Black hole shadows: from LQG to expanding universe: what can they tell us

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Abstract

We perform the shadow calculation of the loop quantum gravity motivated regular black hole recently proposed by Ashtekar, Olmedo and Singh[1] (AOS BH hereafter). In the process, we also construct the rotating loop quantum gravity inspired solution of the originally proposed static spherically symmetric AOS black hole by applying the modified Newman-Janis algorithm. We study the quantum effects on the shadows of both the non-rotating and rotating loop quantum black hole solutions. It is observed that the general shape of the shadow for nonrotating AOS black hole is circular in shape as is expected for its classical counter part too, but the presence of loop quantum gravity inspired modification contracts the shadow radius and the effect reduces with the increase in the mass of the black hole. On a similar note, in the rotating situation, we find contraction in shadow radius due to quantum effects and the tapered nature of the shadow as expected from the classical Kerr case. However, instead of the symmetrical contraction, like non-rotating one, we found more contraction on one side relative to the other when we compare our result with the shadow of the Kerr black hole. As another example of shadow calculation, we find general expressions for asymptotically de Sitter black hole shadow as seen by static and comoving observers, for any spherically symmetric black hole solution, in any space time dimension in generic theories of gravity

Non rotating AOS

In strong gravitational regime, the quantum nature of spacetime is very important to construct a viable theoretical model of the dynamics of gravity. Furthermore, the singularity inside the BH has been a troubling and uncomfortable region. Non-singularly complete solutions, such as regular BHs are one of the suitable candidates to avoid such situations. There exists many regular BH solutions in the literature. However, in most cases, such BH space-times are not obtained as a solution of some underlying theory, neither are they connected to any quantum theory of gravity. On the other hand, it is well know that near the BH singularity the quantum effects are not negligible and must be incorporated within the solution itself. Towards this direction, Loop Quantum Gravity (LQG) turns out to be one of the few successful attempts to understand the quantum nature of gravity. Very recently, Ashtekar, Olmedo and Singh (AOS) found a complete regular static BH spacetime from an effective LQG motivated theory which is a quantum extended version of Kruskal geometry. The usual singular point r = 0 is hidden within a minimum area element determined by "some" underlying microscopic theory.

The spherically symmetric AOS metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + h(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right), \text{ where}$$

$$f(r) = \frac{\left(\frac{r}{r_S}\right)^{2\epsilon}\left(1-\left(\frac{r_S}{r}\right)^{1+\epsilon}\right)\left(2+\epsilon+\epsilon\left(\frac{r_S}{r}\right)^{1+\epsilon}\right)^2}{16\left(1+\frac{\delta_c^2 L_0^2\gamma^2 r_S^2}{16r^4}\right)(1+\epsilon)^4\left((2+\epsilon)^2-\epsilon^2\left(\frac{r_S}{r}\right)^{1+\epsilon}\right)}, \quad g(r)^{-1} = \frac{\left(1+\frac{\delta_c^2 L_0^2\gamma^2 r_S^2}{16r^4}\right)\left(\epsilon+\left(\frac{r}{r_S}\right)^{1+\epsilon}(2+\epsilon)\right)^2}{\left(\left(\frac{r}{r_S}\right)^{1+\epsilon}-1\right)\left(\left(\frac{r}{r_S}\right)^{1+\epsilon}(2+\epsilon)^2-\epsilon^2\right)}$$

$$h(r) = r^2\left(1+\frac{\gamma^2 L_0^2\delta_c^2 r_S^2}{16r^4}\right), \ \delta_b = \left(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\right)^{1/3}, \ L_o\delta_c = \frac{1}{2}\left(\frac{\gamma\Delta^2}{4\pi^2 m}\right)^{1/3}, m = \frac{GM}{c^2} = M; (1+\gamma^2\delta_b^2)^{1/2} = 1+\epsilon$$

δ_b, δ_c are quantum parameters.

Δ is the minimum non-zero eigenvalue of the area operator in LQG, given by $\Delta \sim 5.17\ell_{\rm pl}^2$ and $\gamma \sim 0.2375$ is Barbero Immirzi parameter. L_o is an infrared regulator introduced to make the phase space description well-defined.

