Grover's Quantum Search Algorithm of Causal Multiloop Feynman Integrals.

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1. Introduction

We present the first proof-of-concept application of a quantum algorithm to multiloop Feynman integrals exploiting the Loop-Tree Duality (LTD) and causality. Causality obtained from the LTD formalism is a suitable problem to address with quantum computers. The two on-shell states of Feynman propagators are identified with the two states of a qubit. We modify the original Grover's algorithm for queryng multiple solutions over unstructured databases and implement the algorithm in IBM Quantum[1] and QUTE[2] simulators. The algorithm may also find application and interest in graph theory to solve problems involving directed acyclic graphs.

2. Loop-Tree Duality and Causality

Multiloop multileg integrals and scattering amplitudes are defined, in the Feynman representation, as integrals,

$$\mathcal{A}_F^{(L)} = \int_{\ell_1\ldots\ell_L} \frac{\mathcal{N}(\{\ell_s\}_L, \{p_j\}_P)}{\prod_{i=1}^n (q_{i,0} - q_{i,0}^{(+)})(q_{i,0} + q_{i,0}^{(+)})}\,; \quad \int_{\ell_s} \equiv -\imath\mu^{4-d}\int \frac{\mathrm{d}^d\ell_s}{(2\pi)^d} \tag{1}$$

in the Minkowski space of the L loop momenta. $\int_{\ell_s}$, stands for the integration measure in DREG[3] with d space-time dimensions and μ an arbitrary energy scale, the numerator $\mathcal{N}$ is given by the Feynman rules of the specific theory and $q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + m_i^2 - \imath 0}$ are the energy components of the momenta flowing through the propagators q_i, with $\mathbf{q}_i$ the spacial components and m_i the propagating particle mass.

Causal and noncausal singularities in Eq. (1) arises when Feynman propagators are set on-shell. Explicitly, when the energy component, $q_{i,0}$, takes either value of $\pm q_{i,0}^{(+)}$.

The LTD [1, 2] aims to overcome these and other problems present in scattering amplitudes. The LTD representation of scattering amplitudes is obtained by the iterative application of the Cauchy's residue theorem, integrating out one degree of freedom, selecting the poles with negative imaginary part in the complex plane of the loop momentum when closing the contour from below the real axis which is equivalent to set on-shell certain internal propagators.

The calculation of the nested residues within the LTD leads to a causal representation of scattering amplitudes where it is shown [3, 4] that Eq. (1) is equivalent to

$$\mathcal{A}_D^{(L)} = \int_{\vec{\ell}_1\ldots\vec{\ell}_L} \frac{1}{x_n} \sum_{\sigma\in\Sigma} \mathcal{N}_\sigma \prod_{i=1}^{n-L} \frac{1}{\lambda_{\sigma(i)}^{h_{\sigma(i)}}} + (\lambda^+ \leftrightarrow \lambda^-)\,, \tag{2}$$

with $x_n = \prod_n 2q_{i,0}^{(+)}$, $h_{\sigma(i)} = \pm 1$, and

$$\int_{\vec{\ell}_s} = -\mu^{4-d}\int \frac{d^{d-1}\ell_s}{(2\pi)^{d-1}}\,, \tag{3}$$

the integration measure in the loop three-momentum space. We define the causal propagators in Eq. (2) by

$$\lambda_{\sigma(i)}^{\pm} \equiv \lambda_p^{\pm} = \sum_{i\in p} q_{i,0}^{(+)} \pm k_{p,0}\,, \tag{4}$$

where $\sigma(i)$ stands for the partition p of the set of on-shell energies and the orientation of the energy components of the external momenta, $k_{p,0}$. Setting on-shell all propagators in p leads to a sign of $k_{p,0}$ that defines when $\lambda_p^{\pm}$ becomes singular. Each causal propagator is in a one-to-one correspondence with any possible threshold singularity of the amplitude $\mathcal{A}_F^{(L)}$, which contains overlapped thresholds; the latter known as causal thresholds. The combinations of entangled causal propagators represent causal thresholds that can occur simultaneously which are collected in the set Σ.

The following picture shows the topologies studied. Top, left to right: One-, two-, three-, and four-loops. Down, left to right: t-, s- and u-channels.

