Radion dynamics in multibrane Randall-Sundrum model

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Randall-Sundrum model

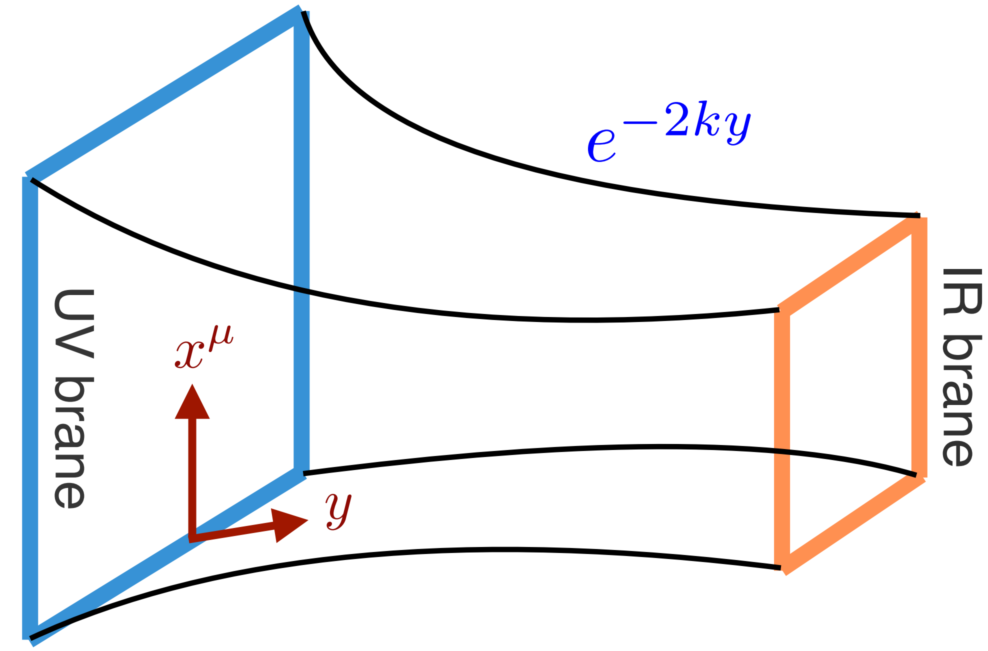
Consider a slice of Anti-de Sitter (AdS) space compacted on an S_1/Z_2 orbifold:

$$ds^2 \quad = \quad e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$-L \leq y \leq L$$

with a reflection symmetry in the y-coordinate of the extra dimension :

$$A(y) \to A(|y|)$$



$$k = \sqrt{-\frac{\Lambda}{12M^3}} \qquad kL \sim 35$$

For an exponential warped factor A(y) = k |y|, the action for the Higgs at the IR brane can be written as:

$$S = \int d^4x e^{-4kL}\left[e^{2kL}\eta_{\mu\nu}\partial^\mu H \partial^\nu H - \lambda\left(H^\dagger H - v^2\right)^2\right]$$

$$\tilde{H} = e^{-kL}H\,, \qquad \tilde{v} = e^{-kL}v \sim 174 \text{ GeV}$$

Randall and Sundrum PRL (1999)

Metric in RS1

Einstein Equation $$R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{2M^3} T_{MN}$$

$$(\mu\nu) \quad : \quad 3\left(A'' - 2A'^2\right) = \frac{1}{2M^3}\left(\Lambda + \lambda_+ \delta(y) + \lambda_- \delta(y - L)\right)$$

$$(55) \quad : \quad 6A'^2 = -\frac{\Lambda}{2M^3}$$

cosmology constant (Λ)

brane tensions (λ_+, λ_-)

We obtain the solutions for the vacuum Einstein equation:

