

Investigating B_c semileptonic decays

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Outline

On B_c exclusive semileptonic modes

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

Conclusions

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General overview

The role of B_c

Why are we so interested in studying such a meson?

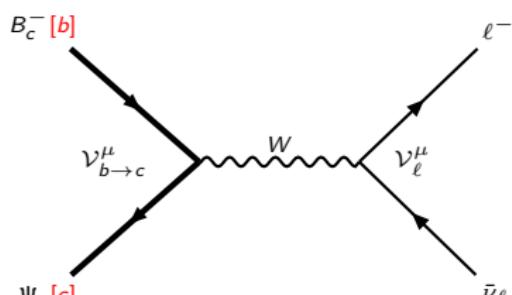
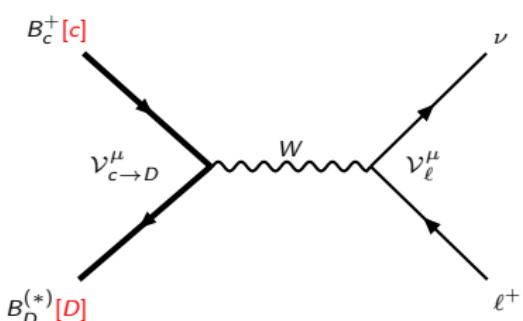
- It has the structure of the heavy quarkonium **but** it decays weakly;
- Considering the semileptonic modes, it allows to investigate and test Lepton Universality.

$$B_c^+ \rightarrow B_D^{(*)} \ell^+ \nu$$

$$D = \{ s, d \} \quad \text{and} \quad \ell = \{ e, \mu \}$$

$$B_c^- \rightarrow \Psi_c \ell^- \bar{\nu}$$

$$\Psi_c = \{ J/\psi, \eta_c \} \quad \text{and} \quad \ell = \{ e, \mu, \tau \}$$



$$\mathcal{V}_{c \rightarrow D}^\mu = -\frac{i e}{\sqrt{2} s_W} V_{cD}^* \gamma^\mu P_L$$

$$\mathcal{V}_{b \rightarrow c}^\mu = -\frac{i e}{\sqrt{2} s_W} V_{cb} \gamma^\mu P_L$$

$$\mathcal{V}_\ell^\mu = -\frac{i e}{\sqrt{2} s_W} \gamma^\mu P_L$$

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

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$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

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The role of B_c

Why are we interested in studying such decays?

There is the possibility of exploiting the heavy quark spin symmetry (HQSS). It relates modes with final pseudoscalar ($J = 0$) and vector meson ($J = 1$).

Why $B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$?

1. Dominant processes:

$$\Gamma \propto |V_{cD}|^2 \quad \text{with} \quad |V_{cs}| > |V_{cd}| > |V_{cb}|$$

2. Small phase-space:

- 2.1 the τ mode is phase-space forbidden;
- 2.2 extrapolation of the spin symmetry relations to the full kinematical range (they should be valid only at zero-recoil).

3. $\mathcal{B}(B_{s,d}^* \rightarrow B_{s,d} \gamma) = 1$:

only radiative decay has been observed \Rightarrow fully differential distributions can be considered.

Why $B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$?

- 1. It is possible to exploit the expansion series in terms of the relative velocity (\tilde{v}) between the heavy quarks bounded in the meson.
- 2. The expansion still preserves the HQSS; this gives relations among matrix elements (FF).

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Conclusions

Exclusive $c \rightarrow \{ s, d \} \bar{\ell} \nu_\ell$ modes:

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

Based on:

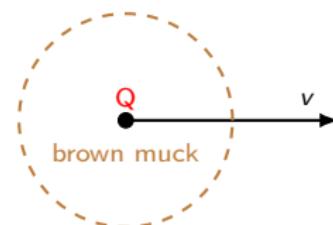
P. Colangelo, F. De Fazio, FL,

Role of $B_c^+ \rightarrow B_{s,d}^{()} \bar{\ell} \nu_\ell$ in the Standard Model and in the search for BSM signals,*
Phys. Rev. D **103** (2021), no. 7 075019, [arXiv:2102.05365].

Applying the Heavy Quark Spin Symmetry (HQSS)

In the infinite heavy quark mass limit $m_Q \gg \Lambda_{\text{QCD}}$ the QCD Lagrangian exhibits a heavy quark spin symmetry, with the decoupling of the heavy quark spin from gluons.

Such systems can be viewed as a **freely** propagating point-like colour source (the heavy quark), dressed by strongly interacting *brown muck*.



$$\langle \Psi'(v') | J(q) | \Psi(v) \rangle = \langle Q'(v'), \pm 1/2 | J(q) | Q(v), \pm 1/2 \rangle \times \underbrace{\langle \text{light}, v', j', m'_j | \text{light}, v, j, m_j \rangle}_{\sim f(v' - v)}$$

In the semileptonic $B_c^+(p) \rightarrow B_a^{(*)}(p', (\epsilon))$ decays induced by the $c \rightarrow a = \{ s, d \}$ transitions, since $m_c \ll m_b$, the energy released to the final hadronic system is much smaller than m_b .

The \bar{b} quark remains **almost** unaffected \Rightarrow the final meson keeps the same B_c four-velocity v :

$$\begin{cases} p_\mu = m_{B_c} v_\mu \\ p'_\mu = m_{B_a^{(*)}} v'_\mu = m_{B_a^{(*)}} v_\mu + k_\mu \end{cases} \Rightarrow \begin{cases} q_\mu = p_\mu - p'_\mu = (m_{B_c} - m_{B_a^{(*)}}) v_\mu - k_\mu \\ v \cdot k \sim \mathcal{O}(1/m_b) \end{cases}$$

Applying the Heavy Quark Spin Symmetry (HQSS)

Doublets (B_c^+, B_c^{*+}) and (B_a, B_a^*)

Heavy (pseudoscalar, vector) mesons are collected in **doublets**.

Its components differ **only** for heavy quark spin orientation.

$$H^{c\bar{b}} = P_+(v) [B_c^{*\mu} \gamma_\mu - B_c \gamma_5] P_-(v) \quad \text{and} \quad H_a^{\bar{b}} = [B_a^{*\mu} \gamma_\mu - B_a \gamma_5] P_-(v)$$

$$[\not{y}, H] = 2H \quad \vee \quad \{\not{y}, H\} = 0 \quad \text{with} \quad H = H^{c\bar{b}} \quad \wedge \quad H = H_a^{\bar{b}} \quad \text{and} \quad P_\pm(v) = \frac{1 \pm \not{y}}{2}$$

Spin transformations

$$H^{c\bar{b}} \rightarrow S_c H^{c\bar{b}} S_b^\dagger \quad H_a^{\bar{b}} \rightarrow (U H^{\bar{b}})_a S_b^\dagger$$

$S_{c,b} \in$ heavy quark spin transformations

$U_a \in$ light quark $SU(3)_F$ transformations

Matrix elements (**must be invariant** under rotations of the \bar{b} spin)

$$\langle B_a^{(*)}(v, k, (\epsilon)) | \bar{q} \Gamma Q | B_c(v) \rangle = -\sqrt{m_{B_c} m_{B_a^{(*)}}} \text{Tr} \left[\bar{H}_a^{\bar{b}} \Omega_a(v, a_0 k) \Gamma H^{c\bar{b}} \right]$$

$$\bar{H}_a^{\bar{b}} = \gamma^0 H_a^{\bar{b}\dagger} \gamma^0 \quad \text{and} \quad \Omega_a(v, a_0 k) = \color{red}{\Omega_{1a} + k a_0 \Omega_{2a}}$$

dimensionless nonperturbative functions

Reducing the number of the independent form factors

Theoretical basis: in the infinite mass limit $m_Q \gg \Lambda_{\text{QCD}}$: the heavy quark spin decouples \Rightarrow
 Theoretical uncertainties reduced: 10 form factors expressed in terms of only 2.

$$B_c \rightarrow P = B_{s,d}$$

$$\langle P(p') | \bar{q} \Gamma Q | B_c(p) \rangle \sim [f_{+,0,T}]^{B_c \rightarrow P}(q^2)$$

$$\langle P(v, k) | \bar{q} \Gamma Q | B_c(v) \rangle \sim \Omega_{1,2}(y)$$

$$B_c \rightarrow V = B_{s,d}^*$$

$$\langle V(p', \epsilon) | \bar{q} \Gamma Q | B_c(p) \rangle \sim [V, A_{0,1,2}, T_{0,1,2}]^{B_c \rightarrow V}(q^2)$$

$$\langle V(v, k, \epsilon) | \bar{q} \Gamma Q | B_c(v) \rangle \sim \Omega_{1,2}(y)$$

$$\Rightarrow \begin{cases} f_+ = \sqrt{\frac{m_P}{m_{B_c}}} [\Omega_1 + (m_{B_c} - m_P) a_0 \Omega_2] \\ f_0 = \sqrt{\frac{m_P}{m_{B_c}}} \frac{1}{m_{B_c}^2 - m_P^2} [(m_{B_c}^2 - m_P^2 + q^2) \Omega_1 + (m_{B_c} + m_P) ((m_{B_c} - m_P)^2 - q^2) a_0 \Omega_2] \\ f_T = \sqrt{\frac{m_P}{m_{B_c}}} (m_{B_c} + m_P) a_0 \Omega_2 \end{cases}$$

$$V = \sqrt{\frac{m_V}{m_{B_c}}} (m_{B_c} + m_V) a_0 \Omega_2$$

$$A_0 = \frac{1}{2\sqrt{m_{B_c} m_V}} [2 m_{B_c} \Omega_1 + (m_{B_c}^2 - m_V^2 + q^2) a_0 \Omega_2]$$

$$A_1 = 2 \frac{\sqrt{m_{B_c} m_V}}{m_{B_c} + m_V} \Omega_1$$

$$A_2 = -\sqrt{\frac{m_V}{m_{B_c}}} (m_{B_c} + m_V) a_0 \Omega_2$$

$$T_0 = 0$$

$$T_1 = 2 \sqrt{\frac{m_V}{m_{B_c}}} [\Omega_1 - m_V a_0 \Omega_2]$$

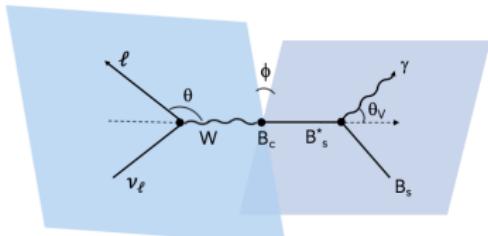
$$T_2 = 2\sqrt{m_{B_c} m_V} a_0 \Omega_2$$

NP Hamiltonian

$$\mathcal{H}_{\text{NP}} = \frac{G_F V_{cD}^*}{\sqrt{2}} \left[(1 + \epsilon_V^\ell) \mathcal{O}_{SM} + \epsilon_S^\ell \mathcal{O}_S + \epsilon_P^\ell \mathcal{O}_P + \epsilon_T^\ell \mathcal{O}_T + \epsilon_R^\ell \mathcal{O}_R \right] + \text{h.c.}$$

Kinematics (B_c rest frame)

$$B_c^+ \rightarrow V (\rightarrow P \gamma) \bar{\ell} \nu_\ell$$



θ_V : \angle btw B_s^* flight direction and photon direction;

θ : \angle btw lepton pair flight direction and W flight direction;

ϕ : \angle btw lepton plane and hadron plane.