Rotating AOS and Shadows

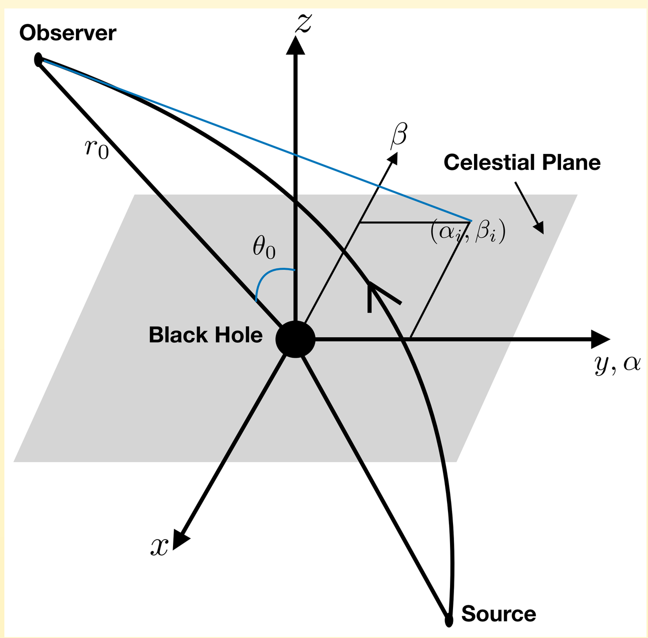
The rotating counterpart[2] of AOS black hole is obtained by using the modified Newman Janis Algorithm (NJA)

$$ds^2 = -Fdt^2 - 2a\sin^2\theta\left(\sqrt{\frac{F}{G}}-F\right)dtd\phi + \frac{H}{g(r)h(r)+a^2}dr^2 + Hd\theta^2 + \sin^2\theta\left[H + a^2\sin^2\theta\left(2\sqrt{\frac{F}{G}}-F\right)\right]d\phi^2$$

$$F = \frac{g(r)h(r)+a^2\cos^2\theta}{(k(r)+a^2\cos^2\theta)^2}H, G = \frac{g(r)h(r)+a^2\cos^2\theta}{H}$$, while H remains undetermined, and $k(r) = \sqrt{\frac{g(r)}{f(r)}}h(r)$

- → Shadow → an image of the photon sphere which is gravitationally lensed by the presence of extremely strong gravitational field around the black hole and projected on the local sky of an observer.
- → Shadow contours correspond to unstable circular null geodesics
- → To find the null geodesics around the AOS BH, use the Hamilton-Jacobi (H-J) equation.

The radial null geodesic equation $\dot{r}^2 + V_{eff}(r) = 0$ obtained from the H-J equation $\frac{\partial S}{\partial\lambda} + \frac{1}{2}g^{\mu\nu}p_\mu p_\nu = 0$ gives the condition to find the radius of the photon sphere as



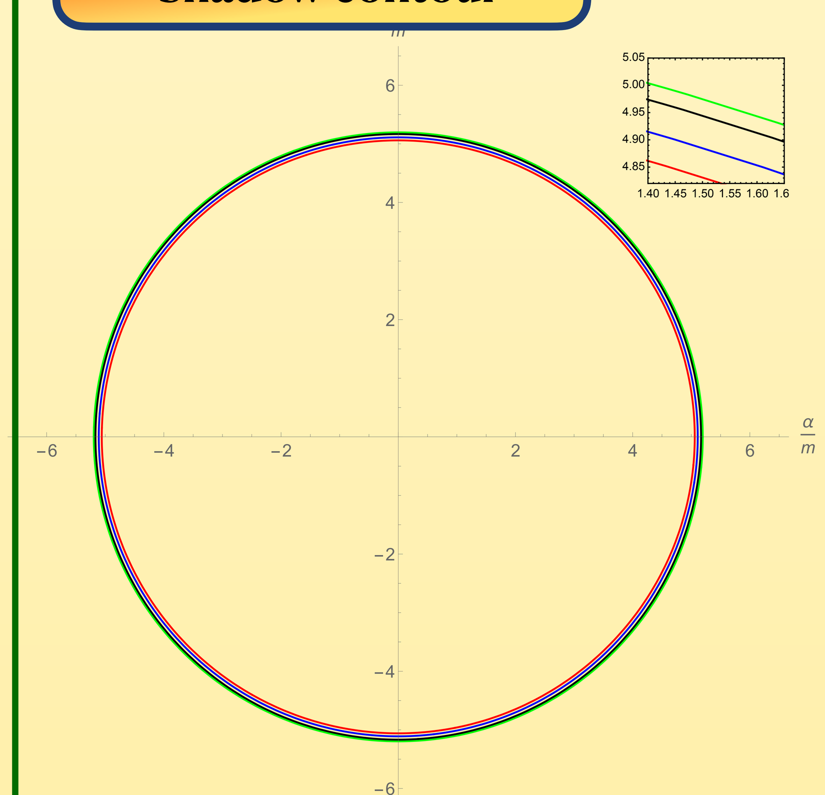
$$V_{eff}(r)|_{r=r_{ph}} = 0, \quad \left.\frac{dV_{eff}(r)}{dr}\right|_{r=r_{ph}} = 0 \quad , \left.\frac{d^2V_{eff}(r)}{dr^2}\right|_{r=r_{ph}} < 0$$