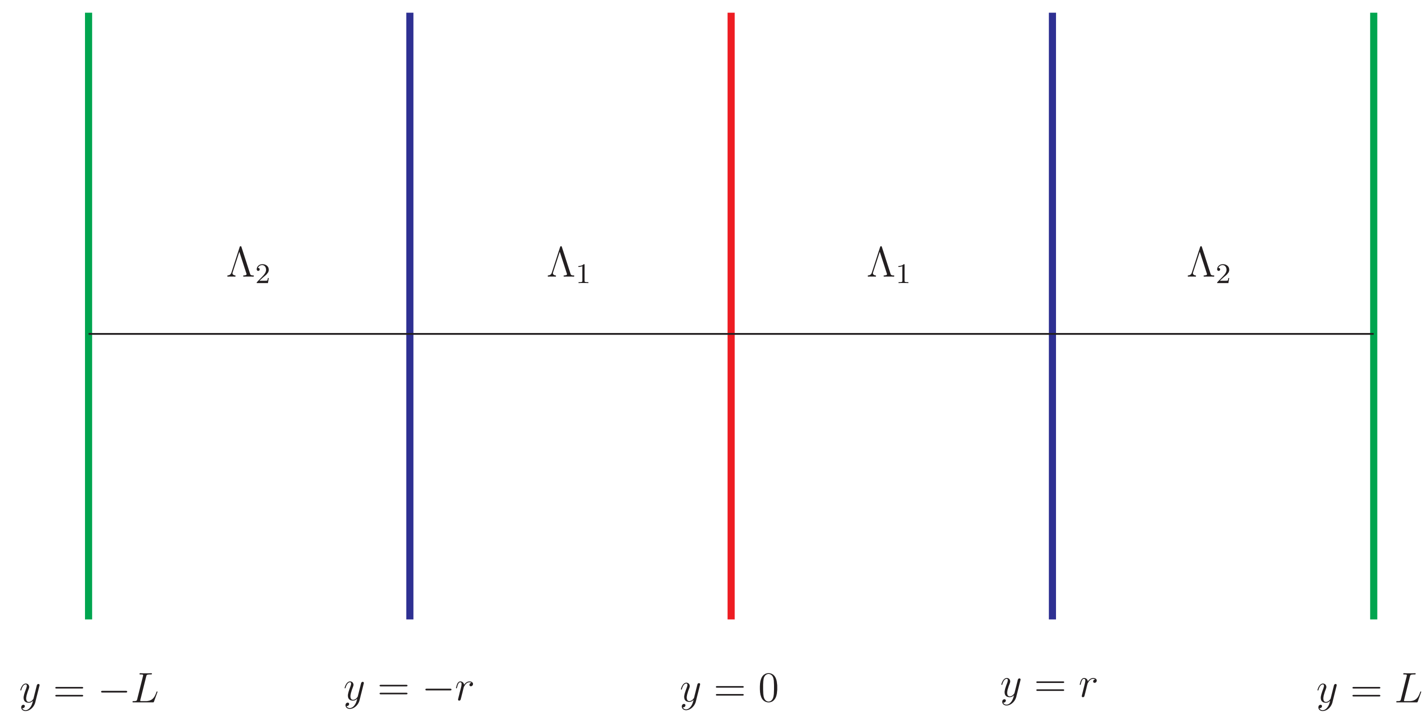
$$A(y) = \sqrt{-\frac{\Lambda}{12M^3}}\,|y| = k|y| \qquad \lambda_+ = -\lambda_- = 12M^3 k$$

A few facts:

First of all, no radion stabilization yet. An orbifold symmetry is imposed and the bulk cosmology constant must be negative. The brane tension is positive at the UV (y=0) but negative at the IR (y=L).

Multiple branes Extension

One needs to add two non-fixed point branes at $y = \pm r$ given that the cosmology constants are different ($\Lambda_1 \neq \Lambda_2$) in the two spatial regions.



$$k_1^2 = -\frac{\Lambda_1}{12M^3}\,, \qquad k_2^2 = -\frac{\Lambda_2}{12M^3}$$

$$\lambda_+ = 12M^3 k_1\,, \quad \lambda_- = -12M^3 k_2$$

$$\lambda_{\pm r} = 12M^3 \frac{k_2 - k_1}{2}$$

In this geometry how many degrees of Freedom for the radion field ?