Fully differential distribution $d\Gamma \equiv d^4\Gamma(B_c \rightarrow V(\rightarrow P \gamma) \bar{\ell} \nu_\ell) / dq^2 d \cos \theta d\phi d \cos \theta_V$

$$\begin{aligned} d\Gamma = \mathcal{N} \times & \{ I_{1s} \sin^2 \theta_V + I_{1c} (3 + \cos 2\theta_V) + (I_{2s} \sin^2 \theta_V + I_{2c} (3 + \cos 2\theta_V)) \cos 2\theta + I_{3s} \sin^2 \theta_V \sin^2 \theta \cos 2\phi + \\ & + I_{4s} \sin 2\theta_V \sin 2\theta \cos \phi + I_{5s} \sin 2\theta_V \sin \theta \cos \phi + (I_{6s} \sin^2 \theta_V + I_{6c} (3 + \cos 2\theta_V)) \cos \theta + \\ & + I_{7s} \sin 2\theta_V \sin \theta \sin \phi + I_{8s} \sin 2\theta_V \sin 2\theta \sin \phi + I_{9s} \sin^2 \theta_V \sin^2 \theta \sin 2\phi \} \end{aligned}$$

$$\mathcal{N} = \frac{3 G_F^2 |V_{CKM}|^2 \mathcal{B}(V \rightarrow P \gamma)}{128 (2\pi)^4 m_{B_c}^2} |P_V| \left(1 - \frac{m_\ell^2}{q^2}\right)^2$$

Angular coefficient functions ($\epsilon_V^\ell, \epsilon_S^\ell, \epsilon_P^\ell, \epsilon_T^\ell, \epsilon_R^\ell$)

$$I_i = |\mathbf{1} + \epsilon_V^\ell|^2 I_i^{\mathbf{SM}} + \sum_X |\epsilon_X^\ell|^2 I_i^{\mathbf{NP}, X} + 2 \sum_X \operatorname{Re}[\epsilon_X^\ell (\mathbf{1} + \epsilon_V^\ell)] I_i^{\mathbf{INT}, X} + 2 \sum_{X,Y} \operatorname{Re}[\epsilon_X^\ell \epsilon_Y^\ell] I_i^{\mathbf{INT}, XY}$$

$$I_7 = 2 \sum_X \operatorname{Im}[\epsilon_X^\ell (\mathbf{1} + \epsilon_V^\ell)] I_7^{\mathbf{INT}, X} + 2 \sum_{X,Y} \operatorname{Im}[\epsilon_X^\ell \epsilon_Y^\ell] I_7^{\mathbf{INT}, XY}$$

$$I_{8,9} = 2 \operatorname{Im}[\epsilon_R^\ell (\mathbf{1} + \epsilon_V^\ell)] I_{8,9}^{\mathbf{INT}, R}$$

$$i = \{ \mathbf{1s}, \mathbf{1c}, \dots, \mathbf{6c} \} \quad \text{and} \quad X, Y \in \{ P, T, R \} \quad \text{with} \quad Y \neq X$$

Helicity amplitudes in SM and beyond

SM

$$H_0 = \frac{(m_{B_c} + m_V)^2 (m_{B_c}^2 - m_V^2 - q^2) A_1 - \lambda A_2}{2 m_V (m_{B_c} + m_V) \sqrt{q^2}} = \sqrt{\frac{m_{B_c}}{m_V}} \frac{m_{B_c}^2 - m_V^2 - q^2}{\sqrt{q^2}} \Omega_1 + \frac{\lambda}{2 \sqrt{m_{B_c} m_V q^2}} a_0 \Omega_2$$

$$H_{\pm} = \frac{(m_{B_c} + m_V)^2 A_1 \mp \sqrt{\lambda} V}{m_{B_c} + m_V} = \sqrt{\frac{m_V}{m_{B_c}}} [2 m_{B_c} \Omega_1 \mp \sqrt{\lambda} a_0 \Omega_2]$$

$$H_t = -\sqrt{\frac{\lambda}{q^2}} A_0 = -\frac{1}{2} \sqrt{\frac{\lambda}{m_{B_c} m_V q^2}} [2 m_{B_c} \Omega_1 + (m_{B_c}^2 - m_V^2 + q^2) a_0 \Omega_2]$$

$$\lambda = \lambda(m_{B_c}^2, m_V^2, q^2)$$

NP

$$H_{\pm}^{\text{NP}} = \frac{(m_{B_c}^2 - m_V^2 \pm \sqrt{\lambda}) (T_1 + T_2) + q^2 (T_1 - T_2)}{\sqrt{q^2}} = \\ = 2 \sqrt{\frac{m_V}{m_{B_c} q^2}} \left[(m_{B_c}^2 - m_V^2 + q^2 \pm \sqrt{\lambda}) \Omega_1 + ((m_{B_c} + m_V) ((m_{B_c} - m_V)^2 - q^2) \pm (m_{B_c} - m_V) \sqrt{\lambda}) a_0 \Omega_2 \right]$$

$$H_L^{\text{NP}} = 4 \left[\frac{\lambda}{m_V (m_{B_c} + m_V)^2} T_0 + 2 \frac{m_{B_c}^2 + m_V^2 - q^2}{m_V} T_1 + 4 m_V T_2 \right] = \\ = \frac{16}{\sqrt{m_{B_c} m_V}} \left[(m_{B_c}^2 + m_V^2 - q^2) \Omega_1 - m_V ((m_{B_c} - m_V)^2 - q^2) a_0 \Omega_2 \right]$$

Form factors (to compute Ω_1 and Ω_2) from:

HPQCD Collaboration, L. J. Cooper, C. T. Davies, J. Harrison, J. Komijani, and M. Wingate,
 $B_c \rightarrow B_{s(d)}$ form factors from lattice QCD,

Phys. Rev. D 102, (2020), no. 1 014513 [arXiv:2003.00914] [Erratum: Phys. Rev. D 103, 099901 (2021)]

Universal functions $\Omega_1(y)$ and $a_0 \Omega_2(y)$

1. Obtained from the form factors $f_{0,+}$ used to describe the processes $B_C \rightarrow B_{s,d}$ computed by lattice QCD.

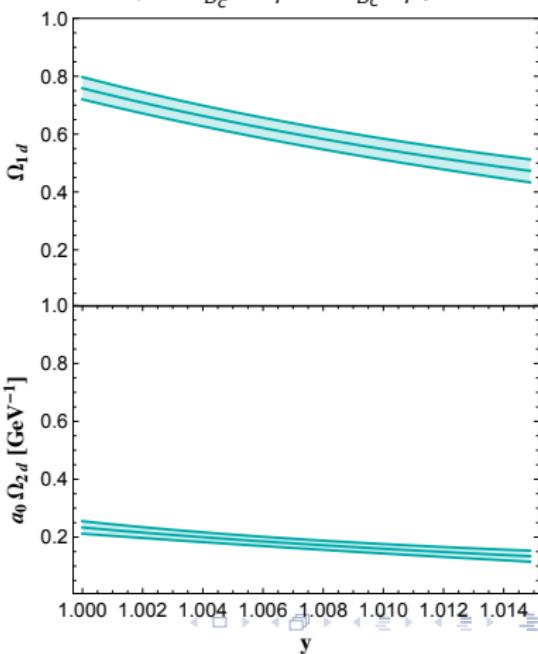
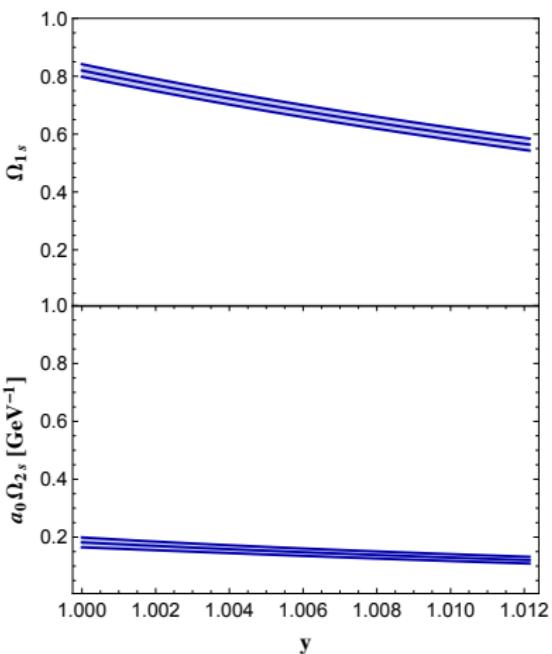
2. Exploiting HQSS:

$B_C \rightarrow B_{s,d}^{(*)}$ are simultaneously described.