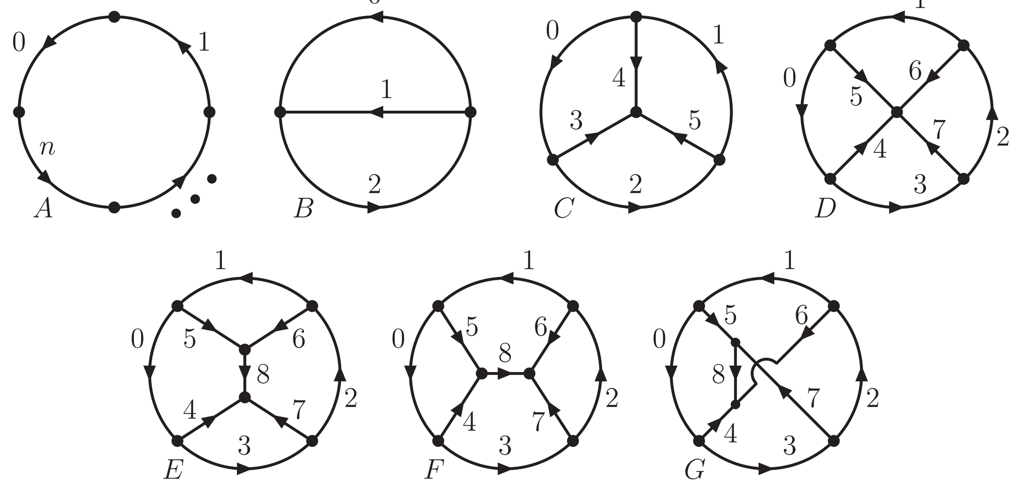


Figure 1: Representative topologies from one loop to up to four loops.

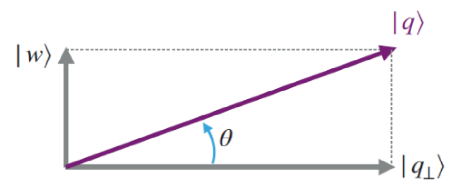
3. Causal Query of Multiloop Feynman Integrals

We construct a modified version of Grover's algorithm [5] for querying the so-called causal configurations.

3.1 Preparation

From the $N = 2^n$ states we have r winning states and $N - r$ orthogonales states

$$|q\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle = \cos\theta|q_\perp\rangle + \sin\theta|w\rangle\,. \tag{5}$$

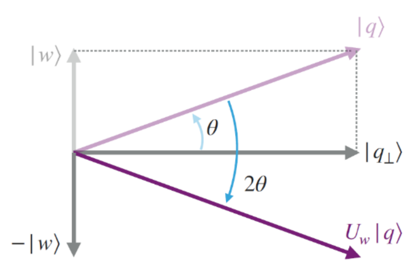


$|q_\perp\rangle$ encodes the noncausal states and $|w\rangle$ the casual states. The angle $\theta = \arcsin\sqrt{r/N}$.

3.2 Amplification

- The oracle operator U_w marks the winning states and the diffusion operator U_q amplifies them

$$U_w = \boldsymbol{I} - 2|w\rangle\langle w|\,, \quad U_q = 2|q\rangle\langle q| - \boldsymbol{I}\,. \tag{6}$$

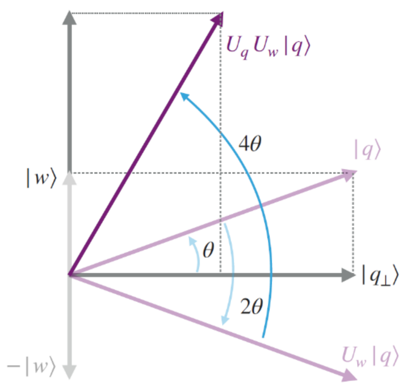


U_w flips the state $|x\rangle$ if $x \in w$, $U_w|x\rangle = -|x\rangle$, and leaves it unchanged otherwise, $U_w|x\rangle = |x\rangle$ if $x \notin w$.

- The iterative application of the U_w and U_q operators for t times leads to

$$(U_qU_w)^t|q\rangle = \cos\theta_t\,|q_\perp\rangle + \sin\theta_t\,|w\rangle\,. \tag{7}$$

where $\theta_t = (2t+1)\,\theta$, reaching the best output when $\sin^2\theta_t \sim 1$.