The metric without stabilization is :

$$A(y) = \begin{cases} k_1|y| & ,0<y<r \\ k_2|y| + (k_1-k_2)r & ,r<y<L \end{cases}$$

Kogan et al NPB (2002)

Graviton-Scalar system

We start with the generic five dimensional action for the graviton coupling to a single bulk scalar field:

Einstein-Hilbert action

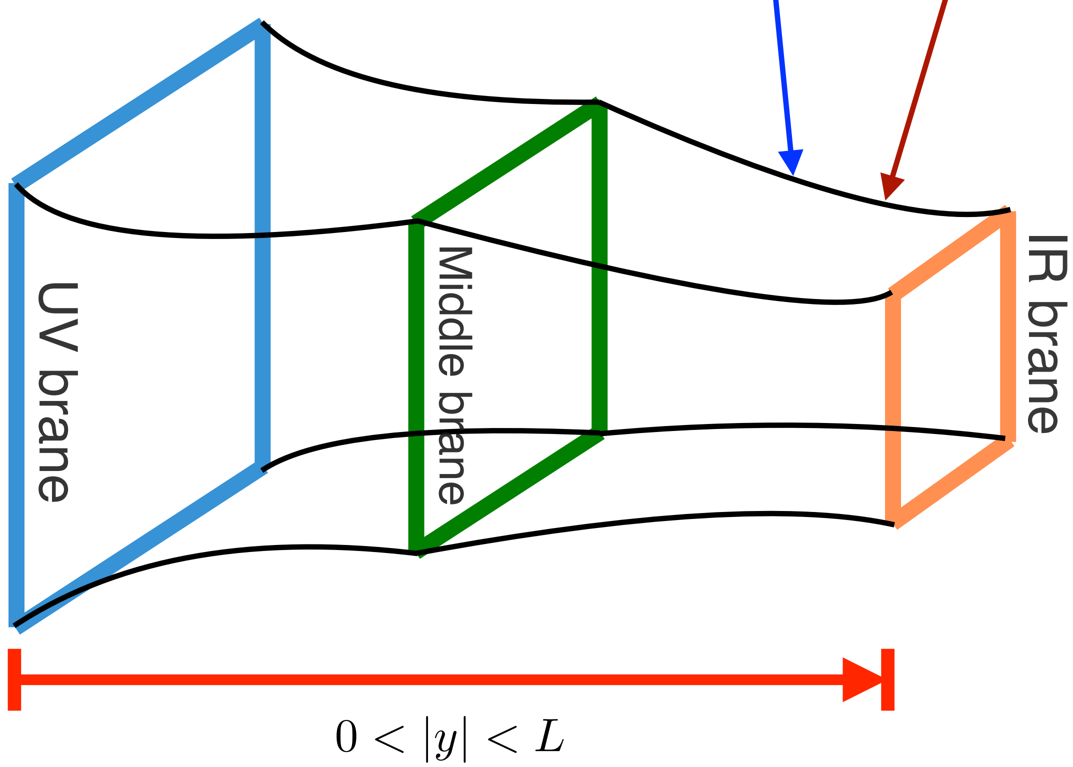
Matter action

$$S = S_{EH} + S_m = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,\mathcal{R} + \int d^5x\sqrt{g}\mathcal{L}_m$$

$$\mathcal{L}_m = \frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi - V(\phi) - \frac{1}{\sqrt{-g_{55}}}\sum_i \lambda_i(\phi)\delta(y-y_i)$$

GW scalar action

$$y_i = \{0\,,\pm r\,,L\}$$



$$V(\phi)\ :\ \Lambda_1,\,\Lambda_2,\,m^2,\cdots$$

$$\lambda_i(\phi)\ :\ \text{brane terms}$$

DeWolfe, Freedman et al PRD (2000)

Csaki, Graesser and Kribs PRD (2001)

