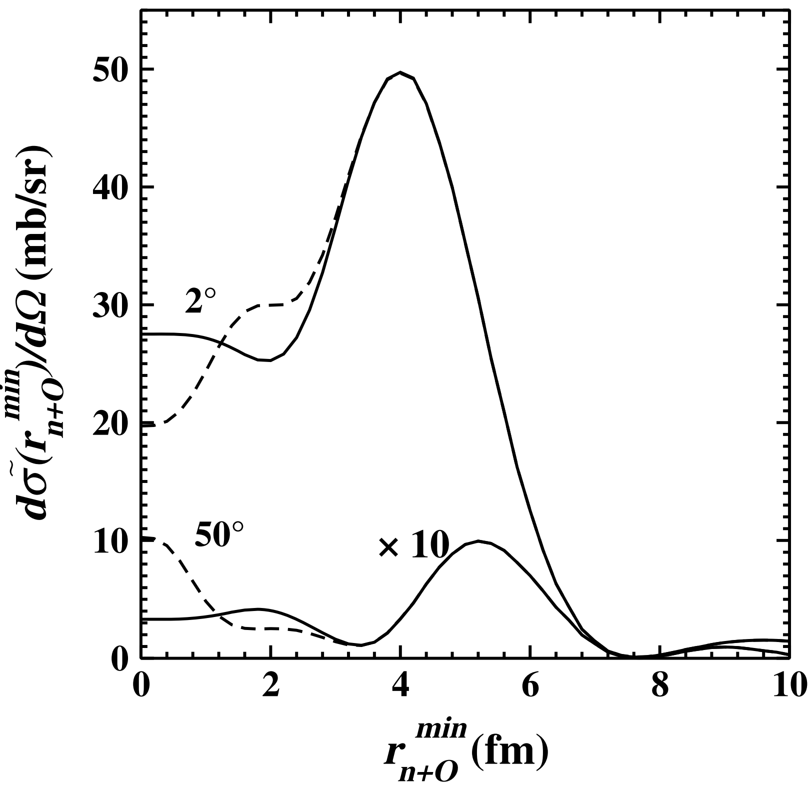
Dashed lines: With WS
Solid lines: With SUSY

