

*Measurement of the Coulomb Sum Rule and Suppression  
of the Longitudinal Quasielastic Cross section  
From an analysis of all available electron scattering data  
on Carbon and Oxygen*

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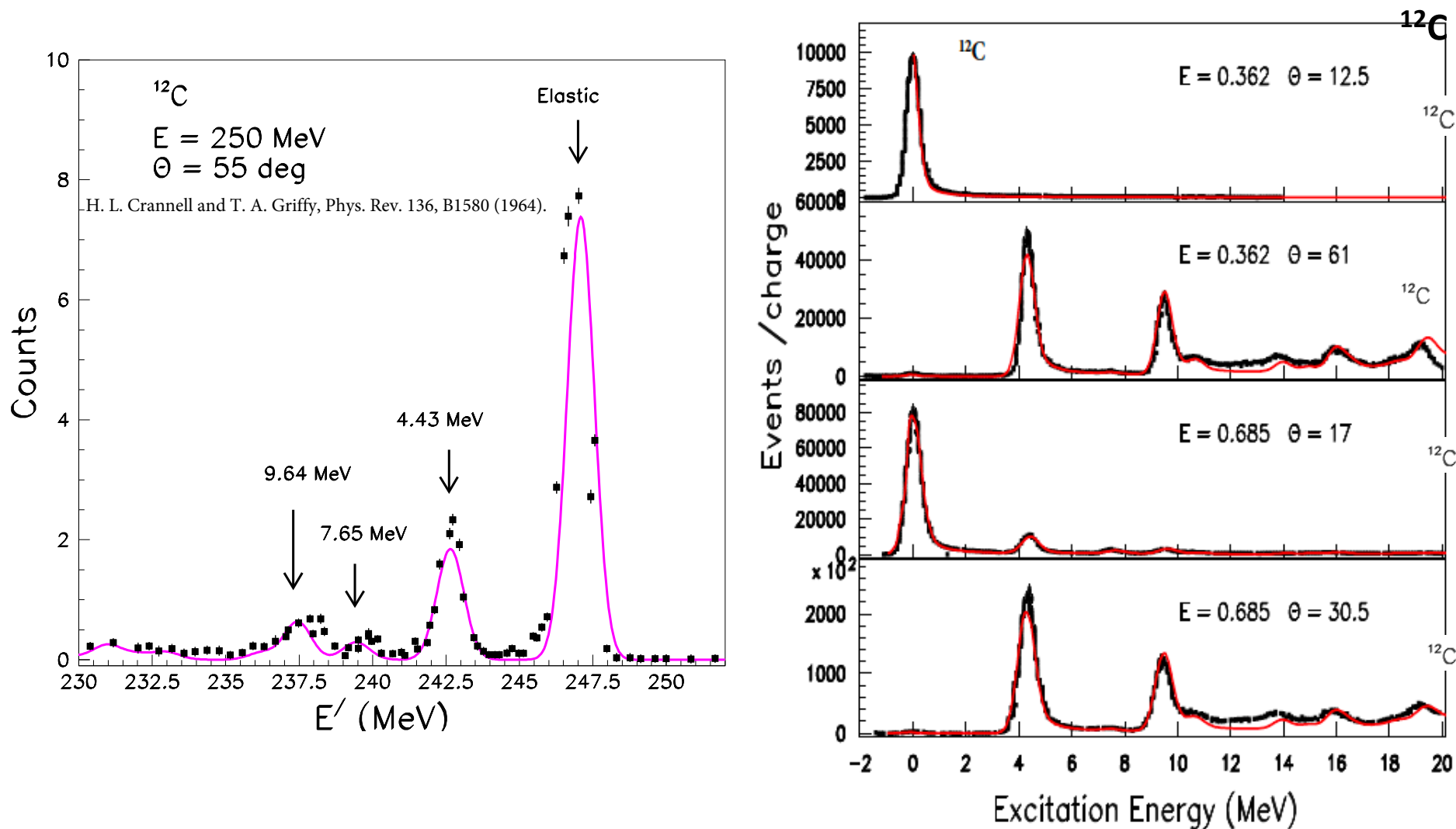
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This talk reports on analysis of all available H, D, Carbon and Oxygen electron scattering data (Analysis will be expanded to all nuclei)

