Anomalous gauge couplings in the light of recent results from muon g-2 and flavor observables

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Abstract

We reassess anomalous triple gauge couplings in the light of the recent $(g-2)_\mu$ measurement at FNAL, the new lattice theory result of $(g-2)_\mu$ and the updated measurements of several B-decay modes. In the framework of SMEFT, three bosonic dimension-6 operators are invoked to parametrize physics beyond the Standard Model and their contributions to such low-energy observables computed. Constraints on the corresponding Wilson coefficients are then derived from fits to the current experimental bounds on the observables and compared with the most stringent ones available from the 13 TeV LHC data in the W^+W^- and $W^\pm Z$ production channels.

Introduction

- The recent FNAL result of the anomalous magnetic moment of the muon, namely $a_\mu \equiv [(g-2)/2]_\mu$ confirms the previous one from BNL measurement and places a combined discrepancy of 4.2σ deviation from the SM when using dispersive techniques for the hadronic vacuum polarisation.
- The use of Lattice-QCD results by the BMW collaboration to calculate the same decreases the discrepancy to a mere 2σ level.
- Also of interest are B physics observables, where neutral current $b \to sll$ transitions have been showing persistent discrepancy from the SM values in recent years with the most recent result being of the R_K anomaly and $BR(B_s \to \mu^+\mu^-)$.
- In the present paper, we reexamine possible anomalous self-interactions of the electroweak gauge bosons in the light of these experiments. Concentrating on three particular dimension-6 terms in the SMEFT Lagrangian that lead to anomalous triple gauge boson couplings (TGCs), we evaluate the corresponding one-loop contributions to both $(g-2)_\mu$ and $(g-2)_e$, another observable that shows a discrepancy, albeit smaller as well as certain electroweak precision measurements.
- Assuming that the aforementioned operators are the leading ones, we show that radiative and rare B and K decays such as $B \to X_s\gamma$, $B_s \to \mu^+\mu^-$, $B \to X_s\ell^+\ell^-$, $B \to K^{(*)}\mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$, $K \to \pi\nu\bar{\nu}$ provide very important constraints.
- Even though the preceding assumption seems quite restrictive at the outset, we find that the ensuing results have interesting implications nonetheless for certain classes of new physics models.
- The most famous of these are Randall-Sundrum-like scenarios with bulk fermions and bosons. The localizations of the light fermions as dictated by the warping, ensures that the overlap integrals for the KK-gauge bosons with the SM fermions are much smaller than those with the SM bosons. This, immediately, leads to an hierarchy in the Wilson coefficients as examined in this analysis.

Effective Lagrangian for the Gauge Sector

- Taking Λ to be the characteristic scale of the UV complete theory, the effective lagrangian at dimension-6 can be written as: $\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i$
- The largest imprint on the observables of interest from the bosonic sector are provided by:

$$\mathcal{O}_{WWW} = \frac{c_{WWW}}{\Lambda^2}\mathrm{Tr}[\hat{W}_\mu^\nu\hat{W}_\nu^\rho\hat{W}_\rho^\mu] \quad \mathcal{O}_W = \frac{c_W}{\Lambda^2}(D_\mu\Phi)^\dagger\hat{W}^{\mu\nu}(D_\nu\Phi) \quad \mathcal{O}_B = \frac{c_B}{\Lambda^2}(D_\mu\Phi)^\dagger\hat{B}^{\mu\nu}(D_\nu\Phi)$$

$$D_\mu\Phi = (\partial_\mu + ig\frac{\sigma^a}{2}W_\mu^a + i\frac{g'}{2}B_\mu)\Phi, \hat{W}{\mu\nu} = ig\frac{\sigma^a}{2}W_{\mu\nu}^a\hat{B}_{\mu\nu} = i\frac{g'}{2}B_{\mu\nu}.$$

- These operators give rise to anomalous triple gauge couplings:

