Overview of Theoretical Luminosity Precision of Future e^+e^- Colliders

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-- see Phys.Lett.B **790** (2019) 314, Eur.Phys.J.C **81** (2021) 1047

Current Situation, Related to LEP

LEP update 2018(2019)

Type of correction / Error	1999	Update 2018
(a) Photonic $O(L_e\alpha^2)$	0.027% [5]	0.027%
(b) Photonic $O(L_e^3\alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7, 8]	0.013% (0.011%(JJ))
(d) Light pairs	0.030% [10]	0.010% [18, 19]
(e) s-channel Z-exchange	0.015% [11, 12]	0.015%
(f) Up-down interference	0.0014% [27]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038% (0.037%(JJ))

JJ <--> Janot and Jadach, Phys. Lett. B 803 (2019) 135319

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Current Situation, Related to LEP

- Implied Upgrade of BHLUMI: 'not urgent' in Review for US DOE OHEP Funding Program
- Recent Work by Banerjee *et al.*, PLB 820 (2021) 136547, T. Engel *et al.*, in arXiv:2203.12557, contacts

Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2\alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18, 19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

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Current Situation, Related to LEP

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Bhabha scattering at NNLO with next-to-soft stabilisation

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ABSTRACT

A critical subject in fully differential QED calculations originates from numerical instabilities due to small fermion masses that act as regulators of collinear singularities. At next-to-next-to-leading order (NNLO) a major challenge is therefore to find a stable implementation of numerically delicate real-virtual matrix elements. In the case of Bhabha scattering this has so far prevented the development of a fixed-order Monte Carlo at NNLO accuracy. In this paper we present a new method for stabilising the real-virtual matrix element. It is based on the expansion for soft photon energies including the non-universal subleading term calculated with the method of regions. We have applied this method to Bhabha scattering to obtain a stable and efficient implementation within the McMule framework. We therefore present for the first time fully differential results for the photonic NNLO corrections to Bhabha scattering.

Current Situation, Related to LEP

References on Virtual Correction to Bremsstrahlung at $O(\alpha^2 L)$

1. S. Jadach et al., Phys. Lett. B 377 (1996) 168.
2. S. Jadach et al., Phys. Lett. B 450 (1999) 262.
3. S. Jadach et al., Comput. Phys. Commun. 70 (1992) 305.
4. S. Jadach et al., Comput. Phys. Commun. 102 (1997) 229.
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Current Situation, Related to LEP

Comparisons:

1. Exact $O(\alpha^2 L) \Rightarrow O(\alpha^2/\pi^2) \sim 5.4 \times 10^{-6}$ is missing
2. BaBaYaga vs Banerjee et al. :

   $E_{min} = 408$ MeV, $20^o < \theta_\pm < 160^o$, $\zeta_{max} = 10^o$

   Agreement: 0.07% (technical?)

   Soft expansion: $\lim_{\xi \to 0} \xi^2 \mathcal{M}^{(\ell)}_{n+1} = \mathcal{E} \mathcal{M}^{(\ell)}_n + \xi \Delta\mathcal{M}_{n+1} + \ldots$,

   next-to-soft term

Current Situation, Related to LEP

Comparisons:

1. BHLUMI: O(α^2L) term implemented => O(α^2/π^2) is missing in $\bar{\beta}^{(r)}_{1U}$, $\bar{\beta}^{(r)}_{1L}$

$$\sigma^{(r)} = \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\frac{1}{n!}\frac{1}{n'!}\int\frac{d^3p_2}{p_2^0}\int\frac{d^3q_2}{q_2^0}\prod_{j=1}^{n}\int_{k_j\notin\Omega_U}\frac{d^3k_j}{k_j^0}\tilde{S}_p(k_j)\prod_{l=1}^{n'}\int_{k'_l\notin\Omega_L}\frac{d^3k'_l}{k'^0_l}\tilde{S}_q(k'_l)$$

$$\times\,\delta^{(4)}\left(p_1-p_2+q_1-q_2-\sum_{j=1}^{n}k_j-\sum_{l=1}^{n'}k'_l\right)e^{Y_p(\Omega_U)+Y_q(\Omega_L)}$$

$$\times\left\{\bar{\beta}_0^{(r)}+\sum_{j=1}^{n}\frac{\bar{\beta}^{(r)}_{1U}(k_j)}{\tilde{S}_p(k_j)}+\sum_{l=1}^{n'}\frac{\bar{\beta}^{(r)}_{1L}(k'_l)}{\tilde{S}_q(k'_l)}+\sum_{n\geq j>k\geq 1}\frac{\bar{\beta}^{(r)}_{2UU}(k_j,k_k)}{\tilde{S}_p(k_j)\tilde{S}_p(k_k)}\right.$$

$$\left.+\sum_{n'\geq l>m\geq 1}\frac{\bar{\beta}^{(r)}_{2LL}(k_l,k_m)}{\tilde{S}_q(k'_l)\tilde{S}_q(k'_m)}+\sum_{j=1}^{n}\sum_{l=1}^{n'}\frac{\bar{\beta}^{(r)}_{2UL}(k_j,k'_l)}{\tilde{S}_p(k_j)\tilde{S}_q(k'_l)}\right\}$$

No semi-soft approximation

Current Situation, Higher Energies

Higher Energies and/or Different Acceptances:

Machine	θ_{min}–θ_{max} (mrad)	$\sqrt{s}$ (GeV)	$\bar{t}/s$	$\sqrt{\bar{t}}$ (GeV)
LEP	28–50	M_Z	3.5×10^{-4}	1.70
FCCee	64–86	M_Z	13.7×10^{-4}	3.37
FCCee	64–86	350	13.7×10^{-4}	13.0
ILC	31–77	500	6.0×10^{-4}	12.2
ILC	31–77	1000	6.0×10^{-4}	24.4
CLIC	39–134	3000	13.0×10^{-4}	108

=> Different $\sqrt{\bar{t}}$

Current Situation, Higher Energies

Higher Energies and/or Different Acceptances: Generalizing our FCCee analysis to higher energies, we get

ILC 500 setup

Type of correction/error	Update 2019	Forecast
(a) Photonic $[\mathcal{O}(L_e\alpha^2)]$ $\mathcal{O}(L_e^2\alpha^3)$	0.033%	0.13×10^{-4}
(b) Photonic $[\mathcal{O}(L_e^3\alpha^3)]$ $\mathcal{O}(L_e^4\alpha^4)$	0.028%	0.27×10^{-4}
(c) Vacuum polariz.	0.022% [34]	1.1×10^{-4}
(d) Light pairs	0.010% [7]	0.4×10^{-4}
(e) Z and s-channel γ exchange	0.5% (0.06%)	1.0×10^{-4}
(f) Up-down interference	0.004% [13]	$<0.1 \times 10^{-4}$
(g) Technical Precision	(0.027%)	0.1×10^{-4}
Total	0.5% (0.078%)	1.6×10^{-4}

Current Situation, Higher Energies

and the forecasts

Forecast			
Type of correction/error	$FCCee_{350}$	ILC_{1000}	$CLIC_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exchange	$1.0 \times 10^{-4(a)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	0.1×10^{-4}	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

(no technical error included).

Summary: We need financial support!