

Flavour anomalies, Light Dark Matter and rare B decays with missing energy in $L_\mu - L_\tau$ model

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Outline of the Talk

- 1 $L_\mu - L_\tau$ model with Scalar Leptoquark
- 2 Symmetry breaking and mass spectra
- 3 Dark Matter Phenomenology
- 4 Constraints from Flavour Sector
- 5 Implications on Rare B decays with \cancel{E}
- 6 Conclusion

Motivation

- There are a few open issues, which can not be addressed in the SM, e.g. Existence of Dark Matter, Non-zero neutrino masses & mixing, Observed Baryon Asymmetry of the Universe, etc
- SM must be extended. **What is the underlying fundamental theory?**
- **No direct evidence of NP either in Energy frontier or Intensity frontier**
- In last few years several anomalies are reported in $b \rightarrow sll$ FCNC transitions, which might possibly suggest the presence of NP
- **Interesting one: Lepton Non-Universality Observable** Sizable discrepancies ($\geq 2\sigma$) reported by the LHCb and Belle Collaborations in the ratio $R_{K^{(*)}}$

$$R_{K^{(*)}} = \frac{\text{Br}(B \rightarrow K^{(*)}\mu\mu)}{\text{Br}(B \rightarrow K^{(*)}ee)}$$

LNU observable	SM prediction	Expt. value	Deviation
$R_K _{q^2 \in [1.0, 6.0]}$	1.0003 ± 0.0001	$0.846^{+0.060+0.016}_{-0.054-0.014}$ (LHCb)	2.5σ
$R_{K^*} _{q^2 \in [0.045, 1.1]}$	0.92 ± 0.02	$0.660^{+0.110}_{-0.070} \pm 0.024$ (LHCb)	2.2σ
$R_{K^*} _{q^2 \in [1.1, 6.0]}$	1.00 ± 0.01	$0.685^{+0.113}_{-0.007} \pm 0.047$ (LHCb)	2.4σ

Model description (Gauged $L_\mu - L_\tau$)

- The SM has accidental $U(1)$ global symmetries like B and L no. conservation
- However, they become anomalous if promoted into a local one
- The anomaly free situation can be obtained if instead of considering B and L separately, some combinations between them, e.g., $B - L$, $L_e - L_\mu$, $L_e - L_\tau$ or $L_\mu - L_\tau$
- For the anomaly cancellation of local $B - L$ models, one requires 3 RHNs with appropriate $B - L$ charges
- Unlike $B - L$ case, the anomaly cancellation does not require any extra chiral fermionic degrees of freedom for $L_\alpha - L_\beta$, as anomalies cancel between different leptonic generations.
- $U(1)_{L_\mu - L_\tau}$ is less constrained, as the extra Z' does not couple to electrons and quarks, \Rightarrow free from any constraints from lepton and hadron colliders
- Another theoretical motivation: it can explain the muon $(g - 2)$ anomaly

Particle Content of $L_\mu - L_\tau$ model

	Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$U(1)_{L_\mu - L_\tau}$	Z_2
Fermions	$Q_L \equiv (u, d)_L^T$	(3, 2, 1/6)	0	+
	u_R	(3, 1, 2/3)	0	+
	d_R	(3, 1, -1/3)	0	+
	$\ell_L \equiv (e, \mu, \tau)_L$	(1, 2, -1/2)	0, 1, -1	+
	$\ell_R \equiv (e, \mu, \tau)_R$	(1, 1, -1)	0, 1, -1	+
	N_e, N_μ, N_τ	(1, 1, 0)	0, 1, -1	-
Scalars	H	(1, 2, 1/2)	0	+
	η	(1, 2, 1/2)	0	-
	ϕ_2	(1, 1, 0)	2	+
	S_1	(3, 1, 1/3)	-1	-
Gauge bosons	$W_\mu^i (i = 1, 2, 3)$	(1, 3, 0)	0	+
	B_μ	(1, 1, 0)	0	+
	V_μ	(1, 1, 0)	0	+

Table: Fields and their charges of the proposed $U(1)_{L_\mu - L_\tau}$ model.

