$|V_{cb}|$, LFU and $SU(3)_F$ symmetry breaking in $B_{(s)} \to D^{(*)}_{(s)} \ell \nu_\ell$ decays using Lattice QCD and Unitarity

[PRD ’21 (2105.02497), PRD ’21 (2105.07851), PRD ’22 (2105.08674), 2109.15248, 2202.10285, …]

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OUTLINE OF THE TALK

1) State of the art;
2) The Dispersive Matrix (DM) Method;
3) Applications to $B_{(s)} \to D^{(*)}_{(s)} \ell \nu_\ell$ ($|V_{cb}|$ & $R(D^{(*)}_{(s)})$);
4) Conclusions.

IN SUPREMÆ DIGNITATIS 1343

UNIVERSITÀ DI PISA

INFN PISA
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Work in collaboration with: G.Martinelli, S. Simula and L.Vittorio

There are mainly two issues for which B decays are interesting.

V_{cb} puzzle:

EXCLUSIVE:

$$|V_{cb}| \times 10^3 = 39.36(68)$$

FLAG Review 2021 [arXiv:2111.09849]

INCLUSIVE:

$$|V_{cb}| \times 10^3 = 42.16(50)$$

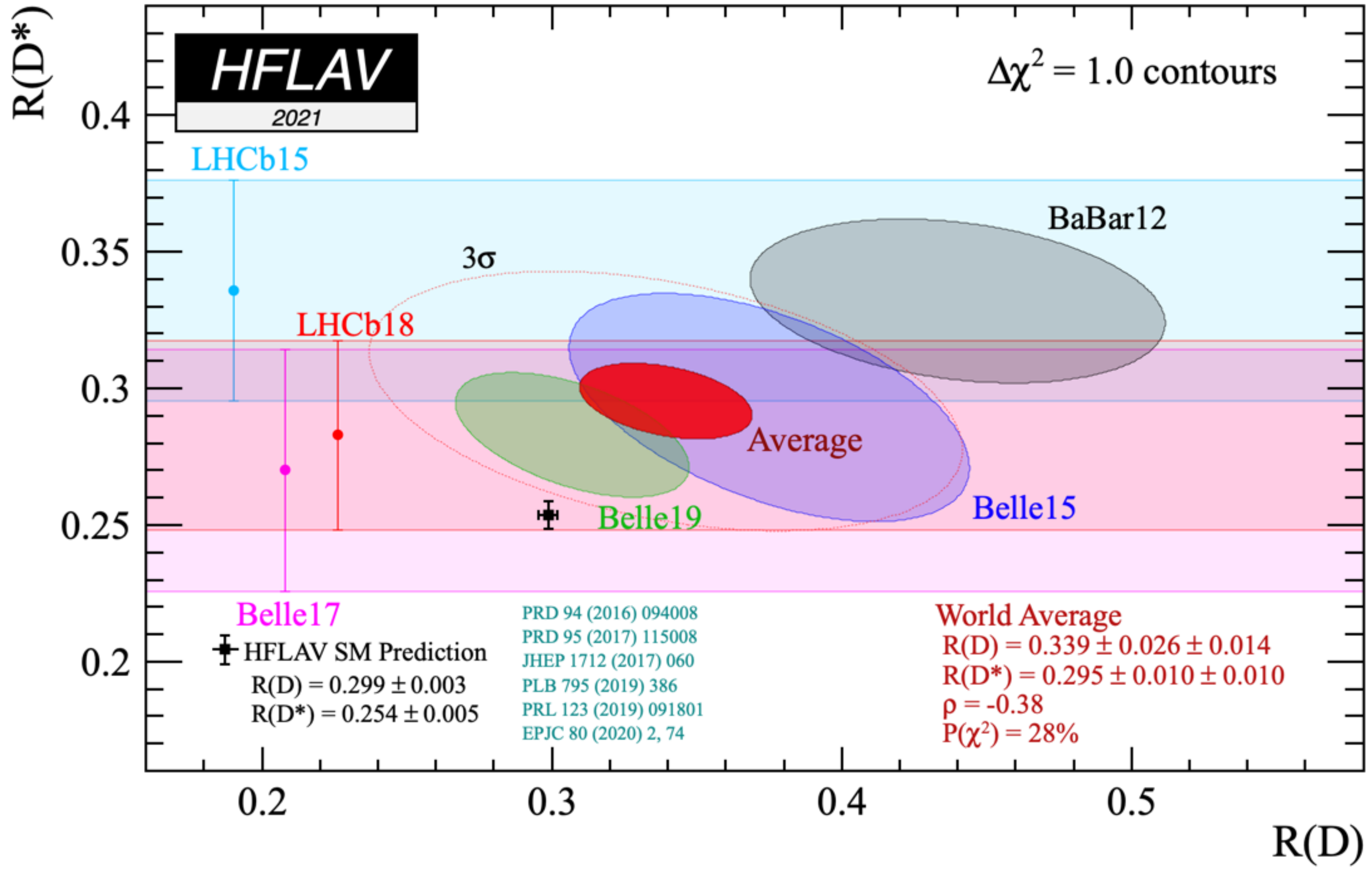
Bordone et al., Phys.Lett.B [2107.00604]

~ 2.7σ difference excl./incl.

$R(D^{(*)})$ anomaly:

$$R(D) = \frac{\mathscr{B}(B \to D\tau\nu_\tau)}{\mathscr{B}(B \to D\ell\nu_\ell)}$$

$$R(D^*) = \frac{\mathscr{B}(B \to D^*\tau\nu_\tau)}{\mathscr{B}(B \to D^*\ell\nu_\ell)}$$



~ 3.4σ *discrepancy between exp.s and "SM"!*

"SM"=mix of theoretical calculations and experimental data to constrain the shape of the hadronic form factors (FFs)

To extract the CKM matrix elements and to test LFU it is necessary to compute the form factors entering the hadronic matrix element as precisely as possible.

Starting from an existing work [L. Lellouch, Nucl. Phys. B 479 (1996)] *we introduced new results* ***PRD '21 (2105.02497).***

$$\mathbf{M} = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \phi_2 f_2 & \dots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-zz_1} & \frac{1}{1-zz_2} & \dots & \frac{1}{1-zz_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \frac{1}{1-z_1 z_2} & \dots & \frac{1}{1-z_1 z_N} \\ \phi_2 f_2 & \frac{1}{1-z_2 z} & \frac{1}{1-z_2 z_1} & \frac{1}{1-z_2^2} & \dots & \frac{1}{1-z_2 z_N} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \frac{1}{1-z_N z_2} & \dots & \frac{1}{1-z_N^2} \end{pmatrix}$$

The idea is to the construct the matrix M.

The matrix depends from the following ***input quantities***:

- The values $z_1, \dots, z_N$ at which the FFs have been computed (e.g. on the lattice);
- The correspondent computed values $f_1, \dots, f_N$ of the FFs in that points;
- The susceptibility χ.

Then, z is the point in correspondence of which we would know the value of f.

$$\boxed{z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1}} \qquad \boxed{t_\pm \equiv (m_B \pm m_D)^2}$$

The properties of this matrix allow us to find bounds on the form factor in the unknown point z !!!

$$\boxed{\det M \geq 0} \longrightarrow \boxed{f_{lo(up)}(z) = \beta(z) \pm \sqrt{\gamma(z)}}$$

Chosen our set of (n+1) input data $\{\chi, f(z_1), \ldots, f(z_n)\}$, the DM method allows to reconstruct the interval of the possible values of the form factor in a generic point z in *a total model independent way and without any truncation* (differently from CLN, BGL…)!

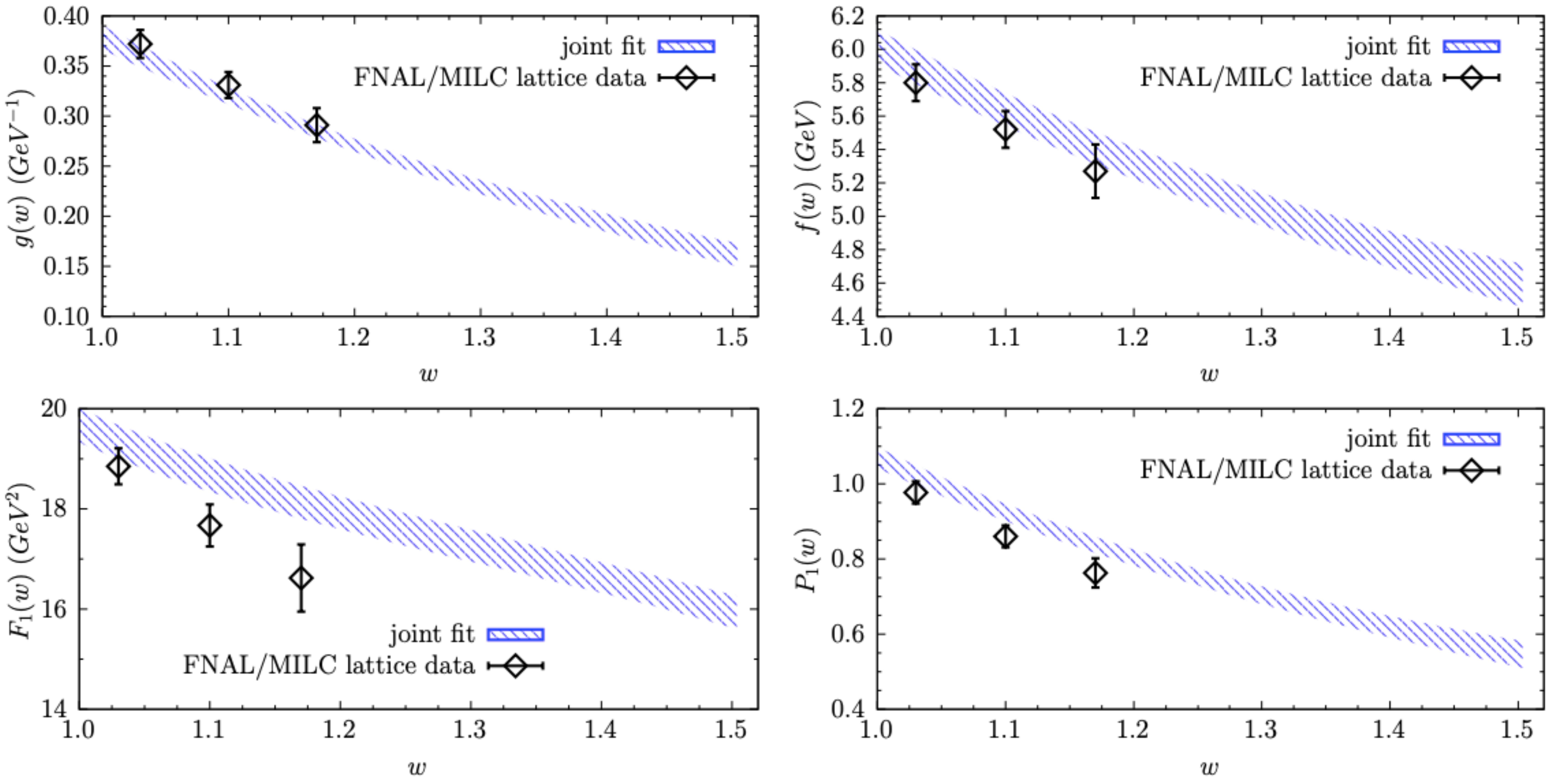
The **obtained band of values represents the results of all possible BGL fits satisfying unitarity** (***the DM result always satisfy unitarity by construction***) **and passing trough the known points**!!!

It contains at least the following three advantages:

1) The method doesn't rely on any assumption about the functional dependence of the FFs on the momentum transferred. Then, in this sense, it is **model independent**;
2) It's **entirely based on first principles**. The susceptibilities are non perturbative and we don't have series expansions;
3) Keeps **theoretical calculations and experimental data separated**.

The last point is crucial to really test the SM predictions!

The FNAL/MILC collaboration recently computed the form factors for $B \to D^* \ell \nu_\ell$ decays



By performing a joint fit using

LQCD pts + Belle + BaBar exp. data

They obtain

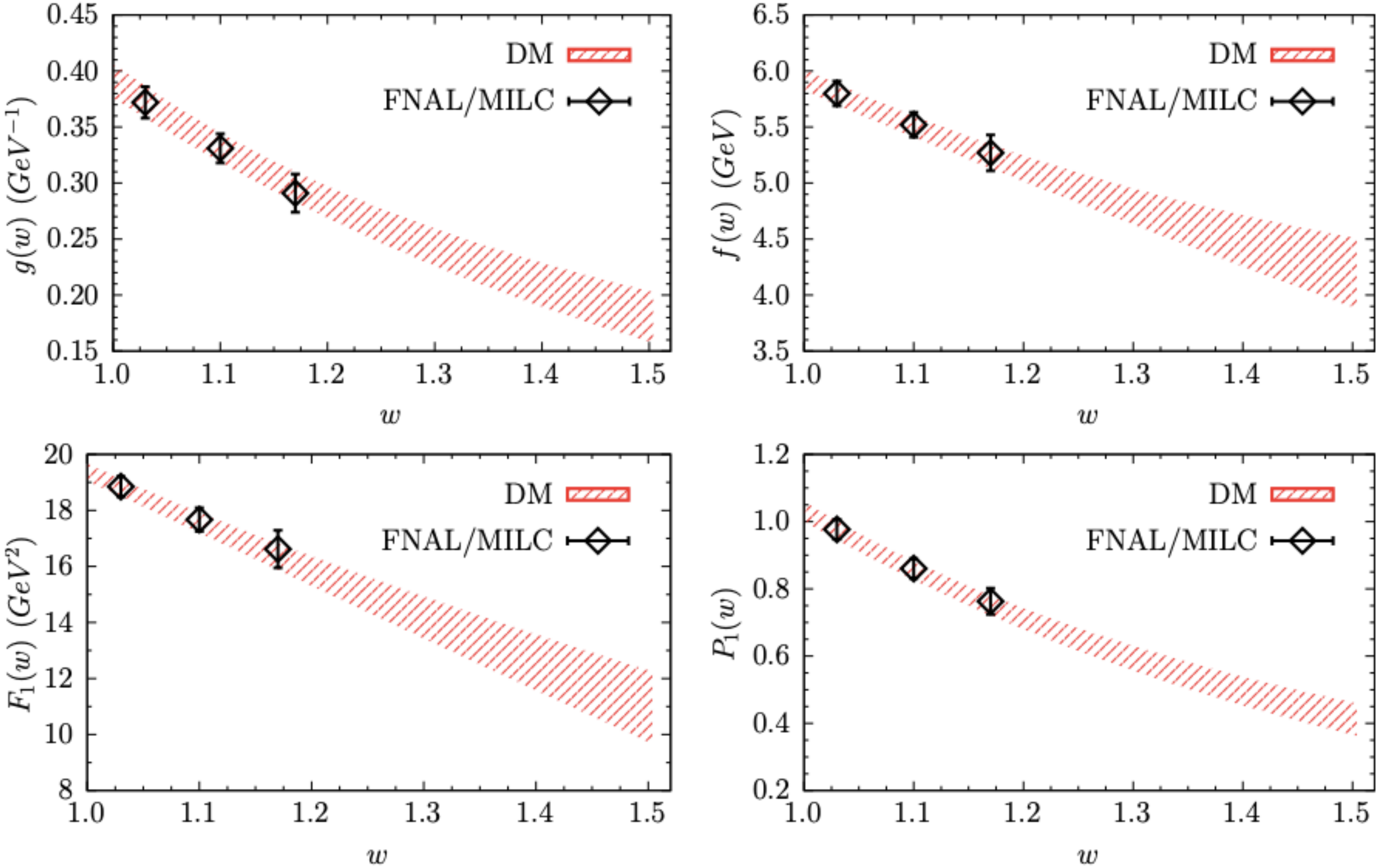
$$|V_{cb}| \cdot 10^3 = 38.40 \pm 0.74$$

$$R(D^*) = 0.2483 \pm 0.0013$$

in tension with $|V_{cb}|_{inc}$ *and* $R^{exp}(D^*)$

But, to which theory do the joint-fit FFs belong? Are $|V_{cb}|$ *and* $R(D^*)$ *pure SM predictions?*

$B \to D^* \ell \nu_\ell$ USING DM METHOD (arXiv:2109.15248)



Our results have been obtained ***ONLY*** *using the 3 lattice QCD data from FNAL/MILC arXiv:2105.14019.*

We used non-perturbative susceptibilities from arXiv:2105.07851 and we took the resonances from Bigi et al., arXiv:1707.09509.