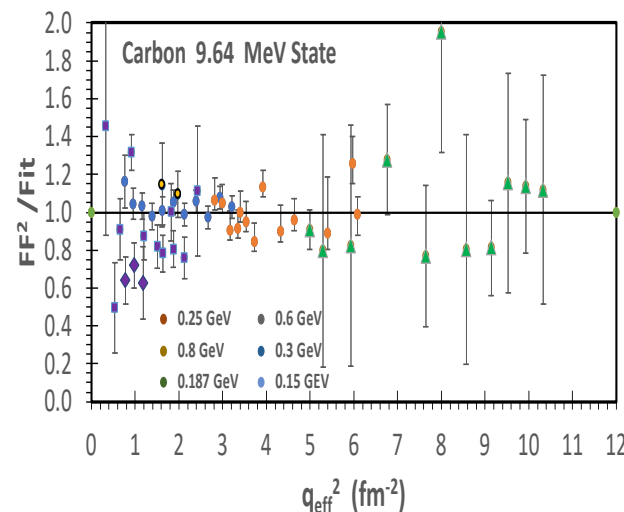
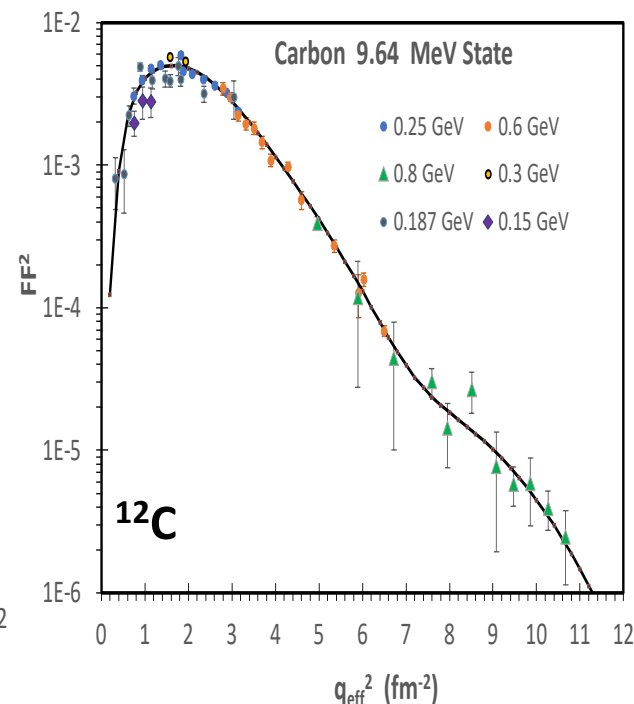
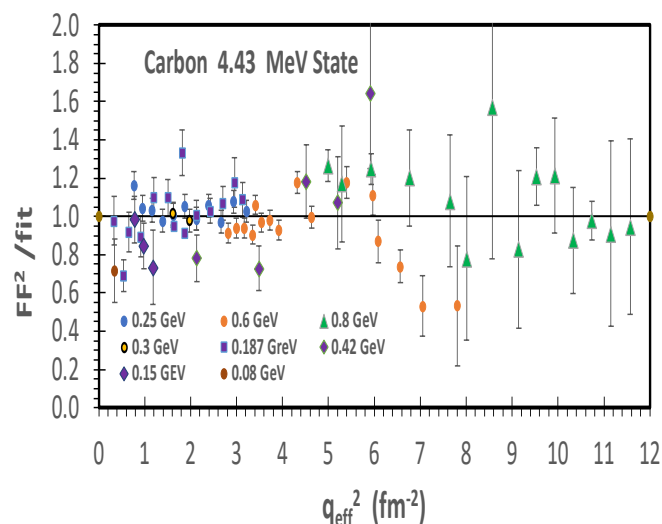
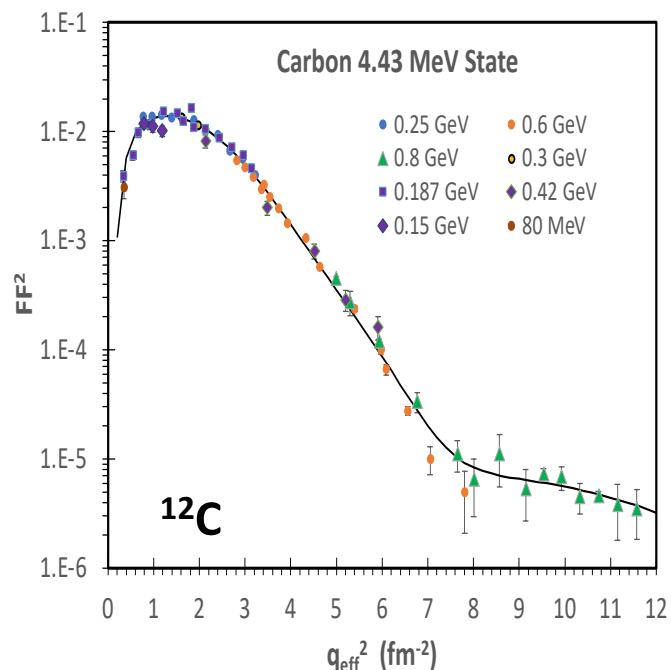
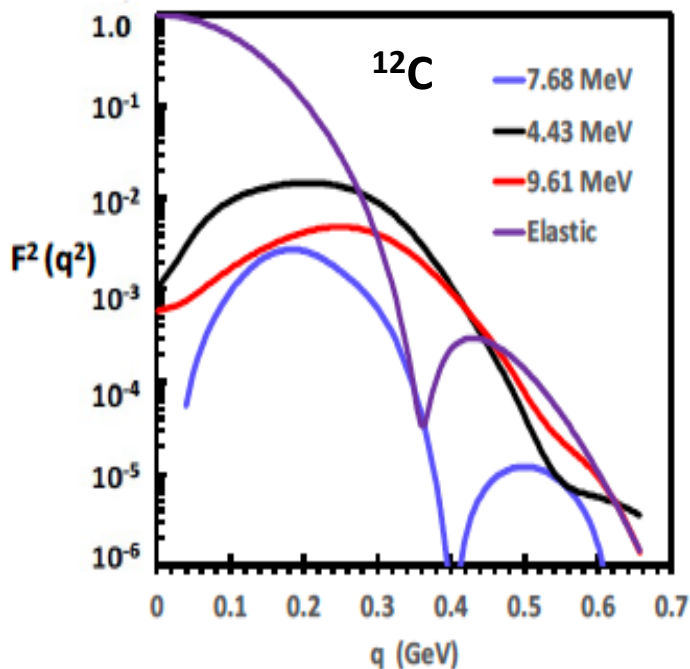
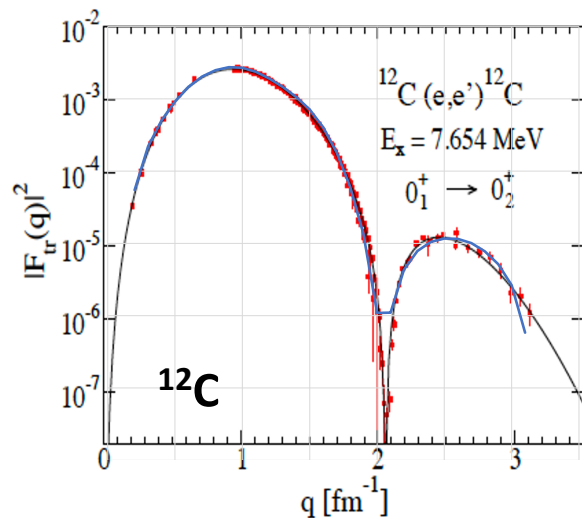
- We update the Bosted-Christy fit to **all of the world's electron scattering data** on H, D and nuclear targets to include the **lowest values of energy transfer  $\nu$  and  $q^2$**  (for carbon we fit about 8000 cross section measurements and 250 measurements for Oxygen). We fit the **QE cross section (including Transverse Enhancement/MEC, +longitudinal low  $q$  suppression)** **resonance and pion production**, **DIS**, nuclear **excitations**, elastic scattering data. Note: **Nuclear excitations are significant at low  $\nu$  and contribute up to 30% to the longitudinal Inelastic Coulomb Sum Rule (CSR)**
- Since the cross sections span a large range of energies and scattering angles, we **extract both the longitudinal RL and transverse RT** contributions.
- We parameterize both the **Enhancement of the Transverse QE cross section** and the **Suppression of the Longitudinal QE cross section**. We extract the most precise Coulomb Sum rule as a function of  $q$  and compare to theoretical calculations.
- 
- The fit can be used **in lieu of data to benchmark Monte Carlo predictions** (e.g. for e-H, e-D and e-<sup>12</sup>C and e-<sup>16</sup>O cross sections, and to is being used **compute radiative corrections for electron scattering** experiments..

# Nuclear excitations in Carbon



We parameterize form-factors for elastic scattering and for 17 longitudinal and transverse nuclear excitation ( $2 < \text{Excitation Energy} < 50 \text{ MeV}$ ) important since they contribute up to 30% to the Coulomb Sum Rule

Examples: Squares of Elastic form factor and the form factors for the first 3 nuclear excitations (all are longitudinal)



We parameterize form-factors for elastic scattering and for 17 longitudinal and transverse nuclear excitations ( $2 < \text{Excitation Energy} < 50 \text{ MeV}$ )

**Cross sections for  
excitations less than 10  
MeV multiplied by (1/6)**

## Nuclear excitation region

**$E_x < 50 \text{ MeV}$**

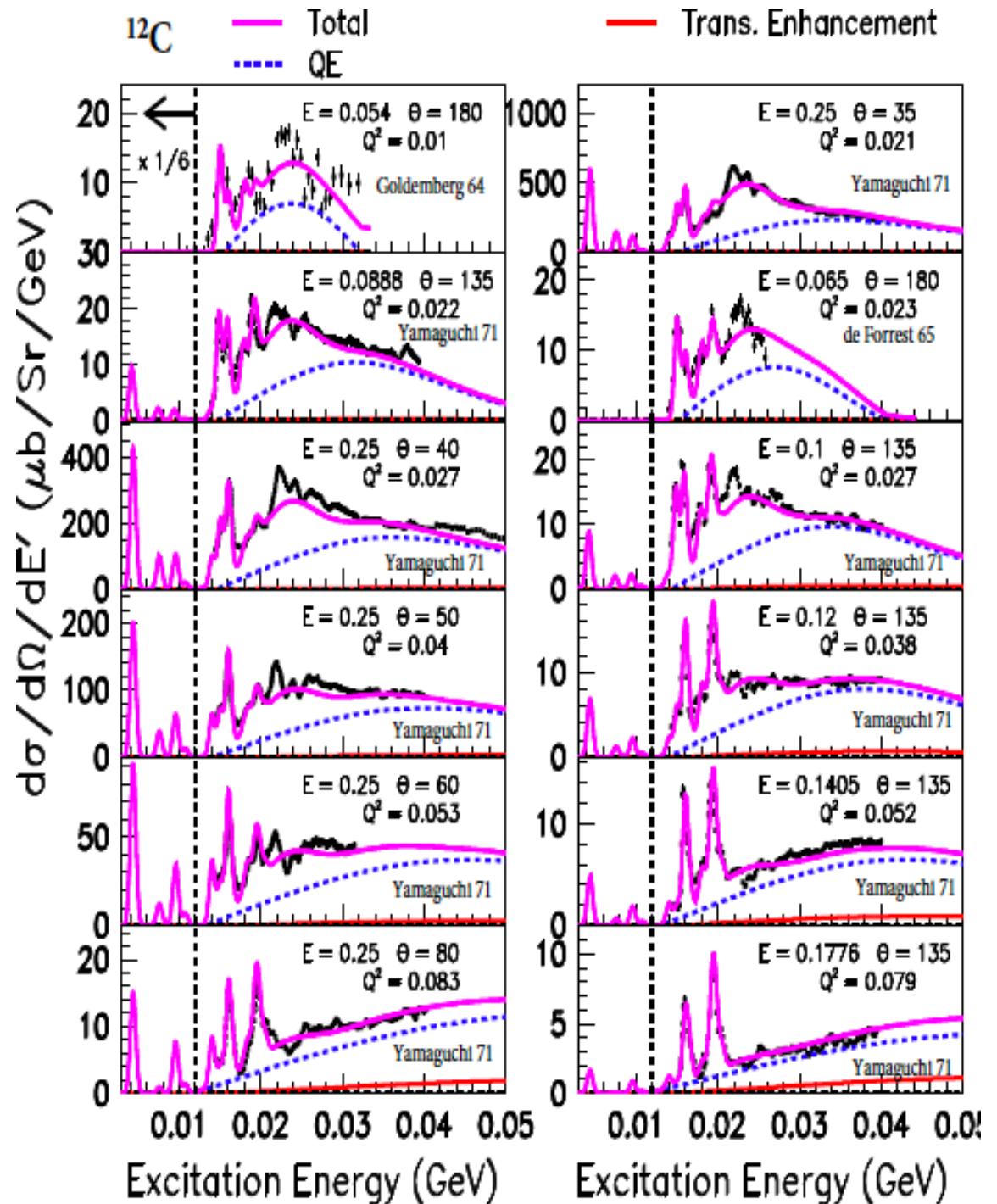
**Comparison of our fit to  
representative e-C12 data for  
 $0.01 < q^2 < 0.08 \text{ GeV}^2$ .**

