Very Special Linear Gravity: A Gauge invariant Graviton mass

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Very Special Relativity (VSR)

Original idea from Cohen and Glashow (2006)[1]

- New Space-time fundamental symmetry:
  Traslations + Special Lorentz's Subgroups
- Same classical consequences of Special Relativity
- The addition of the discrete symmetries P, CP or T enlarges the symmetry group to the full Poincarè group

Small **CP VIOLATIONS** $\Longleftrightarrow$ Small **VSR EFFECTS** [2]

[1] Andrew G Cohen and Sheldon L Glashow. "Very special relativity". Physical review letters, 97(2):021601, 2006.

[2] Andrew G Cohen and Sheldon L Glashow. "A lorentz-violating origin of neutrino mass?" ArXiv, hep-ph/0605036, 2006.

Four parameter $SIM(2)$ Subgroup:

$$T_1 = K_1 + J_2 \; ; \;\; T_2 = K_2 - J_1 \; ; \;\; J_3 \; ; \;\; K_3 \; ,$$

where $\vec{J}$ and $\vec{K}$ are the usual rotations and boosts generators.

$SIM(2)$ Advantages

- Only 1 invariant Tensor: $\eta_{\mu\nu}$
- Directly implies CPT invariance in Quantum Theories

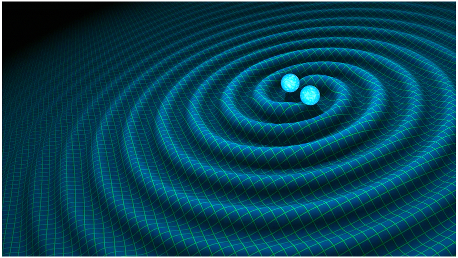
There exists a **preferred spacetime direction**, the light-like vector n_μ, left invariant by $SIM(2)$ transformations

$$n_\mu \underset{SIM(2)}{\longrightarrow} e^{\phi} n_\mu \, , \tag{1}$$

New possibilities for **Lorentz-violating** Lagrangian terms!

Due to Sakharov Conditions[3], the **Discrete Transformations** T and CP are **violated** in cosmology.

Therefore, VSR may become relevant in the propagation of **gravitational waves** from faraway sources.



$\Longrightarrow$ VSR theory of the $h_{\mu\nu}$ spin-2 field, $|h_{\mu\nu}| << \eta_{\mu\nu}$.

[3] A. D. Sakharov, Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe, Pisma Zh. Eksp.Teor. Fiz.5, 32 (1967).

Lagrangian in the form:

$$L = h^{\mu\nu} O_{\mu\nu\alpha\beta} h^{\alpha\beta} \tag{2}$$

Ingredients:

- Metric $\eta_{\mu\nu}$
- Momentum p_μ
- VSR vector $N^\mu = n^\mu/(n \cdot p)$

$$O = 3\,\eta\eta + 9\,pp\eta + 12\,pN\eta + 12\,ppNN\ , \tag{3}$$

Rules:

- Indices Symmetries: $\mu \Longleftrightarrow \nu$, $\alpha \Longleftrightarrow \beta$, $\mu\nu \Longleftrightarrow \alpha\beta$
- Gauge Invariance: $O_{\mu\nu\alpha\beta} p^\alpha = 0$

Equations of Motion (E.o.M.): $O_{\mu\nu\alpha\beta} h^{\alpha\beta} = 0$, where[4]

