Bound-state effects for dark matter models: from the relic density to experimental searches

Simone Biondini

Department of Physics - University of Basel

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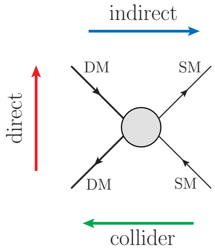
in collaboration with Stefan Vogl and Vladyslav Shtabovenko
arXiv 1907.05766, 2106.06472, 2112.10145

FNSNF Swiss National Science Foundation

University of Basel

Particle interpretation of DM and freeze-out



- Evidence for DM from many compelling (gravitational) observations

- DM as a particle: many candidates (Bertone and Hooper [1605.04909])
- Any model has to comply with

$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

◇ from CMB anysotropies with ΛCDM *Planck Collab. Results 2018*

Thermal freeze-out

- Boltzmann equation for DM (χ)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

- relevant processes $\chi\chi \leftrightarrow \mathrm{SM\,SM}$
- $\langle \sigma v \rangle$: input from particle physics with $v \sim \sqrt{T/M} < 1$

$$\langle \sigma v \rangle \approx \langle a + bv^2 + \dots \rangle \Rightarrow \boxed{\langle \sigma v \rangle^{(0)} \approx \frac{\alpha^2}{M^2}}$$

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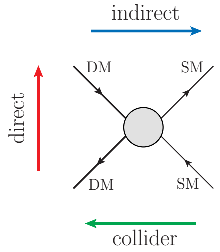

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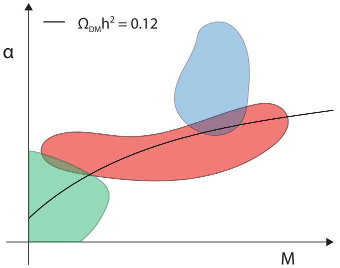
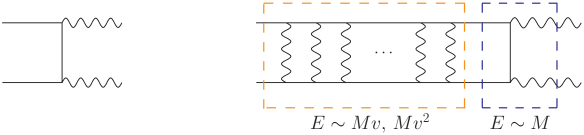


Going towards a realistic picture

- DM and/or coannihilating partners interact with gauge bosons and scalars



- repeated soft interactions: Sommerfeld enhancement and bound states Hisano, Matsumoto, Nojiri [hep-ph/0212022], [hep-ph/0307216]; B. von Harling and K. Petraki [1407.7874]; Beneke, Hellmann, Ruiz-Femenia [1411.6924]

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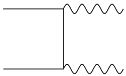

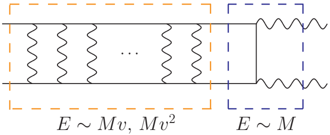

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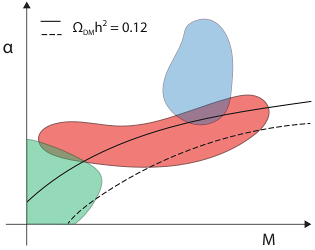
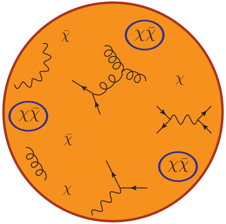
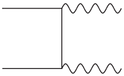
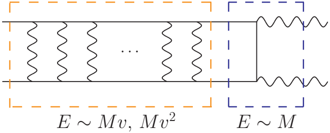




Dark matter bound states

- DM and/or coannihilating partners interact with gauge bosons and scalars





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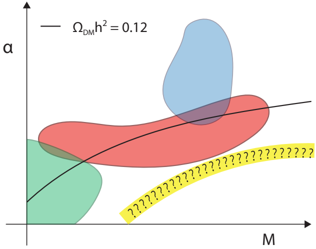
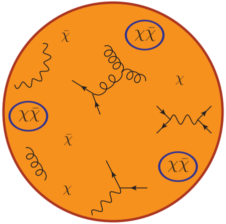
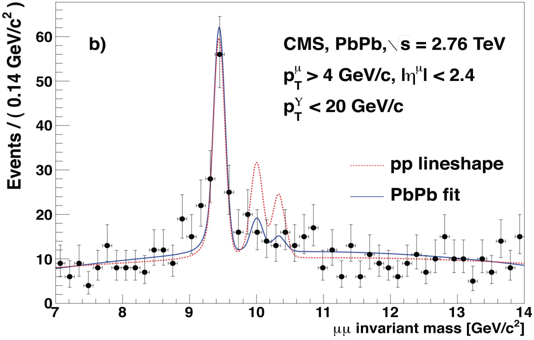


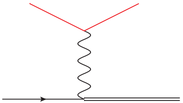


Lessons from heavy quarkonium...