Graviton-Scalar system

The Line Element in the multibrane model that can decouple the graviton and radion is :

$$\begin{aligned} ds^2 &= e^{-2A(y)-2F(x,y)}\left[\eta_{\mu\nu} + 2\epsilon(y)\partial_\mu\partial_\nu f(x)\right. \\ &+ \left. h_{\mu\nu}(x,y)\right] dx^\mu dx^\nu - \left[1+G(x,y)\right]^2 dy^2 \end{aligned}$$

$h_{\mu\nu}$: perturbation of graviton

$F(x,y)$, $G(x,y)$, $\epsilon(y)$: Radion mode

- By varying the action with respect to the 5d metric, one derives Einstein's equation in terms of Ricci tensor:

$$R_{MN} - \frac{1}{2}\, g_{MN} R = \kappa^2\, T_{MN}$$

where the energy-momentum tensor is $T_{MN} = 2\delta\left(\sqrt{g}\,\mathcal{L}_m\right) / \left(\sqrt{g}\,\delta g^{MN}\right)$.

- Minimizing S_m with respect to ϕ gives the scalar EOM, modified by a term $\phi_0'\epsilon'\Box f(x)$.

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Background Equations

The GW scalar develops a y-dependent VEV and $\phi_0(y)$ reacts back on the metric like Λ_1 , Λ_2. The scalar VEV and BG metric form a set of non-linear coupled equations.

$$\phi_0'' = 4A'\phi_0' + \frac{\partial V(\phi_0)}{\partial \phi} + \sum_i \frac{\partial \lambda_i(\phi_0)}{\partial \phi}\delta(y - y_i)\,,$$

$$4A'^2 - A'' = -\frac{2\kappa^2}{3}V(\phi_0) - \frac{\kappa^2}{3}\sum_i \lambda_i(\phi_0)\delta(y - y_i)\,,$$

$$A'^2 = \frac{\kappa^2 {\phi_0}'^2}{12} - \frac{\kappa^2}{6}V(\phi_0)\,.$$

- The 3rd equation is not independent, its differentiation gives the first two.
- The solutions to the coupled equations (with back reaction) can be written in terms of a single super-potential:

$$\phi_0' = \frac{1}{2}\frac{\partial W}{\partial \phi}\ ,\ A' = \frac{\kappa^2}{6}W(\phi_0)\ ,\ V(\phi) = \frac{1}{8}\left[\frac{\partial W(\phi)}{\partial \phi}\right]^2 - \frac{\kappa^2}{6}W(\phi)^2\,.$$

DeWolfe, Freedman et al PRD (2000) Behrndt, Cvetic PLB (2000)

Multi-brane Superpotential

To obtain an exponential metric in the multi-brane model, the superpotential is derived as :

$$W(\phi) = \begin{cases} \frac{6k_1}{\kappa^2} - u\phi^2 \,, & 0 < y < r \\ \frac{6k_2}{\kappa^2} - u\phi^2 \,, & r < y < L \end{cases}$$

The discontinuity in first term is due to $\Lambda_1 \neq \Lambda_2$

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$$\phi_0' = \frac{1}{2}\frac{\partial W}{\partial \phi_0} \quad \Rightarrow \quad \phi_0(y) = \phi_P e^{-uy}$$

$$\Rightarrow \quad L = \log(\phi_P/\phi_T)$$

GW mechanism stabilizes the UV-IR distance

Goldberger and Wise PRL (1999), PLB (2000)

The brane terms are fixed by matching with the discontinuity of A' and ϕ_0' :

$$\lambda_\pm = \pm W(\phi_\pm) \pm W'(\phi_\pm)\left(\phi - \phi_\pm\right) + \gamma_\pm \left(\phi - \phi_\pm\right)^2 \,,$$

$$\lambda_{\pm r} = \frac{1}{2}\left[W(\phi(y))\right]\Big|_{y=r} = \frac{3(k_2 - k_1)}{\kappa^2} \,.$$

where $\lambda_{\pm r}$ has no ϕ_0 dependence since there is no jump for $\phi_0'(y)$ at $y = \pm r$.

