

ABSTRACT

Here we analyze a hybrid scoto-seesaw model based on A_4 discrete symmetry to understand neutrino masses and mixing. The minimal type-I seesaw generates tribimaximal (TBM) neutrino mixing at the leading order. The scotogenic contribution acts as a deviation from this first-order approximation of the lepton mixing matrix to yield the observed non-zero θ_{13} , and to accommodate a potential dark matter (DM) candidate.

MINIMAL SCOTO-SEESAW MODEL

- SM+ N_R + singlet fermion f + scalar doublet η :

$$\mathcal{L} = -Y_N^k \bar{L}^k i\sigma_2 H^* N_R + \frac{1}{2} M_N \bar{N}_R^c N_R + Y_f^k \bar{L}^k i\sigma_2 \eta^* f + \frac{1}{2} M_f \bar{f}^c f + h.c..$$

- A Z_2 symmetry here is responsible for the stability of the DM and only f and η are odd under Z_2 . The atmospheric neutrino mass scale at the tree level. The solar neutrino mass scale is generated at one-loop level, as a result the hierarchy between the atmospheric and solar scale is maintained.

$$M_\nu^{ij} = -\frac{v^2}{M_N} Y_N^i Y_N^j + \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) Y_f^i Y_f^j$$

with

$$\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) = \frac{1}{32\pi^2} \left[\frac{m_{\eta_R}^2 \log(M_f^2/m_{\eta_R}^2)}{M_f^2 - m_{\eta_R}^2} - \frac{m_{\eta_I}^2 \log(M_f^2/m_{\eta_I}^2)}{M_f^2 - m_{\eta_I}^2} \right]$$

where $H = (H^+, H^0)^T$, $\eta = (\eta^+, \eta^0)$, m_{η_R} and η_I are the masses of the neutral component of η .

INTRODUCTION

We begin with a minimal type-I seesaw assisted by A_4 symmetry which helps in reproducing the TBM mixing. The charged lepton RH Majorana neutrino mass matrix is diagonal to start with. The Dirac Yukawa turns out to be solely responsible for generating the TBM mixing. Inclusion of the scotogenic contribution to the neutrino mass helps to generate the observed value of θ_{13} as well as naturally incorporate a DM candidate.

BASICS OF A_4 SYMMETRY

- Standard Model accompanied with non-abelian discrete flavor groups are widely popular to understand the lepton masses and mixings.
- Out of all the discrete groups employed in this purpose A_4 turn out to be the most popular one.
- It is a discrete group of even permutations of four objects with three inequivalent one-dimensional representations (1 , $1'$ and $1''$) and a three-dimensional representation (3).
- The multiplication rules : $1' \otimes 1' = 1''$, $1' \otimes 1'' = 1$, $1'' \otimes 1'' = 1'$, $3 \otimes 3 = 1 + 1' + 1'' + 3_a + 3_s$
- The product rule for this two triplets $((x_1, x_2, x_3)^T$ and $(y_1, y_2, y_3)^T$) :

$$\begin{aligned} 1 &\sim x_1 y_1 + x_2 y_3 + x_3 y_2, \\ 1' &\sim x_3 y_3 + x_1 y_2 + x_2 y_1, \\ 1 &\sim x_2 y_2 + x_1 y_3 + x_3 y_1, \\ 3_s &\sim \begin{pmatrix} 2x_1 y_1 - x_2 y_3 - x_3 y_2 \\ 2x_3 y_3 - x_1 y_2 - x_2 y_1 \\ 2x_2 y_2 - x_1 y_3 - x_3 y_1 \end{pmatrix}, \\ 3_a &\sim \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_1 y_2 - x_2 y_1 \\ x_3 y_1 - x_1 y_3 \end{pmatrix}. \end{aligned}$$

SCOTO-SEESAW MODEL WITH FLAVOR SYMMETRY

- Minimal type-I seesaw (2RHNs) with $A_4 \times Z_4 \times Z_3 \times Z_2$ discrete symmetry to obtain the lepton mass matrices.

- Charged lepton mass matrix:

$$\mathcal{L}_l = \frac{y_e}{\Lambda} (\bar{L} \phi_T) H \alpha_R \Rightarrow M_l = \frac{v_T}{\Lambda} v \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}.$$

- Dirac mass matrix:

$$\mathcal{L}_D = \frac{y_{N1,2}}{\Lambda} (\bar{L} \phi_{s,a}) H N_{R1,2} \Rightarrow M_D = \frac{v}{\Lambda} \begin{pmatrix} 0 & y_{N2} v_a \\ -y_{N1} v_s & y_{N2} v_a \\ y_{N1} v_s & y_{N2} v_a \end{pmatrix}$$

- Majorana mass matrix:

$$\mathcal{L}_N = \frac{1}{2} M_{N1,2} \bar{N}_{R1,2}^c N_{R1,2} \Rightarrow M_R = \begin{pmatrix} M_{R1} & 0 \\ 0 & M_{R2} \end{pmatrix}$$