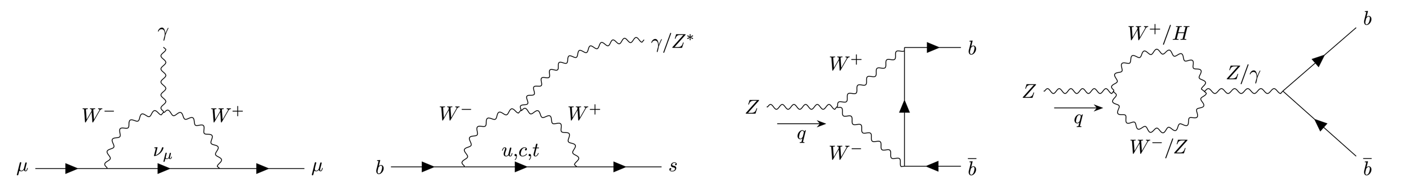
$$\mathcal{L}_{eff}^{WWV} = g_{WWV}\Big\{g_1^V\left(\tilde{W}_{\mu\nu}^-\tilde{W}^{+\nu} - \tilde{W}_{\mu\nu}^+\tilde{W}^{-\nu}\right)V^\mu + \kappa_V\tilde{W}_{\mu\nu}^+\tilde{W}_{\mu\nu}^-\tilde{V}^{\mu\nu} + \frac{\lambda_V}{m_W^2}\tilde{W}_\mu^{+\nu}\tilde{W}_\nu^{-\rho}\tilde{V}_\rho^{\ \mu}\Big\},$$

  with $V \equiv \gamma, Z$. Here, $g_{WW\gamma} = e$, $g_{WWZ} = e\cot\theta$ (with θ being the Weinberg angle) and the field strengths correspond to only the abelian part.
- Within the SM, we have $g_1^V = \kappa_V = 1$, $\Delta g_1^\gamma = 0$ and $\lambda_V = 0$. In other words, $\Delta\kappa_V \equiv \kappa_V - 1$, $\Delta g_1^Z \equiv g_1^Z - 1$ and λ_V suitably define the anomalous couplings, and, post symmetry-breaking, can be related to the Wilson coefficients c_W, c_B and c_{WWW} as follows:

$$\Delta g_1^Z = c_W\frac{m_Z^2}{2\Lambda^2} \quad \Delta\kappa_Z = \left[c_W - s_\theta^2\left(c_W + c_B\right)\right]\frac{m_Z^2}{2\Lambda^2} \quad \Delta\kappa_\gamma = \left(c_W + c_B\right)\frac{m_W^2}{2\Lambda^2} \quad \lambda_\gamma = \lambda_Z = \frac{3m_W^2g^2}{2\Lambda^2}c_{WWW}.$$

  In the above and in the following sections we use the notation $s_\theta = \sin\theta$ and $c_\theta = \cos\theta$.

Contribution to various observables



- At one loop the interaction lagrangian and the expression for the muon anomalous magnetic moment is:

$$\mathcal{L}_{\Delta a_\mu} \supset \frac{g}{\sqrt{2}}\left(\bar{\mu}\gamma^\mu P_L\nu_\mu\right)W_\mu^- + \text{h.c.} + \mathcal{L}_{\text{eff}}^{WW\gamma}, \qquad \Delta a_\mu^{anom.} = \frac{e^2}{48\pi^2 s_\theta^2}\frac{m_\mu^2}{m_W^2}\left\{\Delta\kappa_\gamma\left(\frac{1}{3} + \ln\left[\frac{\Lambda^2}{m_W^2}\right]\right) + \lambda_\gamma\left(\frac{7}{6} - \ln\left[\frac{\Lambda^2}{m_W^2}\right]\right)\right\}.$$

- The anomalous gauge couplings can also contribute to various loop-mediated flavour changing neutral current hadronic decays through a multitude of effective operators such as the electromagnetic dipole or semi-leptonic vector and axial-vector ones, namely

$$\mathcal{Q}_7 = \frac{e}{(4\pi)^2}m_b\left(\bar{s}_L\sigma_{\alpha\beta}b_R\right)F^{\alpha\beta} \qquad \mathcal{Q}_9 = \frac{e^2}{(4\pi)^2}\left(\bar{s}_L\gamma_\alpha b_L\right)(\bar{l}\gamma^\alpha l) \qquad \mathcal{Q}_{10} = \frac{e^2}{(4\pi)^2}\left(\bar{s}_L\gamma_\alpha b_L\right)(\bar{l}\gamma^\alpha\gamma_5 l)$$

  where L and R denote the chirality of the fermionic fields, $\sigma_{\alpha\beta} = i[\gamma_\alpha, \gamma_\beta]/2$ and $F^{\alpha\beta}$ is the electromagnetic field tensor. The $\Delta B, \Delta S = 1$ operator is traditionally written as