Effective Lagrangian

Fierz-Pauli

$$\mathcal{L}_{eff} = \int dy \Big\{ \frac{e^{-2A}}{2\kappa^2} \Big[\frac{e^{-2A}}{4} \left[(\partial_5 h)^2 - \partial_5 h_{\mu\nu}\, \partial_5 h^{\mu\nu} \right] - \mathcal{L}_{FP} \Big] + \mathcal{L}_{mix} + \mathcal{L}_{rad-kin} - \frac{e^{-4A}}{2} \mathcal{L}_{5m} \Big\}$$

$$\mathcal{L}_{mix} = -\frac{e^{-2A}}{2\kappa^2} \Big[\left[G - 2F - e^{2A} \partial_5 \left(\epsilon' f(x) e^{-4A} \right) \right] \left(\partial_\mu \partial_\nu h^{\mu\nu} - \Box h \right) + 3 e^{-2A} \left[F' - A'G - \frac{\kappa^2}{3} \phi_0' \varphi \right] \partial_5 h \Big]$$

Two orthogonal conditions (gauge fixing):

$\mathcal{L}_{mix} = 0$ ⇒

$$F' - A'G - \frac{\kappa^2}{3} \phi_0' \varphi = 0 \qquad (1)$$

$$G - 2F - e^{-2A} \left[\epsilon'' - 4A'\epsilon' \right] f(x) = 0 \qquad (2)$$

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Effective Lagrangian

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$$\mathcal{L}_{eff} = \int dy \Big\{ \frac{e^{-2A}}{2\kappa^2}\Big[\frac{e^{-2A}}{4}\left[(\partial_5 h)^2 - \partial_5 h_{\mu\nu}\,\partial_5 h^{\mu\nu}\right] - \mathcal{L}_{FP}\Big] + \mathcal{L}_{mix} + \mathcal{L}_{rad-kin} - \frac{e^{-4A}}{2}\mathcal{L}_{5m}\Big\}$$

$$\mathcal{L}_{rad-kin} = \frac{1}{2}\int dy e^{-2A}\Big\{\partial_\mu\varphi\partial^\mu\varphi - \frac{6}{\kappa^2}\Big[\partial_\mu F\partial^\mu(F-G) - e^{-2A}\epsilon'\partial_\mu\left[F' - A'G - \frac{\kappa^2}{3}\phi_0'\varphi\right]\partial^\mu f(x)\Big]\Big\}$$

$$\begin{aligned}\mathcal{L}_{5m} &= -\frac{12}{\kappa^2}\left[F'^2 + G^2A'^2 - 2F'GA'\right] \\ &+ \varphi'^2 + G^2\phi_0'^2 - 2(G+4F)\phi_0'\varphi' \\ &+ \left[2(G-4F)\frac{\partial V}{\partial\phi_0}\varphi + \frac{\partial^2 V}{\partial\phi_0^2}\varphi^2\right] \\ &- \sum_i\left[8\frac{\partial\lambda_i}{\partial\phi_0}F\varphi - \frac{\partial^2\lambda_i}{\partial\phi_0^2}\varphi^2\right]\delta(y-y_i)\end{aligned}$$

$\epsilon' f(x)$ behaves like a Lagrange multiplier

radion terms without ∂_μ

The jump $[\epsilon']\,|_{y=\{0,\pm r,L\}} = 0$ HC PRD (2022)

Equation of Motions

In EFT, the variation principle can be applied to a specific perturbation field:

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$$\frac{\delta \mathcal{L}_{eff}}{\delta G} \Rightarrow \phi_0'\varphi' - G\phi_0'^2 - \frac{\partial V}{\partial \phi_0}\varphi = \frac{3}{\kappa^2}\left[4A'(F' - A'G) + \Box\left(Fe^{2A} - A'\epsilon' f(x)\right)\right],$$

$$\frac{\delta \mathcal{L}_{eff}}{\delta F} \Rightarrow \left(\phi_0'\varphi' + \frac{\partial V}{\partial \phi_0}\varphi\right) + \sum_i \left(\lambda_i G + \frac{\partial \lambda_i}{\partial \phi_0}\varphi\right)\delta(y - y_i) = \frac{3}{\kappa^2}\left[F'' - G'A' - 4A'F'\right] - 2GV + \frac{3}{4\kappa^2}e^{2A}\Box\Big(G - 2F - e^{-2A}\left[\epsilon'' - 4A'\epsilon'\right] f(x)\Big),$$

$$\frac{\delta \mathcal{L}_{eff}}{\delta \varphi} \Rightarrow (G' + 4F')\phi_0' + 4A'\varphi' + \sum_i \left(\frac{\partial \lambda_i}{\partial \phi_0}G + \frac{\partial^2 \lambda_i}{\partial \phi_0^2}\varphi\right)\delta(y - y_i) = \varphi'' - \left(2\frac{\partial V}{\partial \phi_0}G + \frac{\partial^2 V}{\partial \phi_0^2}\varphi\right) - \Box\Big(\varphi e^{2A} - \phi_0'\epsilon' f(x)\Big).$$

Scalar EOM

$$\mathcal{L}_{eff} \supset -\frac{3}{\kappa^2}\int dy e^{-4A}\epsilon'\partial_\mu \left[F' - A'G - \frac{\kappa^2}{3}\phi_0'\varphi\right]\partial^\mu f(x)$$

Varying $\mathcal{L}_{eff}$ with respect to $\epsilon' f(x)$ gives back to First Orthogonal Equation.

Equivalence and Correlation

One exact correspondence can be established between the EFT formalism and the linearized Einstein equations:

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$$\frac{\delta \mathcal{L}_{eff}}{\delta G} = 0 \quad \Rightarrow \quad \frac{1}{\kappa^2}\left[e^{2A} R_{\mu\nu}/\eta_{\mu\nu} + R_{55}\right] = \left[e^{2A}\tilde{T}_{\mu\nu}/\eta_{\mu\nu} + \tilde{T}_{55}\right]$$

$$\frac{\delta \mathcal{L}_{eff}}{\delta F} = 0 \quad \Rightarrow \quad \frac{1}{2\kappa^2}\left[2e^{2A} R_{\mu\nu}/\eta_{\mu\nu} - R_{55}\right] = \left[e^{2A}\tilde{T}_{\mu\nu}/\eta_{\mu\nu} - \frac{1}{2}\tilde{T}_{55}\right]$$

- With stabilization, one EOM + two orthogonal equations are independent.

  Four radion fields: F , G , φ , $\epsilon \partial_\mu \partial_\nu f(x)$

- W/O stabilization, only two orthogonal equations are independent.

  For $\phi_0' = 0$, 5d diffeomorphism (keep S_{EH} invariant) can remove $\epsilon f(x)$:

$$\delta \epsilon f(x) = -\zeta \qquad \zeta'(x,y)|_{y=\{0,\pm r, L\}} = 0$$

A Spurious Symmetry

In the presence of stabilization, it is viable to conduct the field redefinition to remove $\epsilon' f(x)$ in EOM:

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$$\tilde{G} = 2\tilde{F}$$

$$\tilde{F}' - A'\tilde{G} - \frac{\kappa^2}{3}\phi_0'\tilde{\varphi} = 0$$

5d diffeomorphism

$$\zeta = \epsilon f(x)$$

$$\tilde{F} = F - A'\epsilon' f(x) e^{-2A}$$

$$\tilde{G} = G - (\epsilon'' - 2A'\epsilon') f(x) e^{-2A}$$

$$\tilde{\varphi} = \varphi - \phi_0'\epsilon' f(x) e^{-2A} \Rightarrow \delta\varphi = 0$$

$\phi_0'\epsilon' = 0$, otherwise 4d Poincare symmetry is broken

For $\phi_0' \neq 0$, $\epsilon' = 0$ is forced after stabilization.