**Shown: Total including  
excitations : solid -----**

**Quasielastic (QE)**  
**contribution: dashed-----**

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of TTransverse QE response dashed-----

## Electron scattering cross sections



# Coulomb Sum rule: Contribution of nuclear excitations

The electron scattering differential cross section can be written in terms of longitudinal ( $R_L(\mathbf{q}, \nu)$ ) and transverse ( $R_T(\mathbf{q}, \nu)$ ) response functions [7]:

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M [A R_L(\mathbf{q}, \nu) + B R_T(\mathbf{q}, \nu)], \quad (1)$$

$$\sigma_M = \frac{\alpha^2 \cos^2(\theta/2)}{4E_0^2 \sin^4(\theta/2)}$$

At very low  $q$ ,  $SL_{\text{inelastic}} = 0$

At high  $q$  expect  $SL_{\text{inelastic}} = 1$

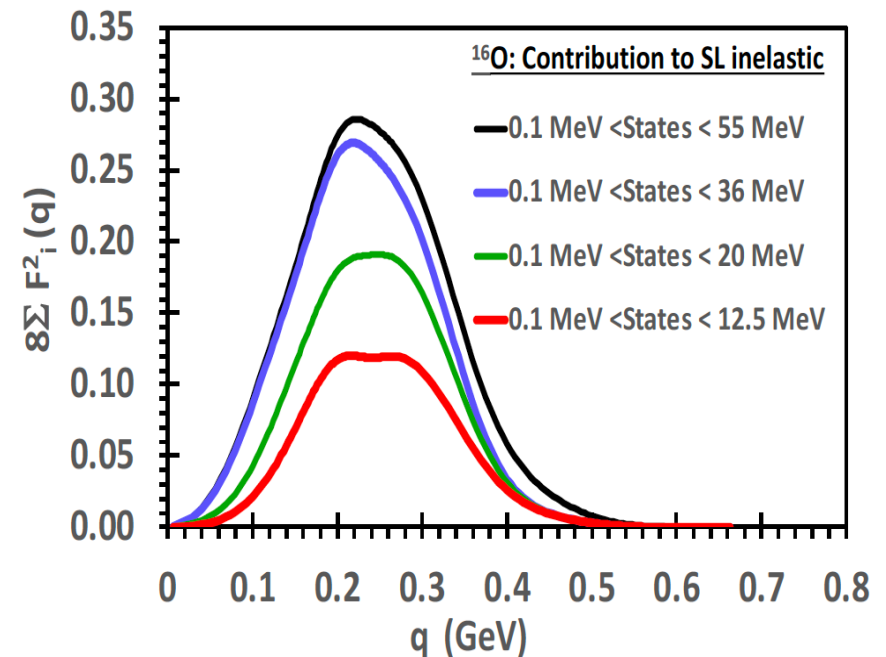
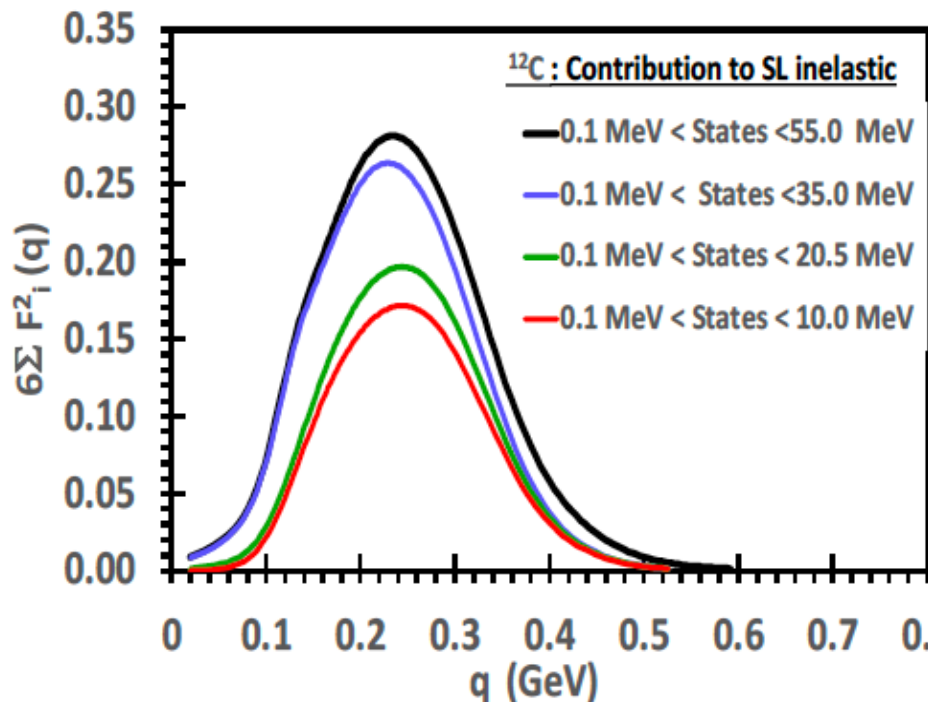
$$CSR(\mathbf{q}) = \int R_L(\mathbf{q}, \nu) d\nu \quad (3)$$

$$= \int R_L^{QE}(\mathbf{q}, \nu) d\nu + GE'_p(q) \times Z^2 \sum_{\text{all}}^L F_i^2(\mathbf{q})$$

$$= GE'_p(q) \times [Z \int V_L^{QE} d\nu + Z^2 \sum_{\text{all}}^L F_i^2(\mathbf{q})]$$

By dividing by  $ZGE'_p(\mathbf{q})$  we obtain the normalized inelastic Coulomb Sum Rule is:

$$SL(\mathbf{q}) = \int V_L^{QE}(\mathbf{q}, \nu) d\nu + Z \sum_{\text{all}}^L F_i^2(\mathbf{q}) \quad (5)$$