Shubhchintak, *Descouvemont, Phys. Letts. B* ***811***, *135874 (2020).*

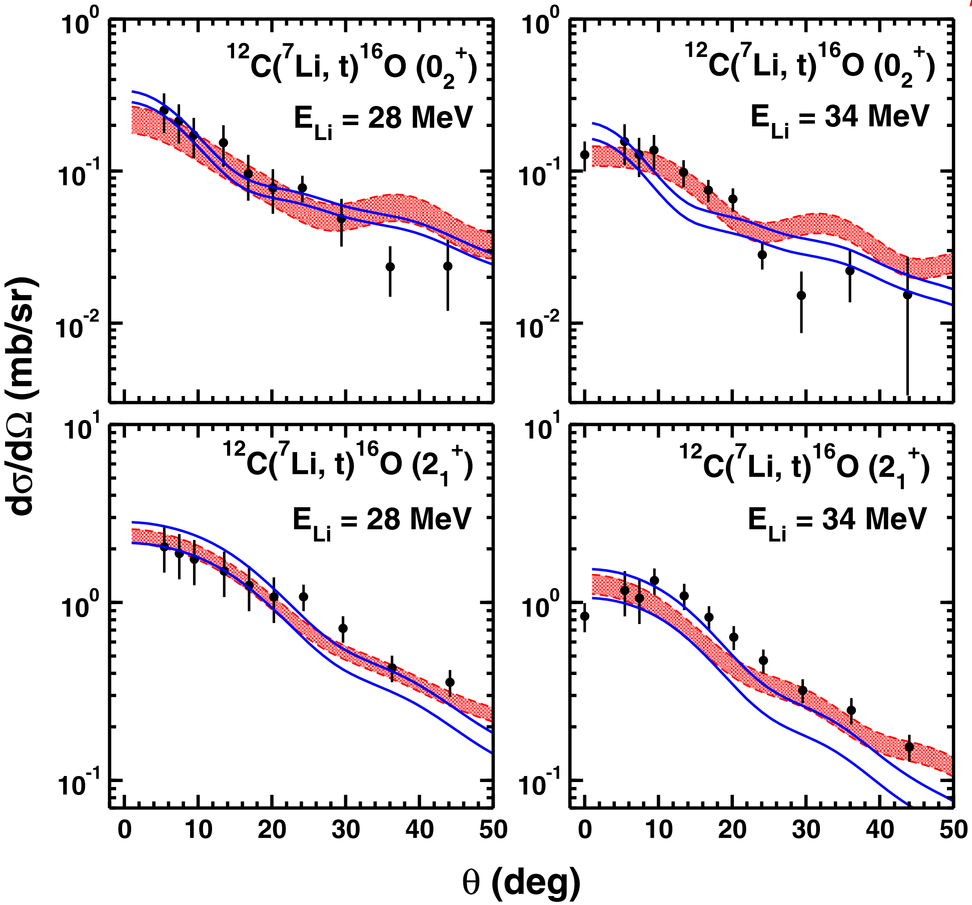
Put a cut, r_{min} over r_A or r_B

$$\tilde{U}^{J\pi}_{\alpha\beta}(0) = U^{J\pi}_{\alpha\beta}$$

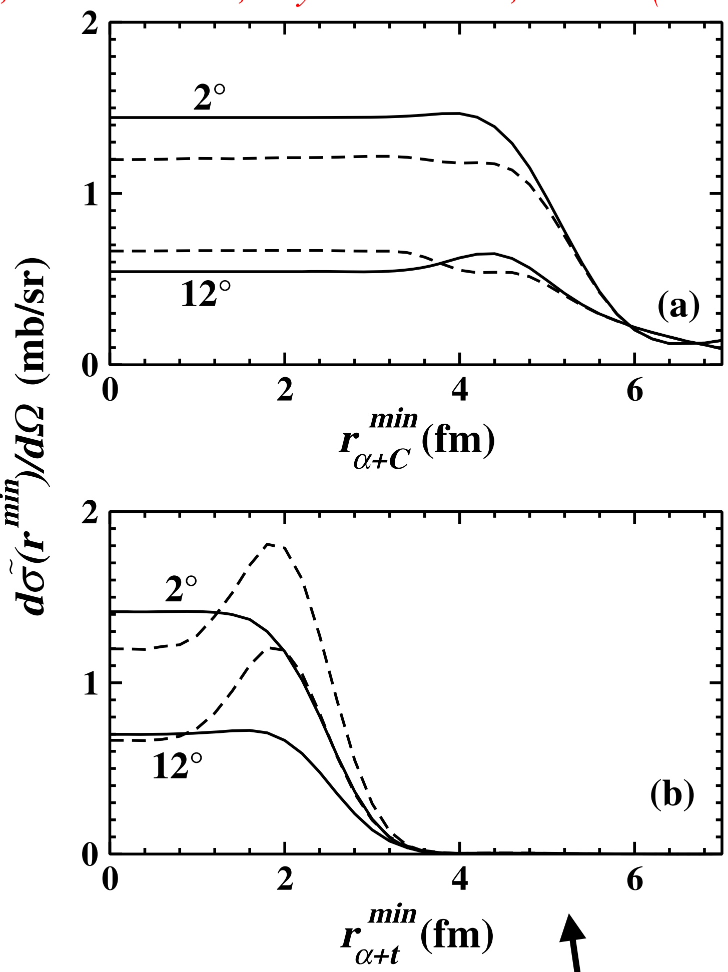
$$\tilde{U}^{J\pi}_{\alpha\beta}(\infty) = 0$$