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}}\Big(C_7Q_7 + C_9Q_9 + C_{10}Q_{10}\Big) + H.c,$$

  with C_7, C_9 and C_{10} being the corresponding Wilson coefficients that factorise the short distance physics.
- While the photonic diagram thus generated mirrors that for $\Delta a_\mu^{\text{anom}}$ and is only logarithmically divergent, for the Z vertex, the anomalous contribution from an individual quark loop is quadratically divergent. However, thanks to the GIM mechanism, the quadratically divergent pieces (as with any other term independent of the internal quark mass) cancel, leaving behind only a logarithmic divergence. Of these, the top-quark contribution dominates overwhelmingly and we can fairly approximate $(x \equiv m_t^2/m_W^2)$

$$C_i \approx (C_i)_{SM} + \left[V_{tb}V_{ts}^*\frac{m_W^2}{\Lambda^2}\ln\left(\frac{\Lambda^2}{m_W^2}\right)\right]\Delta C_i$$

$$\Delta C_7 = \frac{-(c_B + c_W)}{8\,(x-1)^2}\left[2x + \frac{x^3 - 3x^2}{(x-1)}\ln x\right] + \frac{3g^2\,c_{WWW}}{8}\left[\frac{x^2 + x}{(x-1)^2} - \frac{2x^2\ln x}{2\,(x-1)^3}\right],$$

$$\Delta C_9 = \frac{-(c_B + c_W)}{8}x + \frac{3c_W}{16}\frac{1 - 4s_\theta^2}{s_\theta^2}x + \frac{3g^2\,c_{WWW}}{2}\left[\frac{x - 3x^2}{2\,(x-1)^2} + \frac{x^3\ln x}{(x-1)^3}\right], \quad \Delta C_{10} = \frac{-3c_W}{16s_\theta^2}x\,.$$

- The presence of the anomalous gauge couplings also leads to a modification in the electroweak precision variables, whether these be the oblique corrections or the fermion-gauge couplings. Of particular importance is the $Zb\bar{b}$ coupling and the ρ (equivalently, T) parameter. The effective $Zb\bar{b}$ vertex may be parametrized as

$$\mathcal{L}_{Zb\bar{b}} = \frac{e}{s_\theta c_\theta}\left[\left(g_L^b + \delta g_L^b\right)\bar{b}_L\not{Z}b_L + \left(g_R^b + \delta g_R^b\right)\bar{b}_R\not{Z}b_R\right]$$

  where $g_L = \left(-1/2 + s_\theta^2/3\right)$ and $g_R = \left(s_\theta^2/3\right)$ are the SM values of the couplings.

- One part of the corrections to these originates from the wave-function renomalization of the Z boson due to a one-loop oblique correction Π_{ZZ} and is given by

$$(\delta g_{L,R}^b)_{ob} = \frac{-\alpha_{em}}{4\pi\Lambda^2}g_{L,R}\mathcal{A}, \quad \mathcal{A} \equiv \frac{gv\,m_Z}{4c_\theta}\left(\frac{m_H^2}{2m_Z^2} + \frac{2}{3}\right)\left(\frac{c_B}{c_\theta^2} + \frac{c_W}{s_\theta^2}\right)\log\frac{\Lambda^2}{m_H^2} + \frac{m_Z^2}{2s_\theta^2}\log\frac{\Lambda^2}{m_W^2} \times \Bigg(3(1 - 6c_\theta^2)g^2\frac{m_W^2}{m_Z^2}c_{WWW} + \left[4c_\theta^2 - \frac{5}{6}\right]c_W - \frac{1 + 4c_\theta^2 - 36c_\theta^4}{12c_\theta^2}\left[c_\theta^2c_W - s_\theta^2c_B\right]\Bigg)$$

- The second contribution emanates from the direct one-loop correction to the vertex. Applicable only for the left-handed coupling given by

$$(\delta g_L^b)_v = \left[\frac{\alpha_{em}\,|V_{tb}|^2}{16\,\pi\,s_\theta^2}\,\frac{m_Z^2}{2\Lambda^2}\right]\mathcal{B}\log\frac{\Lambda^2}{m_W^2}, \quad \mathcal{B} \equiv \left[c_W - s_\theta^2\left(c_W + c_B\right)\right]\left(-1 + \frac{x}{2} - \frac{1}{6}\frac{m_Z^2}{m_W^2}\right) - c_Wc_\theta^2\left(-\frac{5m_Z^2}{3\,m_W^2} + 3x\right),$$