We obtained a ***pure SM prediction***

$$\boxed{R^{DM}(D^*) = 0.275 \pm 0.008}$$

that compared with HFLAV

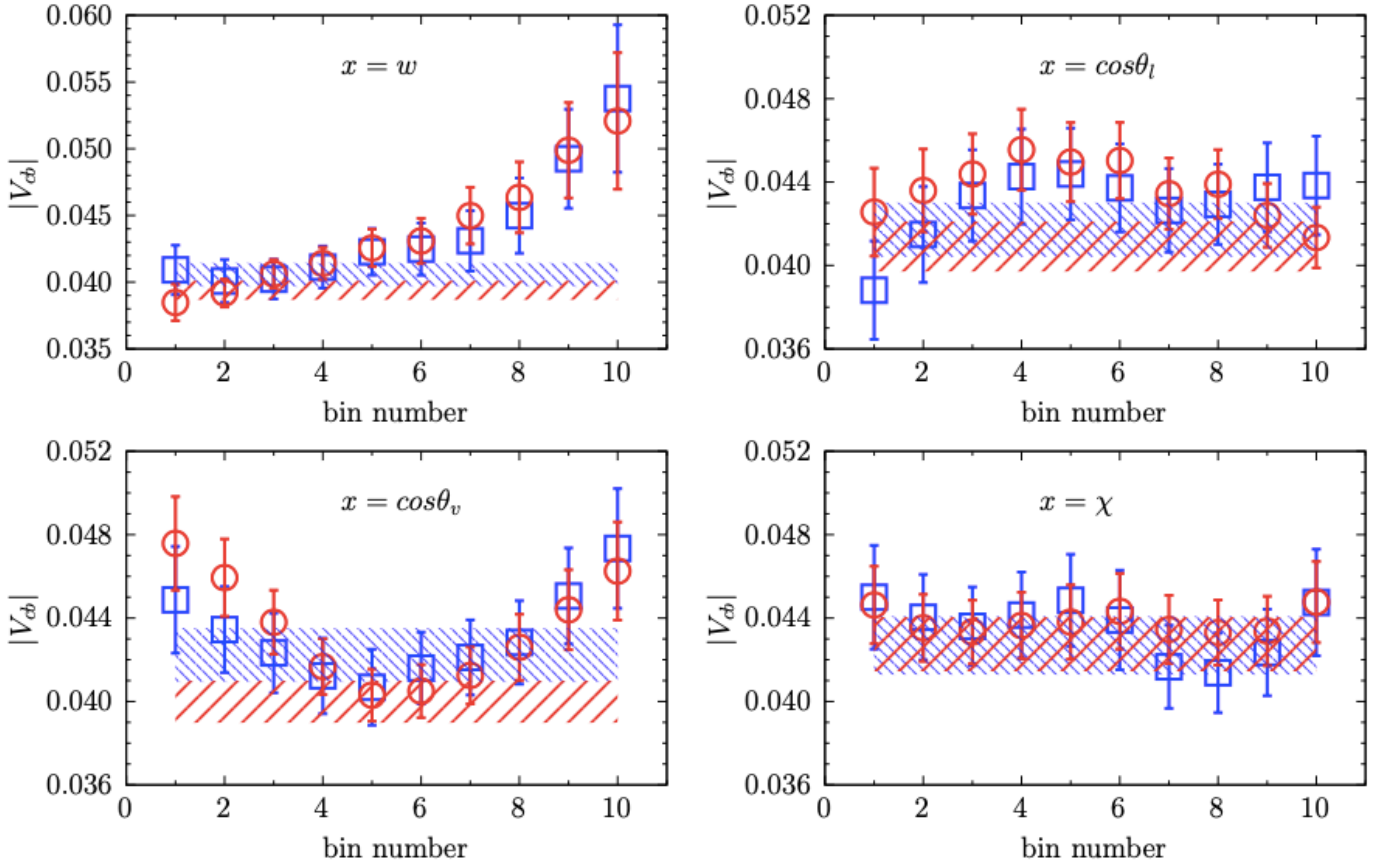
$$\boxed{R^{exp.}(D^*) = 0.295 \pm 0.014}$$

has $\simeq 1.2\sigma$ *difference.*

$|V_{cb}|$ from $B \to D^* \ell \nu_\ell$ USING DM METHOD (arXiv:2109.15248)

blue data: Belle 1702.01521

red data: Belle 1809.03290



To extract $|V_{cb}|$ we don't mix theoretical computation with experimental data

$$|V_{cb}|_i = \sqrt{\frac{(d\Gamma/dx)_i^{exp}}{(d\Gamma/dx)_i^{th}}} \qquad i = 1,\ldots,N_{bins}$$

Then we extract the final result by making (correlated) weighted averages of the bins:

$$|V_{cb}|^{DM}_{excl.} \cdot 10^3 = 41.3 \pm 1.7$$

$$|V_{cb}|_{incl.} \cdot 10^3 = 42.16 \pm 0.50$$ (Bordone et al: arXiv:2107.00604)

Exclusive/inclusive tension reduced to less than 1σ!!!

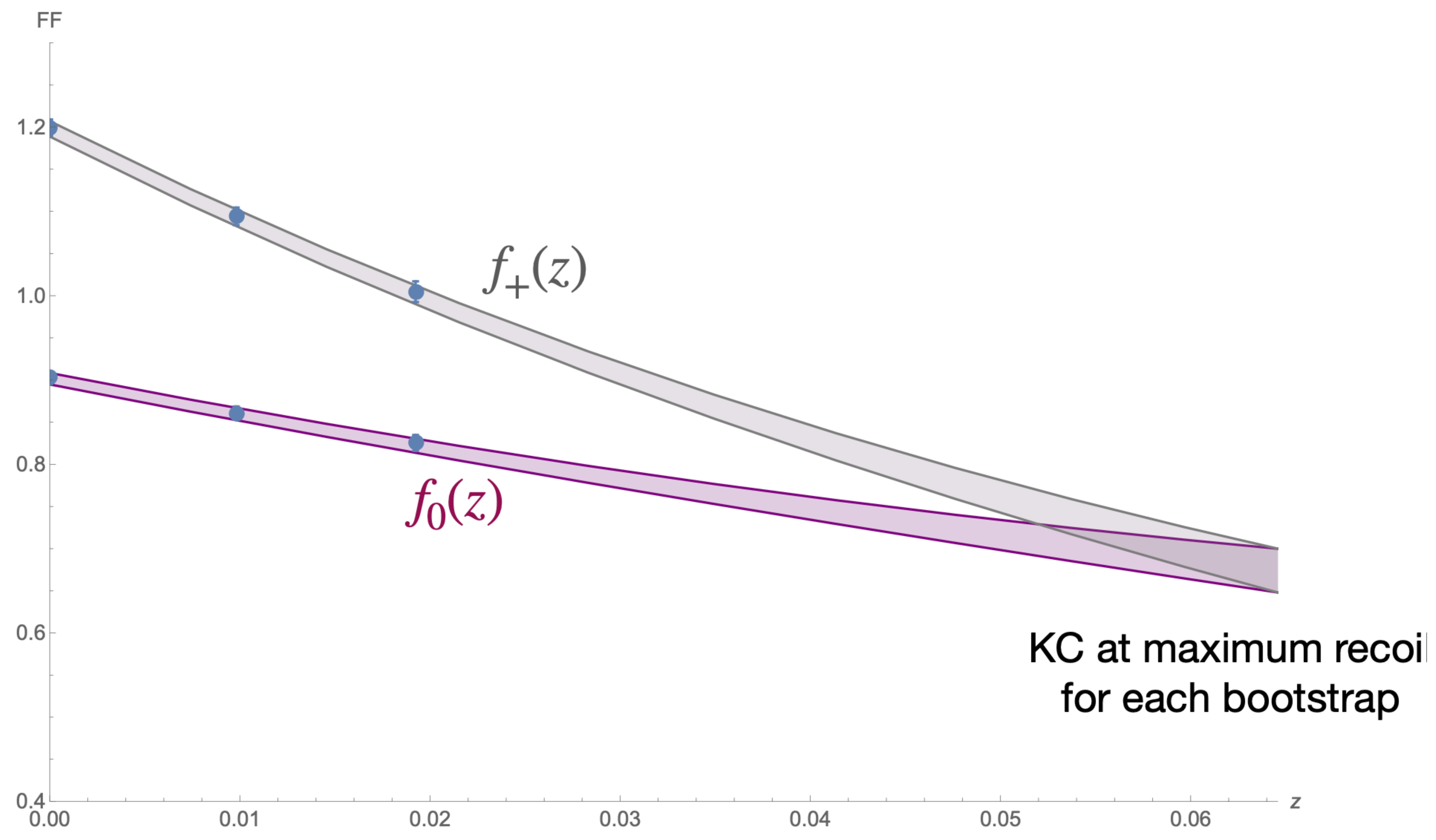
It's crucial to observe that the value of $|V_{cb}|$ exhibits some dependence on the specific w-bin.

Furthermore, the value of $|V_{cb}|$ deviates from a constant fit for $x = cos(\theta_v)$.

These effects seem to be related to a a different w-slope of the theoretical FFs based on lattice FNAL/MILC with respect to the Belle experimental data. This issue has to be further investigated.

In **arXiv:2105.08674**, our DM method has been applied also to B → D decays:

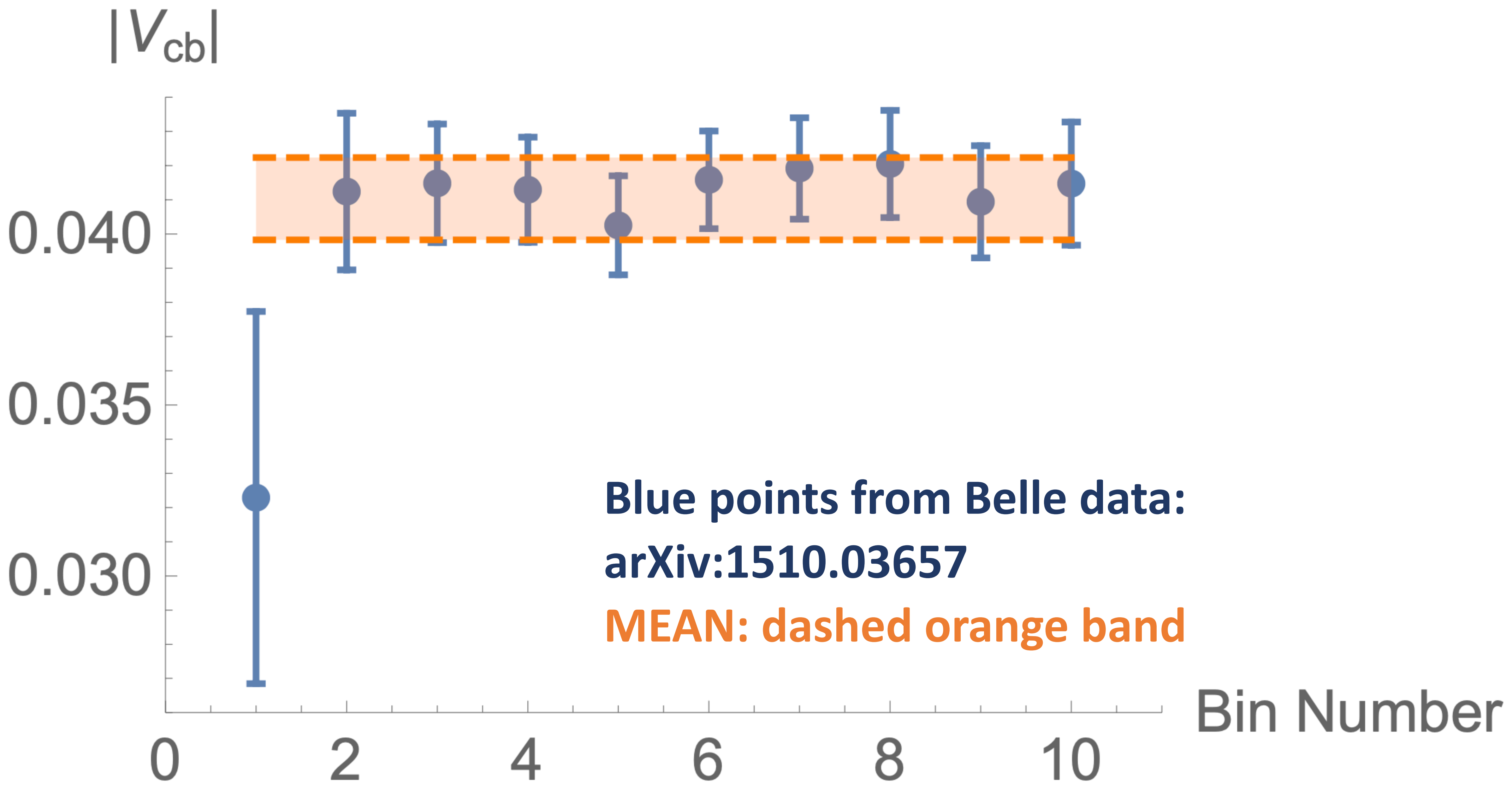
- **3 FNAL/MILC data for each FF**: preliminary results contained in **arXiv:1503.07237**
- **Experimental data from Belle collaboration in 10 bins (arXiv:1510.03657)**