$$\Omega_1(y) = \frac{m_{B_C} + m_P}{2 q^2 \sqrt{m_{B_C} m_P}} \left((m_{B_C} - m_P)^2 (f_0 - f_+) + q^2 f_+ \right)$$

$$a_0 \Omega_2(y) = \frac{1}{2 q^2 \sqrt{m_{B_C} m_P}} \left((m_{B_C}^2 - m_P^2) (f_0 - f_+) + q^2 f_+ \right)$$

$$q^2 = m_{B_C}^2 + m_P^2 - 2 m_{B_C} m_P y$$



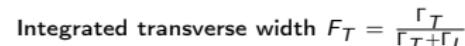
Observables within SM and NP context

Branching fractions in SM

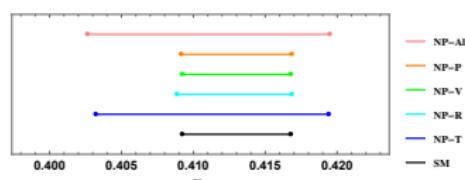
(a, ℓ)	$\mathcal{B}(B_c^+ \rightarrow B_a \ell^+ \nu_\ell)$	$\mathcal{B}(B_c^+ \rightarrow B_a^* \ell^+ \nu_\ell)$
(s, μ)	$1.25(4) \times 10^{-2} x_s$	$3.0(1) \times 10^{-2} x_s$
(s, e)	$1.31(4) \times 10^{-2} x_s$	$3.2(1) \times 10^{-2} x_s$
(d, μ)	$8.3(5) \times 10^{-4} x_d$	$20(1) \times 10^{-4} x_d$
(d, e)	$8.7(5) \times 10^{-4} x_d$	$21(1) \times 10^{-4} x_d$

$$x_s = \left[\frac{|V_{CS}|}{0.987} \right]^2$$

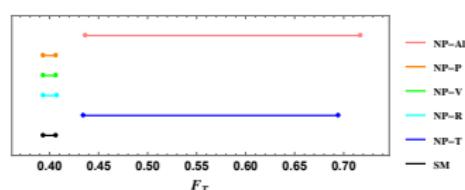
$$x_d = \left[\frac{|V_{cd}|}{0.221} \right]^2$$



$$B_c \rightarrow B_c^* \mu^+ \nu_\mu$$



$$B_c \rightarrow B_d^* \mu^+ \nu_\mu$$



NP parameters from:

D.Becirevic, F.Jaffredo, A.Penuelas, O.Sumensari,
New Physics effects in leptonic and semileptonic
decays,
JHEP 05 (2021) 175. [arXiv:2012.09872]

	$F_T(B_s^*)$	$F_T(B_d^*)$
SM	0.413(4)	0.40(01)
SM + T	0.411(8)	0.56(13)

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Exclusive $b \rightarrow c \ell \bar{\nu}_\ell$ modes:

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

Based on:

P. Colangelo, F. De Fazio, FL,N.Losacco, M. Novoa-Brunet,
Relations among $B_c \rightarrow J/\psi, \eta_c$ form factors,
arXiv:2205.08933.

Expansion of the heavy quark field and the QCD Lagrangian

Positive energy component of the field

To construct the heavy quark expansion, the heavy quark QCD field $Q(x)$ with mass m_Q is written factorizing a fast oscillation mass term:

$$Q(x) = e^{i m_Q v \cdot x} \psi(x) = e^{i m_Q v \cdot x} (\psi_+(x) + \psi_-(x)) \quad \text{with} \quad \psi_\pm(x) = P_\pm(v) \psi(x)$$

v is identified with the heavy meson (quarkonium) 4-velocity, with $v^2 = 1$.

$$\text{EoM} \Rightarrow \psi_-(x) = \frac{1}{2 m_Q + i v \cdot \vec{D}} i \vec{D}_\perp \psi_+(x) \quad \text{with} \quad \vec{D}_{\perp\mu} = D_\mu - (v \cdot D) v_\mu$$

$$\text{Meson R.F.: } v = (1, 0, 0, 0)$$

$$v \cdot \vec{D} = (D_t, 0) \quad \text{and} \quad \vec{D}_{\perp\mu} = (0, D_i) \quad \text{and} \quad \int_V d^3x \bar{\psi}_+ \psi_+ \sim 1$$

We define \tilde{v} as the relative heavy quark 3-velocity in the hadron rest frame, with $\tilde{v} = |\tilde{v}| \ll 1$.
 The power counting of the various operators (objects) is set within NRQCD:

$$[D_t] = [E] \sim [\tilde{v}]^2 \quad \text{and} \quad [D_i] = [p] \sim [\tilde{v}] \quad \text{and} \quad [V] \sim [\tilde{v}]^{-3} \Rightarrow [\psi_+] \sim [\tilde{v}]^{3/2}$$

Hence:

$$Q(x) = e^{i m_Q v \cdot x} \left(1 + \underbrace{\frac{i \vec{D}_\perp}{2 m_Q}}_{\sim \tilde{v}} + \underbrace{\frac{(-iv \cdot \vec{D}) i \vec{D}_\perp}{4 m_Q^2}}_{\sim \tilde{v}^2 \times \tilde{v}} + \dots \right) \underbrace{\psi_+(x)}_{\sim \tilde{v}^{3/2}}$$

Expansion of the heavy quark field and the QCD Lagrangian

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_+ \left(\underbrace{i v \cdot \vec{D}}_{\sim \tilde{v}^2} + \underbrace{\frac{(i \vec{D}_\perp)^2}{2 m_Q}}_{\sim \tilde{v}^2} + \frac{g}{4 m_Q} \sigma \cdot G_\perp + \frac{1}{4 m_Q^2} i \vec{D} (-i v \cdot \vec{D}) i \vec{D} + \dots \right) \psi_+ = \mathcal{L}_0 + \mathcal{L}_1 + \dots$$

\mathcal{L}_0 contains only operators $\mathcal{O}(\tilde{v}^2)$.
 $\mathcal{L}_1 + \dots$ contain subleading terms.

$$G_{\perp \mu\nu} = (g_{\mu\alpha} - v_\mu v_\alpha) (g_{\nu\beta} - v_\nu v_\beta) G^{\alpha\beta} \quad \mapsto \quad \text{in the R.F.: } G_{\perp \mu\nu} = G_{ij}$$

$$E_i = G_{0i} \Rightarrow [E_i] \sim [\tilde{v}]^3 \quad \text{and} \quad B_i = \frac{1}{2} \epsilon_{ijk} G^{jk} \Rightarrow [B_i] \sim [\tilde{v}]^4$$

Leading order \mathcal{L}_0

From \mathcal{L}_0 one obtains the EoM for ψ_+ : $\Rightarrow \left(i v \cdot \vec{D} + \frac{(i \vec{D}_\perp)^2}{2 m_Q} \right) \psi_+ = 0$

The EoM breaks the Flavour Symmetry (m_Q)
 but still preserves the Spin Symmetry.

Subleading terms \mathcal{L}_1

$$\mathcal{L}_1 = \overbrace{\frac{g}{4 m_Q} \bar{\psi}_+ \sigma \cdot G_\perp \psi_+}^{\mathcal{L}_{1,1}} + \overbrace{\frac{1}{4 m_Q^2} \bar{\psi}_+ i \vec{D} (-i v \cdot \vec{D}) i \vec{D} \psi_+}^{\mathcal{L}_{1,2}}$$

$$\mathcal{L}_{1,2} = \frac{1}{4 m_Q^2} \bar{\psi}_+ \left[\frac{(i \vec{D}_\perp)^4}{2 m_Q} + \frac{g}{2} \sigma \cdot G_\perp \frac{(i \vec{D}_\perp)^2}{2 m_Q} + g v^\alpha \vec{D}_\perp^\beta G_{\alpha\beta} - g v^\alpha i \vec{D}_{\perp\lambda} \sigma^{\lambda\beta} G_{\alpha\beta} \right] \psi_+ =$$

$$= \mathcal{L}_{1,2}^{(1)} + \cancel{\mathcal{L}_{1,2}^{(2)}} + \mathcal{L}_{1,2}^{(3)} + \mathcal{L}_{1,2}^{(4)}$$

$\mathcal{L}_{1,2}^{(2)}$ is of higher order in \tilde{v} expansion.

Meson form factors in the effective theory

Current

To obtain the meson form factors in the effective theory, we expand the weak current involving two heavy quarks $\bar{Q}' \Gamma Q$, with Γ a generic Dirac matrix:

$$\bar{Q}' \Gamma Q = J_0 + \left(\frac{J_{0,1}}{2 m_Q} + \frac{J_{1,0}}{2 m_{Q'}} \right) + \left(- \frac{J_{0,2}}{4 m_Q^2} - \frac{J_{2,0}}{4 m_{Q'}^2} + \frac{J_{1,1}}{4 m_Q m_{Q'}} \right) + \mathcal{O}(1/m^3)$$

with

$$J_0 = \bar{\psi}'_+ \Gamma \psi_+$$

$$J_{0,1} = \bar{\psi}'_+ \Gamma i \vec{D}_\perp \psi_+ \quad J_{1,0} = \bar{\psi}'_+ (-i \vec{D}'_\perp) \Gamma \psi_+$$

$$J_{0,2} = \bar{\psi}'_+ \Gamma (i v \cdot \vec{D}) i \vec{D}_\perp \psi_+ \quad J_{2,0} = \bar{\psi}'_+ i \vec{D}'_\perp (i v' \cdot \vec{D}) \Gamma \psi_+ \quad J_{1,1} = \bar{\psi}'_+ (-i \vec{D}'_\perp) \Gamma i \vec{D}_\perp \psi_+$$

Doublets (B_c^+, B_c^{*+}) and ($\eta_c, J/\psi$)

$$H^{c\bar{b}} \equiv H(v) = P_+(v) [B_c^{*\mu} \gamma_\mu - B_c \gamma_5] P_-(v) \quad \text{and} \quad H^{c\bar{c}} \equiv H'(v') = P_+(v') [\Psi^{*\mu} \gamma_\mu - \eta_c \gamma_5] P_-(v')$$

Matrix Elements and Trace Formalism

Local corrections

- Leading order

$$\langle M'(v') | J_0 | M(v) \rangle = -\Delta(w) \text{Tr}[\bar{H}'(v') \Gamma H(v)] \quad \text{with} \quad w = v \cdot v'$$