3.3 Implementation

- We initialise the propagators in $|q\rangle$ in the state $|0\rangle$.
- The uniform superposition is achieved by $|q\rangle = H^{\otimes n}|q\rangle$.
- The register, $|c\rangle$, stores the Boolean clauses c_{ij} that probe if two qubits representing adjacent edges are in the same state,

$$c_{ij} = (q_i = q_j)\,; \qquad \bar{c}_{ij} = (q_i \neq q_j)\,; \qquad i,j \in (0,\ldots,n-1)\,. \tag{8}$$

  and are implemented by CNOT gates.
- The register, $|a\rangle$, encodes the loop clauses that probe if the qubits compared in $|c\rangle$ form a cyclic circuit, is implemented by Toffoli gates.
- The Grover's marker $|out\rangle$ is initialised in the state

$$|-\rangle \equiv \big(|0\rangle - |1\rangle\big)/\sqrt{2}\,.$$

- Finally, the oracle is defined as

$$U_w|q\rangle|c\rangle|a\rangle|out\rangle = (-1)^{f(a,q)}|q\rangle|c\rangle|a\rangle|out\rangle\,. \tag{9}$$

  If all the causal conditions are satisfied $f(a,q) = 1$, the corresponding states are marked; otherwise, $f(a,q) = 0$, they are left unchanged.

4. Benchmark multiloop topologies

We present the explicit implementation of the quantum algorithm in IBM Quantum and QUTE simulators. All the cases under 32 qubits were tested in IBM Quantum simulators. The four loops u-channel topology, with 33 qubits, was tested in QUTE.

4.1 One loop

We present the quantum circuit for one loop case and show the probability distributions of its output.

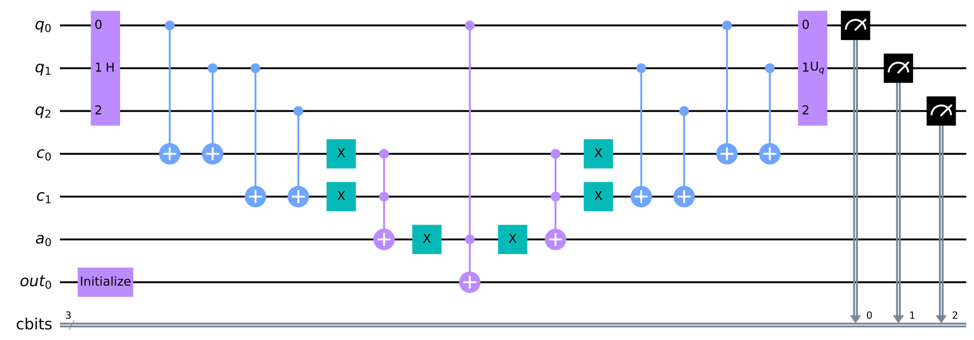


Figure 2: Quantum circuit for the one loop case.

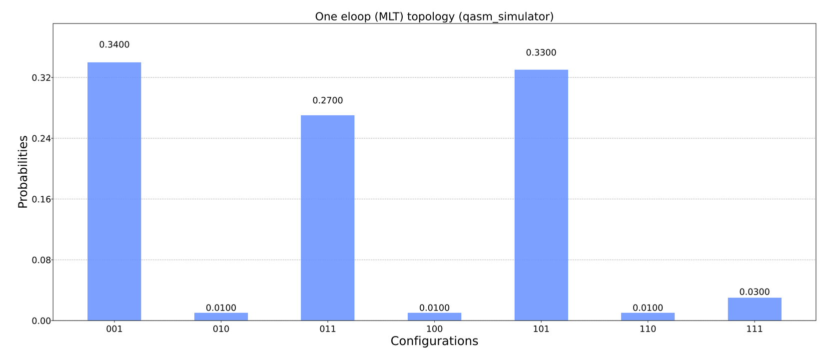


Figure 3: Probability distribution showing the causal configurations amplified. The number of selected states is 6/8.

4.2 Two and three loops

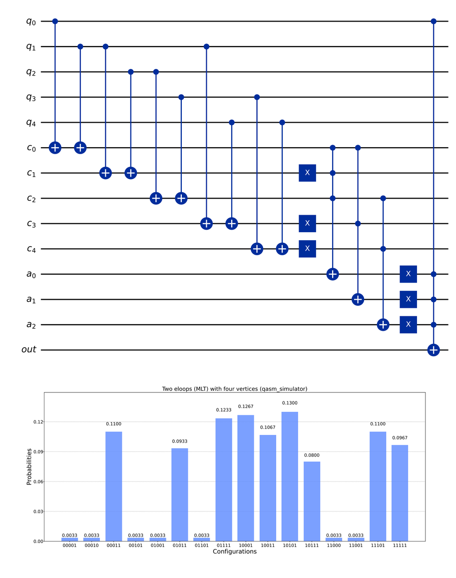


Figure 4: Up: Quantum circuit of the oracle for two loops. Bottom: Probability distributions of the results. Number of states is $9/2^5$