$$R(D) = 0.296(8)$$

HFLAV '21 $\quad R^{exp}(D) = 0.339 \pm 0.030$

$\simeq 1.4\sigma$ *difference*



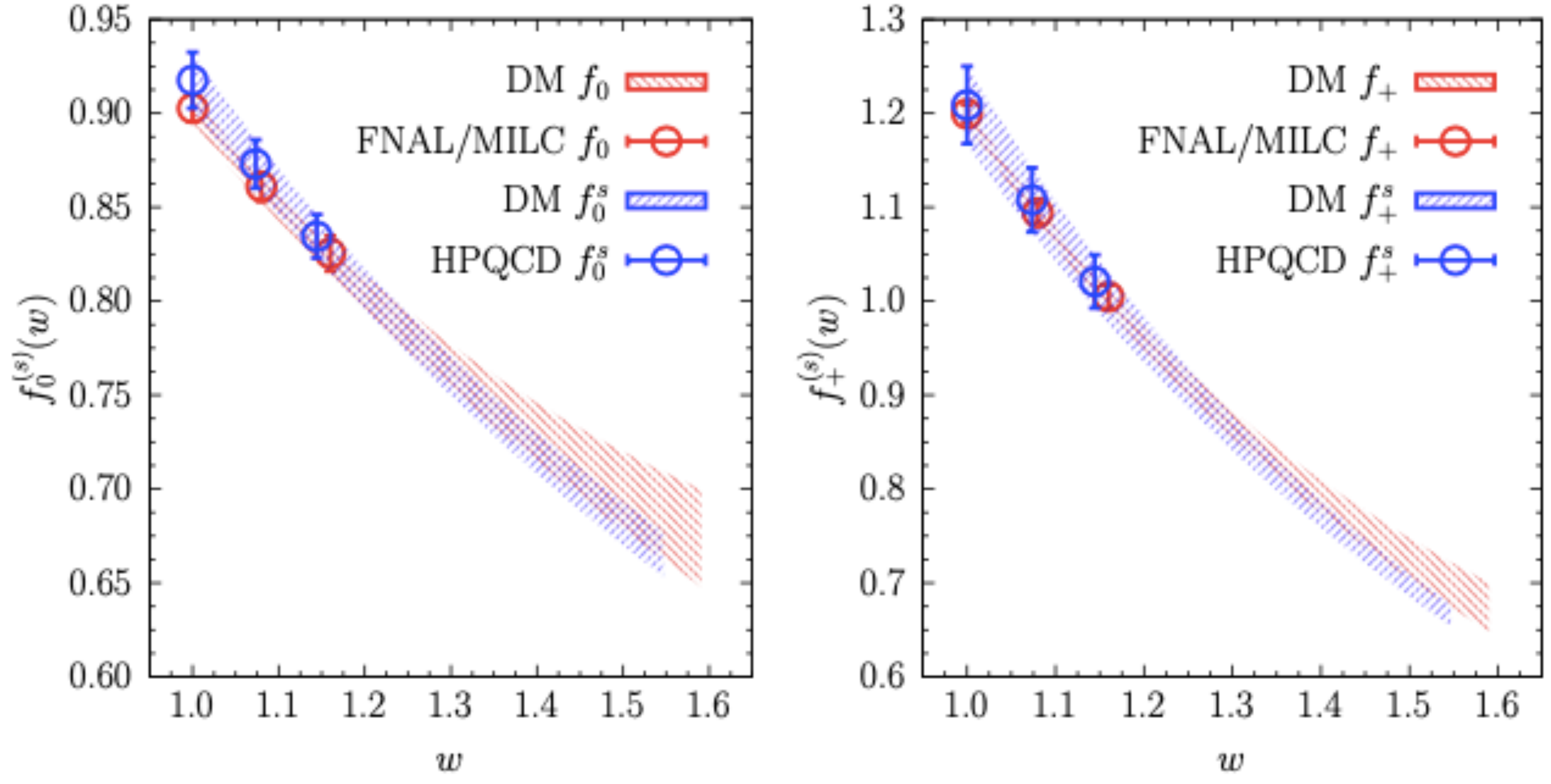
$$|V_{cb}| \times 10^3 = 41.0 \pm 1.2$$

There is a nice consistency with $|V_{cb}|$ *from* $B \to D^*\ell\nu_\ell$

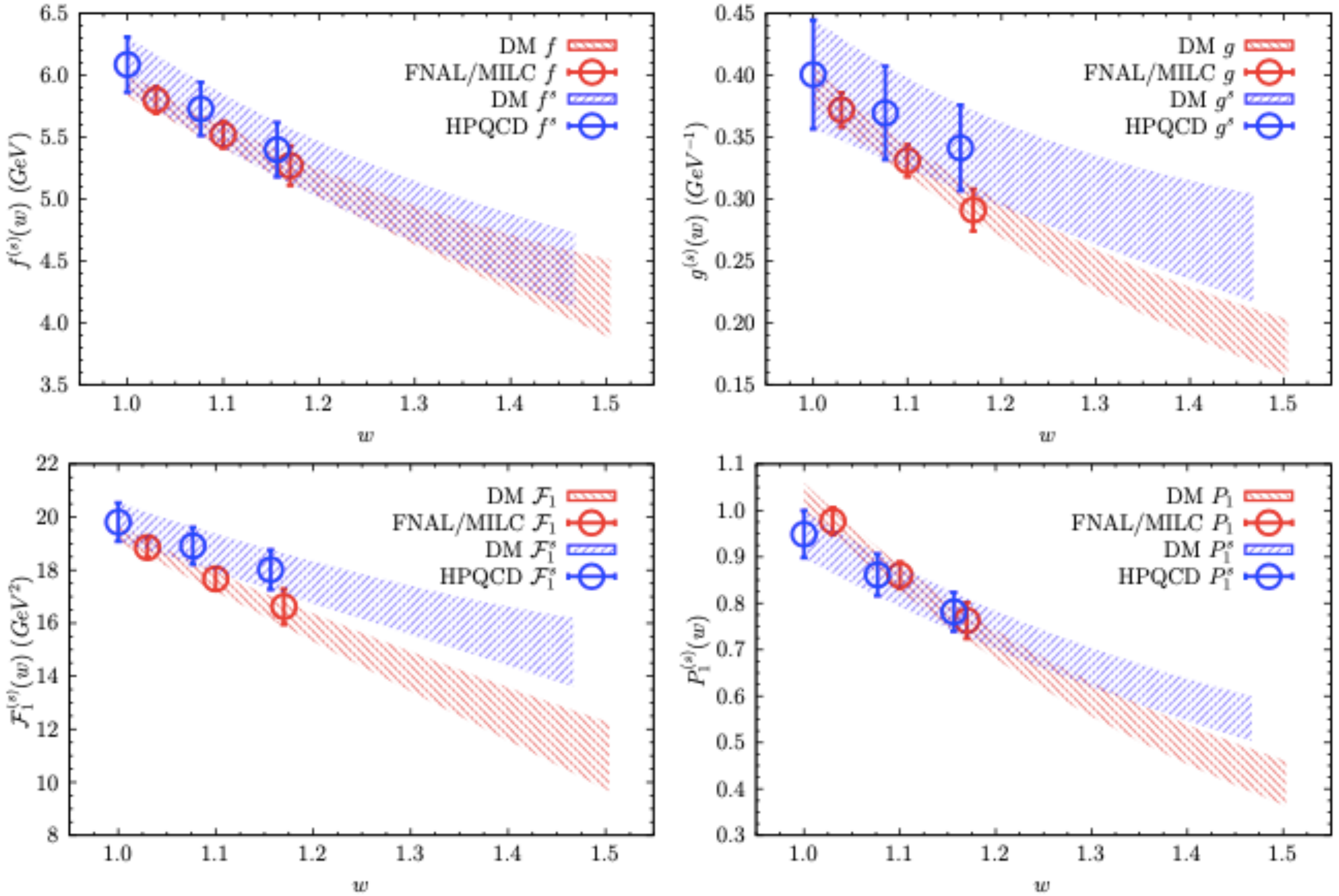
$B_s \to D_s^{(*)} \ell \nu_\ell$ USING DM METHOD and $SU(3)_F$ BREAKING EFFECTS (arXiv:2204.05925)

*Using LQCD computations from **HPQCD** arXiv:1906.00701 and arXiv:2105.11433 we extracted the form factors for $B_s \to D_s^{(*)}$ and we developed a comparison with $B \to D^{(*)}$*

$B_{(s)} \to D_{(s)} \ell \nu_\ell$



$B_{(s)} \to D^*_{(s)} \ell \nu_\ell$



<u>ratios of branching ratios</u>

$$\left.\frac{\mathcal{B}(B_s \to D_s \mu \nu_\mu)}{\mathcal{B}(B \to D \mu \nu_\mu)}\right|_{\text{LHCb}} = 1.09 \pm 0.09 \qquad \left.\frac{\mathcal{B}(B_s \to D_s \mu \nu_\mu)}{\mathcal{B}(B \to D \mu \nu_\mu)}\right|_{\text{DM}} = 1.02 \pm 0.06$$

$$\left.\frac{\mathcal{B}(B_s \to D_s^* \mu \nu_\mu)}{\mathcal{B}(B \to D^* \mu \nu_\mu)}\right|_{\text{LHCb}} = 1.06 \pm 0.10 \qquad \left.\frac{\mathcal{B}(B_s \to D_s^* \mu \nu_\mu)}{\mathcal{B}(B \to D^* \mu \nu_\mu)}\right|_{\text{DM}} = 1.19 \pm 0.11$$

We observe some $SU(3)_F$ breaking effects in $B_{(s)} \to V$ → *More investigation needed!*

$|V_{cb}|^{DM}$ AND OBSERVABLES FROM $B_{(s)} \to D^{(*)}_{(s)} \ell \nu_\ell$

Using the fully theoretical form factors and two sets of experimental data from $\boldsymbol{LHCb}$ *collaboration: arXiv:2001.03225 and arXiv:2003.08453, we extracted estimates for* $|V_{cb}|^{DM}$ *also from* $B_s \to D^{(*)}_s$*. We report a summary of the results obtained from* $B_{(s)} \to D^{(*)}_{(s)} \ell \nu_\ell$*, also for the observables.*

Process	Reference	$V_{cb} \times 10^3$
Inclusive $b \to c$	Bordone et al., arXiv:2107.00604	42.16 ± 0.50
$B \to D$	DM method	41.0 ± 1.2
	FLAG 2021, arXiv:2111.09849	40 ± 1.0
$B \to D^*$	DM method	41.3 ± 1.7
	FLAG 2021, arXiv:2111.09849	39.86 ± 0.88
$B_s \to D_s$	DM method	42.4 ± 2.0
$B_s \to D_s^*$	DM method	41.4 ± 2.6
	HPQCD Coll., arXiv:2105.11433	42.2 ± 2.3

Total Mean	DM method	41.4 ± 0.8

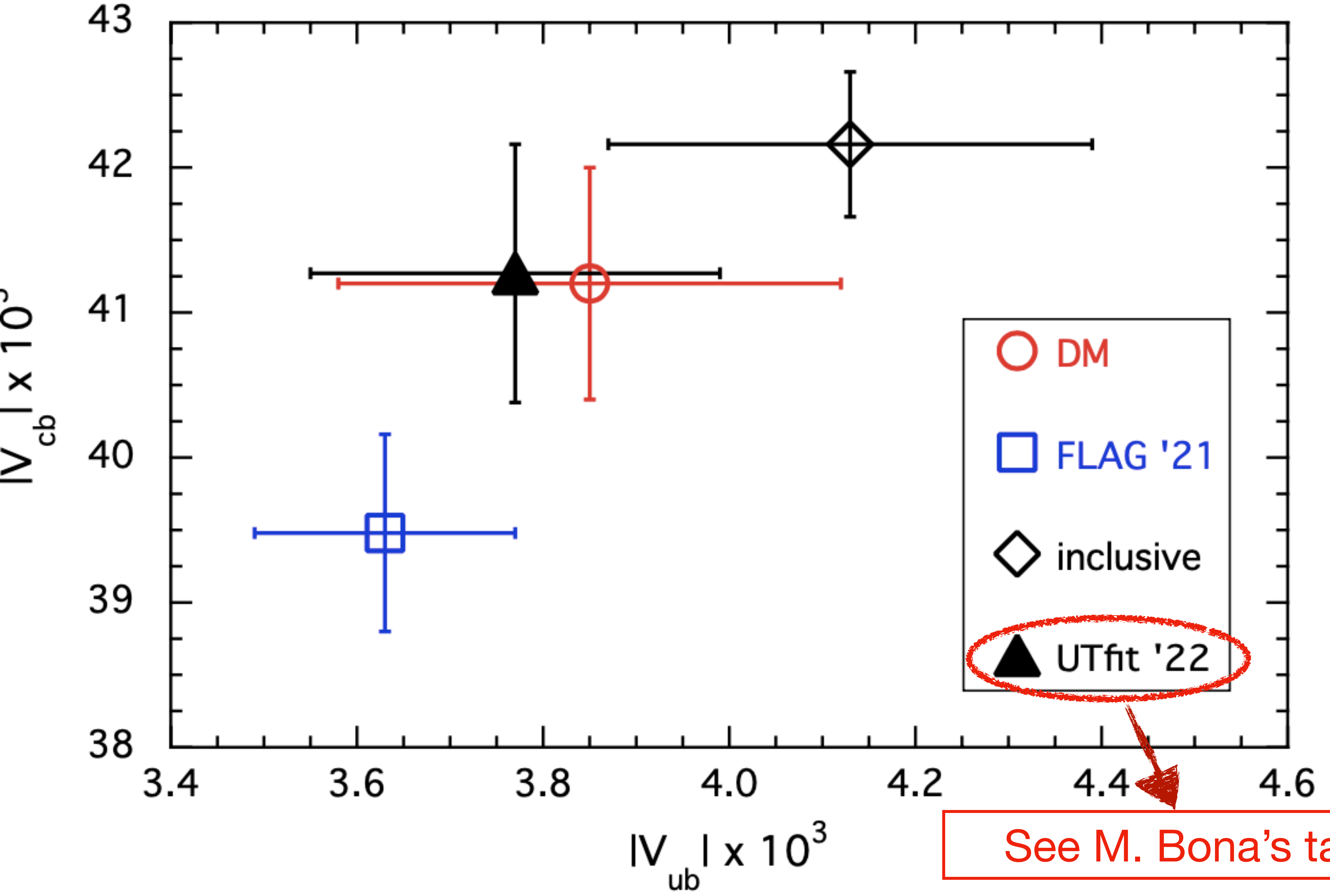
For the first time, there is an indication of a ***sizable reduction of the*** V_{cb} ***puzzle***!!!

Purely theoretical results!

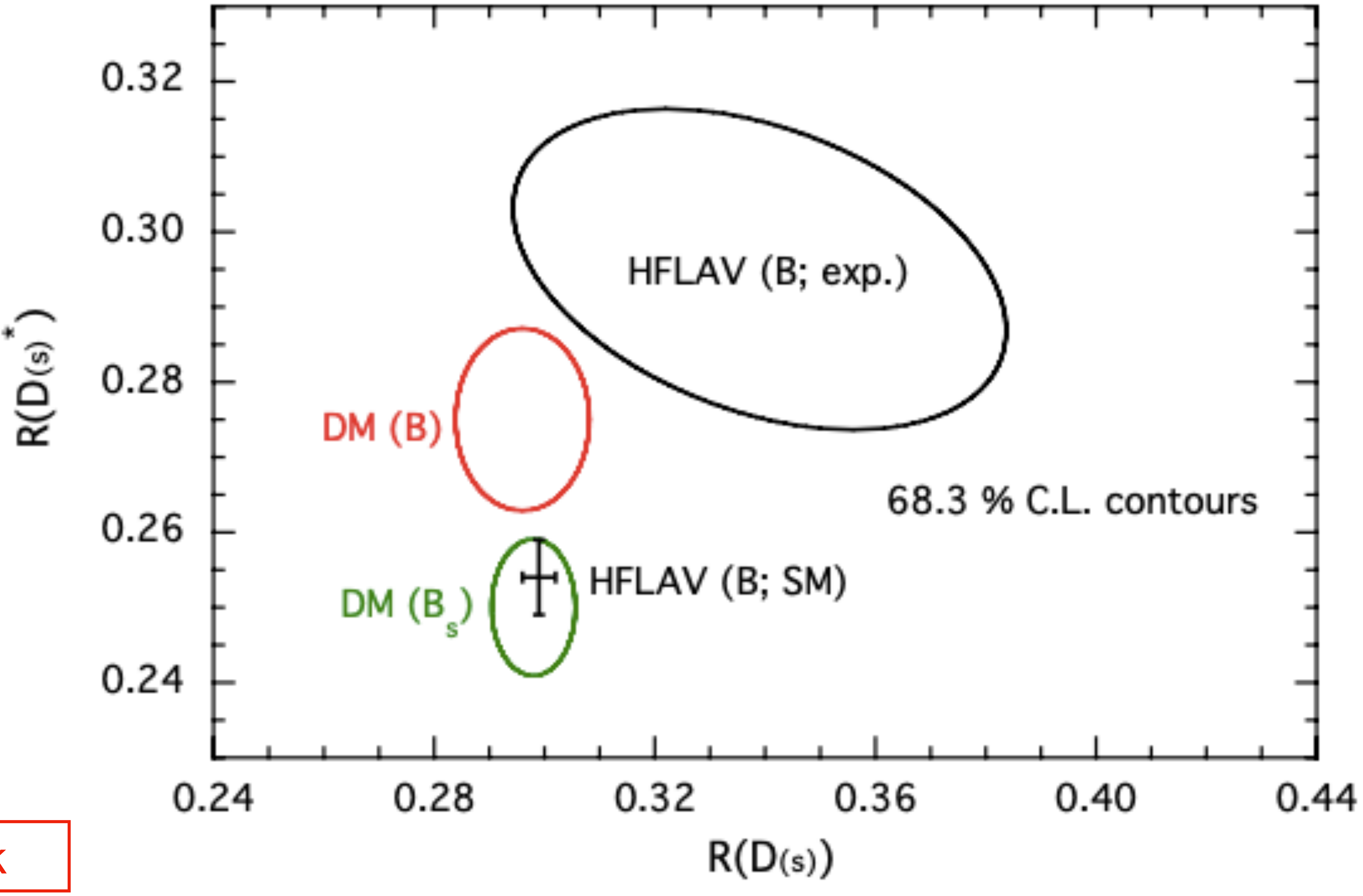
Observable	DM method	Measurements	Difference
$R(D)$	0.296(8)	0.340(27)(13)	$\simeq 1.3\sigma$
$R(D_s)$	0.298(5)	—	—
$R(D^*)$	0.275(8)	0.295(11)(8)	$\simeq 1.3\sigma$
$R(D_s^*)$	0.2497(60)	—	—
$P_\tau(D^*)$	-0.529(7)	$-0.38(^{+21}_{-16})$	$< 0.3\sigma$
$P_\tau(D_s^*)$	-0.520(12)	—	—
$F_L(D^*)$	0.414(12)	0.60(8)(4)	$\simeq 2.0\sigma$
$F_L(D_s^*)$	0.440(16)	—	—

$R(D^*)$ ***anomaly,*** **based on the FNAL/MILC FFs, results to be** ***lighter with respect to the 2.5σ discrepancy stated by HFLAV***!

