Current Status of Resummed Quantum Gravity

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Introduction

- WHAT IS RESUMMATION(IR,UV,CL)?
  - FAMILIAR SUMMATION: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
  - RESUMMATION:

$$\sum_{n=0}^{\infty} C_n \alpha_s^n \begin{cases} = F_{\rm RES}(\alpha_s)\sum_{n=0}^{\infty} B_n \alpha_s^n, & \text{EXACT} \\ \cong G_{\rm RES}(\alpha_s)\sum_{n=0}^{N} B'_n \alpha_s^n, & \text{APROX.} \end{cases}$$

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Introduction

- A LITTLE HISTORY: 1988 ICHEP-Munich Conference Dinner, ONE YEAR BEFORE LEP DATA TAKING THAT LED, BY PRECISION PHYSICS, TO THE 't HOOFT-VELTMAN (1999) EW AND GROSS-WILCZEK-POLITZER (2004) QCD NOBEL PRIZES IN PHYSICS:
  F. Berends and BFLW considered, 'How Accurate Can Exponentiation (RESUMMATION) Really Be?'
- Would It Limit or Enhance Precision for a Given Level of Exactness: LO, NLO, NNLO, ... ?

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Introduction

- 'Two' Realizations in Literature: Jackson-Scharre(JS)(APPROX) vs YFS (EXACT)
- JS → 'limit to precision'
- YFS → 'no limit to precision'
- See 1989 CERN Yellow Book article: Frits was almost convinced, but not completely!
- Today, the analogous discussion continues to new paradigms: precision LHC/FCC physics and quantum gravity: here we focus on the latter.

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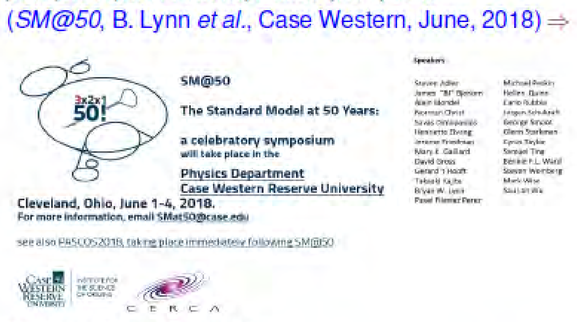
Introduction

- 50 YEARS of $SU_{2L} \times U_1$, S. Weinberg, PRL 19 (1967) 1264[a]; 45 YEARS of QCD, D.J. Gross and F. Wilczek, *ibid.* 30 (1973) 1343, H.D. Politzer, *ibid.* 30 (1973) 1346 (*SM@50*, B. Lynn *et al.*, Case Western, June, 2018) ⇒



Must Keep Historical Perspective

[a] S.L. Glashow, NP 22(1961) 579; A. Salam, in 8th Nobel Sym, 1968, p.367.

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Introduction

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Review of Exact Amplitude-Based Resummation Theory

$$d\bar\sigma_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}} \tilde{\bar\beta}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}, \quad (1)$$

where *new* (YFS-style) *non-Abelian* residuals $\tilde{\bar\beta}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons.

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$QCD \otimes QED$ $\mathcal{EXACT}$ Resummation Theory

Here,

$$\begin{aligned} {\rm SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde B^{nls}_{QCED} \\ D_{\rm QCED} &= \int \frac{d^3k}{k^0}\left(e^{-iky} - \theta(K_{max}-k^0)\right)\tilde S^{nls}_{QCED} \quad (2) \end{aligned}$$

where K_{max} is "dummy" and

$$\begin{aligned} B^{nls}_{QCED} &\equiv B^{nls}_{QCD} + \frac{\alpha}{\alpha_s} B^{nls}_{QED}, \\ \tilde B^{nls}_{QCED} &\equiv \tilde B^{nls}_{QCD} + \frac{\alpha}{\alpha_s} \tilde B^{nls}_{QED}, \\ \tilde S^{nls}_{QCED} &\equiv \tilde S^{nls}_{QCD} + \tilde S^{nls}_{QED}. \quad (3) \end{aligned}$$

"nls"≡ DGLAP-CS synthesization.

Shower/ME Matching: $\tilde{\bar\beta}_{n,m} \to \hat{\tilde{\bar\beta}}_{n,m}$

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Quantum Gravity: New Results and New Issues

- Preliminary Remarks
- Overview of Resummed Quantum Gravity
- Planck Scale Cosmology
- An Estimate of Λ
- An Open Question?
- Einstein-Heisenberg Consistency Condition
- Constraints on SUSY GUTs

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Preliminary Remarks

- IS QUANTUM GRAVITY (Einstein-Hilbert Theory) CALCULABLE IN RELATIVISTIC QFT?
- STRING THEORY: NO. You need superstrings, supersymmetric one-dimensional objects of Planck length size, 1.62×10−33 cm.
- LOOP QUANTUM GRAVITY: NO. You need Planck length size loops that are the fundamental constructs for quantum gravity.
- HORAVA-LIFSHITZ THEORY: NO. You need anisotropic scaling at Planck length scales:
  Time and space differ by a factor of z in scale dimension at Planck length distances with z = 3 in the original proposal–this violates local Lorentz invariance.