- $1/m$ corrections

$$\langle M'(v') | \bar{\psi}'_+ \Gamma i \overleftrightarrow{D}_\alpha \psi_+ | M(v) \rangle = -\text{Tr}[\Delta_\alpha(v, v') \bar{H}' \Gamma H]$$

$$\langle M'(v') | \bar{\psi}'_+ i \overleftrightarrow{D}_\alpha \Gamma \psi_+ | M(v) \rangle = +\text{Tr}[\bar{\Delta}_\alpha(v', v) \bar{H}' \Gamma H]$$

$$\Delta_\alpha(v, v') = \Delta_+(v + v')_\alpha + \Delta_-(v - v')_\alpha - \Delta_3 \gamma_\alpha \quad \bar{\Delta}_\alpha(v', v) = \Delta_\alpha(v', v) [\Delta_{\pm,3} \rightarrow \bar{\Delta}_{\pm,3}]$$

- $1/m^2$ corrections

$$\langle M'(v') | \bar{\psi}'_+ (-i \overleftrightarrow{D}_\alpha) \Gamma i \overleftrightarrow{D}_\beta \psi_+ | M(v) \rangle = -\text{Tr}[\psi_{\alpha\beta}(v, v') \bar{H}' \Gamma H] \quad \text{with} \quad \psi_{\alpha\beta} = \frac{1}{2} [\psi_{\alpha\beta}^S + \psi_{\alpha\beta}^A]$$

$$\begin{aligned} \psi_{\alpha\beta}^S &= \psi_1^S g_{\alpha\beta} + \psi_2^S (v + v')_\alpha (v + v')_\beta + \psi_3^S (v - v')_\alpha (v - v')_\beta + \psi_4^S [(v + v')_\alpha \gamma_\beta + (v + v')_\beta \gamma_\alpha] + \\ &\quad + \psi_5^S [(v - v')_\alpha \gamma_\beta + (v - v')_\beta \gamma_\alpha] + \psi_6^S [(v + v')_\alpha (v - v')_\beta + (v - v')_\alpha (v - v')_\beta] \end{aligned}$$

$$\psi_{\alpha\beta}^A = \psi_1^A [v_\alpha v'_\beta - v_\beta v'_\alpha] + \psi_2^A [(v - v')_\alpha \gamma_\beta - (v - v')_\beta \gamma_\alpha] + \psi_3^A i \sigma_{\alpha\beta} +$$

Non-local corrections

$$+ \psi_4^A [(v + v')_\alpha \gamma_\beta - (v + v')_\beta \gamma_\alpha]$$

- $1/m$ and $1/m^2$ corrections

$$\langle M'(v') | i \int d^4x T[J_0(0), \mathcal{L}_1(x)] | M(v) \rangle = -\frac{\chi_1(w)}{2 m_Q^2} \text{Tr}[\bar{H}' \Gamma H] + \frac{1}{8 m_Q} \text{Tr}[\chi_{2\mu\nu}(v, v') \bar{H}' \Gamma P_+ i \sigma^{\mu\nu} H]$$

$$\langle M'(v') | i \int d^4x T[J_0(0), \mathcal{L}'_1(x)] | M(v) \rangle = -\frac{\tilde{\chi}_1(w)}{2 m_{Q'}^2} \text{Tr}[\bar{H}' \Gamma H] + \frac{1}{8 m_{Q'}} \text{Tr}[\tilde{\chi}_{2\mu\nu}(v', v) \bar{H}' i \sigma^{\mu\nu} P'_+ \Gamma H]$$

$$\chi_{2\mu\nu} = \chi_2^A i \sigma_{\mu\nu} + \chi_2^C (v'_\mu \gamma_\nu - v'_\nu \gamma_\mu) \quad \tilde{\chi}_{2\mu\nu} = \tilde{\chi}_2^A i \sigma_{\mu\nu} + \chi_2^A (v_{\mu'} v_\nu - v_{\nu'} v_\mu)$$

Matrix Elements and Trace Formalism

Local corrections

- Leading order

$$\langle M'(v') | J_0 | M(v) \rangle = -\Delta(w) \text{Tr} [\bar{H}'(v') \Gamma H(v)] \quad \text{with} \quad w = v \cdot v'$$

- $1/m$ corrections

$$\langle M'(v') | \bar{\psi}'_+ \Gamma i \vec{D}_\alpha \psi_+ | M(v) \rangle = -\text{Tr} [\Delta_\alpha(v, v') \bar{H}' \Gamma H]$$

$$\langle M'(v') | \bar{\psi}'_+ i \overleftrightarrow{D}_\alpha \Gamma \psi_+ | M(v) \rangle = +\text{Tr} [\bar{\Delta}_\alpha(v', v) \bar{H}' \Gamma H]$$

$$\Delta_\alpha(v, v') = \Delta_+(v + v')_\alpha + \Delta_-(v - v')_\alpha - \Delta_3 \gamma_\alpha \quad \bar{\Delta}_\alpha(v', v) = \Delta_\alpha(v', v) [\Delta_{\pm,3} \rightarrow \bar{\Delta}_{\pm,3}]$$

- $1/m^2$ corrections

$$\langle M'(v') | \bar{\psi}'_+ (-i \overleftrightarrow{D}_\alpha) \Gamma i \vec{D}_\beta \psi_+ | M(v) \rangle = -\text{Tr} [\psi_{\alpha\beta}(v, v') \bar{H}' \Gamma H] \quad \text{with} \quad \psi_{\alpha\beta} = \frac{1}{2} [\psi_{\alpha\beta}^S + \psi_{\alpha\beta}^A]$$

$$\begin{aligned} \psi_{\alpha\beta}^S &= \psi_1^S g_{\alpha\beta} + \psi_2^S (v + v')_\alpha (v + v')_\beta + \psi_3^S (v - v')_\alpha (v - v')_\beta + \psi_4^S [(v + v')_\alpha \gamma_\beta + (v + v')_\beta \gamma_\alpha] + \\ &\quad + \psi_5^S [(v - v')_\alpha \gamma_\beta + (v - v')_\beta \gamma_\alpha] + \psi_6^S [(v + v')_\alpha (v - v')_\beta + (v - v')_\alpha (v - v')_\beta] \end{aligned}$$

$$\psi_{\alpha\beta}^A = \psi_1^A [v_\alpha v'_\beta - v_\beta v'_\alpha] + \psi_2^A [(v - v')_\alpha \gamma_\beta - (v - v')_\beta \gamma_\alpha] + \psi_3^A i \sigma_{\alpha\beta} +$$

Non-local corrections

$$+ \psi_4^A [(v + v')_\alpha \gamma_\beta - (v + v')_\beta \gamma_\alpha]$$

- $1/m$ and $1/m^2$ corrections

$$\langle M'(v') | i \int d^4x T [J_0(0), \mathcal{L}_1(x)] | M(v) \rangle = -\frac{\chi_1(w)}{2 m_Q^2} \text{Tr} [\bar{H}' \Gamma H] + \frac{1}{8 m_Q} \text{Tr} [\chi_{2\mu\nu}(v, v') \bar{H}' \Gamma P_+ i \sigma^{\mu\nu} H]$$

$$\langle M'(v') | i \int d^4x T [J_0(0), \mathcal{L}'_1(x)] | M(v) \rangle = -\frac{\bar{\chi}_1(w)}{2 m_{Q'}^2} \text{Tr} [\bar{H}' \Gamma H] + \frac{1}{8 m_{Q'}} \text{Tr} [\bar{\chi}_{2\mu\nu}(v', v) \bar{H}' i \sigma^{\mu\nu} P'_+ \Gamma H]$$

$$\chi_{2\mu\nu} = \chi_2^A i \sigma_{\mu\nu} + \chi_2^C (v'_\mu \gamma_\nu - v'_\nu \gamma_\mu) \quad \bar{\chi}_{2\mu\nu} = \bar{\chi}_2^A i \sigma_{\mu\nu} + \chi_2^C (v_\mu \gamma_\nu - v_\nu \gamma_\mu)$$

Matrix Elements and Trace Formalism

Local corrections

- Leading order

$$\langle M'(v') | J_0 | M(v) \rangle = -\Delta(w) \text{Tr} [\bar{H}'(v') \Gamma H(v)] \quad \text{with} \quad w = v \cdot v'$$

- $1/m$ corrections

$$\langle M'(v') | \bar{\psi}'_+ \Gamma i \overleftrightarrow{D}_\alpha \psi_+ | M(v) \rangle = -\text{Tr} [\Delta_\alpha(v, v') \bar{H}' \Gamma H]$$

$$\langle M'(v') | \bar{\psi}'_+ i \overleftrightarrow{D}_\alpha \Gamma \psi_+ | M(v) \rangle = +\text{Tr} [\bar{\Delta}_\alpha(v', v) \bar{H}' \Gamma H]$$

$$\Delta_\alpha(v, v') = \Delta_+(v + v')_\alpha + \Delta_-(v - v')_\alpha - \Delta_3 \gamma_\alpha \quad \bar{\Delta}_\alpha(v', v) = \Delta_\alpha(v', v) [\Delta_{\pm,3} \rightarrow \bar{\Delta}_{\pm,3}]$$

- $1/m^2$ corrections

$$\langle M'(v') | \bar{\psi}'_+ (-i \overleftrightarrow{D}_\alpha) \Gamma i \overleftrightarrow{D}_\beta \psi_+ | M(v) \rangle = -\text{Tr} [\psi_{\alpha,\beta}(v, v') \bar{H}' \Gamma H] \quad \text{with} \quad \psi_{\alpha\beta} = \frac{1}{2} [\psi_{\alpha\beta}^S + \psi_{\alpha\beta}^A]$$

$$\begin{aligned} \psi_{\alpha\beta}^S &= \psi_1^S g_{\alpha\beta} + \psi_2^S (v + v')_\alpha (v + v')_\beta + \psi_3^S (v - v')_\alpha (v - v')_\beta + \psi_4^S [(v + v')_\alpha \gamma_\beta + (v + v')_\beta \gamma_\alpha] + \\ &\quad + \psi_5^S [(v - v')_\alpha \gamma_\beta + (v - v')_\beta \gamma_\alpha] + \psi_6^S [(v + v')_\alpha (v - v')_\beta + (v - v')_\alpha (v - v')_\beta] \end{aligned}$$

$$\psi_{\alpha\beta}^A = \psi_1^A [v_\alpha v'_\beta - v_\beta v'_\alpha] + \psi_2^A [(v - v')_\alpha \gamma_\beta - (v - v')_\beta \gamma_\alpha] + \psi_3^A i \sigma_{\alpha\beta} +$$

$$\text{Non-local corrections} \quad + \psi_4^A [(v + v')_\alpha \gamma_\beta - (v + v')_\beta \gamma_\alpha]$$

