

Study of the heavy bottom baryons in a potential model

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Abstract

The bottom heavy baryons are studied in the framework of a nonrelativistic quark model. We use the hypercentral approach to solve the six-dimensional Schrödinger equation of the baryons. Introducing a potential model, the ground state masses and magnetic moments of the $\Lambda_b, \Sigma_b, \Xi_{bc}, \Xi_{bb}$ heavy baryons are calculated. We investigate the semileptonic decay widths of the Λ_b bottom baryon. Our results are in agreement with the available experimental data and those of other works.

The model

We consider the baryons as a bound state of three quarks in the hypercentral approach. To describe the baryon containing three quarks, we use the Jacobi coordinates

$$\rho = \frac{1}{\sqrt{2}}(r_1 - r_2), \quad \lambda = \frac{1}{\sqrt{6}}(r_1 + r_2 - 2r_3),$$

and

$$m_\rho = \frac{2m_1 m_2}{m_1 + m_2}; \quad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)}$$

One can introduce the hyperspherical coordinates:

$$x = \sqrt{\rho^2 + \lambda^2}, \quad \xi = \tan^{-1}\left(\frac{\rho}{\lambda}\right)$$

Therefore the Hamiltonian will be

$$H = \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + V(\rho, \lambda) = \frac{p^2}{2m} + V(x)$$

The three-body Schrodinger equation in the hypercentral model is obtained as

$$\left[\frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma+1)}{x^2} \right] \psi_\gamma(x) = -2m[E_\gamma - V(x)] \psi_\gamma(x)$$

Where $\psi(x), E_\gamma, x$ and L are the wave function, energy eigenvalues, distance between two particles, and angular quantum number respectively. m is the reduced mass and x is the hyperradius and defined in terms of the Jacobi coordinates:

$$m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda},$$

and potential $V(x)$ is defined as follows

$$V(x) = ax - \frac{c}{x}$$

Where the coefficients of the confining and color-coulombic parts, a and c , are constant. The hyperfine interactions are considered as perturbation. We assume the wave function to be in the form of

$$\psi_\gamma = x^{5/2} \varphi_\gamma$$

to simplify the Schrodinger equation and then solve the equation numerically. For details see Ref. [1].

By using the obtained eigenenergies E_γ , eigenfunctions and also the perturbative hyperfine energy E_{hyp} , the mass of the baryon is obtained as a summation of the quark masses and the energies

$$M = m_1 + m_2 + m_3 + E_\gamma + E_{hyp}$$

Our results for the ground state baryon masses are as follows

$$\Lambda_c = 2259 \text{ MeV}, \Sigma_c = 2471 \text{ MeV}, \Lambda_b = 5639 \text{ MeV}, \Sigma_b = 5859 \text{ MeV}, \Xi_b = 5858 \text{ MeV}, \Omega_b = 5917 \text{ MeV}, \Xi_{bc} = 6989 \text{ MeV}, \Xi_{bb} = 10369 \text{ MeV}.$$

We compared our results for the some of the baryon masses with other results in table 1.

Table 1: Masses of the bottom baryons (in MeV).

State	Prediction	Exp [5]	[6]	[7]
Σ_b	5859	5813	5808	5823
Λ_b	5639	5620	5620	5618
State	Prediction	[8]	[9]	[10]
Ξ_{bc}	6989	6933	6928	7020
Ξ_{bb}	10369	10202	10198	10340

The magnetic moments are computed as

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_i \sigma_i | \phi_{sf} \rangle$$

where $|\phi_{sf}\rangle$ is the spin-flavour wave function of the quark and μ_i is the magnetic moment of i th quark.

Table 2: Spin-flavour wave functions of Σ_b and Λ_b baryons and its magnetic moment in terms of constituent quarks magnetic moments.