$$
\begin{aligned}
O_{\mu\nu\alpha\beta} = \ & \chi \Big(p_\mu p_\nu \eta_{\alpha\beta} - \frac{1}{2} p_\mu p_\alpha \eta_{\nu\beta} - \frac{1}{2} p_\mu p_\beta \eta_{\nu\alpha} + p_\alpha p_\beta \eta_{\mu\nu} - \frac{1}{2} p_\nu p_\beta \eta_{\mu\alpha} - \frac{1}{2} p_\nu p_\alpha \eta_{\mu\beta} \\
& - p^2 \eta_{\mu\nu}\eta_{\alpha\beta} + \frac{1}{2} p^2 \eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2} p^2 \eta_{\mu\beta}\eta_{\nu\alpha} - m_g^2 \eta_{\mu\nu}\eta_{\alpha\beta} + \frac{m_g^2}{2}\eta_{\mu\alpha}\eta_{\nu\beta} + \frac{m_g^2}{2}\eta_{\mu\beta}\eta_{\nu\alpha} \\
& + m_g^2 N_\mu N_\nu p_\alpha p_\beta - \frac{m_g^2}{2} N_\mu N_\alpha p_\nu p_\beta + - \frac{m_g^2}{2} N_\mu N_\beta p_\nu p_\alpha - \frac{m_g^2}{2} N_\nu N_\alpha p_\mu p_\beta - \frac{m_g^2}{2} N_\nu N_\beta p_\mu p_\alpha + m_g^2 N_\alpha N_\beta p_\mu p_\nu \\
& - m_g^2 p^2 N_\mu N_\nu g_{\alpha\beta} + \frac{m_g^2}{2} p^2 N_\mu N_\alpha g_{\nu\beta} + \frac{m_g^2}{2} p^2 N_\mu N_\beta g_{\nu\alpha} + \frac{m_g^2}{2} p^2 N_\nu N_\beta \eta_{\mu\alpha} + \frac{m_g^2}{2} p^2 \eta_{\mu\beta} N_\nu N_\alpha - m_g^2 p^2 N_\alpha N_\beta \eta_{\mu\nu} \\
& + m_g^2 \eta_{\mu\nu} N_\alpha p_\beta + m_g^2 \eta_{\mu\nu} p_\alpha N_\beta - \frac{m_g^2}{2}\eta_{\mu\alpha} N_\nu p_\beta - \frac{m_g^2}{2}\eta_{\mu\alpha} p_\nu N_\beta - \frac{m_g^2}{2}\eta_{\mu\beta} N_\nu p_\alpha - \frac{m_g^2}{2}\eta_{\mu\beta} p_\nu N_\alpha \\
& - \frac{m_g^2}{2}\eta_{\nu\alpha} N_\mu p_\beta - \frac{m_g^2}{2}\eta_{\nu\alpha} p_\mu N_\beta - \frac{m_g^2}{2}\eta_{\nu\beta} N_\mu p_\alpha - \frac{m_g^2}{2}\eta_{\nu\beta} p_\mu N_\alpha + m_g^2 \eta_{\alpha\beta} N_\mu p_\nu + m_g^2 \eta_{\alpha\beta} p_\mu N_\nu \Big).
\end{aligned}
$$

Quite messy!

[4] Alfaro, J., Santoni, A. (2022). Very special linear gravity: A gauge-invariant graviton mass. Physics Letters B, 829, 137080.

Compatible **Gauge Choices** $\partial_\mu h^{\mu\nu} = 0;\ N_\mu h^{\mu\nu} = 0;\ h = h^\mu_\mu = 0.$

New E.o.M. is Klein-Gordon like

$$(p^2 - m_g^2) h_{\mu\nu} = 0 \tag{4}$$

m_g is a **graviton mass**! That could be of interest for

- Universe's Accelerated Expansion[5]
- Dark Matter[6]

Gauge Invariance $\Longleftrightarrow$ only **2 Physical Degrees of Freedom** .

[5] De Felice, A., Gümrükçüoğlu, A. E., Lin, C., Mukohyama, S. (2013). On the cosmology of massive gravity. Classical and Quantum Gravity, 30(18), 184004.

[6] K. Aoki and S. Mukohyama, Massive gravitons as darkmatter and gravitational waves, Physical Review D94,024001 (2016).

In linearized General Relativity we have:

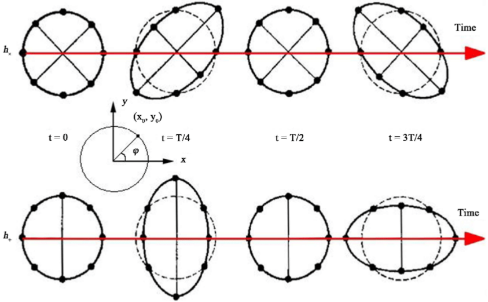


Figure: Lukanenkov, A. V. (2016). Gravitational Telescope. Journal of High Energy Physics, Gravitation and Cosmology, 2(2), 209-225.

In VSLG we have motion also in the **propagation direction**!

$$\delta\xi^z = \delta\xi_0^z + \frac{1}{2}\frac{m_g^2}{E^2}h_{13}\delta\xi_0^x + \frac{1}{2}\frac{m_g^2}{E^2}h_{23}\delta\xi_0^y + \frac{1}{2}\frac{m_g^4}{E^4}h_{33}\delta\xi_0^z\,. \qquad (5)$$

Averaged **Graviton Mass bound**:[7]

$$m_g < 10^{-24} eV \tag{6}$$

VSR effects are proportional to m_g^2/E^2 so roughly[8]...

LIGO/VIRGO $[10Hz, 10kHz]$

$$\frac{m_g^2}{E^2} \sim 10^{-20}$$

LISA $[0.1mHz, 1Hz]$

$$\frac{m_g^2}{E^2} \sim 10^{-10}$$

Still very small but hopefully detectable!

[7] de Rham, C., Deskins, J. T., Tolley, A. J., Zhou, S. Y. (2017). Graviton mass bounds. Reviews of Modern Physics, 89(2), 025004.

[8] $E =$ Energy of a graviton in a monochromatic gravitational wave

VSLG can represent a gauge invariant theory of a massive graviton.

Presence of mass implies new effects proportional to the **small parameter** m_g^2/E^2.

Upcoming Work

Further work to give more precise **numerical predictions** and confront them with the classic ones,
in particular for:

- Early Universe Gravitational Waves
- Non Linear Extension

Thanks for your attention!