- $1/m$ and $1/m^2$ corrections

$$\langle M'(v') | i \int d^4x T [J_0(0), \mathcal{L}_1(x)] | M(v) \rangle = -\frac{\chi_1(w)}{2 m_Q^2} \text{Tr} [\bar{H}' \Gamma H] + \frac{1}{8 m_Q} \text{Tr} [\chi_{2\mu\nu}(v, v') \bar{H}' \Gamma P_+ i \sigma^{\mu\nu} H]$$

$$\langle M'(v') | i \int d^4x T [J_0(0), \mathcal{L}'_1(x)] | M(v) \rangle = -\frac{\bar{\chi}_1(w)}{2 m_{Q'}^2} \text{Tr} [\bar{H}' \Gamma H] + \frac{1}{8 m_{Q'}} \text{Tr} [\bar{\chi}_{2\mu\nu}(v', v) \bar{H}' i \sigma^{\mu\nu} P'_+ \Gamma H]$$

$$\chi_{2\mu\nu} = \chi_2^A i \sigma_{\mu\nu} + \chi_2^C (v'_\mu \gamma_\nu - v'_\nu \gamma_\mu) \quad \bar{\chi}_{2\mu\nu} = \bar{\chi}_2^A i \sigma_{\mu\nu} + \chi_2^C (v_\mu \gamma_\nu - v_\nu \gamma_\mu)$$

Matrix Elements and Trace Formalism

Local corrections

- Leading order

$$\langle M'(v') | J_0 | M(v) \rangle = -\Delta(w) \operatorname{Tr} [\bar{H}'(v') \Gamma H(v)] \quad \text{with} \quad w = v \cdot v'$$

- $1/m$ corrections

$$\langle M'(v') | \bar{\psi}'_+ \Gamma i \overleftrightarrow{D}_\alpha \psi_+ | M(v) \rangle = -\operatorname{Tr} [\Delta_\alpha(v, v') \bar{H}' \Gamma H]$$

$$\langle M'(v') | \bar{\psi}'_+ i \overleftrightarrow{D}_\alpha \Gamma \psi_+ | M(v) \rangle = +\operatorname{Tr} [\bar{\Delta}_\alpha(v', v) \bar{H}' \Gamma H]$$

$$\Delta_\alpha(v, v') = \Delta_+(v + v')_\alpha + \Delta_-(v - v')_\alpha - \Delta_3 \gamma_\alpha \quad \bar{\Delta}_\alpha(v', v) = \Delta_\alpha(v', v) [\Delta_{\pm,3} \rightarrow \bar{\Delta}_{\pm,3}]$$

- $1/m^2$ corrections

$$\langle M'(v') | \bar{\psi}'_+ (-i \overleftrightarrow{D}_\alpha) \Gamma i \overleftrightarrow{D}_\beta \psi_+ | M(v) \rangle = -\operatorname{Tr} [\psi_{\alpha\beta}(v, v') \bar{H}' \Gamma H] \quad \text{with} \quad \psi_{\alpha\beta} = \frac{1}{2} [\psi_{\alpha\beta}^S + \psi_{\alpha\beta}^A]$$

$$\begin{aligned} \psi_{\alpha\beta}^S &= \psi_1^S g_{\alpha\beta} + \psi_2^S (v + v')_\alpha (v + v')_\beta + \psi_3^S (v - v')_\alpha (v - v')_\beta + \psi_4^S [(v + v')_\alpha \gamma_\beta + (v + v')_\beta \gamma_\alpha] + \\ &\quad + \psi_5^S [(v - v')_\alpha \gamma_\beta + (v - v')_\beta \gamma_\alpha] + \psi_6^S [(v + v')_\alpha (v - v')_\beta + (v - v')_\alpha (v - v')_\beta] \end{aligned}$$

$$\psi_{\alpha\beta}^A = \psi_1^A [v_\alpha v'_\beta - v_\beta v'_\alpha] + \psi_2^A [(v - v')_\alpha \gamma_\beta - (v - v')_\beta \gamma_\alpha] + \psi_3^A i \sigma_{\alpha\beta} +$$

$$\text{Non-local corrections} \quad + \psi_4^A [(v + v')_\alpha \gamma_\beta - (v + v')_\beta \gamma_\alpha]$$

- $1/m$ and $1/m^2$ corrections

$$\langle M'(v') | i \int d^4x \mathbb{T} [J_0(0), \mathcal{L}_1(x)] | M(v) \rangle = -\frac{\chi_1(w)}{2 m_Q^2} \operatorname{Tr} [\bar{H}' \Gamma H] + \frac{1}{8 m_Q} \operatorname{Tr} [\chi_{2\mu\nu}(v, v') \bar{H}' \Gamma P_+ i \sigma^{\mu\nu} H]$$

$$\langle M'(v') | i \int d^4x \mathbb{T} [J_0(0), \mathcal{L}'_1(x)] | M(v) \rangle = -\frac{\bar{\chi}_1(w)}{2 m_{Q'}^2} \operatorname{Tr} [\bar{H}' \Gamma H] + \frac{1}{8 m_{Q'}} \operatorname{Tr} [\bar{\chi}_{2\mu\nu}(v', v) \bar{H}' i \sigma^{\mu\nu} P'_+ \Gamma H]$$

$$\chi_{2\mu\nu} = \chi_2^A i \sigma_{\mu\nu} + \chi_2^C (v'_\mu \gamma_\nu - v'_\nu \gamma_\mu) \quad \bar{\chi}_{2\mu\nu} = \bar{\chi}_2^A i \sigma_{\mu\nu} + \bar{\chi}_2^B (v_\mu \gamma_\nu - v_\nu \gamma_\mu)$$

Relations among form factors

$B_c(v) \rightarrow \eta_c(v')$ FFs

$$\langle \eta_c | \bar{c} \gamma_\mu b | B_C \rangle = \sqrt{m_{\eta_c} m_{B_C}} [h_+ (v + v')_\mu + h_- (v - v')_\mu] \quad \langle \eta_c | \bar{c} b | B_C \rangle = \sqrt{m_{\eta_c} m_{B_C}} h_S (1 + w)$$

$$\langle \eta_c | \bar{c} \sigma_{\mu\nu} b | B_c \rangle = -i \sqrt{m_{\eta_c} m_{B_c}} h_T (v_\mu v'_\nu - v_\nu v'_\mu)$$

$$B_c(v) \rightarrow J/\psi(v', \varepsilon') \text{ FFs}$$

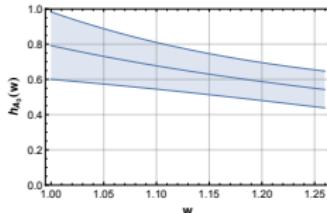
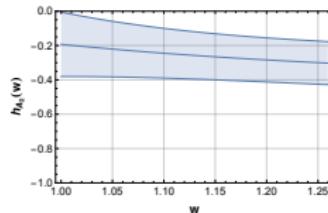
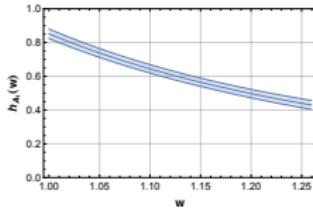
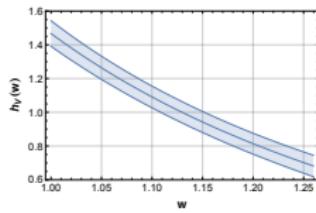
$$\langle J/\psi | \bar{c} \gamma_\mu b | B_c \rangle = i \sqrt{m_{J/\psi} m_{B_c}} \textcolor{brown}{h}_Y \epsilon_{\mu\nu\alpha\beta} \epsilon'^{* \mu} v'^{\alpha} v^{\beta}$$

$$\langle J/\psi | \bar{c} \gamma_\mu \gamma_5 b | B_c \rangle = \sqrt{m_{J/\psi} m_{B_c}} \left[\textcolor{brown}{h}_{A_1} (1+w) \varepsilon'^*_\mu - \textcolor{brown}{h}_{A_2} \varepsilon'^* \cdot v v_\mu - \textcolor{brown}{h}_{A_3} \varepsilon'^* \cdot v v'_\mu \right]$$

$$\langle J/\psi | \bar{c} \sigma_{\mu\nu} b | B_C \rangle = -\sqrt{m_J/m_{B_C}} \epsilon_{\mu\nu\alpha\beta} [h_{T_1} \varepsilon'^{\ast\alpha} (\nu + \nu')^\beta + h_{T_2} \varepsilon'^{\ast\alpha} (\nu - \nu')^\beta + h_{T_3} (\varepsilon'^{\ast} \cdot \nu) \nu^\alpha \nu'^\beta]$$

$$\langle J/\psi | \bar{c} \gamma_5 b | B_c \rangle = -\sqrt{m_{J/\psi} m_{B_c}} \, h_P \, \varepsilon'^* \cdot v$$

$h_V, h_{A_1}, h_{A_2}, h_{A_3}$



FF computed by:

J. Harrison, C. T. H. Davies, A. Lyte,
 $B_c \rightarrow J/\psi$ form factors for the full q^2
range from lattice QCD,
Phys. Rev. D 102 (2020) 094518,
[arXiv:2007.06957]