The shadow is obtained in the celestial plane with parametric plots of α and β

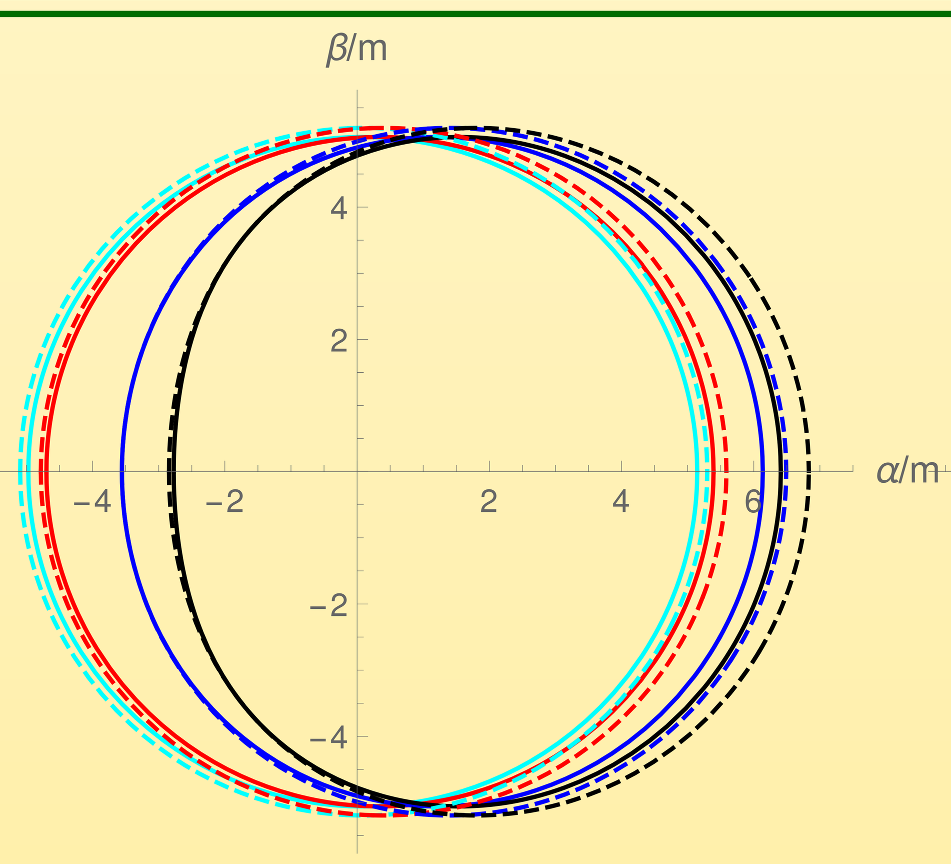
- $\alpha_i = -r_0^2\sin\theta_0\left.\frac{d\phi}{dr}\right|_{(r_0,\theta_0)} = -\frac{\xi}{\sin\theta_0}$
- $\beta_i = r_0^2\left.\frac{d\theta}{dr}\right|_{(r_0,\theta_0)} = \pm\sqrt{\eta + a^2\cos^2\theta_0 - \xi^2\cot^2\theta_0}$

$\xi[= L/E]$ and $\eta[= Q/E^2]$ $\implies$ critical impact parameters determining the motion of the photon.