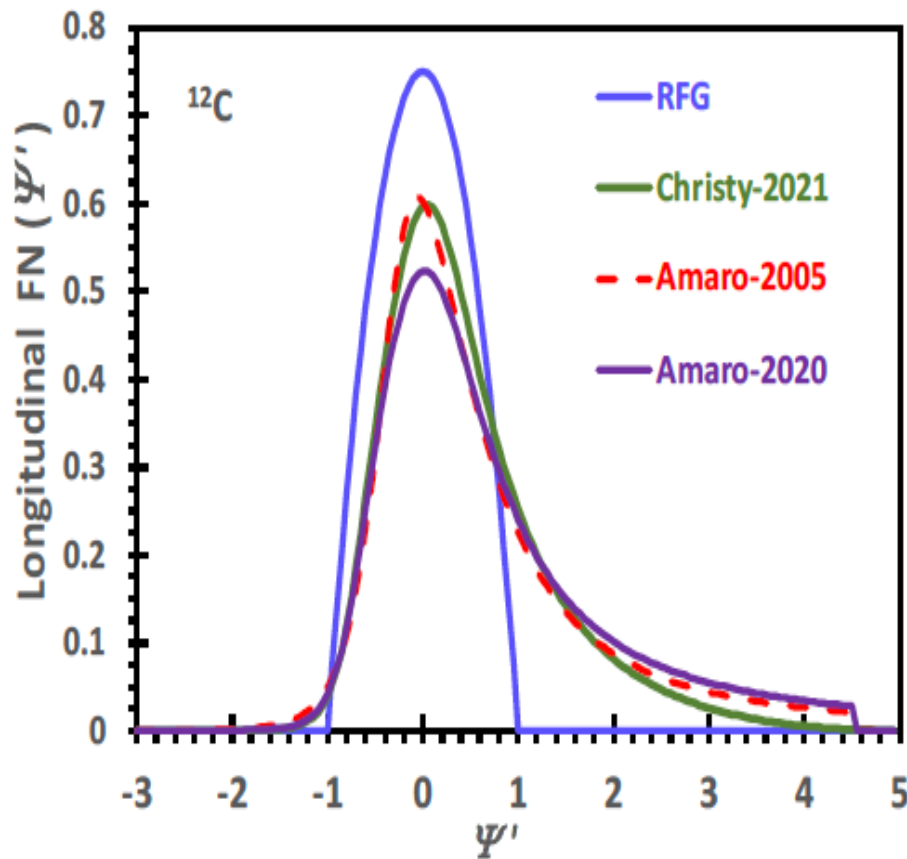
## Modeling QE:

Use superscaling- Fit for the longitudinal scaling function parameters in the overall fit

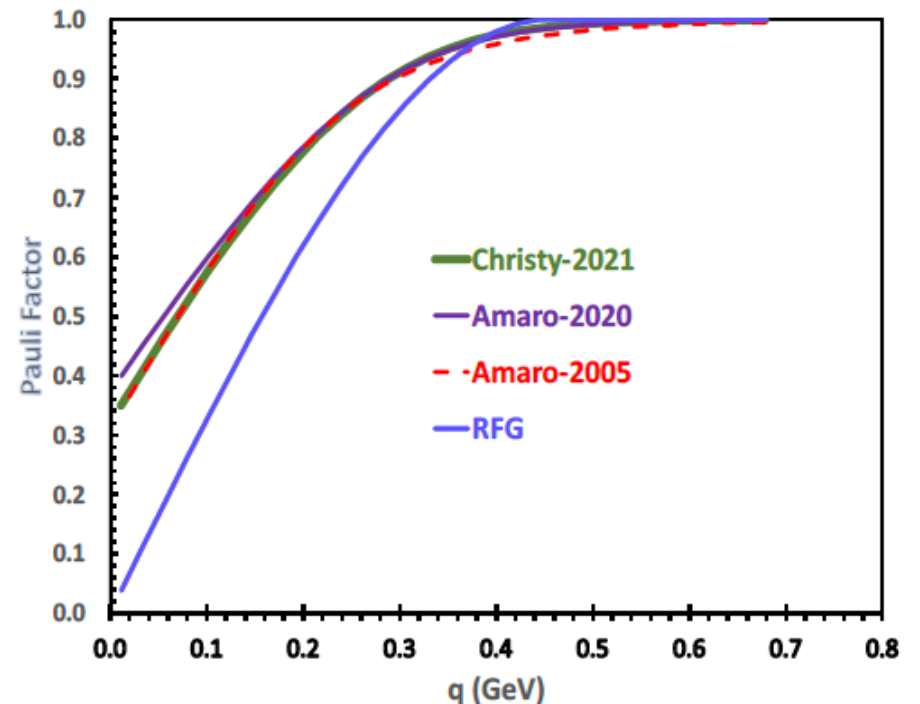
The  $\psi'$  superscaling variable is given by the following expression:

$$\psi' \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda' - \tau'}{\sqrt{(1 + \lambda')\tau' + \kappa\sqrt{\tau'(1 + \tau')}}}, \quad (16)$$

where  $\xi_F \equiv [\sqrt{1 + \eta_F^2} - 1]$ ,  $\eta_F \equiv K_F/M_n$ ,  $\lambda \equiv \nu/2M_n$ ,  $\kappa \equiv |q|/2M_n$  and  $\tau \equiv |Q^2|/4M_n^2 = \kappa^2 - \lambda^2$ .



We include Rosenfelder Pauli suppression





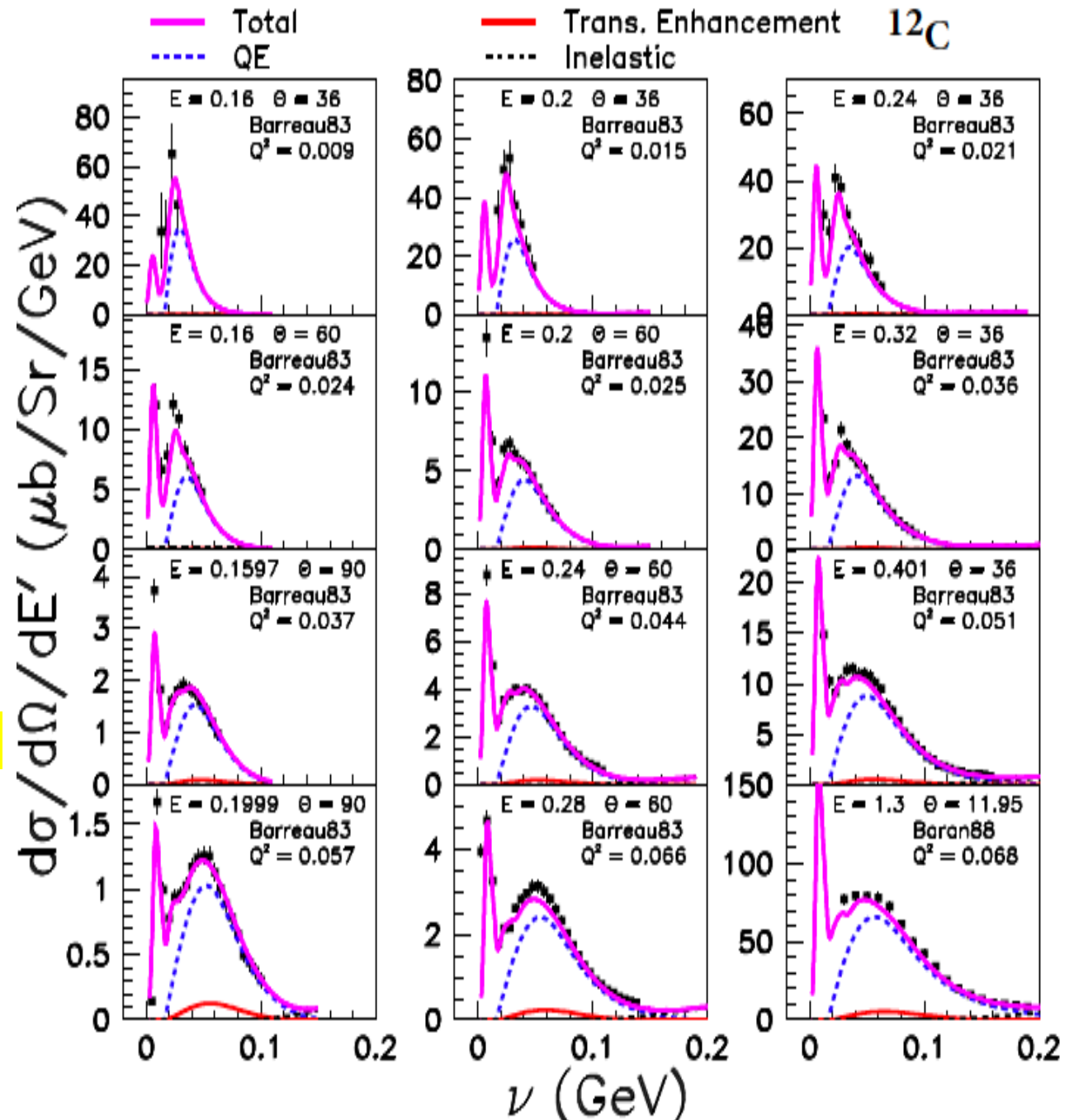
## Quasielastic (QE) Region-I

Comparison of our fit to representative e-C12 data  
For  $\nu < 0.2$  GeV and  $0.01 < q^2 < 0.068$  GeV<sup>2</sup>.