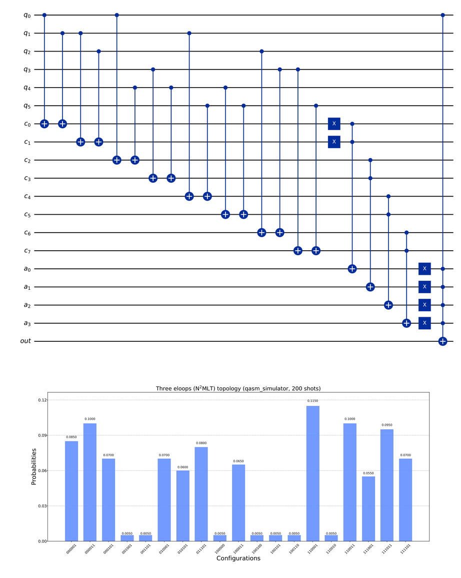


Figure 5: Up: Quantum circuit of the oracle for three loops. Bottom: Probability distributions of the results with the number of selected states $12/2^6$

4.3 Four loops| *s,t,u*-channels

Probability distribution of the results for the four loops, *s,t,* and *u*-channels.

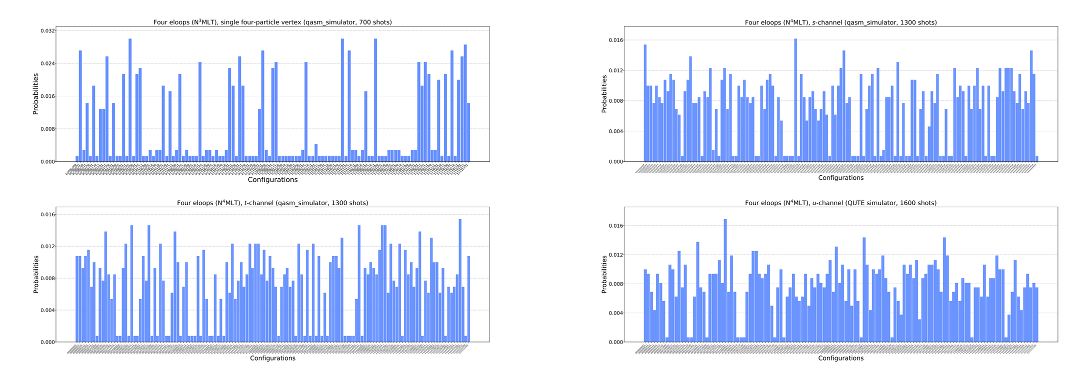


Figure 6: Top left: The number of selected states is $39/2^8$. Bottom left: The number of selected states is $102/2^9$. Bottom right: The number of selected states is $115/2^{10}$

5. Conclusions

- We have successfully identified all the causal singular configurations of selected topologies of multiloop Feynman integrals.
- The algorithm was succesfully implemented in IBM Quantum and QUTE simulators.
- The identification of directed acyclic graphs is a challenging problem beyond particle physics.
- The output of the quantum algorithm is used to bootstrap the causal representation in the LTD of representative multiloop topologies.

References

[1] Stefano Catani, Tanju Gleisberg, Frank Krauss, German Rodrigo, and Jan-Christopher Winter. From loops to trees by-passing Feynman's theorem. *JHEP*, 09:065, 2008.

[2] J. Jesus Aguilera-Verdugo, Felix Driencourt-Mangin, Roger J. Hernández-Pinto, Judith Plenter, Selomit Ramirez-Uribe, Andres E. Renteria Olivo, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla, and Szymon Tracz. Open Loop Amplitudes and Causality to All Orders and Powers from the Loop-Tree Duality. *Phys. Rev. Lett.*, 124(21):211602, 2020.

[3] J. J. Aguilera-Verdugo, F. Driencourt-Mangin, R. J. Hernández-Pinto, J. Plenter, S. Ramirez-Uribe, A. E. Renteria Olivo, G. Rodrigo, G. F. R. Sborlini, W. J. Torres Bobadilla, and S. Tracz. Open Loop Amplitudes and Causality to All Orders and Powers from the Loop-Tree Duality. *Phys. Rev. Lett.*, 124(21):211602, 2020.

[4] J. Aguilera-Verdugo et al. A Stroll through the Loop-Tree Duality. *Symmetry*, 13(6):1029, 2021.

[5] Lov K. Grover. Quantum mechanics helps in searching for a needle in a haystack. *Phys. Rev. Lett.*, 79:325–328, 1997.

[1] https://quantum-computing.ibm.com/
[2] https://qute.ctic.es/
[3] Dimensional Regularisation