Lagrangian of the Model

The Lagrangian of the present model can be written as

$$\begin{aligned}
 \mathcal{L}_G &= -\frac{1}{4} \left(\hat{\mathbf{W}}_{\mu\nu} \hat{\mathbf{W}}^{\mu\nu} + \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} + \hat{V}_{\mu\nu} \hat{V}^{\mu\nu} + 2 \sin \chi \hat{B}_{\mu\nu} \hat{V}^{\mu\nu} \right), \\
 \mathcal{L}_f &= -\frac{1}{2} M_{ee} \overline{N_e^c} N_e - \frac{f_\mu}{2} \left(\overline{N_\mu^c} N_\mu \phi_2^\dagger + \text{h.c.} \right) - \frac{f_\tau}{2} \left(\overline{N_\tau^c} N_\tau \phi_2 + \text{h.c.} \right) \\
 &\quad - \frac{1}{2} M_{\mu\tau} \left(\overline{N_\mu^c} N_\tau + \overline{N_\tau^c} N_\mu \right) - \sum_{l=e,\mu,\tau} \left(Y_{ll} (\overline{\ell_L}) i \tilde{\eta} N_{lR} + \text{h.c.} \right) \\
 &\quad - \sum_{q=d,s,b} \left(y_{qR} \overline{d_{qR}^c} S_1 N_\mu + \text{h.c.} \right), \\
 \mathcal{L}_{G-f} &= -g_{\mu\tau} \overline{\mu} \gamma^\mu \mu \hat{V}_\mu + g_{\mu\tau} \overline{\tau} \gamma^\mu \tau \hat{V}_\mu - g_{\mu\tau} \overline{\nu_\mu} \gamma^\mu (1 - \gamma^5) \nu_\mu \hat{V}_\mu \\
 &\quad + g_{\mu\tau} \overline{\nu_\tau} \gamma^\mu (1 - \gamma^5) \nu_\tau \hat{V}_\mu - g_{\mu\tau} \overline{N_\mu} \hat{V}_\mu \gamma^\mu \gamma^5 N_\mu + g_{\mu\tau} \overline{N_\tau} \hat{V}_\mu \gamma^\mu \gamma^5 N_\tau, \\
 \mathcal{L}_S &= \left| \left(i\partial_\mu - \frac{g}{2} \boldsymbol{\tau}^a \cdot \hat{\mathbf{W}}_\mu^a - \frac{g'}{2} \hat{B}_\mu \right) \eta \right|^2 + \left| \left(i\partial_\mu - \frac{g'}{3} \hat{B}_\mu + g_{\mu\tau} \hat{V}_\mu \right) S_1 \right|^2 \\
 &\quad + \left| \left(i\partial_\mu - 2g_{\mu\tau} \hat{V}_\mu \right) \phi_2 \right|^2 - V(H, \eta, \phi_2, S_1).
 \end{aligned}$$

Scalar potential

- The scalar potential V is expressed as

$$\begin{aligned} V(H, \eta, \phi_2, S_1) = & V(H) + \mu_\eta^2 \eta^\dagger \eta + \lambda_{H\eta} (H^\dagger H) (\eta^\dagger \eta) + \lambda_\eta (\eta^\dagger \eta)^2 \\ & + \lambda'_{H\eta} (H^\dagger \eta) (\eta^\dagger H) + \frac{\lambda''_{H\eta}}{2} \left[(H^\dagger \eta)^2 + \text{h.c.} \right] + \mu_\phi^2 (\phi_2^\dagger \phi_2) + \lambda_\phi (\phi_2^\dagger \phi_2)^2 \\ & + \mu_S^2 (S_1^\dagger S_1) + \lambda_S (S_1^\dagger S_1)^2 + \left[\lambda_{H\phi} (\phi_2^\dagger \phi_2) + \lambda_{HS} (S_1^\dagger S_1) \right] (H^\dagger H) \\ & + \lambda_{S\phi} (\phi_2^\dagger \phi_2) (S_1^\dagger S_1) + \lambda_{\eta\phi} (\phi_2^\dagger \phi_2) (\eta^\dagger \eta) + \lambda_{S\eta} (S_1^\dagger S_1) (\eta^\dagger \eta). \end{aligned}$$

- SSB occurs when the scalars get their VEVs: $\langle \phi_2 \rangle = \frac{v_2}{\sqrt{2}}$, $\langle H \rangle = \frac{v}{\sqrt{2}}$,
 $SU(2)_L \times U(1)_Y \times U(1)_{L_\mu - L_\tau} \implies SU(2)_L \times U(1)_Y \implies U(1)_{em}$
- We have $\mu_\eta^2, \mu_S^2 > 0$ and the masses of the SLQ and inert doublet η are

$$\begin{aligned} M_{S_1}^2 &= 2\mu_S^2 + \lambda_{HS} v^2 + \lambda_{S\phi} v_2^2, \\ M_{\eta_c}^2 &= \mu_\eta^2 + \lambda_{H\eta} v^2/2 + \lambda_{\eta\phi} v_2^2/2, \\ M_{\eta_{r,i}}^2 &= \mu_\eta^2 + (\lambda_{H\eta} + \lambda'_{H\eta} \pm \lambda''_{H\eta}) v^2/2 + \lambda_{\eta\phi} v_2^2/2. \end{aligned}$$