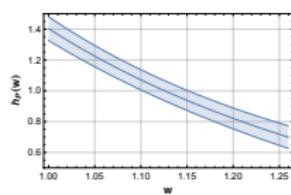
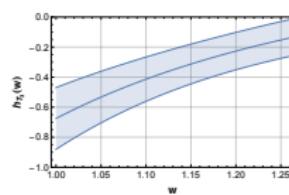
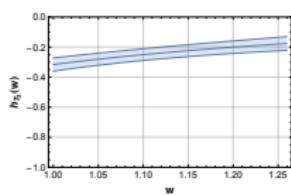
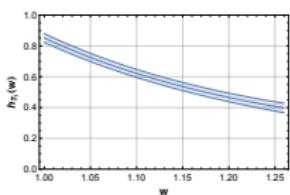
Relations among form factors

$$h_{T_1} = \frac{1}{2} [(1+w) h_{A_1} + (1-w) h_V]$$

$$h_{T_3} = h_{A_3} - h_V$$

$$h_{T_2} = \frac{1+w}{2(m_b + 3m_c)} [(m_b - 3m_c) h_{A_1} + 2m_c (h_{A_2} + h_{A_3}) - (m_b - m_c) h_V]$$

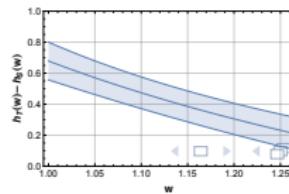
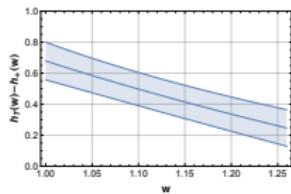
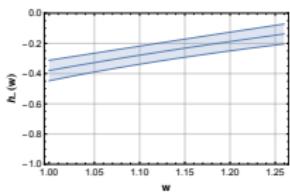
$$h_P = \frac{1}{m_b + 3m_c} [(1+w)(m_b h_{A_1} + 2m_c h_V) - (m_b + (2-w)m_c) h_{A_2} - (w m_b - (1-2w)m_c) h_{A_3}]$$



$$h_- = \frac{m_b - m_c}{2(m_b + 3m_c)} (1+w) [3 h_{A_1} - h_{A_2} - h_{A_3} - 2 h_V]$$

$$h_T - h_+ = - \frac{m_b + m_c}{2(m_b + 3m_c)} (1+w) [3 h_{A_1} - h_{A_2} - h_{A_3} - 2 h_V]$$

$$h_T - h_S = - \frac{m_b + m_c}{m_b + 3m_c} [3 h_{A_1} - h_{A_2} - h_{A_3} - 2 h_V]$$



FF obtained by us:
 P. Colangelo, F. De Fazio,
 FL,N.Losacco, M.
 Novoa-Brunet,
 Relations among
 $B_C \rightarrow J/\psi, \eta_c$ form factors,
 arXiv:2205.08933.

Outline

On B_c exclusive semileptonic modes

$$B_c^+ \rightarrow B_{s,d}^{(*)}(\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

Conclusions

Conclusions

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

1. They represent further processes described by $c \rightarrow \{ s, d \}$ $\ell^+ \nu_\ell$ transitions to investigate NP effects;
2. It is possible to reduce the hadronic uncertainties, using:
 - 2.1 computations of the hadronic form factors f_+ and f_0 ;
 - 2.2 HQSS;
3. In addition, looking at $B_c^+ \rightarrow B_d^* \bar{\ell} \nu_\ell$, NP may produce large deviations from SM predictions.

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

1. Lattice QCD results for $B_c \rightarrow J/\psi$ SM form factors can be used as an input for:
 - 1.1 $B_c \rightarrow J/\psi$ to predict NP pseudoscalar and tensor form factors;
 - 1.2 $B_c \rightarrow \eta_c$ to relate the form factors to the previous ones;
2. The effort is to efficiently control the hadronic uncertainties affecting the predictions for semileptonic $B_c \rightarrow J/\psi, \eta_c$ decays in the SM and beyond.

Back-up

Back-up

$$B_c^+ \rightarrow B_{s,d}^{(*)}(\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

Exclusive $c \rightarrow \{ s, d \} \bar{\ell} \nu_\ell$ modes:

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

Based on:

P. Colangelo, F. De Fazio, FL,

Role of $B_c^+ \rightarrow B_{s,d}^{()} \bar{\ell} \nu_\ell$ in the Standard Model and in the search for BSM signals,*
Phys. Rev. D **103** (2021), no. 7 075019, [arXiv:2102.05365].

$$B_c^+ \rightarrow B_s^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

$$B_c^- \rightarrow J/\psi(\eta_c) \ell^+ \bar{\nu}_\ell$$

○○○○○○○

Angular coefficient functions for $B_c \rightarrow V(\rightarrow P\gamma)\bar{\ell}\nu_\ell$

	SM
I_{1s}	$2m_\ell^2 H_t^2 + H_0^2(m_\ell^2 + q^2)$
I_{1c}	$\frac{1}{8}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$
I_{2s}	$H_0^2(m_\ell^2 - q^2)$
I_{2c}	$\frac{1}{8}(H_+^2 + H_-^2)(q^2 - m_\ell^2)$
I_3	$H_+ H_-(q^2 - m_\ell^2)$
I_4	$\frac{1}{2}H_0(H_+ + H_-)(q^2 - m_\ell^2)$
I_5	$H_t(H_+ + H_-)m_\ell^2 + H_0(H_+ - H_-)q^2$
I_{6s}	$-4H_t H_0 m_\ell^2$
I_{6c}	$\frac{1}{2}(H_+^2 - H_-^2)q^2$
$I_{7,8,9}$	0

	NP, R	INT, R
I_{1s}	$2m_\ell^2 H_t^2 + H_0^2(m_\ell^2 + q^2)$	$-2m_\ell^2 H_t^2 - H_0^2(m_\ell^2 + q^2)$
I_{1c}	$\frac{1}{8}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$	$-\frac{1}{4}H_+H_-(m_\ell^2 + 3q^2)$
I_{2s}	$H_0^2(m_\ell^2 - q^2)$	$-H_0^2(m_\ell^2 - q^2)$
I_{2c}	$\frac{1}{8}(H_+^2 + H_-^2)(q^2 - m_\ell^2)$	$\frac{1}{4}H_+H_-(m_\ell^2 - q^2)$
I_3	$H_+H_-(q^2 - m_\ell^2)$	$\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 - q^2)$
I_4	$\frac{1}{2}H_0(H_+ + H_-)(q^2 - m_\ell^2)$	$\frac{1}{2}H_0(H_+ + H_-)(m_\ell^2 - q^2)$
I_5	$H_t(H_+ + H_-)m_\ell^2 - H_0(H_+ - H_-)q^2$	$-H_t(H_+ + H_-)m_\ell^2$
I_{6s}	$-4H_tH_0m_\ell^2$	$4H_tH_0m_\ell^2$
I_{6c}	$-\frac{1}{2}(H_+^2 - H_-^2)q^2$	0
I_7	0	$-H_t(H_+ - H_-)m_\ell^2$
I_8	0	$\frac{1}{2}H_0(H_+ - H_-)(m_\ell^2 - q^2)$
I_9	0	$\frac{1}{2}(H_+^2 - H_-^2)(m_\ell^2 - q^2)$

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

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$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

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Angular coefficient functions for $B_c \rightarrow V (\rightarrow P \gamma) \bar{\ell} \nu_\ell$

	NP, T	INT, T
I_{1s}	$\frac{1}{16}(H_L^{\text{NP}})^2(q^2 + m_\ell^2)$	$-\frac{1}{2}H_L^{\text{NP}}H_0m_\ell\sqrt{q^2}$
I_{1c}	$\frac{1}{2}((H_+^{\text{NP}})^2 + (H_-^{\text{NP}})^2)(3m_\ell^2 + q^2)$	$-(H_+^{\text{NP}}H_+ + H_-^{\text{NP}}H_-)m_\ell\sqrt{q^2}$
I_{2s}	$\frac{1}{16}(H_L^{\text{NP}})^2(q^2 - m_\ell^2)$	0
I_{2c}	$\frac{1}{2}((H_+^{\text{NP}})^2 + (H_-^{\text{NP}})^2)(m_\ell^2 - q^2)$	0
I_3	$-4H_+^{\text{NP}}H_-^{\text{NP}}(q^2 - m_\ell^2)$	0
I_4	$-\frac{1}{4}H_L^{\text{NP}}(H_+^{\text{NP}} + H_-^{\text{NP}})(q^2 - m_\ell^2)$	0
I_5	$\frac{1}{2}H_L^{\text{NP}}(H_+^{\text{NP}} - H_-^{\text{NP}})m_\ell^2$	$-\frac{1}{8}[H_L^{\text{NP}}(H_+ - H_-) + 8H_+^{\text{NP}}(H_t + H_0) + 8H_-^{\text{NP}}(H_t - H_0)]m_\ell\sqrt{q^2}$
I_{6s}	0	$\frac{1}{2}H_L^{\text{NP}}H_t m_\ell\sqrt{q^2}$
I_{6c}	$2((H_+^{\text{NP}})^2 - (H_-^{\text{NP}})^2)m_\ell^2$	$-(H_+^{\text{NP}}H_+ - H_-^{\text{NP}}H_-)m_\ell\sqrt{q^2}$
I_7	0	$-\frac{1}{8}[H_L^{\text{NP}}(H_+ + H_-) - 8H_+^{\text{NP}}(H_t + H_0) + 8H_-^{\text{NP}}(H_t - H_0)]m_\ell\sqrt{q^2}$
$I_{8,9}$	0	0