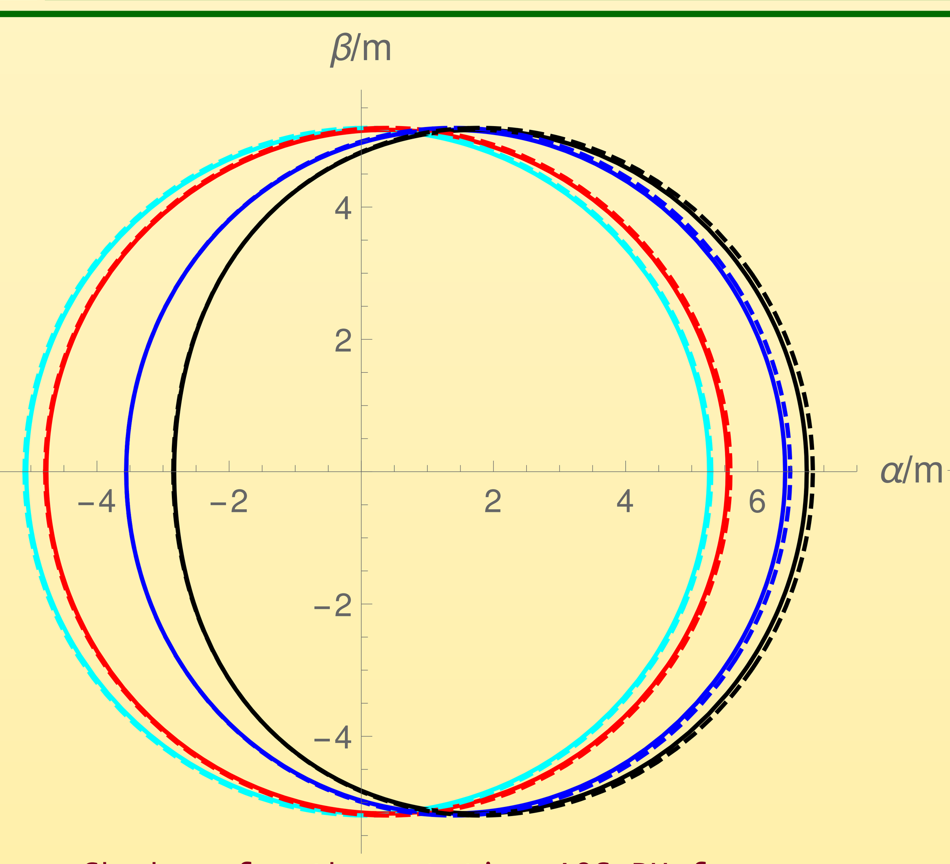
Shadow contour



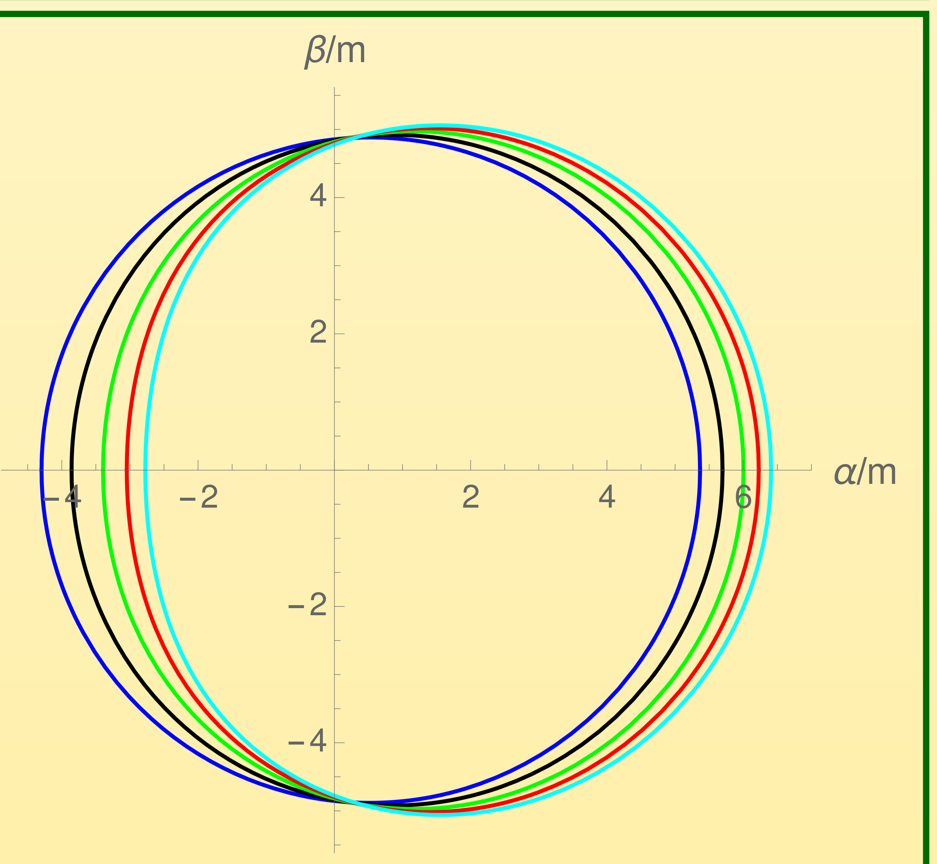
Shadows for the non rotating AOS BH for different values of the quantum parameters. The green circle corresponds to the standard Schwarzschild BH shadow with mass $1\ell_{pl}$ and The red, blue and black circles correspond to the quantum case with masses 1, 2 and 10 Planck lengths respectively.