CONCLUSIONS



See M. Bona's talk



CKM Matrix element	DM method estimate
$V_{cb} \times 10^3$	41.4 ± 0.8
$V_{ub} \times 10^3$	3.85 ± 0.27

See Ludovico Vittorio's talk for the DM application to heavy-to-light decays

- ***The Dispersive Matrix method is very effective and precise in its prediction;***
- ***Model independent analysis keeping theory and experiments separated;***
- ***Reduction of $|V_{cb}|$ puzzle and $R(D^*)$ anomaly.***

Back-up slides

The imaginary part of the longitudinal and transverse polarization functions are related to their derivatives with respect q^2 by

$$\chi_{0^+}(q^2) = \frac{\partial}{\partial q^2}\left[q^2 \Pi_{0^+}(q^2)\right] = \frac{1}{\pi}\int_0^\infty dz \frac{z Im \Pi_{0^+}}{(z-q^2)^2}, \qquad \chi_{1^-}(q^2) = \frac{1}{2}\left(\frac{\partial}{\partial q^2}\right)^2 \left[q^2 \Pi_{1^-}(q^2)\right] = \frac{1}{\pi}\int_0^\infty dz \frac{z Im \Pi_{1^-}}{(z-q^2)^3},$$

where for a generic current J

$$Im \Pi_{0^+,1^-} = \frac{1}{2}\sum_n \int d\mu(n)(2\pi)^4 \delta^{(4)}(q-p_n)\,|\langle 0|J|n\rangle|\,.$$

We can restrict our attention to a subset of hadronic states and thus produce, using analyticity, a strict inequality

$$\boxed{\frac{1}{2\pi i}\int_{|z|=1}\frac{dz}{z}\,|\phi(z,q^2)f(z)|^2 \leq \chi(q^2),}$$

where f is a generic form factor and ϕ an associated kinematical function which may contain subtraction of resonances.

We can introduce the inner product

$$\langle g\,|\,h\rangle = \frac{1}{2\pi i}\int_{|z|=1}\frac{dz}{z}\bar{g}(z)h(z)\,.$$

Using this formalism, the inequality can be simply written as

$$0 \le \langle \phi f\,|\,\phi f\rangle \le \chi(q^2)\,.$$

Introducing the function $g_t(z) = \dfrac{1}{1-\bar{z}(t)z}$ and using the definition of the inner product we can define

$$\langle g_t\,|\,\phi f\rangle = \phi(z(t),q^2)f(z(t)), \qquad \langle g_{t_m}\,|\,g_{t_n}\rangle = \frac{1}{1-z(t_l)\bar{z}(t_m)}\,.$$

$z(t)$ is such that

$$\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}$$

where for $D \to K$

$t_\pm = (m_D \pm m_K)^2$

We can use these three quantities to build a matrix!!

$$\mathbf{M} = \begin{pmatrix} \langle\phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_n}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle \end{pmatrix}.$$

The values $t_1, t_2, \ldots, t_n$ correspond to the squared 4-momenta at which the FFs have been computed while the first element is the quantity directly related to the susceptibility $\chi(q^2)$. The point t is the unknown point where we want to extract the value of the FF.

The positivity of the inner product guarantees that

$$\det M \geq 0.$$

This condition leads to a constraints on the form factor f computed in the generic unknown point t

$$f_{lo}(t) \leq f(t) \leq f_{up}(t),$$

where

α and $\Delta_1(t)$ are determinants of minors of M depending only on kinematical factors.

$$\boxed{f_{lo(up)}(t) = f(t) \mp \frac{1}{\alpha\phi}\sqrt{\Delta_1(t)\Delta_2^f}}$$

Δ_2^f depends on the FF and on χ but not on t. It is a crucial quantity because, depending on the susceptibility, it contains information on the unitarity!

The crucial point is that $\Delta_1(t) > 0\ \forall t \longrightarrow \Delta_2^f$ must be positive!

If $\Delta_2^f > 0$ the unitarity is always satisfied $\forall t$!

$f_{lo(up)}$ are defined if unitarity is satisfied. Then, the bounds that we can obtain imposing $\Delta_2^f > 0$ in the Dispersive Matrix method always satisfy unitarity!!!

We build a multivariate Gaussian distribution of N_{boot} bootstrap events both for the form factors extracted from the three-point functions and for the susceptibilities (***properly correlated*** if we have access to the data of the simulations) in our numerical simulation and covariance matrix $\Sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$.

To take into account in our analysis the Kinematical Constraint for each of the N_{boot} events we define

$$f^*_{lo}(0) = min[f_{+,lo}(0), f_{0,lo}(0)],$$

$$f^*_{up}(0) = max[f_{+,up}(0), f_{0,up}(0)] .$$

$$\longrightarrow \quad f^*_{lo}(0) \leq f(0) \leq f^*_{up}(0) .$$

If we consider $f(0)$ to be uniformly distributed in this range, we can generate N_0 values, obtaining a sample of $\bar{N}_{boot} = N_{boot} \times N_0$, that we can add to the input data set as a new point at $t_{n+1} = 0$.

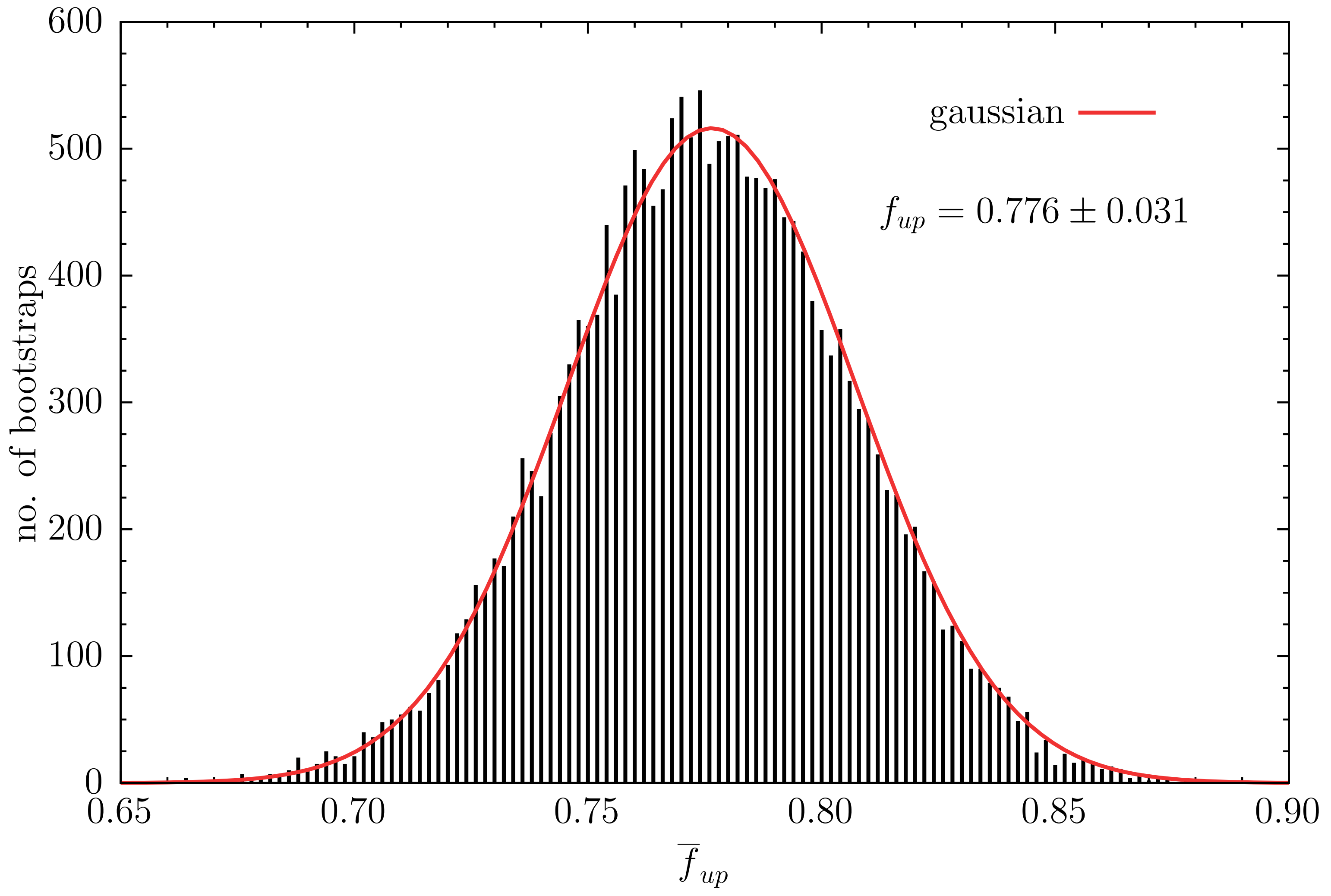
$$\mathbf{M}_C = \begin{pmatrix} \langle\phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_n}\rangle & \langle\phi f|g_{t_{n+1}}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle & \langle g_t|g_{t_{n+1}}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle & \langle g_{t_1}|g_{t_{n+1}}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle & \langle g_{t_n}|g_{t_{n+1}}\rangle \\ \langle g_{t_{n+1}}|\phi f\rangle & \langle g_{t_{n+1}}|g_t\rangle & \langle g_{t_{n+1}}|g_{t_1}\rangle & \cdots & \langle g_{t_{n+1}}|g_{t_n}\rangle & \langle g_{t_{n+1}}|g_{t_{n+1}}\rangle \end{pmatrix} .$$

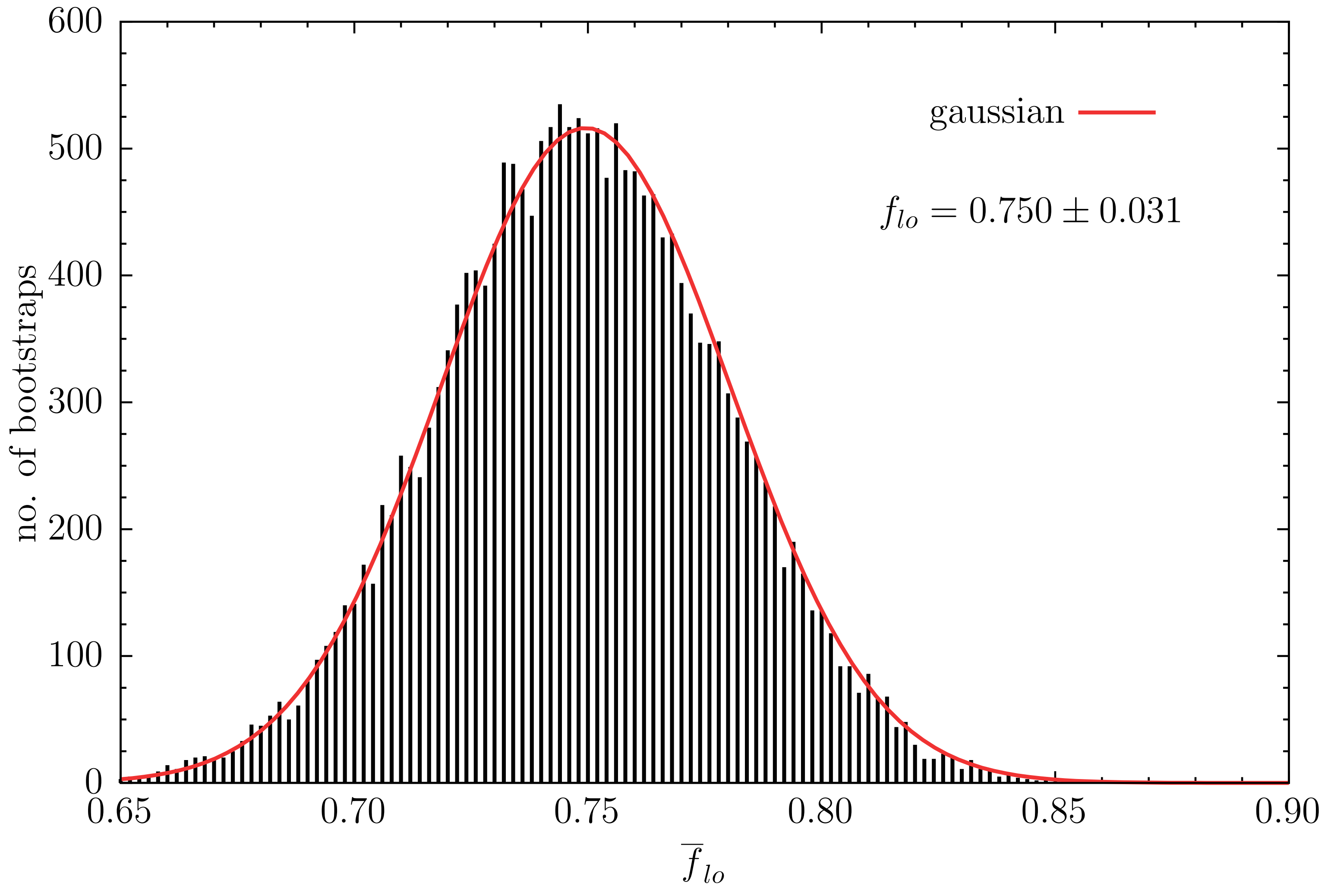
We can do the analysis as before using now a further information that takes into account the KC. This can be done for each of the N_0 events.

At the end of this second analysis we recombine the N_0 events choosing

$$\bar{f}_{lo}(t) = min[f^1_{lo}(t), \ldots, f^{N_0}_{lo}(t)],$$

$$\bar{f}_{up}(t) = max[f^1_{up}(t), \ldots, f^{N_0}_{up}(t)] .$$





To recombine the N_{boot} events we generate the corresponding histograms and fit them with a Gaussian Ansatz. We can then extract for every value of t average values $f_{lo(up)}(t)$, standard deviations $\sigma_{lo(up)}$ and the corresponding correlation $\rho_{lo,up}(t) = \rho_{up,lo}(t)$.