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Preliminary Remarks

- New Approach: Exact Amplitude-Based Resummation of Feynman's Formulation of Einstein's Theory – Resummed Quantum Gravity (RQG)
- RESULT (1): UV Finiteness!
- RESULT (2): Constraints on SUSY GUT's
- RESULT (3): Prediction for the Cosmological Constant Λ with Relatively Small Theoretical Uncertainty.
- RESULT (4): Consistent with Weinberg's Asymptotic Safety Ansatz, as realized by Exact Field Space Renormalization Group Program of Reuter *et al.*

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Preliminary Remarks

- RESULT (5): Consistent with Kreimer's Leg Renormalizability Results ...
- Today we give highlights on the status and outlook for this new RQG approach.

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Overview of Resummed Quantum Gravity

SM ⇔ Many Massive Point Particles.
Feynman: spin is an inessential complication – checked. We replace $L^{\mathcal{G}}_{SM}(x)$ with that a free physical Higgs field, $\varphi(x)$, with a rest mass 125 GeV(ATLAS,CMS) ⇒ the representative model **{R.P. Feynman, Acta Phys. Pol. 24 (1963) 697;** *Feynman Lectures on Gravitation*, **eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971). }**

$$\begin{aligned} \mathcal{L}(x) &= \frac{1}{2\kappa^2}R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g} \\ &= \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar h_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar h_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\bar h_{\mu'\sigma,\sigma'}\right\} \\ &\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right] \\ &\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar h^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar h^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots \quad (1) \end{aligned}$$

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Overview of Resummed Quantum Gravity

YFS resum the propagators in the NON-ABELIAN gauge theory of QG:
⇒ from the YFS formula

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'_F(p)}, \quad (2)$$

we find for Quantum Gravity, proceeding as above, the analogue of

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4}\frac{-i\eta^{\mu\nu}}{(\ell^2-\lambda^2+i\epsilon)}\frac{-ie(2ik_\mu)}{(\ell^2-2\ell k+\Delta+i\epsilon)}\frac{-ie(2ik'_\nu)}{(\ell^2-2\ell k'+\Delta'+i\epsilon)}\Big|_{k=k'} \quad (3)$$

as $-B''_g(k)$ with

$$B''_g(k) = -2i\kappa^2k^4\frac{\int d^4\ell}{16\pi^4}\frac{1}{\ell^2-\lambda^2+i\epsilon}\frac{1}{(\ell^2+2\ell k+\Delta+i\epsilon)^2} \quad (4)$$

for $\Delta = k^2 - m^2$ ⇒ for a scalar field

$$i\Delta'_F(k)|_{\rm YFS-resummed} = \frac{ie^{B''_g(k)}}{(k^2-m^2-\Sigma'_s+i\epsilon)}.$$

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Overview of Resummed Quantum Gravity

⇒
Expand theory with the 'improved Born' propagators

$$iP_{\alpha_1\cdots;\alpha'_1\cdots}\Delta'_F(k)|_{YFS-resummed,\Sigma'_s=0} = \frac{iP_{\alpha_1\cdots;\alpha'_1\cdots}e^{B''_g(k)}}{(k^2-m^2+i\epsilon)} \quad (5)$$

where in the DEEP UV we get

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \quad (6)$$

⇒ ALL PROPAGATORS FALL FASTER THAN ANY POWER OF $|k^2|$ ⇒ QG IS FINITE (SEE MPL**A17** (2002) 2371;hep-ph/0607198)!

BAYLOR 1

Overview of Resummed Quantum Gravity

CONTACT WITH ASYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G(k) = G_N/(1+\frac{k^2}{a^2})$$

  ⇒ FIXED POINT BEHAVIOR FOR $k^2 \to \infty$,
  IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONANNO & REUTER IN PRD**62**(2000) 043008.
- OUR RESULTS ⇒ AN ELEMENTARY PARTICLE HAS NO HORIZON. THIS AGREES WITH BONANNO & REUTER THAT A BLACK HOLE WITH A MASS LESS THAN $M_{cr} \sim M_{Pl}$ HAS NO HORIZON.
  BASIC PHYSICS:
  $G(k)$ VANISHES FOR $k^2 \to \infty$.

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An Estimate of Λ

- In Phys. Dark Univ. **2** (2013) 97, using (5) and (6) we get rigorous cut-off independent values for the fixed points g_*, λ_* and the following estimate of Λ:

$$\begin{aligned} \rho_\Lambda(t_0) &\cong \frac{-M_{Pl}^4(1+c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64}\sum_j\frac{(-1)^{F_j}n_j}{\rho_j^2} \\ &\quad \times \frac{t_{tr}^2}{t_{eq}^2}\times\left(\frac{t_{eq}^{2/3}}{t_0^{2/3}}\right)^3 \\ &\cong \frac{-M_{Pl}^2(1.0362)^2(-9.194\times10^{-3})}{64}\frac{(25)^2}{t_0^2} \\ &\cong (2.4\times10^{-3}eV)^4, \quad (12) \end{aligned}$$

  where the age of the universe is $t_0 \cong 13.7\times10^9$ yrs.
- Compare: $\rho_\Lambda(t_0)|_{\rm expt} \cong ((2.37\pm0.05)\times10^{-3}eV)^4$.

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Constraints on SUSY GUTS

⟹

- Compensate by either (A) adding new susy families with scalars lighter than fermions or (B) allowing the gravitino mass to go to ~.05 M_{GUT} ~ 2x 10^{15} GeV.
- For approach (A),
  new quarks and leptons at
  M_{High}~ 3.4(3.3) x 10^3 TeV, 100 TeV too small!
  scalar partners at ~.5TeV =M_{Low} NEW LHC LIMITS??

BAYLOR Conclusion: IR-improvement enhances QG physics