Sensitivity to the bound state wave functions



***Shubhchintak**, Descouvemont, Phys. Lett. B **811**, 135874 (2020).*



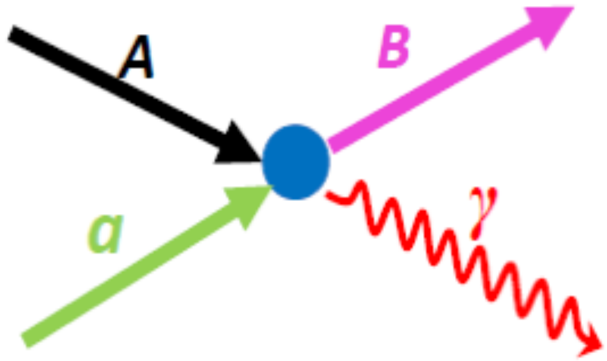
Nucleus	State	Beam energy (MeV)	SFs (WS)	SFs (SUSY)
^{17}O	$1/2^+$	25.4	1.73	1.22
	$1/2^+$	36.0	2.08	1.63
^{16}O	0_2^+	28	0.18 ± 0.04	0.19 ± 0.02
^{16}O	0_2^+	34	0.24 ± 0.04	0.25 ± 0.03
^{16}O	2_1^+	28	0.17 ± 0.02	0.15 ± 0.02
^{16}O	2_1^+	34	0.16 ± 0.02	0.14 ± 0.03

Similar tests with the cutoff for the Peripherality. Important for ANC extraction

***Shubhchintak**, Descouvemont, Phys. Rev. C **100**, 034611 (2019).*

ANC & radiative capture

For the peripheral radiative capture reactions $\sigma \propto (\mathrm{ANC})^2$



The overlap function (I) For $B \rightarrow A + a$

$$I_{lsj}(\mathbf{r}_{Aa}) = \langle \phi_A \phi_a | \phi_B \rangle = \text{angular part} \times \boxed{I_{lsj}(r_{Aa})}$$

Radial overlap function

ANC

$$I_{lsj}(r_{Aa}) = C_{lsj}\, W_{-\eta, l+\frac{1}{2}}(2\kappa r_{Aa})/r_{Aa}$$

For $r_{Aa} >> R_n$

$$M = \langle \phi_B | \hat{O}(r_{Aa}) | \phi_A \phi_a \psi_i^+(\mathbf{r}_{Aa}) \rangle = \langle I_{B(Aa)}(\mathbf{r}_{Aa}) | \hat{O}(r_{Aa}) | \psi_i^+(\mathbf{r}_{Aa}) \rangle$$

$$\sigma \propto |\mathrm{M}|^2$$
$$\propto (\mathrm{ANC})^2$$

Two body Potential model

$$I_{lsj}(r_{Aa}) = \sqrt{S_{lsj}}\,\phi_{lsj}(r_{Aa})$$

S is the spectroscopic factor of the final bound state.

The tail of the bound state wave function:

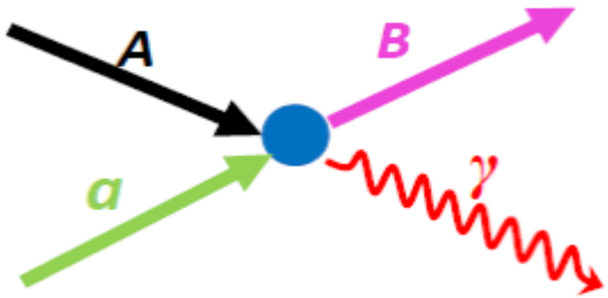
$$\phi_{lsj}(r_{Aa}) \approx b_{lsj}\,W_{\eta,l+\frac{1}{2}}(2\kappa r_{Aa})/r_{Aa} \qquad \text{For } r_{Aa} > R_n$$

b is the single particle ANC and it depends upon potential.