By combining the flat distribution that we have between f_{lo} and f_{up} with a multivariate Gaussian distribution necessary to mediate over the whole set of bootstrap events, we can obtain the final values for the form factor $f(t)$ and its variance $\sigma_f^2(t)$ using

$$\boxed{f(t) = \frac{f_{lo}(t) + f_{up}(t)}{2},}$$

$$\boxed{\sigma_f^2(t) = \frac{1}{12}[f_{up}(t) - f_{lo}(t)]^2 + \frac{1}{3}[\sigma_{lo}^2(t) + \sigma_{up}(t)^2 + \rho_{lo,up}(t)\sigma_{lo}(t)\sigma_{up}(t)]\,.}$$

- Production of a pseudoscalar meson (*i.e. D*):

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{24\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left[|\vec{p}_D|^3 \left(1 + \frac{m_\ell^2}{2q^2}\right) \boxed{f^+(q^2)|^2} + m_B^2 |\vec{p}_D| \left(1 - \frac{m_D^2}{m_B^2}\right)^2 \frac{3m_\ell^2}{8q^2} \boxed{f^0(q^2)|^2} \right]$$

- Production of a vector meson (*i.e. D**):

$$\frac{d\Gamma_\tau}{dw} = \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw} \begin{cases} \dfrac{d\Gamma_{\tau,1}}{dw} = \left(1 - \dfrac{m_\tau^2}{q(w)^2}\right)^2 \left(1 + \dfrac{m_\tau^2}{2q(w)^2}\right) \times \dfrac{d\Gamma}{dw} \\ \dfrac{d\Gamma}{dw} = \dfrac{\eta_{EW}^2 G_F^2 m_{D^*}^2 |V_{cb}|^2}{48\pi^3 m_B} \sqrt{w^2-1} \left[2\, q^2(w) \left(\boxed{f(w)^2} + m_B^2 m_{D^*}^2 \, (w^2-1) \boxed{g(w)^2} \right) + \boxed{\mathcal{F}_1(w)}^2 \right] \\ \dfrac{d\Gamma_{\tau,2}}{dw} = \dfrac{\eta_{EW}^2 |V_{cb}|^2 G_F^2 m_B^5}{32\pi^3} \dfrac{m_\tau^2 (m_\tau^2 - q(w)^2)^2 r^3 (1+r)^2 (w^2-1)^{3/2} \boxed{P_1(w)^2}}{q(w)^6} \end{cases}$$

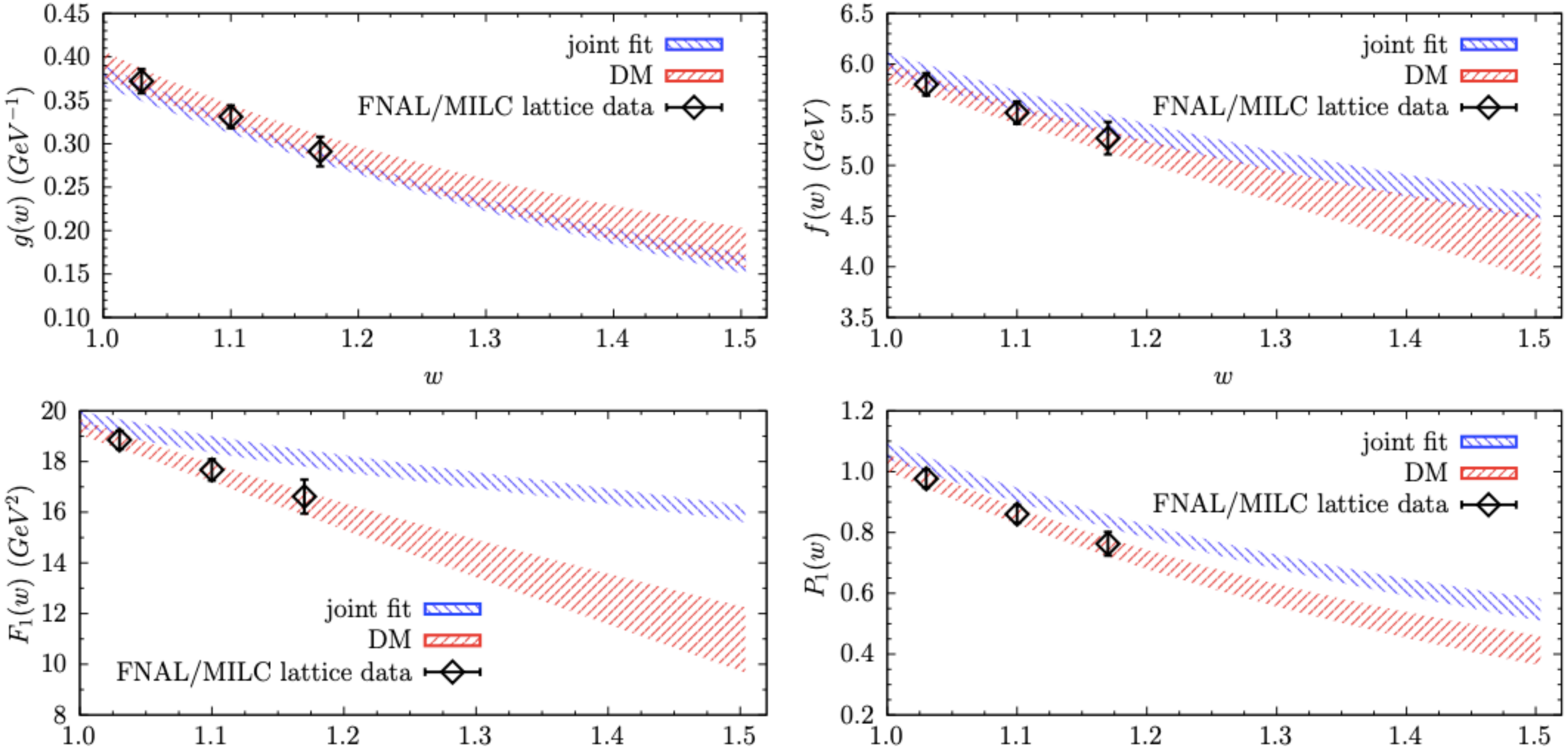
Relation between the momentum transfer and the recoil:

$$q^2 = m_B^2 + m_P^2 - 2 m_B m_P \, w$$

Two FFs coupled to the lepton:

$$f_0(w) \ \text{(pseudoscalar)}, \ P_1(w) \ \text{(vector)}$$

COMPARISON JOINT FIT AND DM RESULTS



Joint fit (Exp.+Th.)

$R(D^*) = 0.2483 \pm 0.0013$

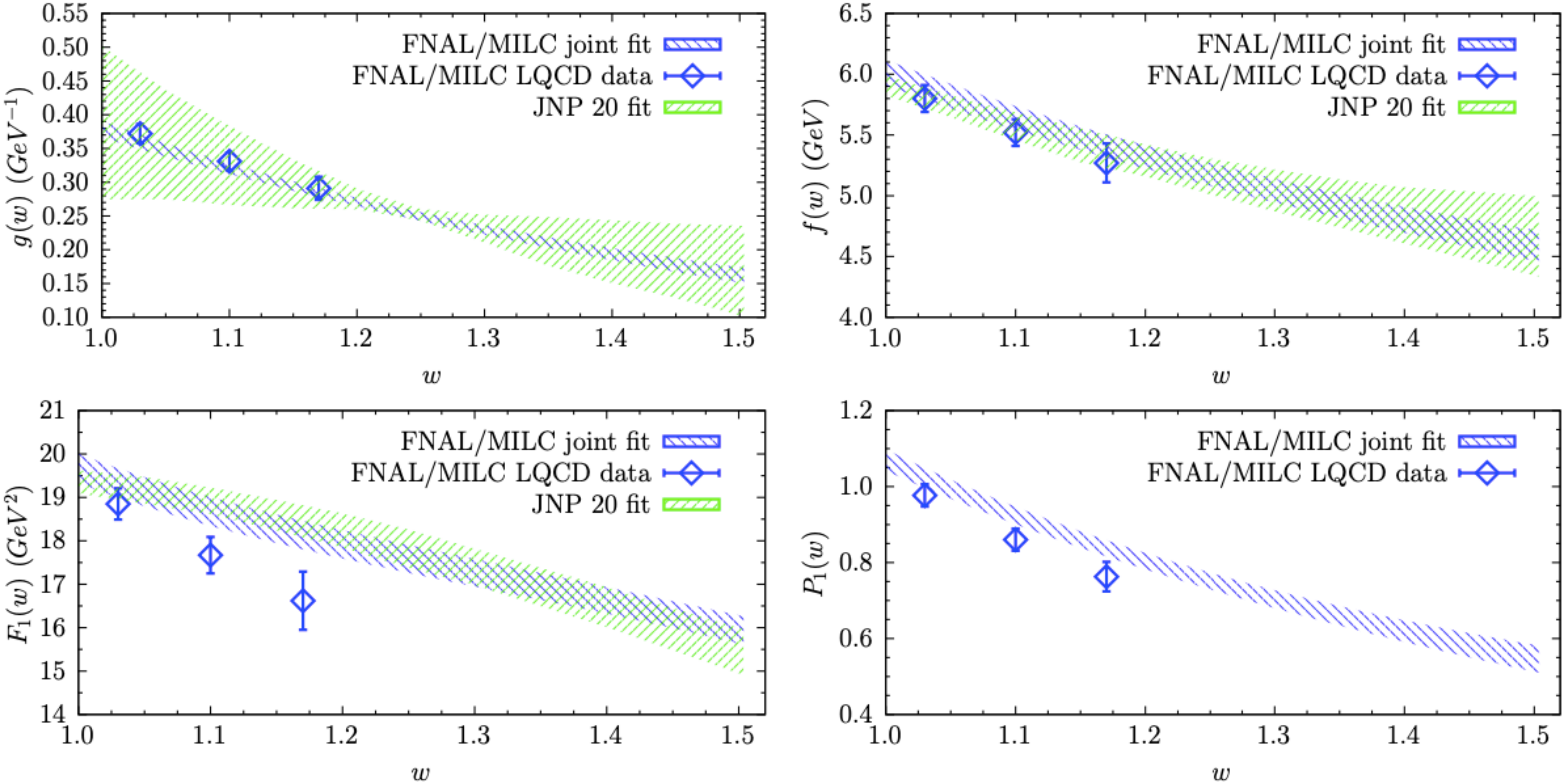
DM

$R(D^*) = 0.275 \pm 0.008$

$$R(D^*) = \frac{\int_{m_\tau^2}^{q^2_{max}} dq^2 \left[\left.\frac{d\Gamma}{dq^2}\right|_{m_\ell=0} + \frac{m_\tau^2}{q^2}\frac{d\tilde{\Gamma}}{dq^2} \right]}{\int_0^{q^2_{max}} \left.\frac{d\Gamma}{dq^2}\right|_{m_\ell=0}}$$

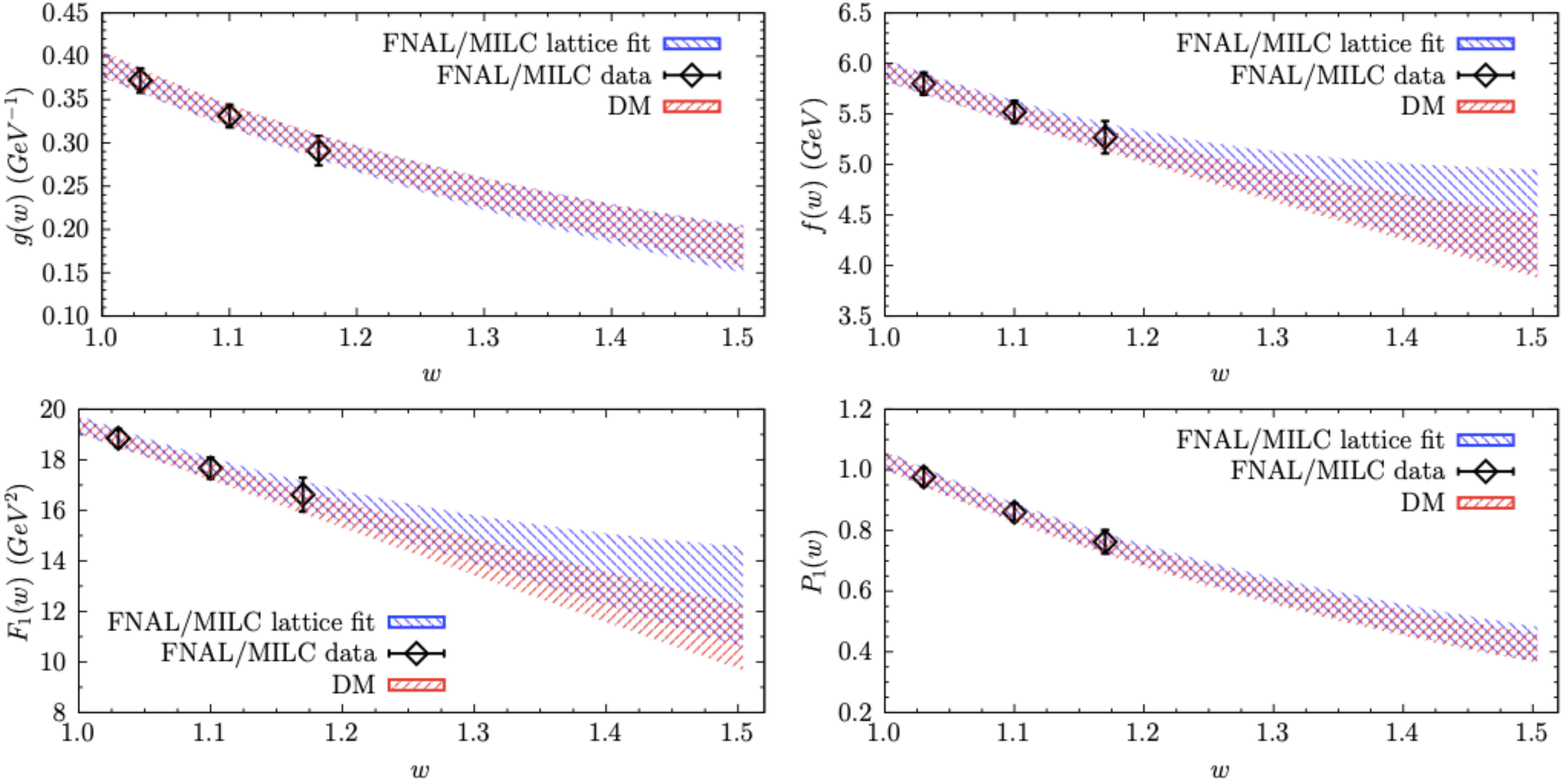
At small q^2 and then large w the FF F_1 dominates

FNAL/MILC AND JNP JOINT FITS



FNAL/MILC joint fit (arXiv:2105.14019) uses Belle+BaBar data and new FNAL/MILC LQCD points

JNP 20 fit (Jaiswal et al. JHEP '20) uses Belle data + old FNAL/MILC LQCD point $h_{A_1}(1)$



If we consider the quadratic BGL fit of LQCD points only made by FNAL/MILC we obtain an overall consistency. There is some difference in $F_1(w_{max})$ that impacts $R(D^*)$

$R(D^*) = 0.265 \pm 0.013$ $\quad$ $R(D^*) = 0.275 \pm 0.008$

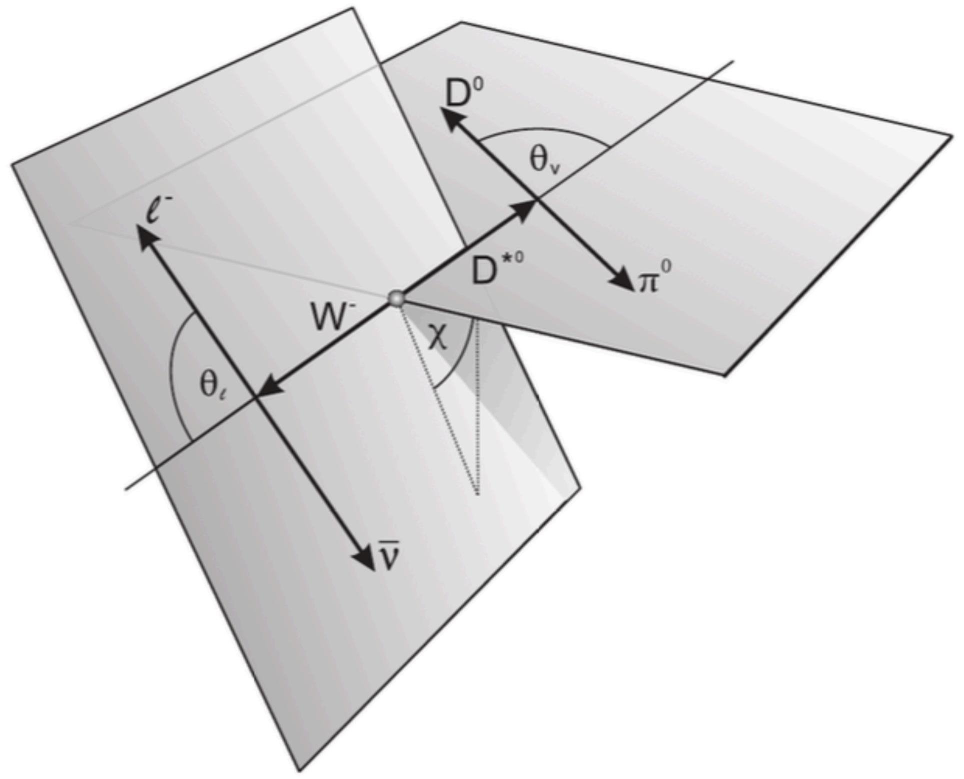
Starting from the FFs bands, we use the experimental data to compute bin-per-bin estimates of Vcb.