Shown: Total including excitations solid -----

Quasielastic (QE) contribution dashed -----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----

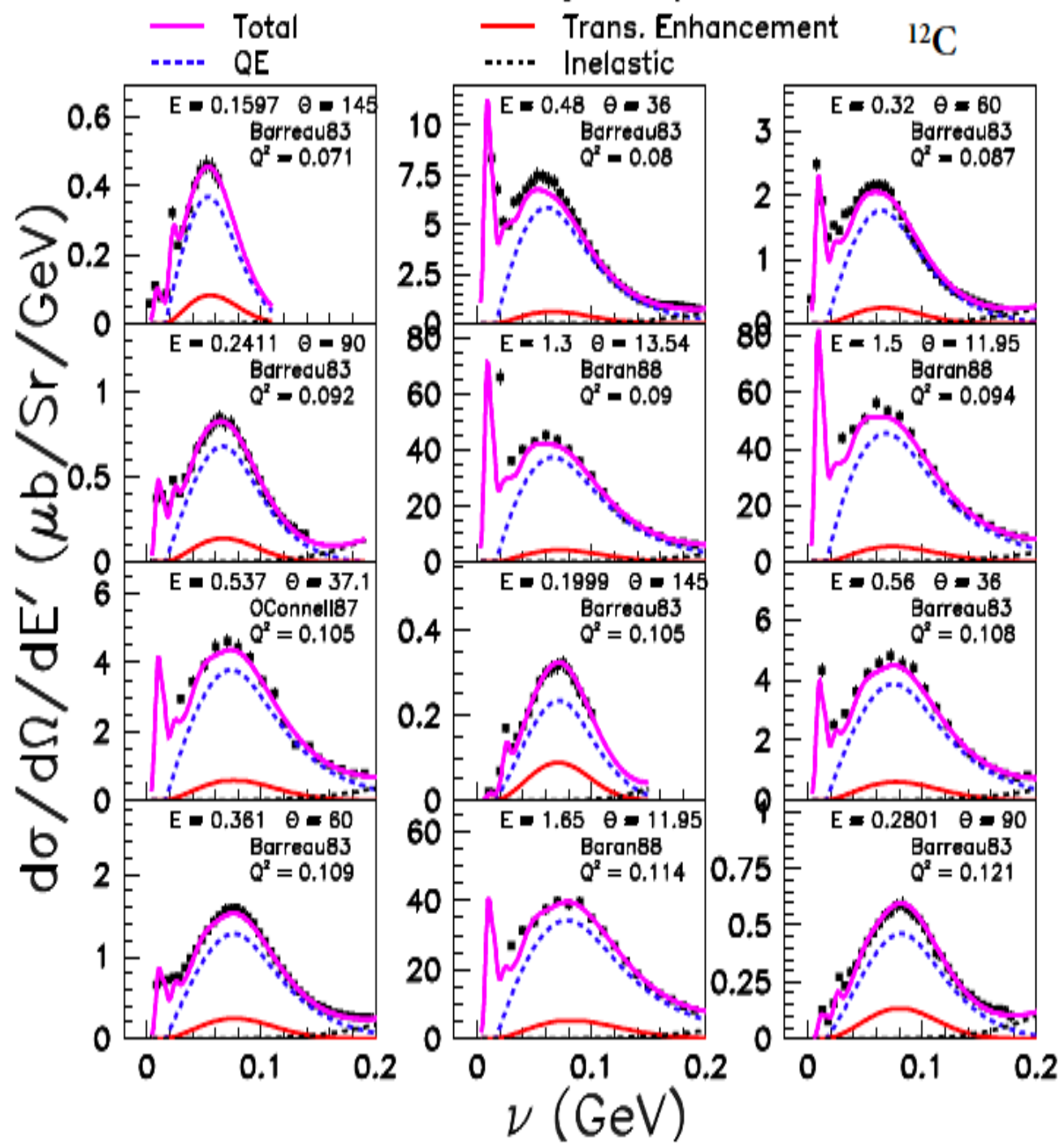


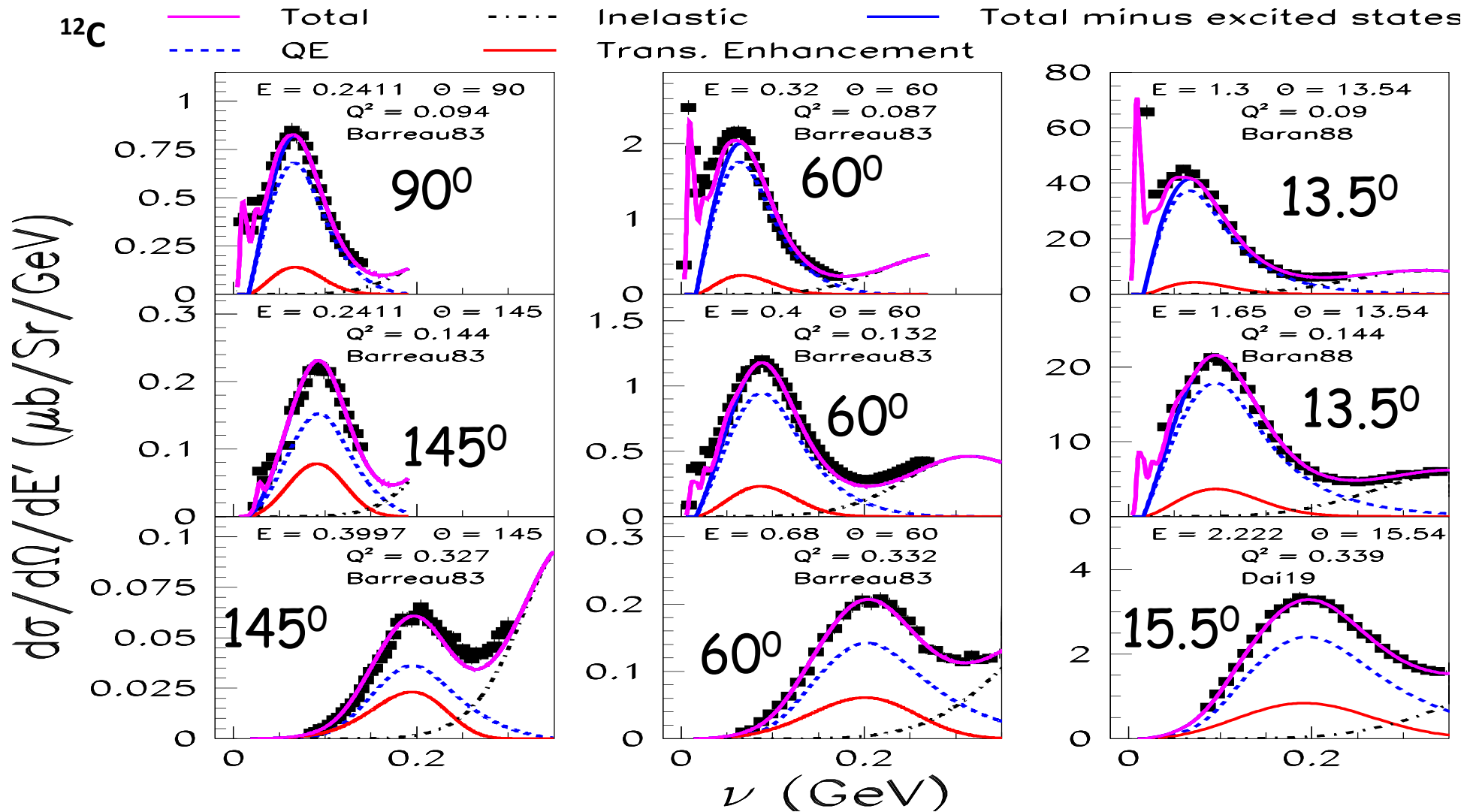


## Quasielastic (QE) Region II

Comparison of our fit to representative e-C12 data for  $< 0.2 \text{ GeV}$  and  $0.071 < q^2 < 0.121 \text{ GeV}^2$ .