Shadows for the rotating AOS BH for different spin parameters and m = $1\ell_{pl}$. Cyan (a = 0.05m), Red (a = 0.2m), Blue (a = 0.7m), Black (a = 0.9m). The dashed contours represent the Kerr case while the solid contours represent the rotating AOS BH.



Shadows for the rotating AOS BH for different spin parameters and m = 10 ℓ_{pl}. Cyan (a = 0.05m), Red (a = 0.2m), Blue (a = 0.7m), Black (a = 0.9m). The dashed contours represent the Kerr case while the solid contours represent the rotating AOS BH.



Shadows for the effective quantum metric with various values of inclination angles θ_0 for $a = 0.9m$ with $m = 1\ell_{pl}$

Color codes: Blue $\theta_0 = 17°$, Black $\theta_0 = 30°$, Green $\theta_0 = 45°$, Red $\theta_0 = 60°$, Cyan $\theta_0 = 90°$.

m	$\delta_c \times 10^{-36}$	δ_b	ϵ	$r_{ps}\times 10^{-35}$	$R_{sh}\times 10^{-35}$
$1\ell_{pl}$	0	0	0	4.848000000000000069	8.3969823150
$1\ell_{pl}$	2.929195935615	2.5241163490137	0.165921709351	4.991546565555	8.175394616783
$2\ell_{pl}$	2.324904354808	2.0033924738572	0.1074254268738	9.87045034976057	0.1651663773526
$10\ell_{pl}$	1.359612314621	1.171591026032	0.0379906834505	0.487671184162043	0.83499096523578

Values of different parameters for the non-rotating case. The first row contains the numbers for Schwarzschild black hole with mass m = $1\ell_{pl}$ and the corresponding rows contain the values for the AOS black hole with masses $1\ell_{pl}$, $2\ell_{pl}$ and $10\ell_{pl}$ respectively.

Conclusion

- Shape of the shadow for non-rotating AOS does not change (circular in shape) when compared with the Schwarzschild BH, the presence of LQG inspired modification contracts the radius which becomes less with the increase in the mass.
- In rotating situation, we find contraction in shadow radius due to quantum effects. Instead of symmetrical contraction, like non-rotating one, here we have more contraction on the right hand side relative to the left hand side when one compares with shadow for vanishing quantum parameters.
- For a fixed mass of BH contraction increases on the right hand side while the same decreases on the left side as we increase the rotation parameter.
- For a fixed rotation of BH the contraction decreases as the mass of BH increases. Moreover, it approaches towards shadow of the Kerr BH as one increases the mass.

Shadows of BH in an expanding universe

Spacetime in most of the shadow calculations is assumed to be time independent ➡ static or stationary observer will see a time-independent shadow.

We live in an expanding universe ➡ Q. How shadow changes with time?

Problem to be tackled: dependence of the shadow on the momentary position of the observer will no longer be expressed by the formulas for a static or stationary black hole.