Gauge mixing

- For the mixing of $U(1)_Y$ and $U(1)_{L_\mu-L_\tau}$ gauge bosons, we consider the $GL(2, R)$ transformation

$$\begin{pmatrix} \bar{B}_\mu \\ \bar{V}_\mu \end{pmatrix} = \begin{pmatrix} 1 & \sin \chi \\ 0 & \cos \chi \end{pmatrix} \begin{pmatrix} \hat{B}_\mu \\ \hat{V}_\mu \end{pmatrix}.$$

- Thus, the mass matrix of gauge fields in the basis $(W_\mu^3, \bar{B}_\mu, \bar{V}_\mu)$ as

$$M_G^2 = \begin{pmatrix} \frac{1}{8}g^2v^2 & -\frac{1}{8}gg'v^2 & \frac{1}{8}gg'\tan\chi v^2 \\ -\frac{1}{8}gg'v^2 & \frac{1}{8}g'^2v^2 & -\frac{1}{8}g'^2\tan\chi v^2 \\ \frac{1}{8}gg'\tan\chi v^2 & -\frac{1}{8}g'^2\tan\chi v^2 & 2g_{\mu\tau}^2 \sec^2\chi v^2 \end{pmatrix}.$$

- Diagonalization of M_G^2 gives the masses of the physical gauge bosons

$$M_Z^2 = M_{Z_{SM}}^2 \cos^2 \alpha - \delta M^2 \sin 2\alpha + M_{\bar{V}}^2 \sin^2 \alpha,$$

$$M_{Z'}^2 = M_{Z_{SM}}^2 \sin^2 \alpha + \delta M^2 \sin 2\alpha + M_{\bar{V}}^2 \cos^2 \alpha,$$

$$\alpha = \frac{1}{2} \tan^{-1} \left[\frac{2 \delta M^2}{M_{\bar{V}}^2 - M_{Z_{SM}}^2} \right].$$

Scalar and Fermion mixing

- The CP-even scalars h and h_2 as well as the heavy fermion states N_μ and N_τ mix with the mixing matrices given as

$$M_H^2 = \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H\phi} v v_2 \\ \lambda_{H\phi} v v_2 & 2\lambda_\phi v_2^2 \end{pmatrix}, \quad M_N = \begin{pmatrix} \frac{1}{\sqrt{2}} f_\mu v_2 & M_{\mu\tau} \\ M_{\mu\tau} & \frac{1}{\sqrt{2}} f_\tau v_2 \end{pmatrix}.$$

One can diagonalize the above mass matrices using a 2×2 rotation matrix

$$U_\zeta^T M_H^2 U_\zeta = \text{diag} [M_{H_1}^2, M_{H_2}^2], \quad U_\beta^T M_N U_\beta = \text{diag} [M_{N_-}, M_{N_+}],$$

$$\text{with } \zeta = \frac{1}{2} \tan^{-1} \left(\frac{\lambda_{H\phi} v v_2}{\lambda_\phi v_2^2 - \lambda_H v^2} \right), \quad \beta = \frac{1}{2} \tan^{-1} \left(\frac{2M_{\mu\tau}}{(f_\tau - f_\mu)(v_2/\sqrt{2})} \right).$$

- The lightest fermion mass eigenstate N_- considered as probable DM candidate, and M_{H_1} as the SM Higgs

M_{S_1}	M_+	M_{H_1}	M_{H_2}	$\sin \beta$	$\sin \zeta$	χ	$\alpha \times 10^4$
1200	500	125	500	1/2	$10^{-3} - 10^{-2}$	10^{-3}	4.83 - 4.85

Table: Values of the model parameters used in the analysis (masses are in GeV).