NB: the experimental data do NOT enter in the determination of the bands of the FFs!!!

To do it, it is sufficient to compare the two sets of measurements of the differential decay widths

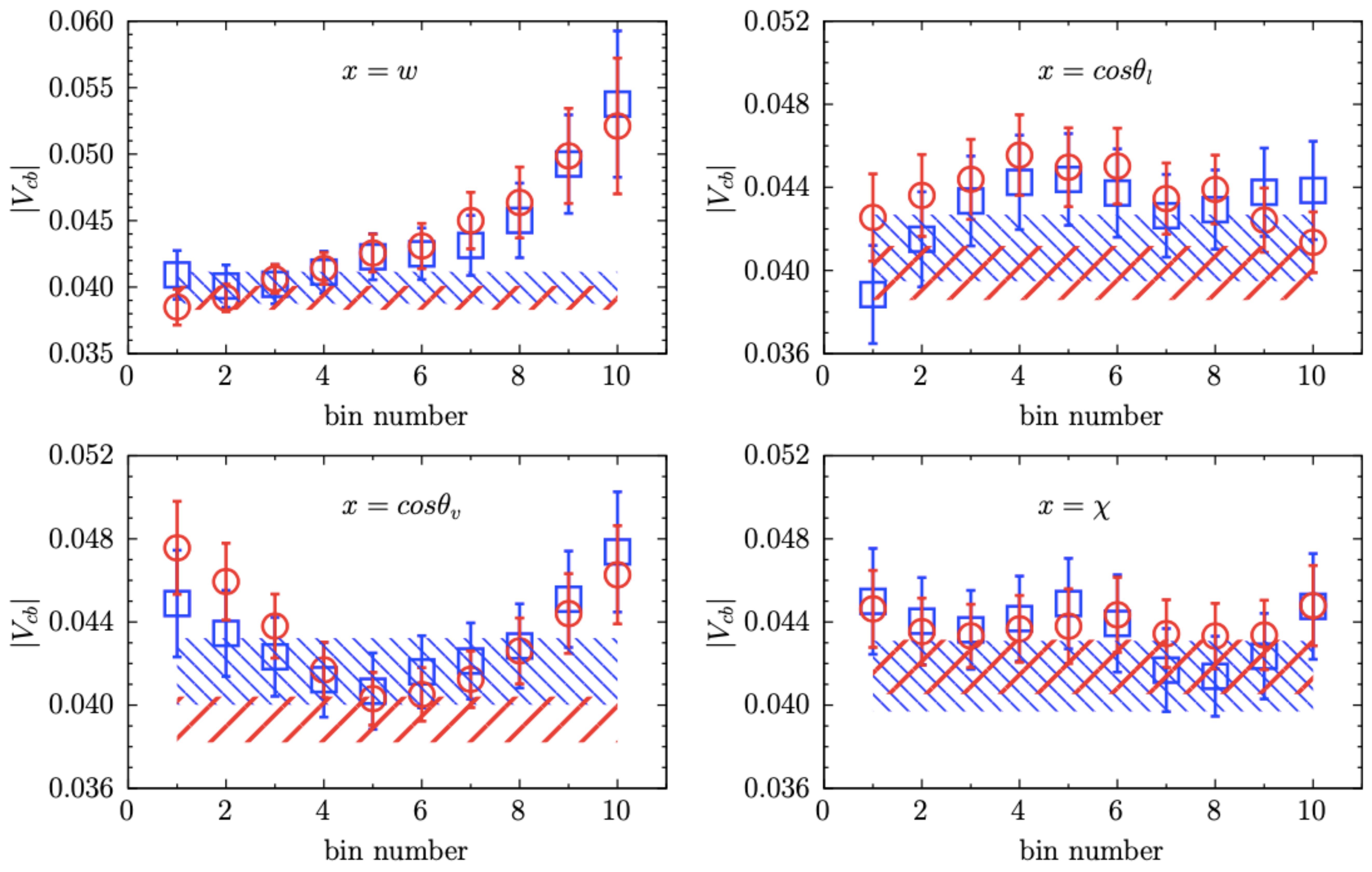
$$d\Gamma/dx \qquad x = w, \cos\theta_l, \cos\theta_v, \chi$$

by the Belle Collaboration **(arXiv:1702.01521, arXiv:1809.03290)** with their *theoretical estimate*, computed through the unitarity bands shown before.



$$\begin{aligned}\frac{d\Gamma(B\to D^*(\to D\pi)\ell\nu)}{dw\,d\cos\theta_\ell\,d\cos\theta_v\,d\chi} &= \frac{G_F^2|V_{cb}|^2\eta_{EW}^2}{4(4\pi)^4}3m_Bm_{D^*}^2\sqrt{w^2-1}\\ &\times B(D^*\to D\pi)\{(1-\cos\theta_\ell)^2\sin^2\theta_v|H_+|^2\\ &+(1+\cos\theta_\ell)^2\sin^2\theta_v|H_-|^2+4\sin^2\theta_\ell\cos^2\theta_v|H_0|^2\\ &-2\sin^2\theta_\ell\sin^2\theta_v\cos 2\chi H_+H_-\\ &-4\sin\theta_\ell(1-\cos\theta_\ell)\sin\theta_v\cos\theta_v\cos\chi H_+H_0\\ &+4\sin\theta_\ell(1+\cos\theta_\ell)\sin\theta_v\cos\theta_v\cos\chi H_-H_0\},\end{aligned}$$

We find:



To mediate (for each kinematical variable) the various Vcb estimates:

$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}}, \qquad \sigma^2_{|V_{cb}|} = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}}$$

Red points: arXiv:1809.03290
MEAN: red band

Blue squares: arXiv:1702.01521
MEAN: blue band

In *some* cases, there is an evident underestimation of the weighted mean value!
This problem is well-known and has been deeply studied in ***Nucl.Instrum.Meth.A 346 (1994) 306-311***

Alternative strategy?

We *suppose* that there is a calibration error in the data. Thus, calling *x* one of the four kinematical variables of interest, we compute the quantity **(dΓ/dx)/Γ** by using the experimental data by Belle.

In this way, we reduce this error since all the points enter in the evaluation of Γ!

By computing the correlations between the various bins of (dΓ/dx)/Γ, we define a **new experimental covariance matrix** as

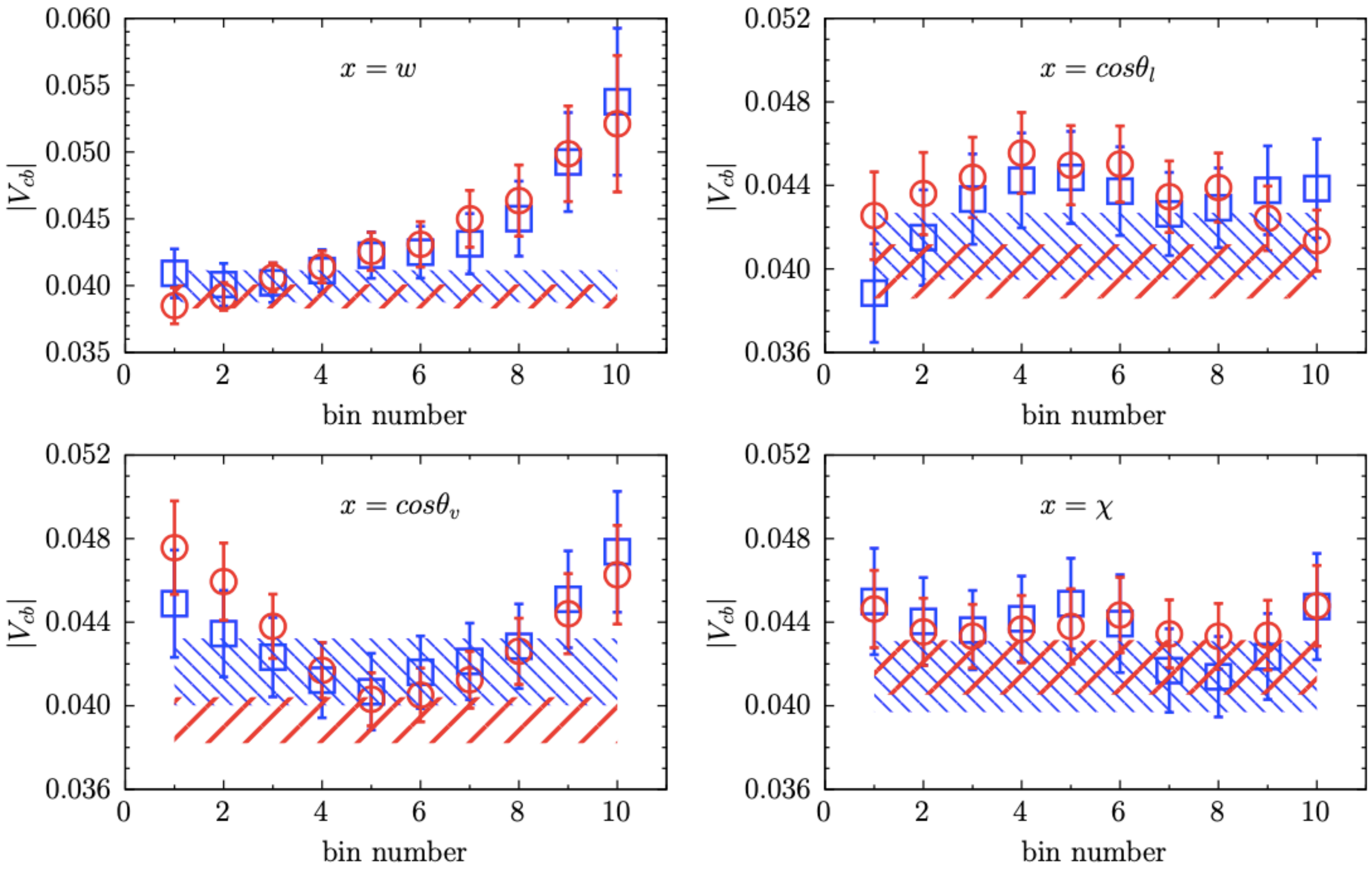
$$C_{ij}|_{exp,NEW} = \underbrace{\boxed{\rho_{ij}|_{ratio}}}_{\text{Correlation of } (d\Gamma/dx)/\Gamma} \times \underbrace{\boxed{\sigma_{i,exp}}\boxed{\sigma_{j,exp}}}_{\text{Experimental errors}}$$

This procedure has two advantages:

1. The new covariance matrix will be free of calibration errors
2. The fact that (for a fixed kinematical variable) the ten bins are ***not* independent** is taken into account

a similar effect has also been discussed in **arXiv:2105.14019**

BEFORE the modification of the correlations



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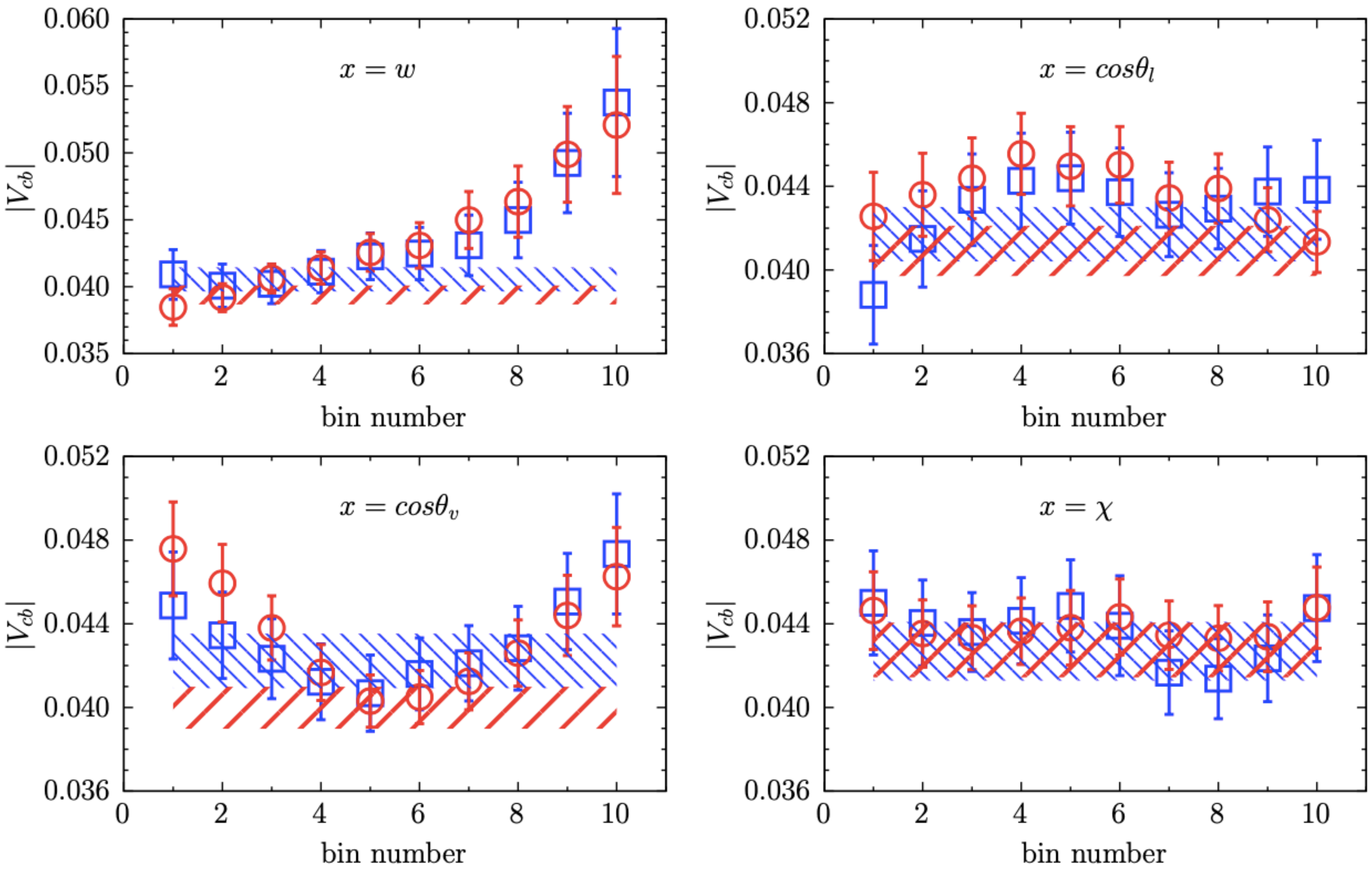
EXCLUSIVE V_{cb} DETERMINATION TROUGH UNITARITY (FNAL/MILC CASE)

AFTER the modification of the correlations



Red points: arXiv:1809.03290
MEAN: red band

Blue squares: arXiv:1702.01521
MEAN: blue band

$$|V_{cb}| = (41.3 \pm 1.7) \cdot 10^{-3}$$

What is the **main improvement** with respect to BGL parametrization?