- The flavon fields get VEVs along $\langle \phi_T \rangle = (v_T, 0, 0)$, $\langle \phi_s \rangle = (0, v_s, -v_s)$ and $\langle \phi_a \rangle = (v_a, v_a, v_a)$
- Neutrino mass obtained from type-I seesaw:

$$(M_\nu)_T = -M_D M_R^{-1} M_D^T \Rightarrow -\begin{pmatrix} B & B & B \\ B & A+B & -A+B \\ B & -A+B & A+B \end{pmatrix}$$

where $A = \frac{v^2 v_s^2 y_{N1}^2}{\Lambda^2 M_{N1}}$, $B = \frac{v^2 v_a^2 y_{N2}^2}{\Lambda^2 M_{N2}}$
 • $U_{TB}^T (M_\nu)_T U_{TB} = :$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -3B & 0 \\ 0 & 0 & -2A \end{pmatrix} \text{ where } U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

- Scotogenic contribution to the neutrino mass:

$$\mathcal{L}_S = \frac{y_s}{\Lambda^2} (\bar{L} \phi_S) \xi i \sigma_2 \eta^* f + \frac{1}{2} M_f \bar{f}^c f + h.c..$$

which yields $(M_\nu)_S = \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) M_f Y_f^i Y_f^j$

- Total neutrino mass $M_\nu = (M_\nu)_T + (M_\nu)_S$

$$= \begin{pmatrix} -B+C & -B & -B-C \\ -B & -(A+B) & A-B \\ -B-C & A-B & -A-B+C \end{pmatrix}.$$

RESULTS

- After rotation by U_{TB}

$$\begin{aligned} M'_\nu &= U_{TB}^T M_\nu U_{TB} \\ &= \frac{1}{2} \begin{pmatrix} 3C & 0 & -\sqrt{3}C \\ 0 & -6B & 0 \\ -\sqrt{3}C & 0 & -4A+C \end{pmatrix}, \end{aligned}$$

- Full diagonalization : $(U_{TB} U_{13})^T M_\nu U_{13} U_{TB} = \text{diag}(m_1 e^{i\gamma_1}, m_2 e^{i\gamma_2}, m_3 e^{i\gamma_3})$, where

$$U_{13} = \begin{pmatrix} \cos \theta & 0 & \sin \theta e^{-i\phi} \\ 0 & 1 & 0 \\ -\sin \theta e^{i\phi} & 0 & \cos \theta \end{pmatrix},$$

$$\tan \phi = \frac{\alpha \sin \phi_{AC}}{1 - \alpha \cos \phi_{AC}}, \quad \tan 2\theta = \frac{\sqrt{3}}{\cos \phi + 2\alpha \cos(\phi_{AC} + \phi)}$$

- Comparing with U_{PMNS} we obtain:

$$\sin \theta_{13} e^{-i\delta_{CP}} = \sqrt{\frac{2}{3}} e^{-i\phi} \sin \theta, \quad \tan^2 \theta_{12} = \frac{1}{2 - 3 \sin^2 \theta_{13}}$$

- Neutrino masses and associated phases:

$$\begin{aligned} m_{1,3} &= |C| [(1 - \alpha \cos \phi_{AC} \mp P)^2 + (Q \pm \alpha \sin \phi_{AC})^2]^{1/2} \\ m_2 &= |C| 3\beta, \\ \phi_1 &= \tan^{-1} \left(\frac{Q \pm \alpha \sin \phi_{AC}}{1 - \alpha \cos \phi_{AC} \mp P} \right), \quad \phi_2 = \phi_{BC}, \end{aligned}$$

where we write $A = |A| e^{i\phi_A}$, $B = |B| e^{i\phi_B}$, $C = |C| e^{i\phi_C}$, $\alpha = |A|/|C|$, $\beta = |B|/|C|$, $\phi_{AC} = \phi_A - \phi_C$ and $\phi_{BC} = \phi_B - \phi_C$.

- With 3σ allowed range of neutrino data: $2.3 \leq \alpha \leq 2.9$ (NH) and $1.7 \leq \alpha \leq 2.8$ (IH)
 $0.3 \leq \beta \leq 1$ (NH) and $1.1 \leq \beta \leq 1.9$ (IH)
 $2.6 \leq \phi_{AC} \leq 5.1$ (NH) and $0 \leq \phi_{AC} \leq 2$ (IH)
- Predictions : θ_{23} in the upper octant; $0.06(0.115)eV \leq \Sigma m_i \leq 0.12(0.15)eV$ for NH (IH); $0.001(0.01)eV \leq m_{\beta\beta} \leq 0.05(0.05)eV$ for NH (IH), δ_{CP} between 0.3-1.4 for NH (0.5-0.9 for IH) disallowed.

REFERENCES

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DARK MATTER

There exists thee potential dark matter candidates in this construction. These are the dark fermion f and real and imaginary components of η , namely

η_R and η_I . If η_R is considered to the dark matter candidate the region $m_{\eta_R} \leq 550$ GeV is a viable allowed region.

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