Dark Matter Relic Abundance

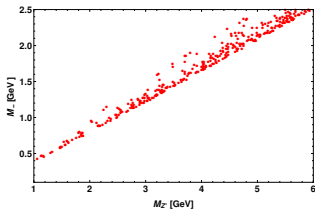
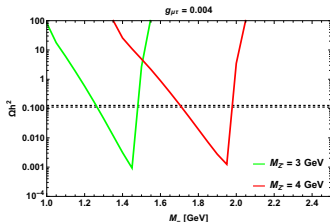
- The relic density of the light DM (N_-) is computed via freeze-out mechanism through the following decay channels:

$$\begin{aligned} N_- \bar{N}_- &\rightarrow \mu \bar{\mu}, \tau \bar{\tau}, \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau \quad (s \text{ channel } Z' \text{ and } \eta \text{ portal}) \\ &\rightarrow d \bar{d}, s \bar{s} \quad (t \text{ channel } SLQ(S_1) \text{ portal}) \end{aligned}$$

- DM relic density is computed by

$$\Omega h^2 = \frac{2.14 \times 10^9 \text{ GeV}^{-1}}{g_*^{1/2} M_{Pl}} \frac{1}{J(x_f)}, \quad J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma v \rangle(x)}{x^2} dx.$$

where $x = M_-/T$ and x_f is the freeze out parameter.



Detection prospects

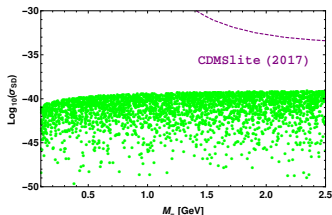
- SLQ portal spin-dependent (SD) cross section can arise from the effective interaction

$$\mathcal{L}_{\text{eff}}^{\text{SD}} \simeq \frac{y_{qR}^2 \cos^2 \beta}{4(M_{S_1}^2 - M_-^2)} \bar{N} \gamma^\mu \gamma^5 N \bar{q} \gamma_\mu \gamma^5 q,$$

and the computed cross section is given as

$$\sigma_{\text{SD}} = \frac{\mu_r^2}{\pi} \frac{\cos^4 \beta}{(M_{S_1}^2 - M_-^2)^2} [y_{dR}^2 \Delta_d + y_{sR}^2 \Delta_s]^2 J_n(J_n + 1).$$

- The WIMP-nucleon cross section via (Z, Z') portal and (H_1, H_2) portal is found to be very small and insensitive to direct detection experiments.



Constraints from Flavour sector

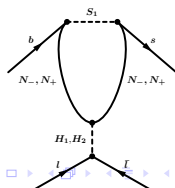
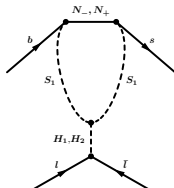
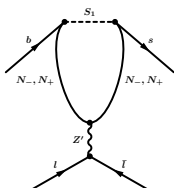
- Model parameters of LQ and Z' couplings can be constrained using $R_{K^{(*)}}$ and $\text{Br}(B \rightarrow X_s \gamma)$.
- The effective Hamiltonian mediating $b \rightarrow sl^+ l^-$ is

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^6 C_i(\mu) O_i + \sum_{i=7,9,10} \left(C_i(\mu) O_i + C'_i(\mu) O'_i \right) \right],$$

$$O_7^{(l)} = \frac{e}{16\pi^2} \left[\bar{s} \sigma_{\mu\nu} (m_s P_{L(R)} + m_b P_{R(L)}) b \right] F^{\mu\nu},$$

$$O_9^{(l)} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma^\mu P_{L(R)} b) (\bar{l} \gamma_\mu l), \quad O_{10}^{(l)} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \gamma^\mu P_{L(R)} b) (\bar{l} \gamma_\mu \gamma_5 l),$$

- Following one loop diagrams provide non-zero contribution to the rare $b \rightarrow sl$ processes (2nd and 3rd diagrams $\propto m_q M_\pm / M_{S_1}^2$)



- Z' exchange penguin diagram gives the transition amplitude of $b \rightarrow sll$ process

$$\mathcal{M} = \frac{1}{2^5 \pi^2} \frac{y_{qR}^2 g_{\mu\tau}^2}{(q^2 - M_{Z'}^2)} \mathcal{V}_{sb}(\chi_-, \chi_+) \left[\bar{u}(p_B) \gamma^\mu (1 + \gamma_5) u(p_K) \right] \left[\bar{\nu}(p_2) \gamma_\mu u(p_1) \right],$$