Why important? BH at the centre of our galaxy/centres of nearby galaxies the effect of the cosmological expansion is tiny. But, for galaxies at a larger distance the influence on the angular diameter of the shadow is large.

Two different classes of observers: static (spatial position is fixed) and comoving (with the cosmic expansion).

The shadow with respect to the comoving observer: perform a coordinate transformation to go from the asymptotically de Sitter BH metric to a form of expanding universe metric with a black hole embedded in it. (McVittie type transformation) ➡ requires expressing the BH metric in an isotropic form[3].

Not always possible to find a closed form solution for any general BH metric in an isotropic form, let alone in a McVittie type coordinate system.

Question: is it possible to find an expression for the shadow with respect to a comoving observer for BH metric without a closed form solution for the isotropic transformation function? One can indeed find the comoving shadow even without knowing the exact transformation functions as mentioned above.

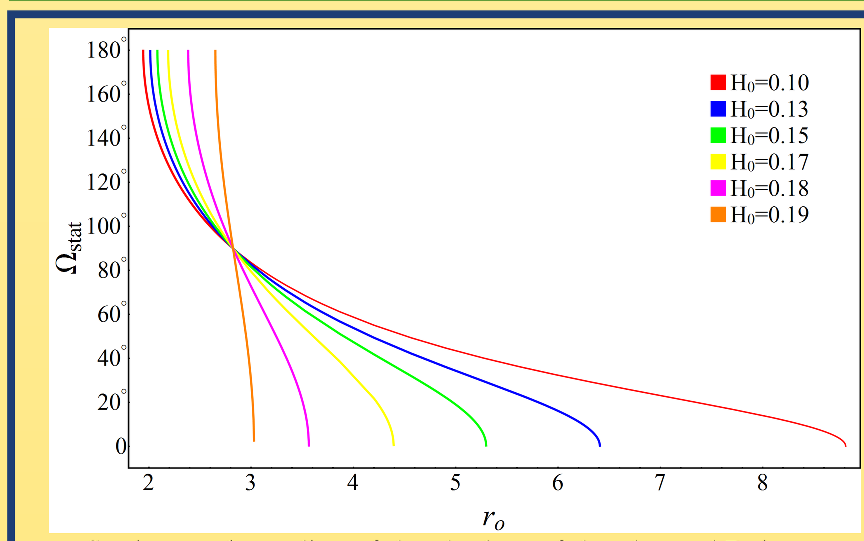
Velocity of comoving observer with the cosmic expansion with respect to the static observer $v_{\rm comov} = \sqrt{1-\frac{f(r_0)}{f(r)}}$

Comoving velocity does not depend explicitly on the transformation functions or the scaling function, but only depends on the metric function f(r). $f(r_0) = f(r)|_{\Lambda=0}$ and $ds^2 = -h(r)dt^2 + f(r)dr^2 + r^2 d\Omega_{d-2}^2$

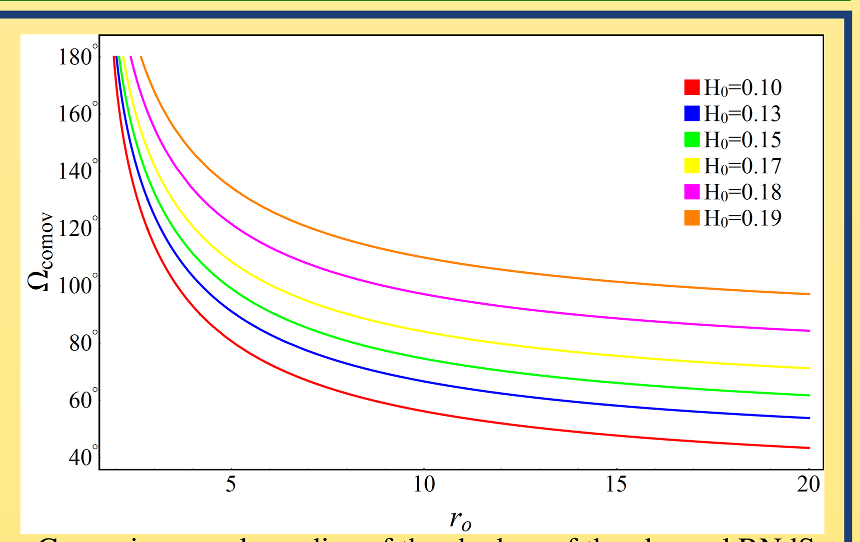
General expression[4] for the angular radius of the shadow of an asymptotically deSitter BH w.r.t. a comoving observer

$$\sin^2\Omega_{\rm comov} = \frac{f_0(r_o)}{f(r_o)}\frac{\chi_p h(r_o)/r_o^2}{\left(1\pm\sqrt{1-\frac{f_0(r_o)}{f(r_o)}}\sqrt{1-\frac{\chi_p h(r_o)}{r_o^2}}\right)^2}$$