$$\boxed{C = S^{1/2}\,b}$$

ANC from transfer measurement

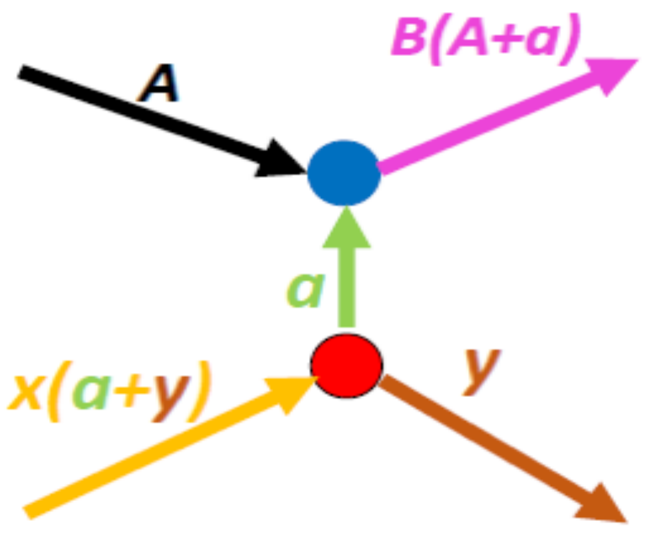
ANC of $a + A \longrightarrow B$



Using transfer reaction A(x,y)B

$$\frac{d\sigma}{d\Omega} = (S^{B}_{Aal_Bj_B})(S^{x}_{yal_xj_x})\left(\frac{d\sigma}{d\Omega}\right)^{DWBA}$$

$$\frac{d\sigma}{d\Omega} = (C^{B}_{Aal_Bj_B})^2(C^{x}_{yal_xj_x})^2\frac{(d\sigma/d\Omega)^{DWBA}}{b^2_{Aal_Bj_B}\,b^2_{yal_xj_x}}$$



Lithium Problems

Abundace of ^{7}Li

BBN: $^{7}Li/H = (4.56\text{-}5.34) \times 10^{-10}$ *J. Cosm. Astrophys.* ***10***, *050 (2014)*

Observed: $^{7}Li/H = 1.58^{+0.35}_{-0.28} \times 10^{-10}$ *Astron. Astrophys.* ***522***, *A26 (2010)*

First Lithium Puzzle

Hartos, Bertulani, ***Shubhchintak****, Mukhamedzhanov, Hou, Astrophys. J 862, 62 (2018).*

Isotopic ratio of $^{6}Li/\,^{7}Li$,

BBN: $^{6}Li/\,^{7}Li \sim 10^{-5}$ *J. Cosm. Astrophys.* ***10***, *050 (2014).*

Observed: $^{6}Li/\,^{7}Li \sim 5 \times 10^{-2}$ *APJ* ***644***, *229 (2006).*

Second Lithium Puzzle

?

$\alpha + d \rightarrow \mathbf{^{6}Li} + \gamma$ Reaction

First successful Experiment by LUNA at two Big Bang energies 94 and 134 keV

M. Anders et al., PRL. 113, 042501 (2014).

Photon's angular distribution in $d(\alpha, \gamma)^6$Li

Red lines : Method 1 (Fix ANC by using Spectroscopic factor)
Green lines : Method 2 (Fix ANC from the phase equivalent potential)