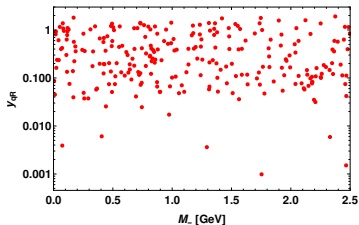
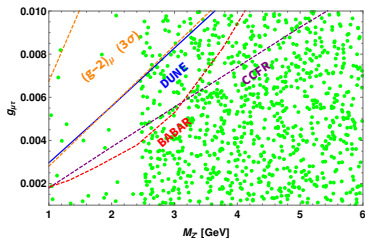
which provides additional primed Wilson coefficient

$$C_9^{\text{NP}} = \frac{\sqrt{2}}{2^4 \pi G_F \alpha_{\text{em}} V_{tb} V_{ts}^*} \frac{y_{qR}^2 g_{\mu\tau}^2}{(q^2 - M_{Z'}^2)} \mathcal{V}_{sb}(\chi_-, \chi_+),$$

$\mathcal{V}_{sb}(\chi_-, \chi_+)$ is the loop function and $\chi_\pm = M_\pm^2 / M_{S_1}^2$.

- As only C_9^{NP} involves, $B_s \rightarrow \mu\mu(\tau\tau)$ won't play any role in constraining the new parameters.
- Absence of $Z' \mu\tau$ coupling \Rightarrow LFV decays like $B \rightarrow K^{(*)} \mu\tau$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow 3\mu$ are not allowed

- Thus, using R_K/R_{K^*} and $\text{Br}(B \rightarrow X_s \gamma)$ observables, the $g_{\mu\tau}$, $M_{Z'}$ and the y_{qR} , M_- allowed regions are shown below



- The allowed range of all the four new parameters consistent with flavor phenomenology

Parameters	y_{qR}	$g_{\mu\tau}$	M_- (GeV)	$M_{Z'}$ (GeV)
Allowed range	0 – 2.0	0 – 0.01	0 – 2.5	1 – 6

Table: The allowed regions of y_{qR} , $g_{\mu\tau}$, M_- and $M_{Z'}$ parameters.

Footprints on $b \rightarrow s + \cancel{E}$ decay modes

- In SM, $b \rightarrow s +$ missing energy can be described by the $b \rightarrow s\nu\bar{\nu}$
- The effective Hamiltonian in SM

$$\mathcal{H}_{\text{eff}} = \frac{-4G_F}{\sqrt{2}} V_{tb} V_{ts}^* (C_L^\nu \mathcal{O}_L^\nu + C_R^\nu \mathcal{O}_R^\nu) + h.c.,$$

where

$$\mathcal{O}_L^\nu = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_R \gamma_\mu b_L) (\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu), \quad \mathcal{O}_R^\nu = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_L \gamma_\mu b_R) (\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu),$$

$$C_L^\nu = -X(x_t)/\sin^2 \theta_w, \quad X(x_t) = X_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t),$$

- The branching ratios of $B_{(s)} \rightarrow K^*(\phi)\nu\bar{\nu}$ and their corresponding experimental limits are

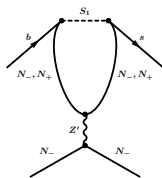
Decay process	BR in the SM	Experimental limit
$B^0 \rightarrow K^0 \nu_l \bar{\nu}_l$	$(4.53 \pm 0.267) \times 10^{-6}$	$< 2.6 \times 10^{-5}$
$B^+ \rightarrow K^+ \nu_l \bar{\nu}_l$	$(4.9 \pm 0.288) \times 10^{-6}$	$< 1.6 \times 10^{-5}$
$B^0 \rightarrow K^{*0} \nu_l \bar{\nu}_l$	$(9.48 \pm 0.752) \times 10^{-6}$	$< 1.8 \times 10^{-5}$
$B^+ \rightarrow K^{*+} \nu_l \bar{\nu}_l$	$(1.03 \pm 0.06) \times 10^{-5}$	$< 4.0 \times 10^{-5}$
$B_s \rightarrow \phi \nu_l \bar{\nu}_l$	$(1.2 \pm 0.07) \times 10^{-5}$	$< 5.4 \times 10^{-3}$

- In this model, the additional process involved is

$$b \rightarrow s + \text{missing energy} = b \rightarrow s\nu\nu + b \rightarrow sN_-N_-$$

Footprints on $b \rightarrow s + \cancel{E}$ decay modes

- The relevant one-loop diagram for $b \rightarrow s N_- N_-$ is



- Thus, e.g., the amplitude of $B \rightarrow KN_- N_-$ process from the Z' exchanging diagram is

$$\mathcal{M} = C^{\text{NP}}(q^2) [\bar{u}(p_B) \gamma^\mu (1 + \gamma_5) u(p_K)] [\bar{v}(p_2) \gamma_\mu u(p_1)]$$

where

$$C^{\text{NP}}(q^2) = \frac{1}{2^5 \pi^2} \frac{y_{qR}^2 g_{\mu\tau}^2 \cos 2\beta \cos \alpha \sec \chi}{q^2 - M_{Z'}^2} \mathcal{V}_{sb}(\chi_-, \chi_+),$$