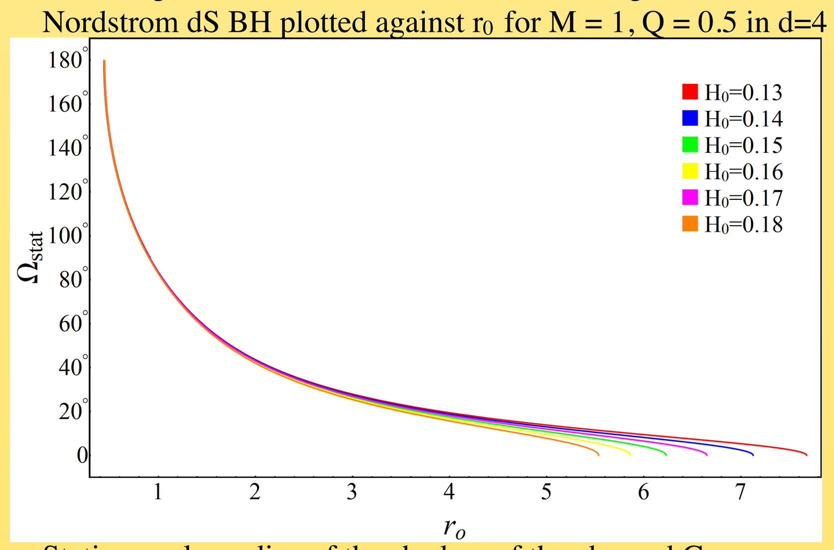
χ_p: one of the constants of motion for unstable photon



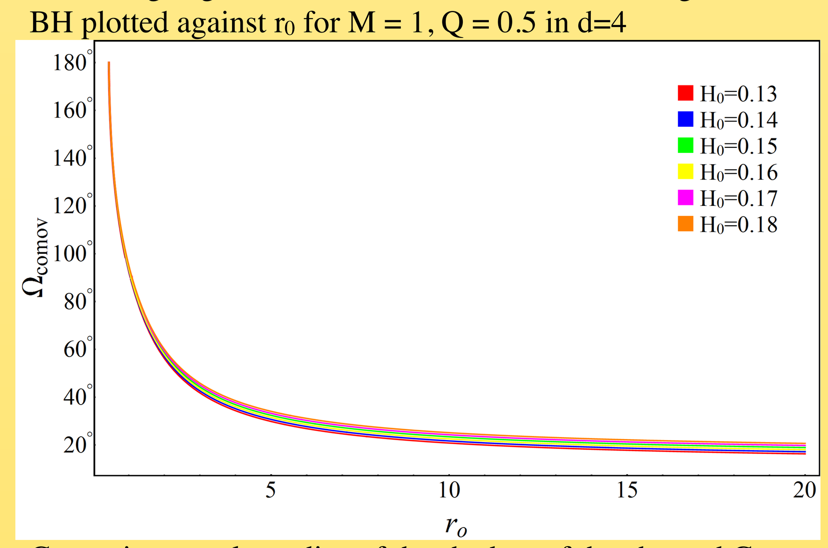
Static angular radius of the shadow of the charged Reissner Nordstrom dS BH plotted against r_0 for M = 1, Q = 0.5 in d=4



Comoving angular radius of the shadow of the charged RNdS BH plotted against r_0 for M = 1, Q = 0.5 in d=4



Static angular radius of the shadow of the charged Gauss Bonnet dS BH for M = 1, Q = 0.5, α = 0.31 in d=5



Comoving angular radius of the shadow of the charged Gauss Bonnet dS BH for M = 1, Q = 0.5, α = 0.31 in d=5

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