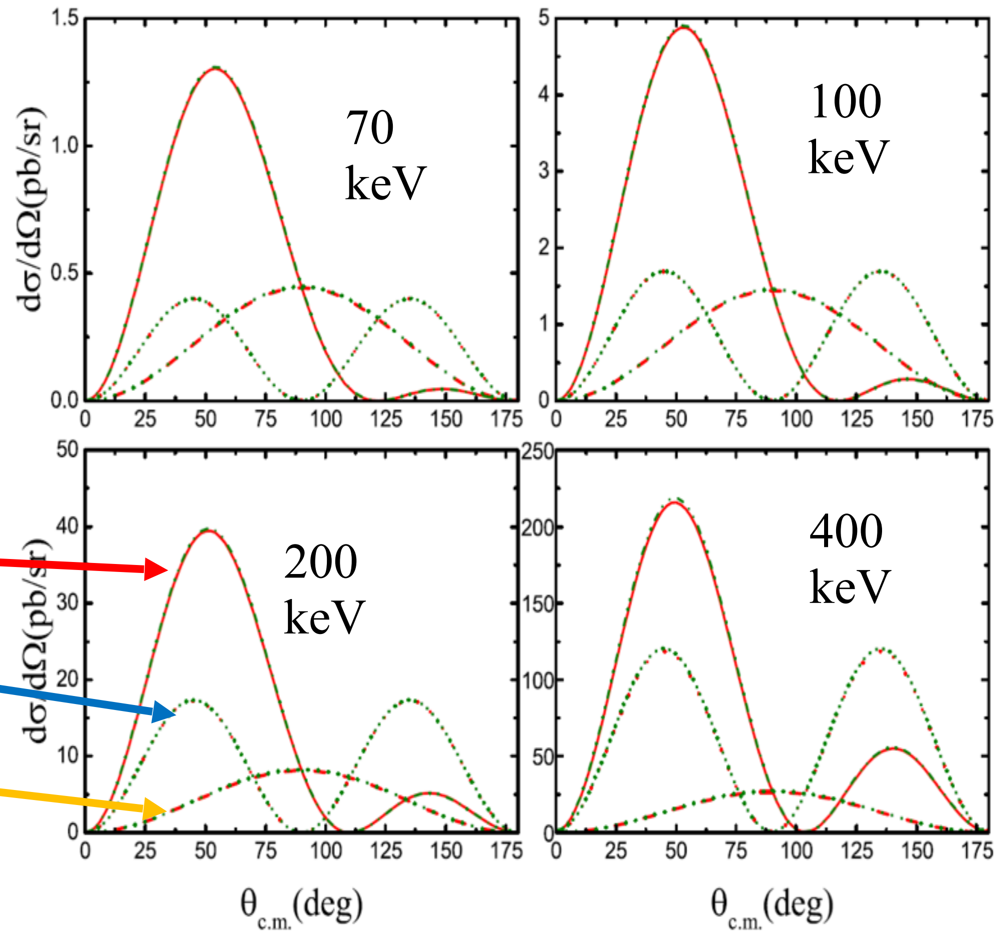
ANC C = 2.32 ± 0.12 fm$^{-1/2}$

Phys. Rev. C 48, 2390 (1993)

Total with Interference

Quadrupole (E2)

Dipole (E1)



*Mukhamedzhanov, **Shubhchintak**, Bertulani, Phys. Rev. C **93**, 045805 (2016).*

Astrophysical S-factor of $d(\alpha,\gamma)^6Li$

*Mukhamedzhanov, **Shubhchintak**, Bertulani, Phys. Rev. C **93**, 045805 (2016).*

E <100 keV, E1 dominates and at higher energies E2 dominates.

$$S(E) = E\, e^{2\pi\eta}\sigma(E)$$

Blue Square boxes, LUNA:

PRL 113, 042501 (2014).

Black dots: J. Kiener et al. PRC 44, 2195 (1991)