	INT, PR	INT, RT	INT, PT
I_{1s}	$-2H_t^2 \frac{m_\ell q^2}{m_Q + m_q}$	$\frac{1}{2}H_0H_L^{\text{NP}}m_\ell\sqrt{q^2}$	0
I_{1c}	0	$(H_+^{\text{NP}}H_- + H_-^{\text{NP}}H_+)m_\ell\sqrt{q^2}$	0
$I_{2s,2c,3,4,8,9}$	0	0	0
I_5	$-H_t(H_+ + H_-) \frac{m_\ell q^2}{2(m_Q + m_q)}$	$\frac{1}{8}[H_L^{\text{NP}}(H_- - H_+) + 8H_+^{\text{NP}}(H_t + H_0) + 8H_-^{\text{NP}}(H_t - H_0)]m_\ell\sqrt{q^2}$	$-H_t(H_+^{\text{NP}} + H_-^{\text{NP}}) \frac{(q^2)^{3/2}}{m_Q + m_q}$
I_{6s}	$2H_tH_0 \frac{m_\ell q^2}{m_Q + m_q}$	$-\frac{1}{2}H_tH_L^{\text{NP}}m_\ell\sqrt{q^2}$	$H_tH_L^{\text{NP}} \frac{(q^2)^{3/2}}{2(m_Q + m_q)}$
I_{6c}	0	$(H_+^{\text{NP}}H_- - H_-^{\text{NP}}H_+)m_\ell\sqrt{q^2}$	0
I_7	$H_t(H_+ - H_-) \frac{m_\ell q^2}{2(m_Q + m_q)}$	$-\frac{1}{8}[H_L^{\text{NP}}(H_- + H_+) - 8H_+^{\text{NP}}(H_t + H_0) + 8H_-^{\text{NP}}(H_t - H_0)]m_\ell\sqrt{q^2}$	$-H_t(H_+^{\text{NP}} - H_-^{\text{NP}}) \frac{(q^2)^{3/2}}{m_Q + m_q}$

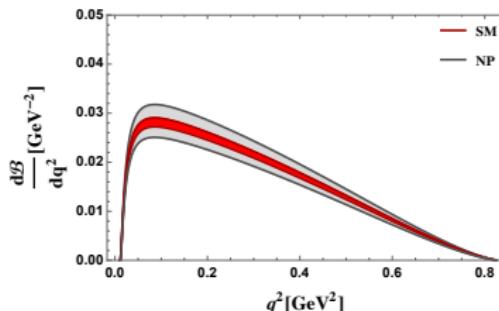
$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

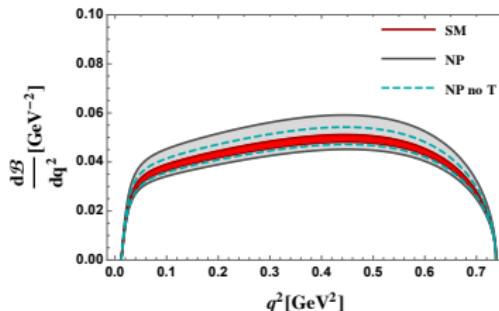
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Differential Branching Ratios

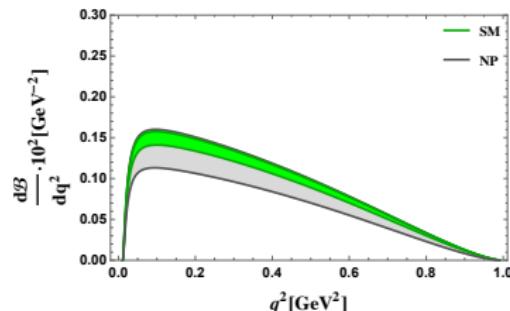
$$B_c \rightarrow B_s \mu^+ \nu_\mu$$



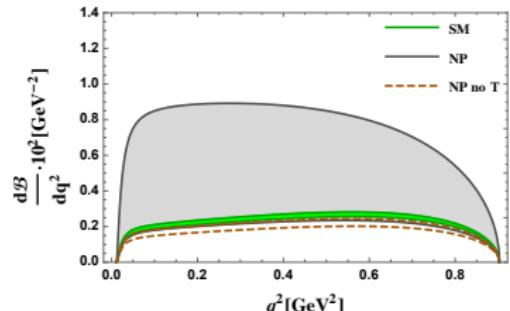
$$B_c \rightarrow B_s^* \mu^+ \nu_\mu$$



$$B_c \rightarrow B_d \mu^+ \nu_\mu$$



$$B_c \rightarrow B_d^* \mu^+ \nu_\mu$$

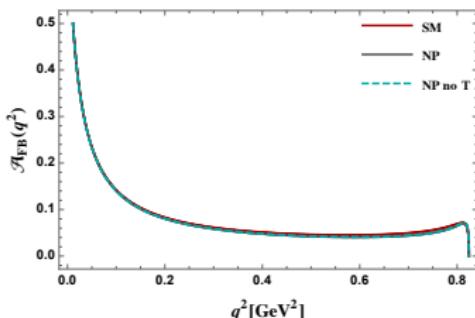


$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

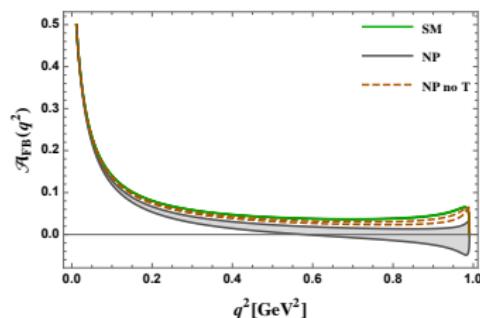
$$B_c^- \rightarrow J/\psi(\eta_c) \ell^+ \bar{\nu}_\ell$$

Forward-Backward Lepton Asymmetry

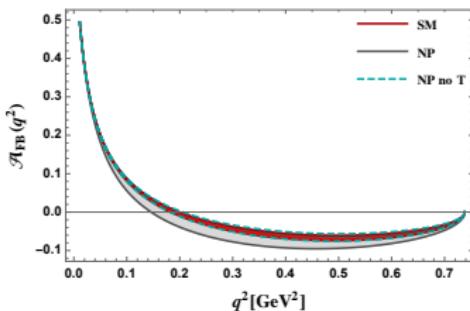
$$B_c \rightarrow B_s \mu^+ \nu_\mu$$



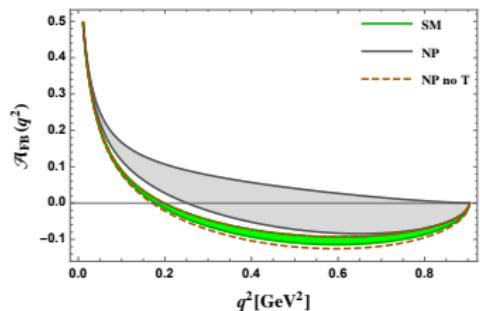
$$B_c \rightarrow B_d \mu^+ \nu_\mu$$



$$B_c \rightarrow B_s^* \mu^+ \nu_\mu$$



$$B_c \rightarrow B_d^* \mu^+ \nu_\mu$$



Back-up

$$B_c^+ \rightarrow B_{s,d}^{(*)}(\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

Exclusive $b \rightarrow c \ell \bar{\nu}_\ell$ modes:

$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

Based on:

P. Colangelo, F. De Fazio, FL,N.Losacco, M. Novoa-Brunet,
Relations among $B_c \rightarrow J/\psi, \eta_c$ form factors,
arXiv:2205.08933.

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

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$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

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Expansion of the heavy quark field and the QCD Lagrangian

Lagrangian

$$\mathcal{L}_{\mathbf{QCD}} = \bar{Q} (i \vec{D} - m_Q) Q$$

Remember that

$$Q(x) = e^{i m_Q v \cdot x} \psi(x) = e^{i m_Q v \cdot x} (\psi_+(x) + \psi_-(x)) \quad \text{with} \quad \psi_\pm(x) = P_\pm(v) \psi(x)$$

So

$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_+ (i v \cdot \vec{D}) \psi_+ + \bar{\psi}_+ i \vec{D}_\perp \psi_- + \bar{\psi}_- i \vec{D}_\perp \psi_+ - \bar{\psi}_- (i v \cdot \vec{D} + 2 m_Q) \psi_-$$

and looking at the EoM,

$$\frac{\partial \mathcal{L}_{\mathbf{QCD}}}{\partial \bar{\psi}_\pm} = \partial_\mu \frac{\partial \mathcal{L}_{\mathbf{QCD}}}{\partial (\partial_\mu \bar{\psi}_\pm)} \quad \Rightarrow \quad \begin{cases} \psi_+ & \rightarrow (i v \cdot \vec{D}) \psi_+ + i \vec{D}_\perp \psi_- = 0 \\ \psi_- & \rightarrow i \vec{D}_\perp \psi_+ - (i v \cdot \vec{D} + 2 m_Q) \psi_- = 0 \end{cases}$$

we find

$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}_+ \left(i v \cdot \vec{D} + i \vec{D}_\perp \frac{1}{2 m_Q + i v \cdot \vec{D}} i \vec{D}_\perp \right) \psi_+ = \dots = \mathcal{L}_0 + \mathcal{L}_1 + \dots$$

Current

$$\bar{Q} \Gamma Q = \bar{\psi}'_+ \left[1 - \frac{i \vec{D}'_\perp}{2 m_{Q'}} - \frac{i \vec{D}'_\perp (i v' \cdot \vec{D})}{(2 m_Q)^2} \right] \Gamma \left[1 + \frac{i \vec{D}_\perp}{2 m_Q} - \frac{(i v \cdot \vec{D}) i \vec{D}_\perp}{(2 m_Q)^2} \right] \psi_+ + \dots$$

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

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$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

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1/m relations

x -dependence: $|M(x)\rangle = e^{-i\tilde{\Lambda} v \cdot x} |M(0)\rangle$

$$\mathcal{M}_0(x) = e^{-i(\tilde{\Lambda} v - \tilde{\Lambda}' v') \cdot x} \mathcal{M}_0(0)$$

where, for a $[Q_1 \bar{Q}_2]$ meson

$$\tilde{\Lambda} = m_H - m_{Q_1} - m_{Q_2}$$

The small binding energy scale $\tilde{\Lambda}$
can display a residual
heavy quark mass dependence.