Baryon	Spin-flavour wave function	μ_B
Σ_b^+	$\frac{1}{\sqrt{18}} [2u \uparrow b \downarrow u \uparrow + 2u \uparrow u \uparrow b \downarrow + 2b \downarrow u \uparrow u \uparrow - u \uparrow u \downarrow b \uparrow - u \uparrow b \uparrow u \downarrow - u \downarrow b \uparrow u \uparrow - b \uparrow u \downarrow u \uparrow - b \uparrow u \uparrow u \downarrow - u \downarrow u \uparrow b \uparrow]$	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_b$
Σ_b^0	$\frac{1}{\sqrt{6}} [u \uparrow d \downarrow b \uparrow + d \uparrow u \downarrow b \uparrow + b \uparrow d \downarrow u \uparrow + b \uparrow u \downarrow d \uparrow - 2u \uparrow b \downarrow d \uparrow - 2d \uparrow b \downarrow u \uparrow + u \downarrow d \uparrow b \uparrow + d \downarrow u \uparrow b \uparrow - 2b \downarrow d \uparrow u \uparrow - 2b \downarrow u \uparrow d \uparrow + u \downarrow b \uparrow d \uparrow + d \downarrow b \uparrow u \uparrow - 2u \uparrow d \uparrow b \downarrow - 2d \uparrow u \uparrow b \downarrow + b \uparrow d \uparrow u \downarrow + b \uparrow u \uparrow d \downarrow + u \uparrow b \uparrow d \downarrow + d \uparrow b \uparrow u \downarrow]$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_b$
Σ_b^-	$\frac{1}{\sqrt{18}} [2d \uparrow b \downarrow d \uparrow + 2d \uparrow d \uparrow b \downarrow + 2b \downarrow d \uparrow d \uparrow - d \uparrow d \downarrow b \uparrow - d \uparrow b \uparrow d \downarrow - d \downarrow b \uparrow d \uparrow - b \uparrow d \downarrow d \uparrow - b \uparrow d \uparrow d \downarrow - d \downarrow d \uparrow b \uparrow]$	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_b$
Λ_b^0	$\frac{\sqrt{6}}{6} [u \downarrow d \uparrow b \uparrow - d \downarrow u \uparrow b \uparrow + u \downarrow b \uparrow d \uparrow - d \downarrow b \uparrow u \uparrow - u \uparrow d \downarrow b \uparrow + d \uparrow u \downarrow b \uparrow - b \uparrow d \downarrow u \uparrow + b \uparrow u \downarrow d \uparrow + b \uparrow d \uparrow u \downarrow - b \uparrow u \uparrow d \downarrow - u \uparrow b \uparrow u \downarrow + d \uparrow b \uparrow u \downarrow]$	μ_b

The magnetic moments of the bottom baryon are obtained as

$$\Lambda_b = -0.06 \mu_N, \Sigma_b^+ = 2.45 \mu_N, \Sigma_b^0 = 0.62 \mu_N, \Sigma_b^- = -1.00 \mu_N, \Xi_{bb}^0 = -2.00 \mu_N$$

Semileptonic transition of Λ_b baryon

In the heavy-quark limit, all the form factors describing the semileptonic decays are proportional to the universal function $\xi(\omega)$ known as the Isgur-Wise (IW) function.

$$F_1 = G_1 = \xi \\ F_2 = G_2 = F_3 = G_3 = 0$$

To consider the $\Lambda_b \rightarrow \Lambda_c$ we use the following form of the IW function obtained by Jenkins, Manohar and Wise:

$$\xi(\omega) = 0.99 \exp[-1.3(\omega - 1)]$$

The differential decay width has the form

$$\frac{d\Gamma}{d\omega} = \frac{2}{3} m_{\Lambda_c}^4 m_{\Lambda_b} A \xi^2(\omega) \sqrt{\omega^2 - 1} [3\omega(\eta + \eta^{-1}) - 2 - 4\omega^2]$$

Where $\eta = \frac{m_{\Lambda_b}}{m_{\Lambda_c}}$ and $A = G_F^2 / (2\pi)^3 |V_{cb}|^2 B(\Lambda_c \rightarrow ab)$.

$B(\Lambda_c \rightarrow ab)$ is the branching ratio for the decay $\Lambda_c \rightarrow a + b$ through which Λ_c is detected.

For the considered universal function $\xi(\omega)$ we get the slope as

$$\rho^2 = \left. \frac{d\xi(\omega)}{d\omega} \right|_{\omega=1} = 1.28$$

The decay width can be calculated as

$$\Gamma = \int_{\omega=1}^{\omega_{max}} \frac{2}{3} m_{\Lambda_c}^4 m_{\Lambda_b} A \xi^2(\omega) \sqrt{\omega^2 - 1} [3\omega(\eta + \eta^{-1}) - 2 - 4\omega^2] d\omega$$

where ω_{max} is defined as

$$\omega_{max} = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c}^2}{2m_{\Lambda_b} m_{\Lambda_c}}$$

using the obtained baryon masses we get

$$\Gamma = 8.87B \times 10^{10} \text{ s}^{-1}$$

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