Shown: Total including excitations solid -----  
 Quasielastic (QE) contribution dashed -----  
 Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----

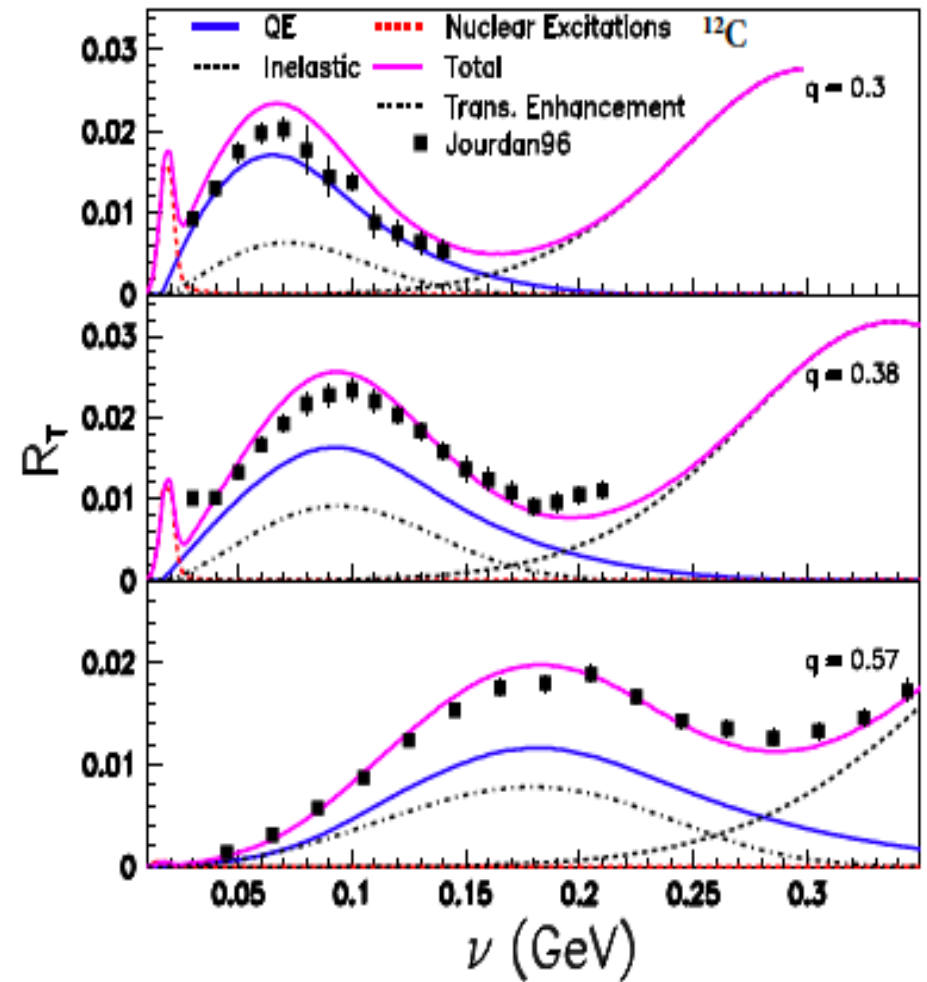
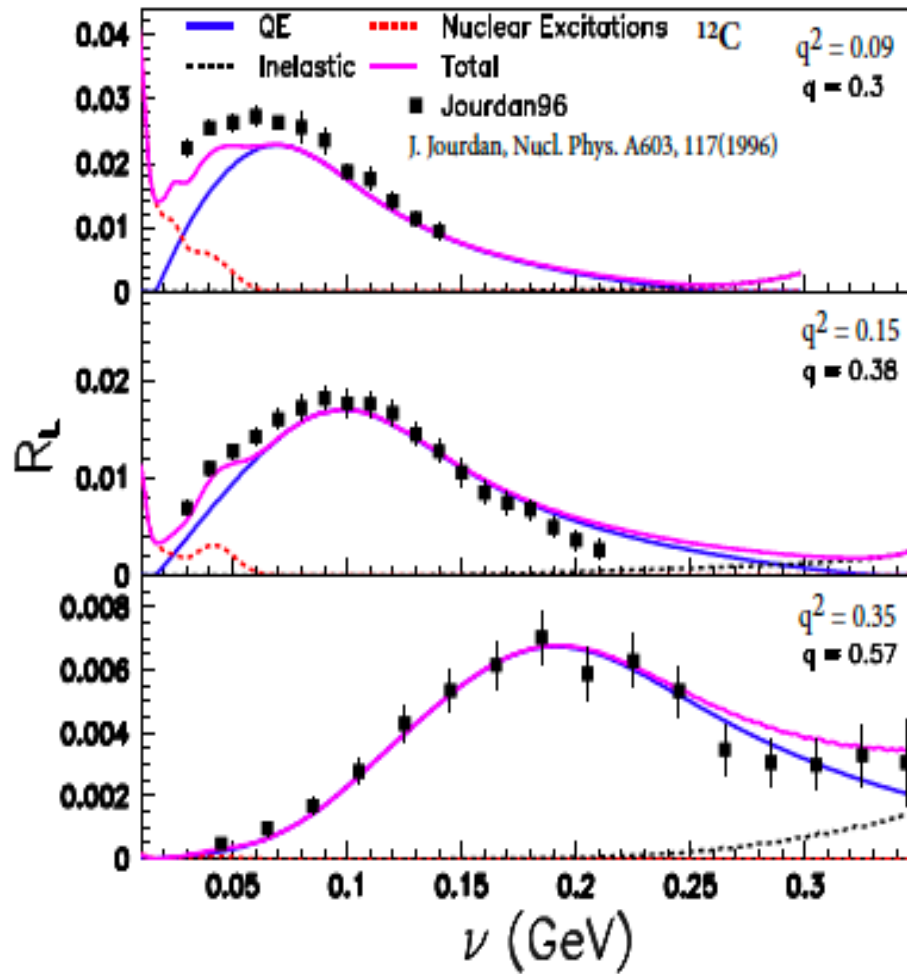




The overall fit provides  $R_L$  and  $R_T$  at all values of  $q$

Shown are **large and small angle cross sections** at the same  $q$  that provide the major contribution to the extraction of  $R_L$  and  $R_T$  at

**$q^2=0.09, 0.15$  and  $0.35 \text{ GeV}^2$  ( $q=0.3, 0.38$  and  $0.57 \text{ GeV}$ )**



**Comparison of our  $R_L$  and  $R_T$  from our universal fit to ( $\sim 8000$  cross sections) to previous extraction by Jourdan at  $q^2=0.09, 0.15$  and  $0.35 \text{ GeV}^2$ . ( $q=0.3, 0.38$  and  $0.57 \text{ GeV}$ )**

**Our extraction is more reliable since we include all of the world's data in the fit**

- **At low  $q$**  the contribution of the **nuclear excitations** important.
- The **superscaling fit function** describes the QE distribution at higher  $\nu$ .
- **Resonance region** is modeled with **Fermi Smeared H and D data**.