Trace formalism

$$\langle M'(v') | \bar{\psi}'_+ \Gamma(i v \cdot \vec{D}) \psi_+ | M(v) \rangle = -((1+w) \Delta_+ + (1-w) \Delta_- + \Delta_3) \text{Tr}[\bar{H}' \Gamma H] = -\phi_K \text{Tr}[\bar{H}' \Gamma H]$$

$$\langle M'(v') | \bar{\psi}'_+ (-i v' \cdot \vec{D}) \Gamma \psi_+ | M(v) \rangle = -((1+w) \bar{\Delta}_+ + (1-w) \bar{\Delta}_- + \bar{\Delta}_3) \text{Tr}[\bar{H}' \Gamma H] = -\phi'_K \text{Tr}[\bar{H}' \Gamma H]$$

or, using $i \partial_\alpha (\bar{\psi}'_+ \Gamma \psi_+) = \bar{\psi}'_+ (i \vec{D}_\alpha) \Gamma \psi_+ + \bar{\psi}'_+ \Gamma (i \vec{D}_\alpha) \psi_+$ we get

$$\Delta_+ - \bar{\Delta}_+ = \frac{\tilde{\Lambda} - \tilde{\Lambda}'}{2} \Delta \quad \Delta_- + \bar{\Delta}_- = \frac{\tilde{\Lambda} + \tilde{\Lambda}'}{2} \Delta \quad \Delta_3 - \bar{\Delta}_3 = 0$$

Final solutions

$$\Delta_+ = \frac{(\tilde{\Lambda} w - \tilde{\Lambda}') \Delta - 2 \Delta_3 + \phi_K + \phi'_K}{2(w+1)}$$

$$\Delta_- = \frac{(\tilde{\Lambda} w - \tilde{\Lambda}') \Delta - (\phi_K - \phi'_K)}{2(w-1)}$$

$$\bar{\Delta}_+ = \frac{(\tilde{\Lambda}' w - \tilde{\Lambda}) \Delta - 2 \bar{\Delta}_3 + \phi_K + \phi'_K}{2(w+1)}$$

$$\bar{\Delta}_- = \frac{(\tilde{\Lambda}' w - \tilde{\Lambda}) \Delta + (\phi_K - \phi'_K)}{2(w-1)}$$

Imposing $\phi_K - \phi'_K = (\tilde{\Lambda} - \tilde{\Lambda}') \Delta$ we get

$$\Delta_+ = \frac{\tilde{\Lambda}(w-1) \Delta + 2 \Delta_3 + 2 \phi_K}{2(w+1)}$$

$$\bar{\Delta}_+ = \frac{\tilde{\Lambda}'(w-1) \Delta + 2 \Delta_3 + 2 \phi'_K}{2(w+1)}$$

$$\Delta_- = \frac{\tilde{\Lambda}}{2} \Delta \quad \bar{\Delta}_- = \frac{\tilde{\Lambda}'}{2} \Delta$$

the divergences in Δ_- and $\bar{\Delta}_-$ are cancelled.

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

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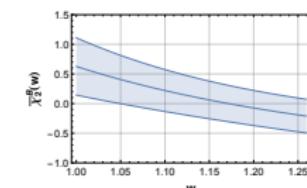
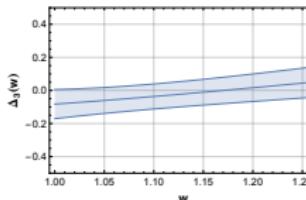
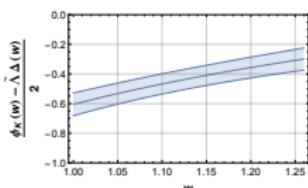
$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

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How to obtain the relations among the FF at $\mathcal{O}(1/m_Q)$

Some definitions:

$$F_1 \equiv \frac{\phi_K - \Delta \tilde{\Lambda}}{2} + \Delta_3 \quad K \equiv 3 \chi_2^A + 2(w-1) \chi_2^C \quad F_2 \equiv \frac{\phi_K - \Delta \tilde{\Lambda}}{2} - \Delta_3 \quad \tilde{K} \equiv 3 \bar{\chi}_2^A - 2(w-1) \bar{\chi}_2^B$$



$B_c \rightarrow J/\psi$ FFs (SM)

$$\begin{aligned} h_V &= \Delta - \frac{4F_1 - K}{4m_b} - \frac{\bar{\chi}_2^A + 2(F_1 + F_2)}{4m_c} & h_{A_3} &= \Delta - \frac{4F_1 - K}{4m_b} - \frac{1}{2m_c} \left[\frac{wF_1 - (2-w)F_2}{1+w} + \frac{\bar{\chi}_2^A + 2\bar{\chi}_2^B}{2} \right] \\ h_{A_1} &= \Delta + \frac{1}{m_b} \left[\frac{1-w}{1+w} F_1 + \frac{K}{4} \right] + \frac{1}{2m_c} \left[\frac{1-w}{1+w} (F_1 + F_2) + \frac{\bar{\chi}_2^A}{2} \right] & h_{A_2} &= \frac{1}{2m_c} \left[\frac{F_1 + 3F_2}{1+w} + \bar{\chi}_2^B \right] \end{aligned}$$

$B_c \rightarrow J/\psi$ FFs (NP) and $B_c \rightarrow \eta_c$ FFs (SM and NP)

$$\begin{aligned} h_+ &= \Delta + \frac{K}{4m_b} + \frac{\tilde{K}}{4m_c} & h_- &= -\left(\frac{1}{m_b} - \frac{1}{m_c} \right) F_1 & h_T &= \Delta - \frac{4F_1 - K}{4m_b} - \frac{4F_1 - \tilde{K}}{4m_c} \\ h_S &= \Delta + \frac{1}{m_b} \left[\frac{1-w}{1+w} F_1 + \frac{K}{4} \right] + \frac{1}{m_c} \left[\frac{1-w}{1+w} F_1 + \frac{\tilde{K}}{4} \right] & h_{T_1} &= \Delta + \frac{K}{4m_b} - \frac{\bar{\chi}_2^A}{4m_c} \\ h_{T_2} &= -\frac{F_1}{m_b} + \frac{F_1 + F_2}{2m_c} & h_{T_3} &= -\frac{1}{2m_c} \left[\bar{\chi}_2^B - \frac{F_1 + 3F_2}{1+w} \right] & h_P &= \Delta - \frac{4F_1 - K}{4m_b} - \frac{4F_2 + \tilde{K} - 2\bar{\chi}_2^A}{4m_c} \end{aligned}$$

$$B_c^+ \rightarrow B_{s,d}^{(*)} (\rightarrow B_{s,d} \gamma) \bar{\ell} \nu_\ell$$

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$$B_c^- \rightarrow J/\psi(\eta_c) \ell \bar{\nu}_\ell$$

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\$B_c \rightarrow J/\psi\$ SM FF: Relations at leading order

\$B_c(p) \rightarrow J/\psi(p', \varepsilon')\$: QCD basis

$$\langle J/\psi | \bar{Q}' \gamma_\mu Q | B_c \rangle = - \frac{2i V^{B_c \rightarrow J/\psi}(q^2)}{m_{B_c} + m_{J/\psi}} \epsilon_{\mu\nu\alpha\beta} \varepsilon'^*\nu p^\alpha p'^\beta$$

$$\begin{aligned} \langle J/\psi | \bar{Q}' \gamma_\mu \gamma_5 Q | B_c(p) \rangle &= \epsilon^* \cdot q \frac{2 m_{J/\psi}}{q^2} q_\mu A_0^{B_c \rightarrow J/\psi}(q^2) + (m_{B_c} + m_{J/\psi}) \left(\epsilon_\mu^* - \frac{\epsilon^* \cdot q}{q^2} q_\mu \right) A_1^{B_c \rightarrow J/\psi}(q^2) + \\ &\quad - \frac{\epsilon^* \cdot q}{m_{B_c} + m_{J/\psi}} \left((p + p')_\mu - \frac{m_{B_c}^2 - m_{J/\psi}^2}{q^2} q_\mu \right) A_2^{B_c \rightarrow J/\psi}(q^2) \end{aligned}$$

\$B_c(v) \rightarrow J/\psi(v', \varepsilon')\$: HQET basis

$$\langle J/\psi | \bar{c} \gamma_\mu b | B_c \rangle = i \sqrt{m_{J/\psi} m_{B_c}} h_V(w) \epsilon_{\mu\nu\alpha\beta} \varepsilon'^*\mu v'^\alpha v^\beta$$

$$\langle J/\psi | \bar{c} \gamma_\mu \gamma_5 b | B_c \rangle = \sqrt{m_{J/\psi} m_{B_c}} [h_{A_1}(w)(1+w) \varepsilon_\mu'^* - h_{A_2}(w) \varepsilon'^* \cdot v v_\mu - h_{A_3}(w) \varepsilon'^* \cdot v v'_\mu]$$

Relations among FF: \$\{V(q^2), A_i(q^2)\} \sim \{h_V(w), h_{A_i}(w)\}\$

$$V = \frac{m_{B_c} + m_{J/\psi}}{2 \sqrt{m_{B_c} m_{J/\psi}}} h_V \quad A_1 = \frac{\sqrt{m_{B_c} m_{J/\psi}}}{m_{B_c} + m_{J/\psi}} (w + 1) h_{A_1} \quad A_2 = \frac{m_{B_c} + m_{J/\psi}}{2 \sqrt{m_{B_c} m_{J/\psi}}} \left(\frac{m_{J/\psi}}{m_{B_c}} h_{A_2} + h_{A_3} \right)$$

$$A_0 = \frac{1}{2 \sqrt{m_{B_c} m_{J/\psi}}} \left(m_{B_c} (w + 1) h_{A_1} - (m_{B_c} - w m_{J/\psi}) h_{A_2} - (w m_{B_c} - m_{J/\psi}) h_{A_3} \right)$$

$$q^2 = 0 \Rightarrow w = w_{\max} \equiv (m_{B_c}^2 + m_{J/\psi}^2)/2 m_{B_c} m_{J/\psi}$$

$$0 = \frac{m_{B_c} + m_{J/\psi}}{2 m_{J/\psi}} A_1(0) - \frac{m_{B_c} - m_{J/\psi}}{2 m_{J/\psi}} A_2(0) - A_0(0) = -\frac{1}{2} \sqrt{\frac{m_{B_c}}{m_{J/\psi}}} w_{\max} h_{A_1}(w_{\max}) \rightarrow -\frac{1}{2} \sqrt{\frac{m_{B_c}}{m_{J/\psi}}} w_{\max} \Delta(w_{\max})$$