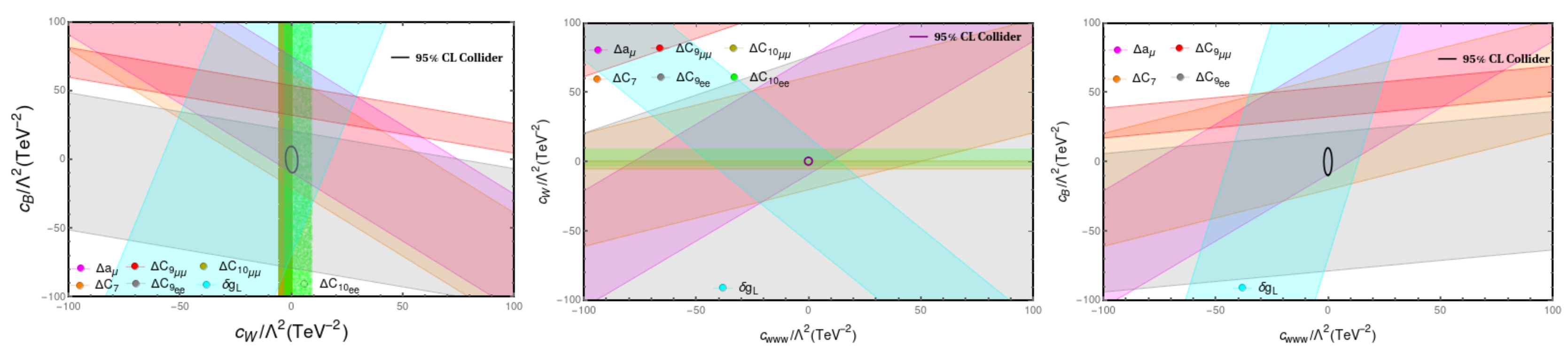
Results

- Some of the important experimental values of the observables pertaining to our analysis are:

Current limits	
Observable($\mathcal{F}$)	1σ limit
$\Delta a_\mu^{\text{DISP}}$(WP20)	$251 \pm 59 \times 10^{-11}$
$\Delta a_\mu^{\text{BMW}}$(BMW)	$107 \pm 69 \times 10^{-11}$
ΔC_7	-0.03 ± 0.03
$\Delta C_{9\mu\mu}$	-1.03 ± 0.13
$\Delta C_{10\mu\mu}$	0.41 ± 0.23
ΔC_{9ee}	0.70 ± 0.60
ΔC_{10ee}	-0.50 ± 0.50
δg_L	0.0016 ± 0.0015
δg_R	0.019 ± 0.007

Calculation	Descriptor	$(c_B, c_W, c_{WWW})/\Lambda^2$ [TeV^{-2}]	χ^2
WP20	SM	(0,0,0)	101.76
	2-param B.F.	$(41.78, -1.52, 0)$	29.75
	3-param B.F.	$(37.99, -1.48, -15.20)$	28.29
BMW	SM	(0,0,0)	86.121
	2-param B.F.	$(38.64, -1.67, 0)$	25.383
	3-param B.F.	$(38.58, -1.67, -0.29)$	25.382

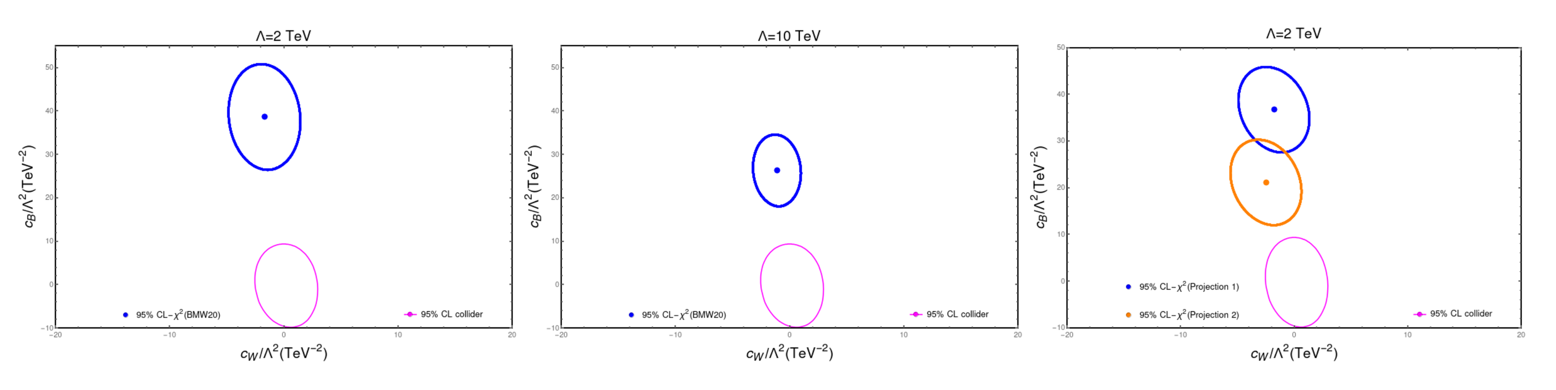
- Using the expressions for the observables and the corresponding experimental values, we can plot:



- The Δa_μ-allowed band, as calculated using the WP20 result does not include the SM point, reflecting the fact that the data does not agree with the SM value at the 2σ level. For the lattice result (BMW), though, it is indeed included (showed by purple band).
- The band for Δa_e sits on the opposite side of the origin, owing to the sign of the discrepancy. However, being suppressed by m_e^2, the required sizes of the Wilson coefficients are too large to be meaningful.
- δg_L has a relatively weaker dependence on c_B than on c_W leading to the slightly tilted band. δg_R receives a small correction only from the correction to the Z self-energy, and the ensuing bounds are too weak to be relevant.
- Since ΔC_7, just like Δa_μ, parametrizes the coupling of a fermion current to the photon, both are proportional (in the absence of a nonzero c_{WWW}) to the combination $(c_B + c_W)$ and the ensuing bands are parallel to each other.
- ΔC_{10}, being dependent on c_W alone, leads to a relatively narrow vertical band in this plane. Most restrictive of all the observables, the difference in its value as calculated from the electronic and muonic channels exert opposite pulls leading to the two parallel bands. Although both bands overlap with the collider limit, the muonic one has a greater sensitivity to c_W and, hence, its partial overlap presents a comparatively stronger constraint on the allowed region, favouring negative values for c_W. This leads to an interesting possibility wherein ΔC_{10} is the dominant flavour-blind Wilson coefficient parametrizing new physics effects in FCNC B decays. A sizable range of c_W values compatible with the LHC limits exists that could, then, ameliorate the discrepancies in the aforementioned B decay observables (excluding LFU ones). Not contributing to ΔC_{10}, a similar-sized c_B would lead to only tiny changes in the low-energy observables and would be primarily constrained by collider experiments.
- Similar to the preceding observable, the opposing experimental numbers for ΔC_9 from the two (e and μ) channels lead to two bands.
- The χ^2 function for the combined fit to these observables can be defined as:

$$\chi^2(c_B, c_W, c_{WWW}; \Lambda) = \sum_i\left(\frac{\mathcal{F}_i^{\text{exp}} - \mathcal{F}_i^{\text{th}}}{\sigma_i}\right)^2,$$

- The best-fit point would then be given by the minimum of the χ^2 and parameter points leading to $\chi^2 \leq \chi^2_{min} + \delta\chi^2$ being inseparable from the best-fit point at a confidence level determined by $\delta\chi^2$.
- The first two plots in the figure below shows the χ^2 analysis for the BMW case with $\Lambda = 2$ and $\Lambda = 10$ TeV respectively.
- The third plot represents estimated projections in future when (a) Same deviation as BMW but with errors reduced by four times represented by blue ellipse, and (b) assuming no deviation from SM and errors reduced by four times represented by orange ellipse.



Summary and Outlook

- Our study indicates that the limits on low-energy observables, taken individually, lead to weak bounds on the bosonic SMEFT Wilson coefficients when compared with the existing LHC limits, except for the bounds on c_W/Λ^2 emanating from the limits on ΔC_{10} which are comparable and consistent with the collider results.
- On the other hand, a global fit in the $(c_W/\Lambda^2, c_B/\Lambda^2)$ plane, while imposing significantly stronger constraints on the WCs, exhibits disagreement with the LHC results.
- One has to note that the assertion of the 1-loop contributions being at most logarithmically divergent is based on the assumption that all higher divergences are cancelled by contributions from higher dimensional operators. In the event that such cancellations are inexact, the residual contributions, of quadratic (or higher) order in the cutoff Λ may lead to substantially stronger constraints on the WCs.
- The LHC limits have used cross-sections that also include terms quadratic in the TGCs, whereas we consider contributions to the concerned observables only upto a linear order in the same. Had we included quadratic contributions, we would have obtained stronger bounds as well.
- Notwithstanding the caveats, the fact that ΔC_{10} also favours c_W/Λ^2 values that are very close to the origin indicates that any explicit new physics model designed to explain the discrepancies (e.g., models which give rise to lepton flavour universality violating (LFUV) or a combination of LFUV and LFU 4-fermion operators) should either induce $\mathcal{O}_W$ with a suppressed (or vanishing) Wilson coefficient or, otherwise, one must account for the WC c_W generated therein, in addition to other parameters, while performing a fit to the concerned observables.