A Spurious Symmetry

- For $\phi_0' \neq 0$ and $\epsilon' = 0$ $(\epsilon'(r) = 0)$, only ζ-symmetry is broken, but 4d diffeomorphism (required by graviton) is conserved.

  Thus ONE degree of freedom for radion is permitted. ✓

- For $\phi_0' = 0$, relaxing the BC to be $\epsilon'(r) \neq 0$ $(\epsilon'|_{y=\{0,L\}} = 0)$:

$$\delta S =$$

$$\frac{3}{\kappa^2} \int dx^5 \left(e^{-4A} \partial_\mu \tilde{F} \left[\epsilon'' - 2A'\epsilon' \right] + \frac{A'}{2} \frac{d}{dy} \left[\epsilon'^2 e^{-6A} \right] \partial_\mu f(x) \right) \partial^\mu f(x)$$

  $\tilde{F} \sim e^{2A}$ and $A' \sim$ constant, this is a nonzero surface term.

  However no radion stabilization

- If $\phi_0' \neq 0$ and $\epsilon' \neq 0$ (forbidden choice), the radion kinetic term will depend on the bulk value of $\epsilon(y)$ due to breaking of Poincare symmetry.

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Radion EOM

The radion EOM (respecting 4d Poincare symmetry) corresponds to the combination of $e^{2A} R_{\mu\nu} (\eta_{\mu\nu})^{-1} + R_{55}$ in Einstein equation:

$$
\begin{aligned}
& 3\left(F'' - A'G'\right) f(x) + 3\left[F e^{2A} - A'\epsilon'(y)\right] \partial_\mu \partial^\mu f(x) \\
= \quad & 2\kappa^2 \phi_0' \varphi' + \frac{\kappa^2}{3} \sum_i \left[3\lambda_i(\phi_0) G f(x) + 3 \frac{\partial \lambda_i}{\partial \phi} \varphi\right] \delta(y - y_i)
\end{aligned}
$$

with $\epsilon = 0, \; G = 2F$ and satisfying the junction conditions:

$$
[F' f(x)]\,|_i = \frac{\kappa^2}{3} \left(\lambda_i \, G(y) f(x) + \frac{\partial \lambda_i}{\partial \phi} \varphi(x, y) \right)
$$

that can be rewritten in terms of the jumps for A' and ϕ_0' :

$$
[A']\,|_i = \frac{\kappa^2}{3} \lambda_i \left(\phi_0\right) , \qquad [\phi_0']\,|_i = \frac{\partial \lambda_i}{\partial \phi} (\phi_0)
$$

$$
\Rightarrow \quad \left([F']|_i - [A']|_i G(y)\right) f(x) = \tfrac{\kappa^2}{3} [\phi_0']|_i \varphi(x, y)
$$

Consistent with the First Orthogonal Equation

Csaki, Graesser and Kribs PRD (2001)

Radion Mass

The stabilization of multibrane model is similar to the RS1. The wave function is corrected by back-reaction, with $l = \kappa\phi_P/\sqrt{2}$:

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$$F = \begin{cases} e^{2k_1|y|}\left[1 + l^2 f_1(y)\right] & , 0 < y < r \\ e^{2k_2|y|+2r(k_1-k_2)}\left[1 + l^2 f_2(y)\right] & , r < y < L \end{cases}$$

Only the BC of φ needs to be imposed. At UV and IR the BC reduces to be $\left(f'_{1,2} + \frac{2}{3}ue^{-2uy}\right)|_{y=\{0,L\}} = 0$, while at y = r, we require $f'_1(r-\varepsilon) = f'_2(r+\varepsilon)$. These BC choices satisfy the Hermitian conditions.