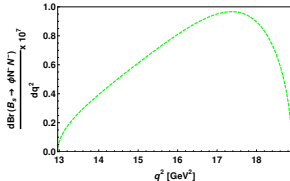
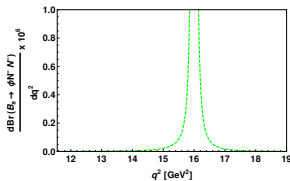
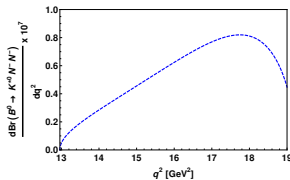
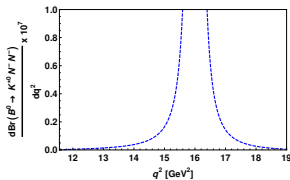
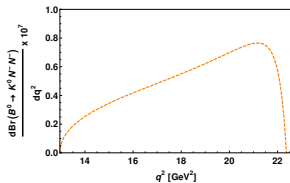
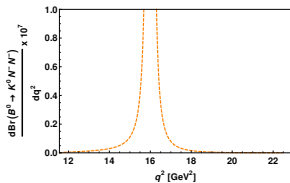
Predicted Results for $b \rightarrow s + \cancel{E}$ decay modes

- We use two sets of benchmark values of new parameters, allowed by both the DM and flavor phenomenology

Benchmark	y_{qR}	$g_{\mu\tau}$	M_- (GeV)	$M_{Z'}$ (GeV)
Benchmark-I	2.0	0.002	1.7	4
Benchmark-II	2.0	0.008	1.8	4.8

Table: Benchmark values of y_{qR} , M_- , $g_{\mu\tau}$ and $M_{Z'}$ parameters used in our analysis.

Predicted Results for $b \rightarrow s + \cancel{E}$ decay modes



Predicted Results for $b \rightarrow s + \cancel{E}$ decay modes

- For Benchmark-I, there is a singularity at $q^2 = M_{Z'}^2$, i.e., $q^2 = 16 \text{ GeV}^2$. To avoid it, we use the cuts at $(M_{Z'} - 0.002)^2 \leq q^2 \leq (M_{Z'} + 0.002)^2 \text{ GeV}^2$.

$\text{Br}(b \rightarrow s \cancel{E})$	Benchmark-I	Benchmark-II	Experimental Limit
$\text{Br}(B^0 \rightarrow K^0 \cancel{E})$	0.645×10^{-5}	0.457×10^{-5}	$< 2.6 \times 10^{-5}$
$\text{Br}(B^+ \rightarrow K^+ \cancel{E})$	0.697×10^{-5}	0.516×10^{-5}	$< 1.6 \times 10^{-5}$
$\text{Br}(B^0 \rightarrow K^{*0} \cancel{E})$	1.271×10^{-5}	0.981×10^{-5}	$< 1.8 \times 10^{-5}$
$\text{Br}(B^+ \rightarrow K^{*+} \cancel{E})$	1.381×10^{-5}	1.066×10^{-5}	$< 4.0 \times 10^{-5}$
$\text{Br}(B_s \rightarrow \phi \cancel{E})$	1.618×10^{-5}	1.24×10^{-5}	$< 5.4 \times 10^{-3}$

Table: The predicted branching ratios of $b \rightarrow s \cancel{E}$ processes for two different benchmark values of new parameters.

Conclusion

- We explored GeV scale dark matter and flavour anomalies in a $U(1)_{L_\mu-L_\tau}$ gauge extension of SM with an additional $(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$ Scalar LQ.
- Relic density is investigated in Z' portal and the spin-dependent WIMP-nucleon cross section in SLQ-portal.
- In flavor sector, the model parameters are constrained from experimental limits on R_K/R_{K^*} and $B \rightarrow X_s \gamma$.
- We have shown the impact on rare B meson decays to missing energy, the observation of these modes would provide strong hints for the existence of light fermionic dark matter.
- This simple gauge extension provides an ideal platform to address the phenomenological perspectives of dark matter and flavor anomalies simultaneously.

Thank You