## Pauli Suppression

**Factor.** We use the Rosenfelder method.

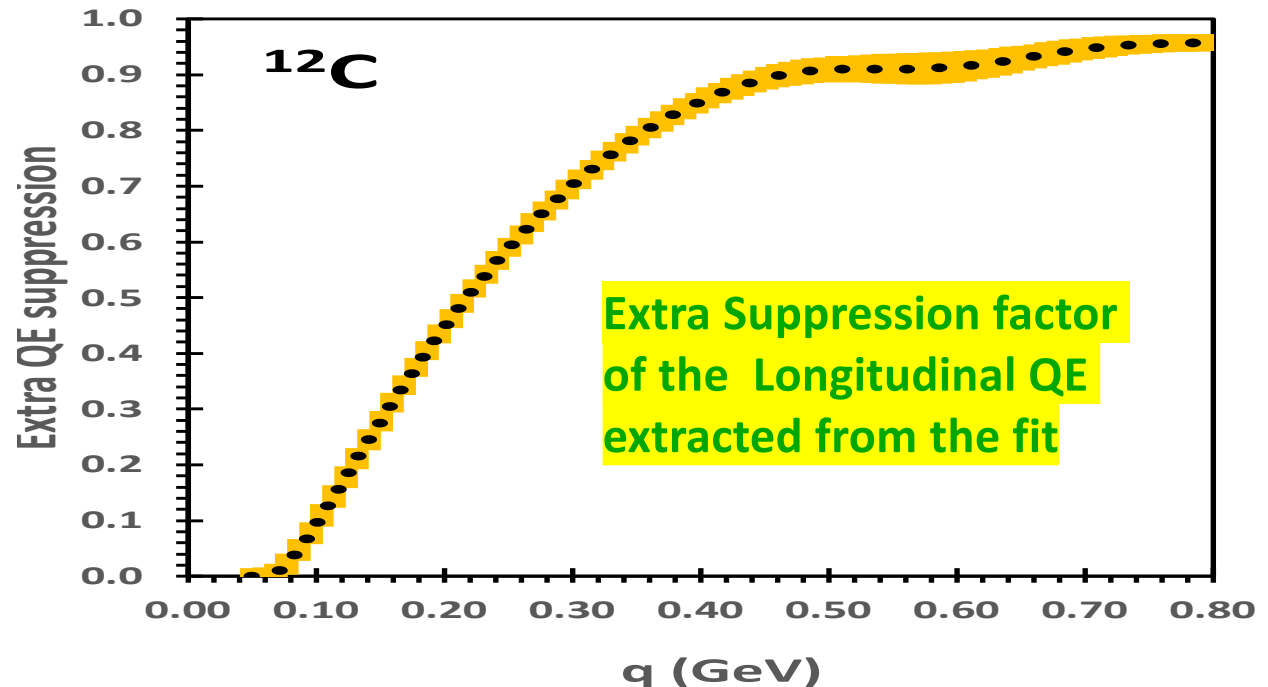
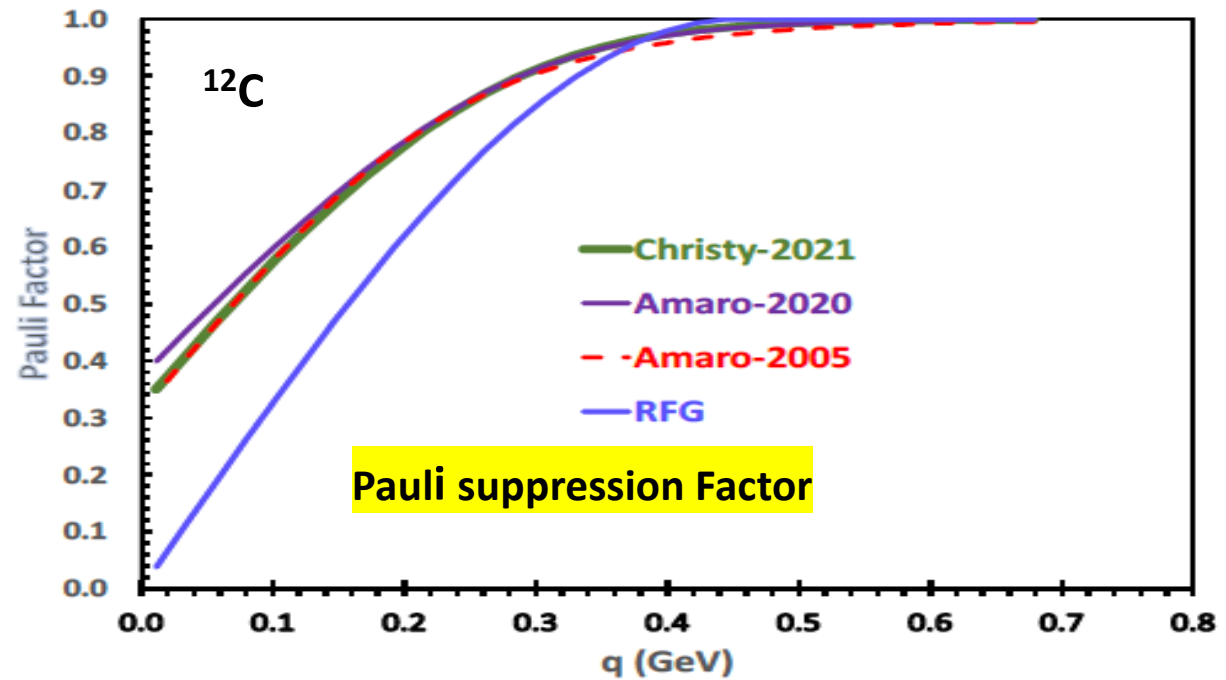
$P_1$  = Pauli factor for the Christy 2021 QE superscaling model.

$P_2$  = Pauli factor for another QE model (e.g. Amaro-2020)

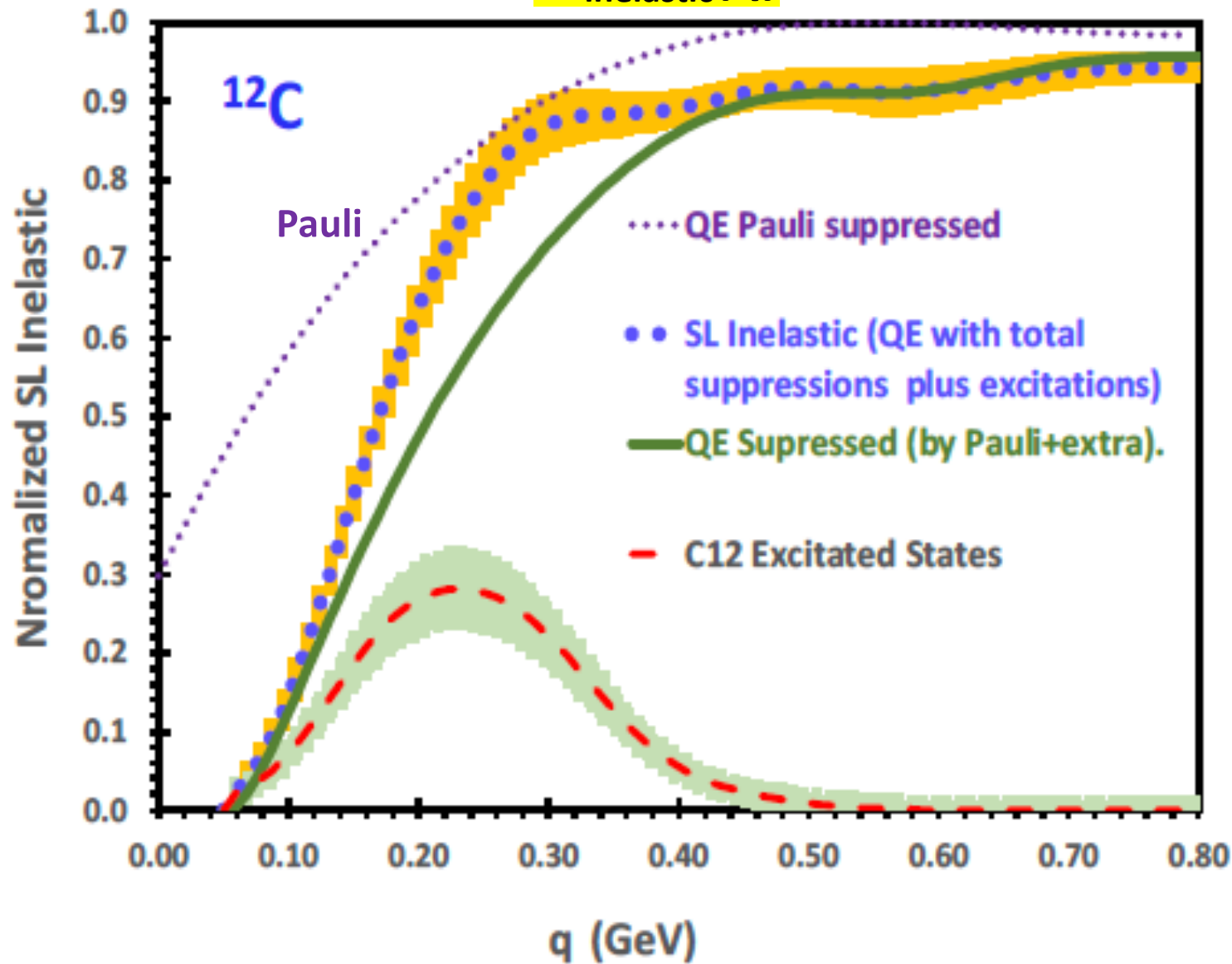
$ES_1$  = The Extra Suppression of the longitudinal QE cross section (in addition to Pauli) extracted from the fit