Basics of BGL: the hadronic FFs corresponding to definite spin-parity can be represented as an expansion, originating from unitarity, analyticity and crossing symmetry, in terms of the conformal variable *z*, for instance

$$g(z) = \frac{1}{\sqrt{\chi_{1^-}(q_0^2)}} \frac{1}{\phi_g(z, q_0^2)\, P_{1^-}(z)} \sum_{n=0}^{\infty} a_n\, z^n$$

And *unitarity*?

$$\sum_{n=0}^{\infty} a_n^2 \leq 1$$

Boyd, Grinstein and Lebed, Phys. Lett. B353, 306 (1995)
Boyd, Grinstein and Lebed, Nucl. Phys. B461, 493 (1996)
Boyd, Grinstein and Lebed, Phys. Rev. D 56, 6895 (1997)

Let us use the BGL parametrization to fit *g(w)* by using the final FNAL/MILC computations of that FF.

How does unitarity work in this case? Two possible fits (3 points as inputs…):

LINEAR

$$a_0^2 + a_1^2 \leq 1$$

100% *of generated bootstraps passes the unitarity filter*

QUADRATIC

$$a_0^2 + a_1^2 + a_2^2 \leq 1$$

12% *of generated bootstraps passes the unitarity filter*

w	g(w)
1.03	**0.372 (14)**
1.10	**0.331 (13)**
1.17	**0.291 (17)**

correlation matrix

1	**0.928**	**0.657**
	1	**0.832**
		1

Unitarity is not built-in!!!

The consequence is that a truncated BGL fit might be distorted by events which do not fulfill unitarity…

To summarize, there are two important improvements in the DM method with respect to BGL parametrization:

- No series expansion to describe the FFs → ***NO TRUNCATION ERRORS!***

  This effect is particularly relevant for semileptonic decays characterized by a very large q^2 range:

  $B \to \pi \ell \nu$ **Maximum q^2 = 26.46 GeV^2**

  $\Lambda_b \to p \ell \nu$ **Maximum q^2 = 21.9 GeV^2**

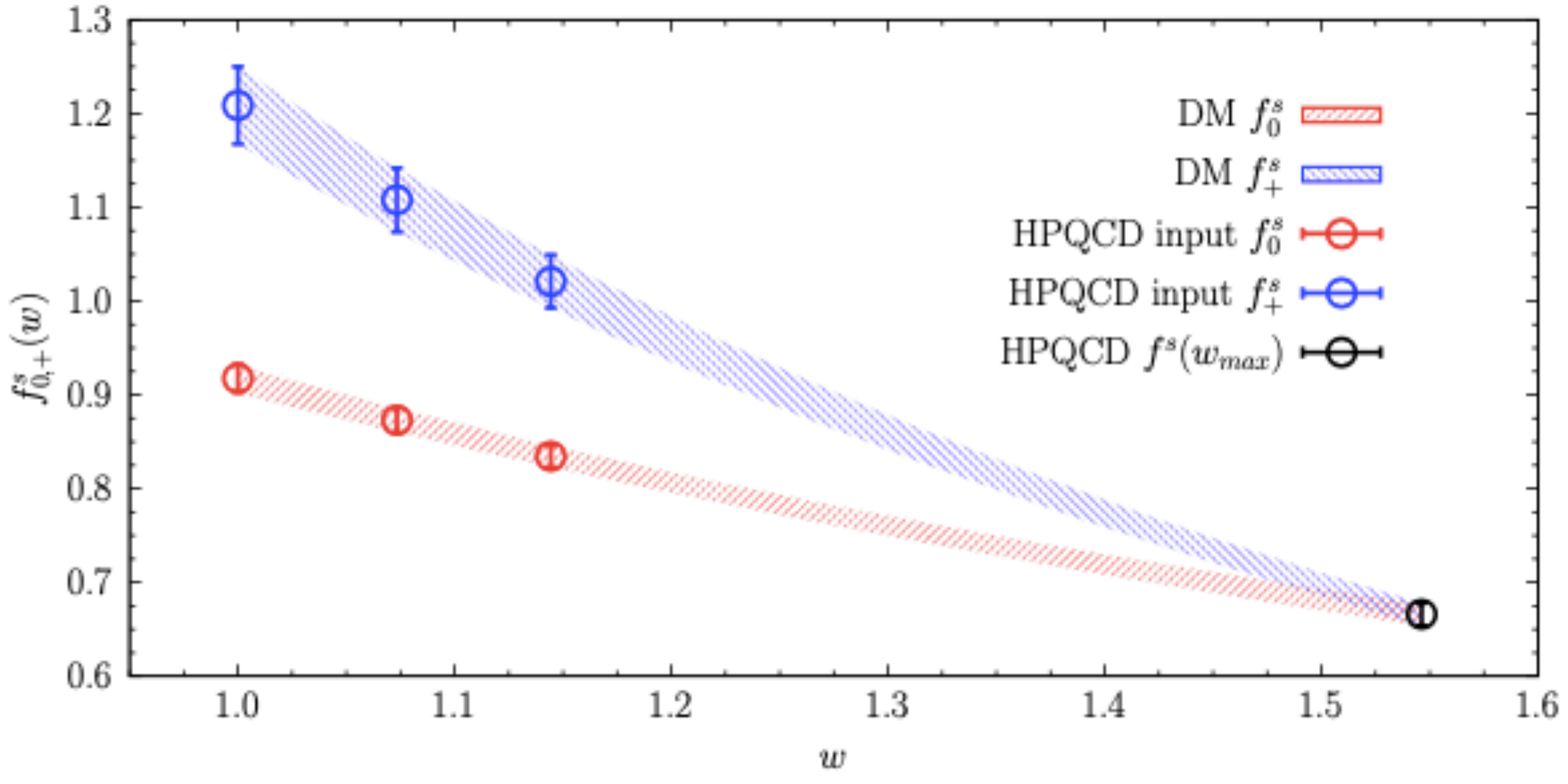
- Unitarity check of FFs data completely independent of the parameterization

The DM approach reproduces exactly the known data and it allows to extrapolate the form factor in the whole kinematical range in a parameterization-independent way **providing a band of values representing the results of all possible BGL fits satisfying unitarity and passing through the known points** (important for estimating uncertainties)

Lattice QCD form factors from **HPQCD arXiv:1906.00701** ($B_s \to D_s$) **and arXiv:2105.11433** ($B_s \to D_s^*$) in the form of BCL fits in the whole kinematical range.

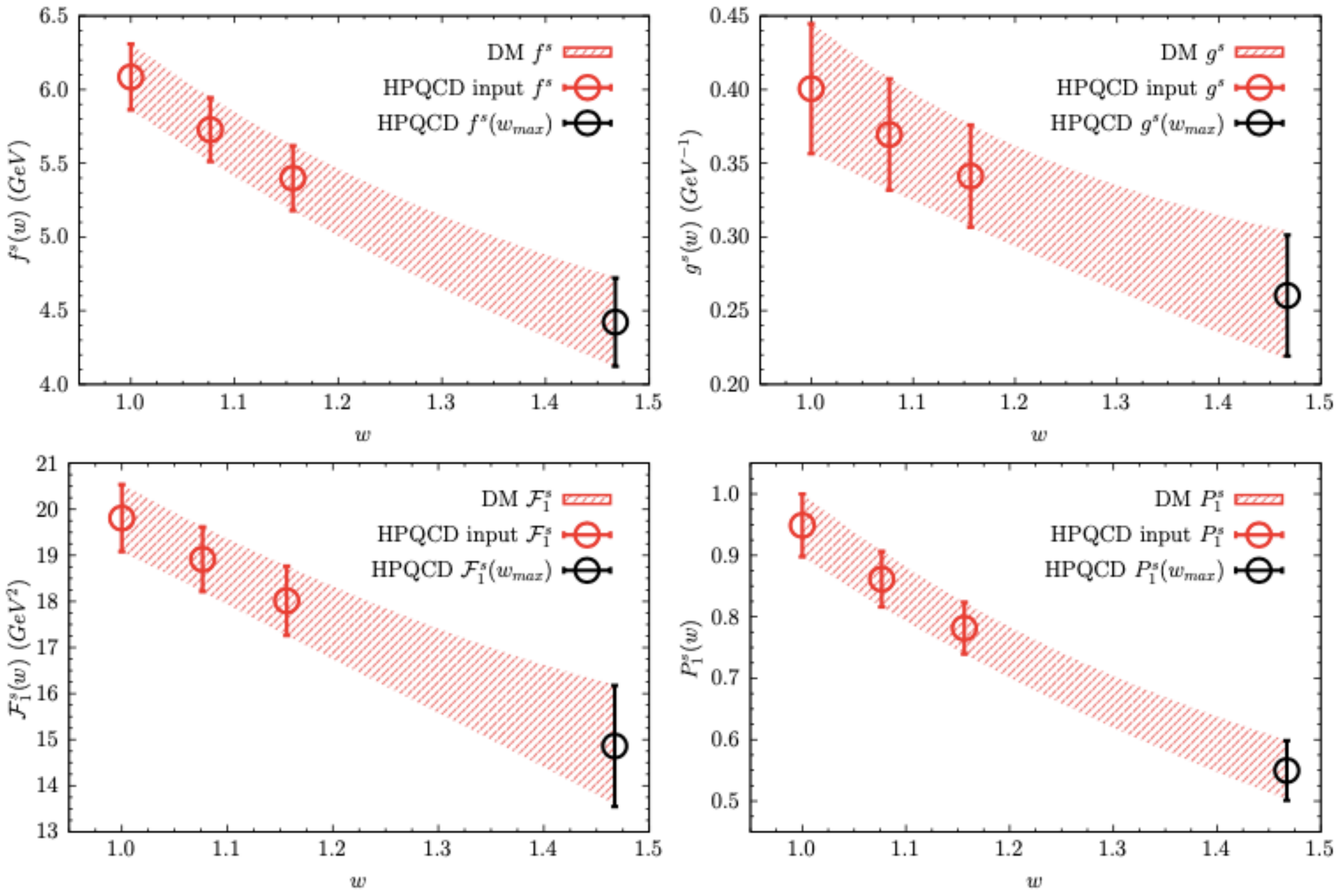
We extract 3 data points for the FFs at small values of the recoil and we apply the DM approach

$B_s \to D_s \ell \nu_\ell$



* nice agreement in the whole kinematical range

$B_s \to D_s^* \ell \nu_\ell$



We had two sets experimental data from LHCb collaboration: arXiv:2001.03225 and arXiv:2003.08453 coming from two different runs. We made three analysis:

1) ratios of the branching ratios [2001.03225].

$$\frac{\mathscr{B}(B_s \to D_s \mu \nu_\mu)}{\mathscr{B}(B \to D \mu \nu_\mu)} = 1.09 \pm 0.05_{stat} \pm 0.06_{syst} \pm 0.05_{inputs} = 1.09 \pm 0.09$$

$$\frac{\mathscr{B}(B_s \to D_s^* \mu \nu_\mu)}{\mathscr{B}(B \to D^* \mu \nu_\mu)} = 1.06 \pm 0.05_{stat} \pm 0.07_{syst} \pm 0.05_{inputs} = 1.06 \pm 0.10$$

Using the PDG values for $\mathscr{B}(B \to D^{(*)} \mu \nu_\mu)$ and the B_s meson lifetime one gets

$$\Gamma^{LHCb}(B_s \to D_s \mu \nu_\mu) = (1.08 \pm 0.10) \cdot 10^{-14} GeV$$

$$\Gamma^{LHCb}(B_s \to D_s^* \mu \nu_\mu) = (2.34 \pm 0.26) \cdot 10^{-14} GeV$$

to be compared with

$$\Gamma^{DM}(B_s \to D_s \mu \nu_\mu)/|V_{cb}|^2 = (6.04 \pm 0.23) \cdot 10^{-12} GeV$$

$$\Gamma^{DM}(B_s \to D_s^* \mu \nu_\mu)/|V_{cb}|^2 = (1.29 \pm 0.11) \cdot 10^{-11} GeV$$