$$\begin{aligned} m^2 &= \frac{4u^2(2k_2+u)l^2}{3k_2} e^{-2[(k_2+u)L+(k_1-k_2)r]} \\ &- C\, l^2 e^{-2[(2k_2+u)L+2(k_1-k_2)r]} \end{aligned}$$

$$C \simeq \frac{4u^2\,(2k_2+u)}{3k_1k_2}\left[(k_2-k_1)\,e^{2k_1 r} - k_2\right]$$

$(\text{for } \ 0 \ll r \ll L)$

The C term is negligible due to a large warped suppression

Conclusion

- In the multiple branes extension, two non-fixed point branes at $y = \pm r$ are present. Because there is only one degree of freedom of radion, the middle branes need to be rigid for a static solution.

- The BC of $\epsilon'(r)$ can not be used to create another degree of freedom, due to symmetry breaking. After stabilization, $\epsilon'(y) = 0$ is forced.

- In terms of effective Lagrangian, one can apply the variation principle to a specific perturbation field. This approach is demonstrated to be equivalent to the linearized Einstein equation.

- After applying the Goldberger-Wise mechanism similar to RS1, we show the radion mass is below the cut off scale of the IR brane.

Back up Slides

Diffeomorphism

For an infinitesimal coordinate shift, the metric transforms accordingly:

$$\delta g_{MN} = -\xi^K \, \partial_K g^{(0)}_{MN} - \partial_M \xi^K \, g^{(0)}_{KN} - \partial_N \xi^K \, g^{(0)}_{MK}$$

Since diffeomorphism retains the metric in its original structure (i.e. keep S_{EH} invariant), the transformation is of the specific form:

$$\xi^\mu(x,y) \quad = \quad \hat{\xi}^\mu(x) + \eta^{\mu\nu}\partial_\nu \zeta(x,y)$$

$$\xi^5(x,y) \quad = \quad e^{-2A}\,\zeta'(x,y)$$

The component fields will transform as following:

$$\delta h_{\mu\nu} = -\partial_\mu \hat{\xi}_\nu - \partial_\nu \hat{\xi}_\mu \,, \quad \delta F = -A'\zeta' e^{-2A}$$

$$\delta \epsilon f(x) = -\zeta \,, \qquad \delta G = -\left(\zeta'' - 2A'\zeta'\right) e^{-2A}$$

$\hat{\xi}^\mu$ represents the usual 4d diffeomorphism and the fifth coordinate shift is subject to the constraint $\zeta'(x,y)|_{y=\{0,\pm r,L\}} = 0$.

Radion Kinetic term

The radion kinetic term gets three parts: 1) involving only F and G 2) with one epsilon 3) involving only GW scalar:

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S_m

$$\mathcal{L}_{rad-kin} = \frac{1}{2}\int dy e^{-2A}\left\{\partial_\mu \varphi \partial^\mu \varphi - \frac{6}{\kappa^2}\left[\partial_\mu F \partial^\mu (F-G) - e^{-2A}\epsilon' \partial_\mu \left[F' - A'G - \frac{\kappa^2}{3}\phi_0' \varphi\right] \partial^\mu f(x)\right]\right\}$$

$S_{EH} + S_m$

Cosmological solution

To discuss the cosmological expansion, the metric needs to include the time evolution:

$$ds^2 = n(t,y)^2 dt^2 - a(t,y)^2 dx^2 - b(t,y)^2 dy^2$$

$$a(t,y) = a_0(t) e^{-A}(1+\delta a)\,,\; n(t,y) = e^{-A}(1+\delta n)$$
$$b(t,y) = 1 + \delta b$$

Averaging $G_{55} = \kappa^2 T_{55}$ with respect to y = r brane, one derives:

$$\left(\frac{\dot a_0}{a_0}\right)^2 + \frac{\ddot a_0}{a_0} = \frac{e^{-2A}}{3}\frac{\kappa^2 k_1 k_2}{k_1 - k_2}(\rho - 3p) + \frac{e^{-2A}}{3}\kappa^2 \phi_0'^2 \delta b$$

ρ and p are the matter density and pressure at the brane.

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