$ES_2$  = Extra Suppression for another QE model

$ES_2 = ES_1 (P_1/P_2)$

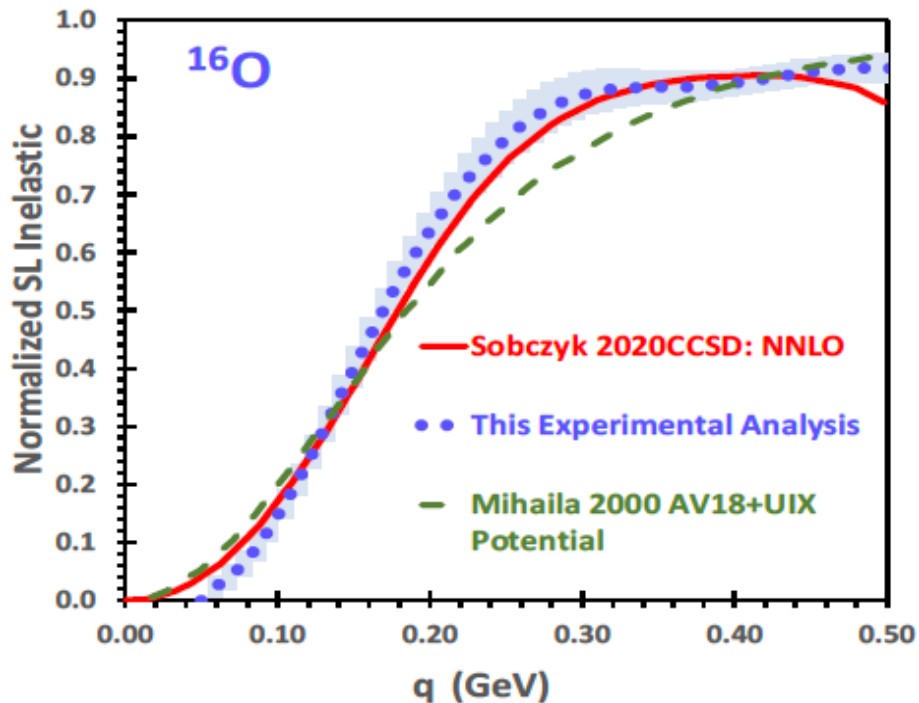
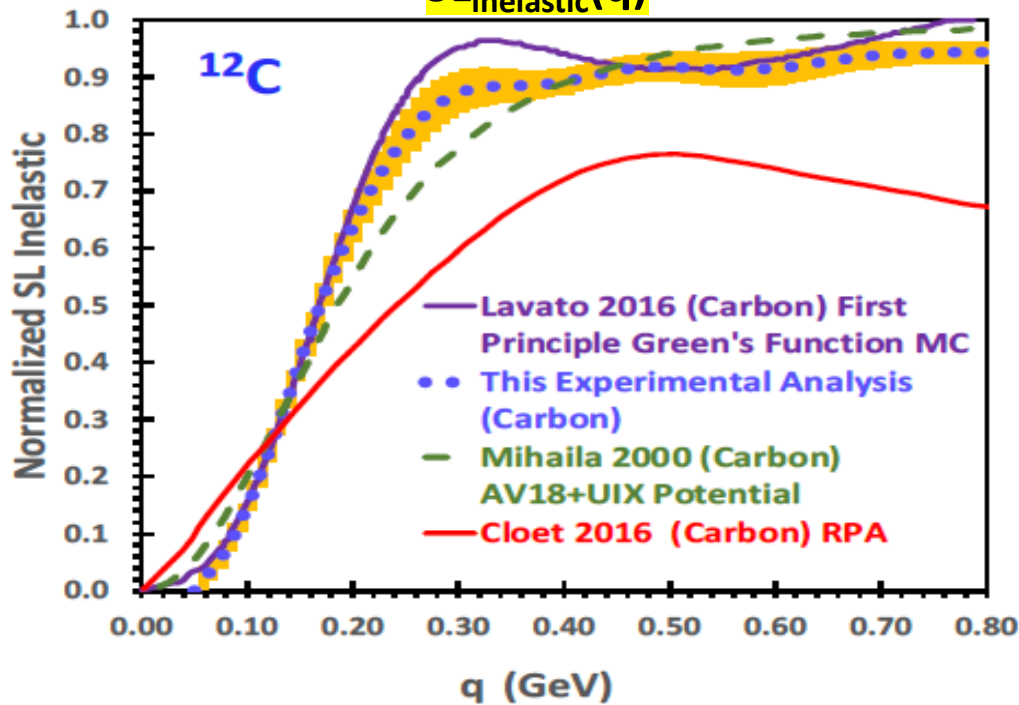


$$SL_{\text{inelastic}}(q)$$



The different contribution from our fit to the extracted Inelastic Coulomb sum Rule:  $SL_{\text{inelastic}}(q)$

$SL_{\text{inelastic}}(q)$



## Comparison to theory for Coulomb sum Rule: $SL_{\text{inelastic}}(q)$ $^{12}\text{C}$

1. Reasonable agreement with Lavato 2016 First Principle Green's function MC.
2. Poor agreement with Mihaila 2010 Coupled Cluster AV18-UIX potential
3. Poor agreement with Cloet 2016 (RPA)

A. Lovato et. al, Phys. Rev. Lett. 117, 082501 (2016)

Ian C. Cloet, Wolfgang Bentz, Anthony W. Thomas, Phys. Rev. Lett. 116, 032701 (2016) (arxiv.org/abs/1506.05875).

Bogdan Mihaila and Jochen H. Heisenberg, Phys. Rev. Lett. 84 (2000) 1403. 2009.01761 [nucl-th]

## Comparison to theory for Coulomb sum Rule: $SL_{\text{inelastic}}(q)$ $^{16}\text{O}$

1. Reasonable agreement with Sobczyk 2000 CCSD:NNLO
2. Poor agreement with Mihaila 2010 Coupled Cluster AV18-UIX potential

J. E. Sobczyk, B. Acharya, S. Bacca, and G. Hagen Phys.Rev.C 102 (2020) 064312 (arXiv: 2009.01761 [nucl-th])



# Conclusions

- **We fit all existing e-H, e-D, e- $^{12}\text{C}$  and e- $^{16}\text{O}$  data including elastic, nuclear excitations, Quasielastic, Resonance and Inelastic region.**
- **Fit provides a benchmark** to test electron and neutrino MC generators. (all parameters will be published).
- **The contributions of nuclear excitations** is important at low  $q$  and should be added to electron and neutrino MC generators.
- For the QE Longitudinal structure function, we find that the QE cross section is suppressed by **an Extra Suppression in addition to Pauli blocking**. We provide a parameterization of the Extra Suppression.
- We extract the inelastic Coulomb sum rule and compare to theoretical calculations. Since all available e- $^{12}\text{C}$  data is included in the fit, **this is the best extraction of the Inelastic Coulomb Sum Rule from all the world's data on  $^{12}\text{C}$ .**