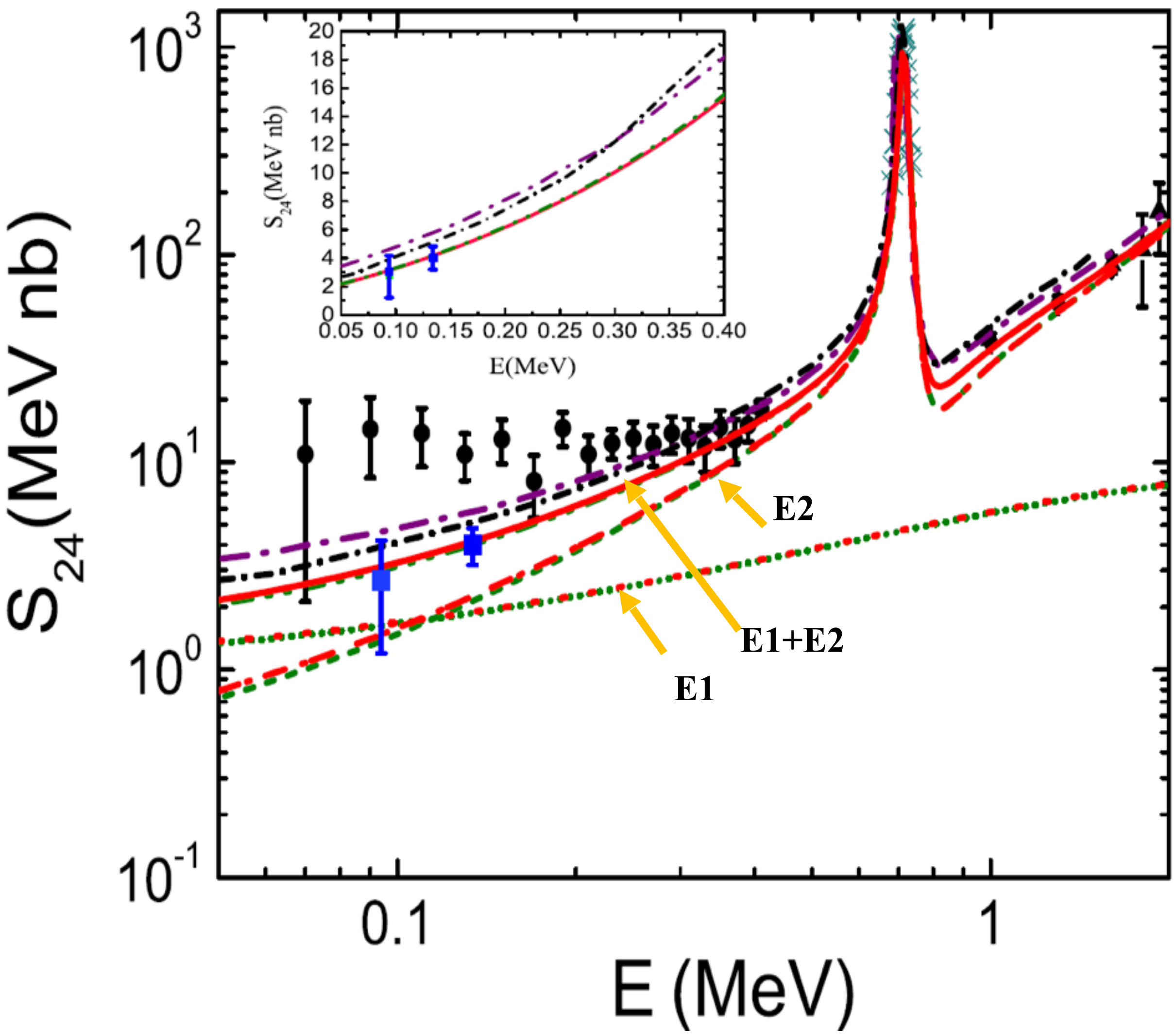
Abundance:

^{6}Li/^{7}Li: $(1.5 \pm 0.3) \times 10^{-5}$

Observed in nine stars is

$\sim 5\times10^{-2}$

*APJ **644**, 229 (2006).*



BBN: Wagoner, Ap. J. Suppl. Ser. 18, 247 (1969).
Kawano, NASA Technical Reports Server (NTRS): Hampton, VA, USA, 1992.

Summary

- Importance of transfer for astrophysics: SF, ANC, Γ …

- Utility of R-matrix and Lagrange mesh methods to transfer reactions in DWBA

- $^{16}O(d, p)^{17}O$, $^{16}O(d, n)^{17}F$, $^{12}C(^{7}Li, t)^{16}O$

- Effects of remnant terms, post-prior equivalence in DWBA

- Sensitivity of transfer cross sections to bound state wave functions using shallow and deep potentials.

- Application of ANC to d(α, γ)^{6}Li in context of 2nd Li problem