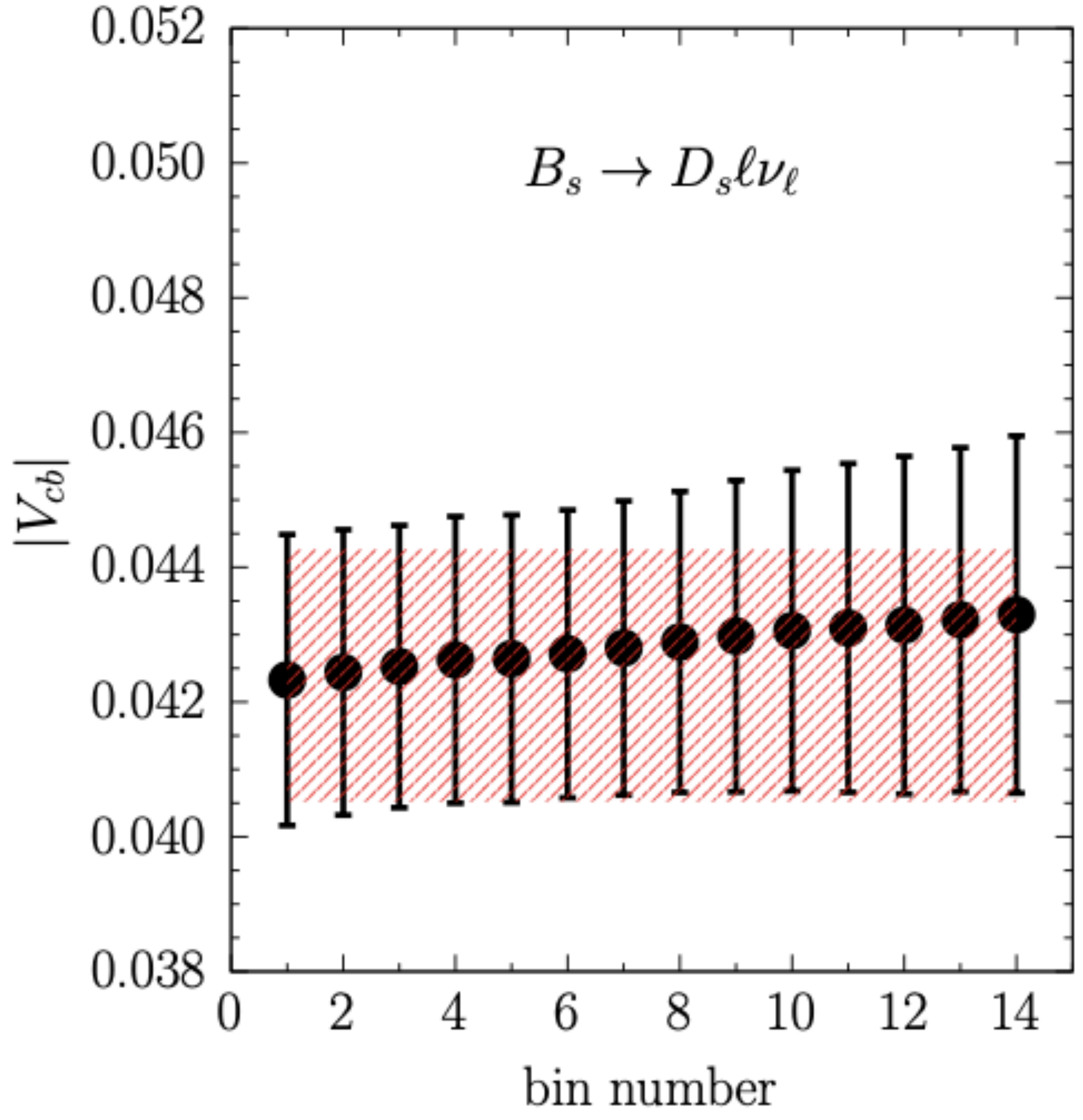
↓

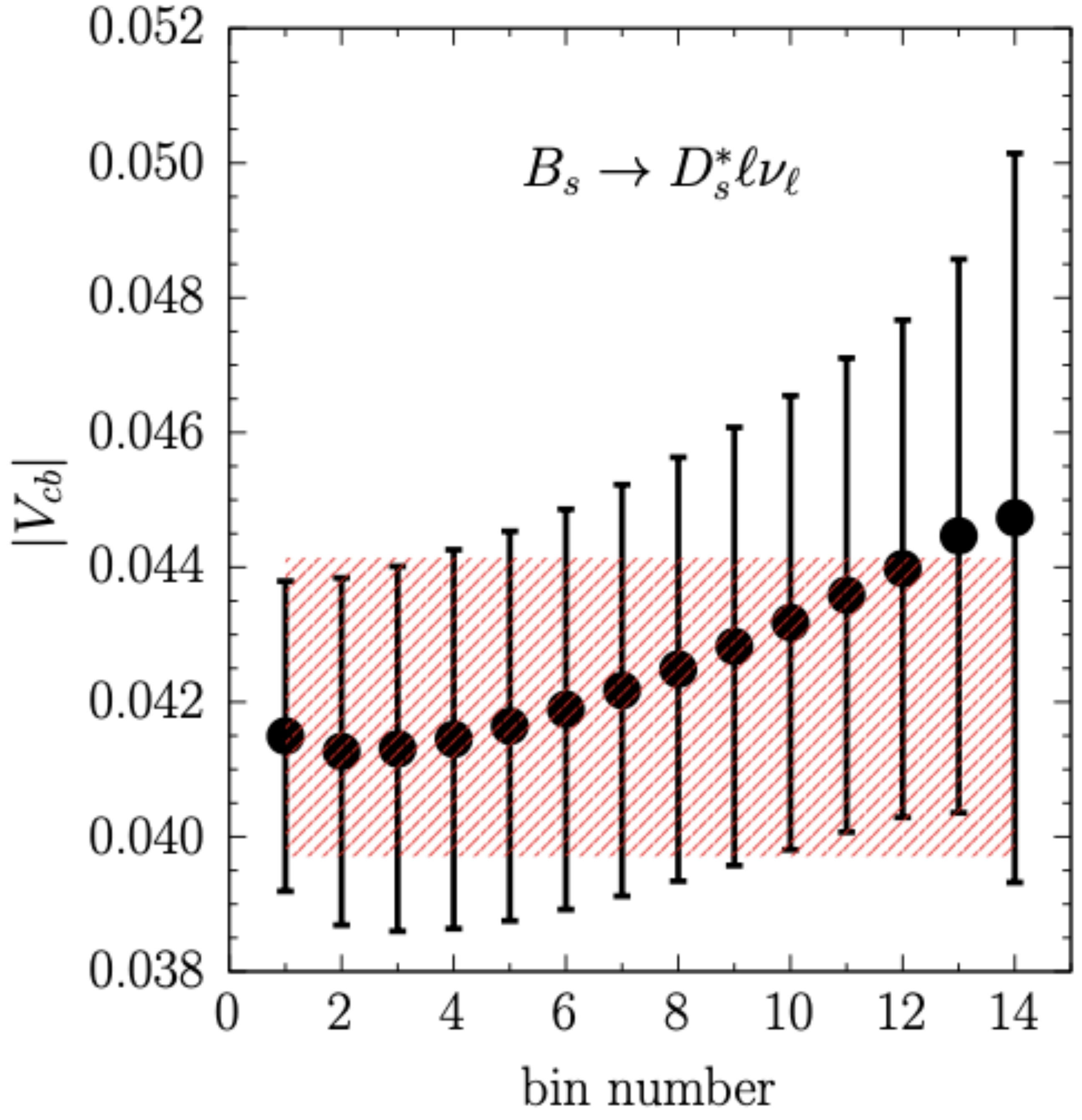
decays	$\lvert V_{cb}\rvert^{DM} \cdot 10^3$
$B_s \to D_s \ell \nu_\ell$	42.3 ± 2.1
$B_s \to D_s^* \ell \nu_\ell$	41.0 ± 2.8

2) Differential decay rates reconstructed from the LHCb fits of $p_\perp$ distributions (BGL/CLN parameterizations for the FFs) carried out in arXiv:2001.03225

bin-per-bin analysis: $|V_{cb}|_j = \sqrt{\dfrac{d\Gamma^{LHCb}/dw_j}{d\Gamma^{DM}/dw_j}} \qquad j = 1,\ldots,N_{bins}$

we adopted $N_{bins} = 14\ w - bins$





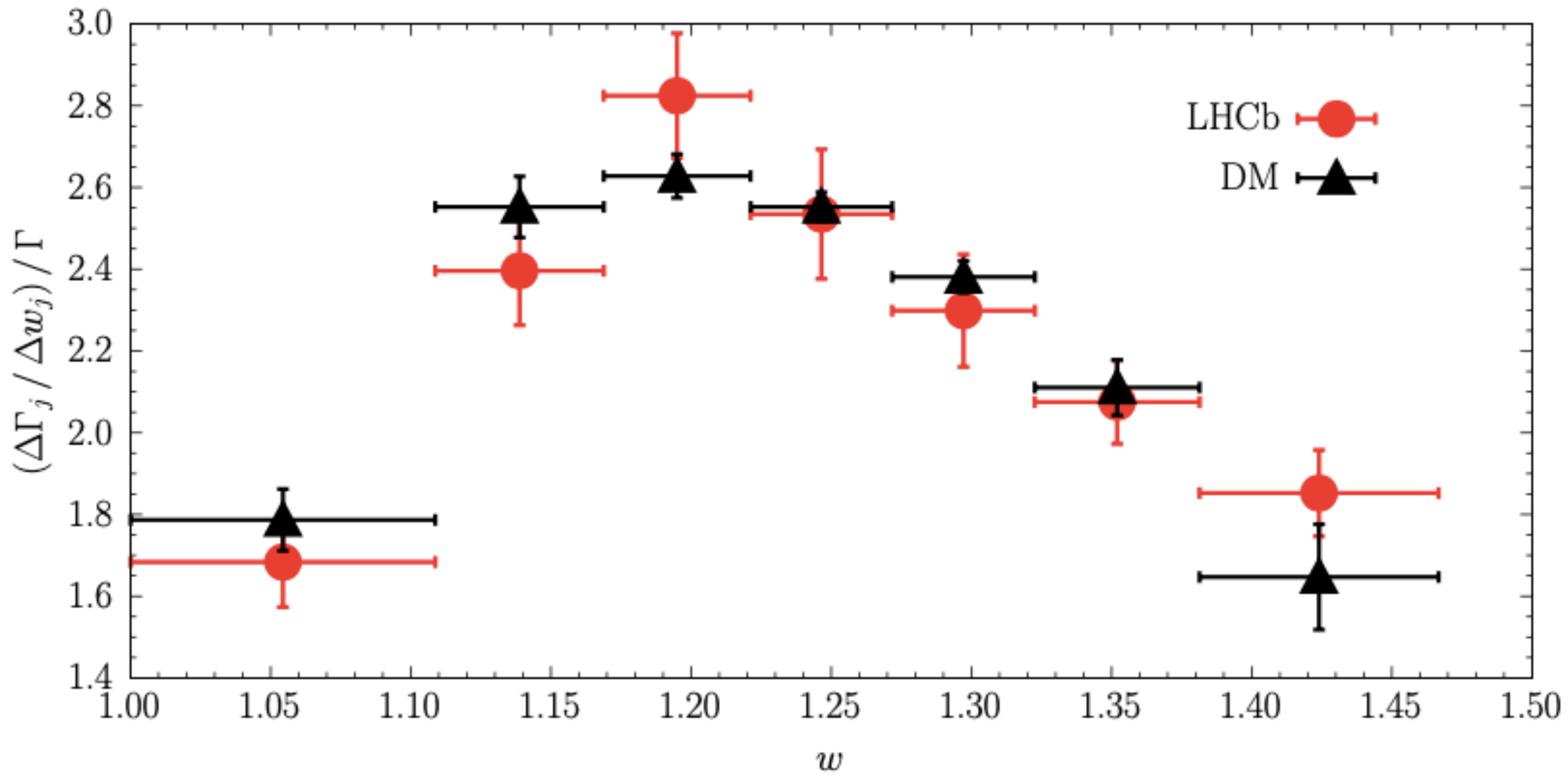
decays	$\lvert V_{cb}\rvert^{DM} \cdot 10^3$
$B_s \to D_s \ell \nu_\ell$	42.4 ± 1.9
$B_s \to D_s^* \ell \nu_\ell$	41.9 ± 2.2

$|V_{cb}|^{LHCb} \cdot 10^3 = 42.3 \pm 1.7$

2) LHCb ratios from arXiv:2003.08453

$$\Delta r_j = \frac{d\Gamma_j(B_s \to D_s^* \mu \nu_\mu)}{\Gamma(B_s \to D_s^* \mu \nu_\mu)} \quad j = 1,...,7$$

j	1	2	3	4	5	6	7
w-bin	1.000 - 1.1087	1.1087 - 1.1688	1.1688 - 1.2212	1.2212 - 1.2717	1.2717 - 1.3226	1.3226 - 1.3814	1.3814 - 1.4667
Δw_j	0.1087	0.0601	0.0524	0.0505	0.0509	0.0588	0.0853
$\Delta r_j^{\mathrm{LHCb}}$	0.183(12)	0.144(8)	0.148(8)	0.128(8)	0.117(7)	0.122(6)	0.158(9)
Δr_j^{DM}	0.1942(82)	0.1534(45)	0.1377(28)	0.1289(18)	0.1212(20)	0.1241(40)	0.1405(110)



consistency within ~1σ

⇓

shape of theoretical FFs is consistent with the one of the experimental data

To extract $|V_{cb}|$, we evaluated the integrated experimental differential decays rate for each bin

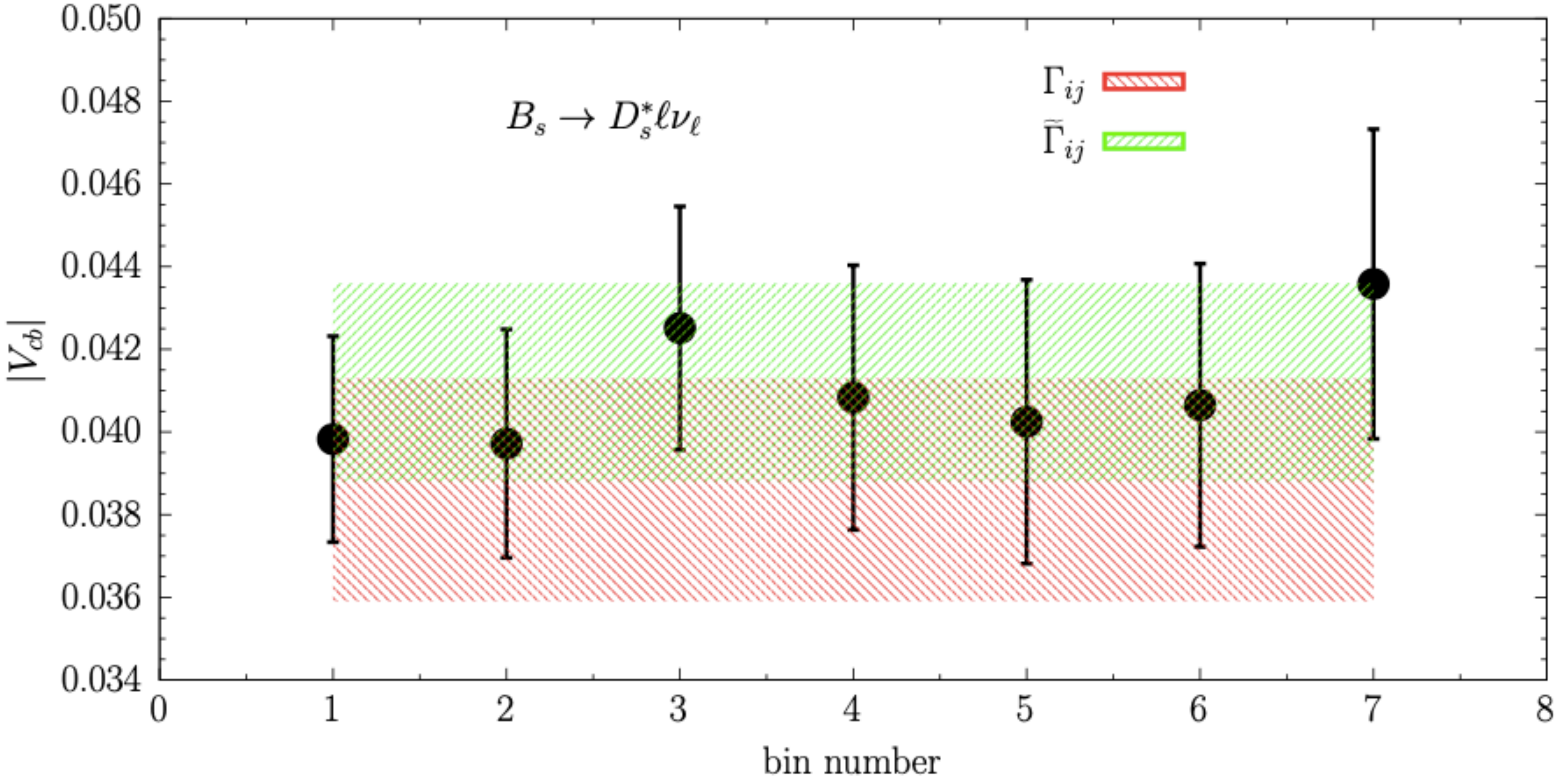
$$\Delta\Gamma_j^{exp} = \Delta r_j^{LHCb} \cdot \Gamma^{LHCb}(B_s \to D_s^* \mu\nu_\mu) \quad j = 1,\ldots,7$$

and the covariance matrix: $\Gamma_{ij}^{exp} = R_{ij}^{LHCb}\left[\bar{\Gamma}^2 + \sigma_{\bar{\Gamma}}^2\right] + \Delta r_i^{LHCb}\Delta r_j^{LHCb}\sigma_{\bar{\Gamma}}^2$

general property: $\sum_{i,j=1}^{N_{bins}} \Gamma^{exp} = \sigma_{\bar{\Gamma}}^2$ $\longleftrightarrow$ $\sum_{j=1}^{N_{bins}} \Delta r_j^{LHCb} = 1$ and $\sum_{i,j=1}^{N_{bins}} R_{ij}^{LHCb} = 0$

- D'Agostini effect (NIMA '94): negative bias on constant fits to data affected by an overall normalization uncertainty;

  It depends upon $\sigma_{\bar{\Gamma}}$ and $\Delta r_i^{LHCb} \neq \Delta r_j^{LHCb}$



Modified covariance matrix

$$\tilde{\Gamma}_{ij}^{exp} = R_{ij}^{LHCb}\left[\bar{\Gamma}^2 + \sigma_{\bar{\Gamma}^2}\right] + \sigma_{\bar{\Gamma}}^2 / N_{bins}^2$$

$$\sum_{i,j=1}^{N_{bins}} \tilde{\Gamma}_{ij}^{exp} = \sum_{i,j=1}^{N_{bins}} \Gamma_{ij}^{exp} = \sigma_{\bar{\Gamma}}^2$$

Correlated weighted averages

$|V_{cb}| \cdot 10^3 = 38.6 \pm 2.7$

$|V_{cb}| \cdot 10^3 = 41.2 \pm 2.4$