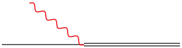
- many bound states may appear in the spectrum
- their existence depends on the temperature
  →**dissociation** and **recombination** processes
- bound-states calculations can be performed in NREFT/pNREFTs

Matsui and Satz (1986); Laine, Philipsen, Romatschke and Tassler [hep-ph/0611300]; Brambilla, Ghiglieri, Petreczky, Vairo [0804.0993]





$(\chi\chi)_{\rm bound} + f \to (\chi\chi)_{\rm open} + f$

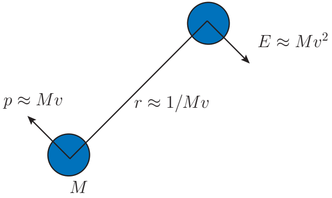


$(\chi\chi)_{\rm bound} + A^\mu \to (\chi\chi)_{\rm open}$

NREFTs and pNREFTs for dark matter

Apply an EFT approach

- Non-relativistic scales: $M \gg Mv \gg Mv^2$ (for bound states with Coulomb potential $v \sim \alpha$)
- Thermal scales: πT and $m_D \approx \alpha^{1/2} T$, if weakly-coupled plasma $\pi T \gg m_D$



$$\mathcal{L}_{\rm RT} = \frac{1}{2}\bar{\chi}(i\not{D} - M)\chi$$

$$\mathcal{L}_{\rm NREFT} = \psi^\dagger \left(iD^0 - \frac{\boldsymbol{D}^2}{2M} \right) \psi + \frac{c}{M^2}\psi^\dagger \chi \chi^\dagger \psi + \dots$$

$$\mathcal{L}_{\rm pNREFT} = \int d^3r \left\{ \phi^\dagger \left[i\partial_0 - H + g\boldsymbol{r}\cdot\boldsymbol{E} \right] \phi \right\} + \cdots$$

G. T. Bodwin, E. Braaten, and G. Lepage (1997); A.Pineda and J. Soto (1997)

- work with the natural d.o.f. at a given scale; **systematic expansions ($\alpha, r, 1/M$)**

  $E \sim Mv^2$ non-relativistic pairs $(\psi\chi^\dagger) \to \phi_s + \phi_b$ (bound states and scattering states)

$$V_\phi = V(r, T, m_D) + i\Gamma(r, T, m_D)$$

- the imaginary part of the potential encodes the effect of bound-state formation and dissociation

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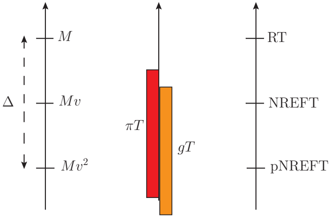

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Coloured mediators: simplified models

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\chi} + \mathcal{L}_{\eta} + \mathcal{L}_{\mathrm{int}} \,, \quad \eta \text{ is a color triplet}$$

$$\mathcal{L}_{\chi} = \frac{1}{2}\bar{\chi} i \partial\!\!\!/ \chi - \frac{M}{2}\bar{\chi}\chi \,, \quad \mathcal{L}_{\eta} = (D^{\mu}\eta)^{\dagger}(D_{\mu}\eta) - M_{\eta}^{2}\eta^{\dagger}\eta - \lambda_{2}\left(\eta^{\dagger}\eta\right)^{2}$$

$$\mathcal{L}_{\mathrm{int}} = -\mathrm{y}\,\eta^{\dagger}\bar{\chi}P_{R}q - \mathrm{y}^{*}\,\bar{q}P_{L}\chi\eta - \lambda_{3}\eta^{\dagger}\eta H^{\dagger}H$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{S} + \mathcal{L}_{F} + \mathcal{L}_{\mathrm{int}} \,, \quad F \text{ is a color triplet}$$

$$\mathcal{L}_{S} = \frac{1}{2}\partial_{\mu}S\partial^{\mu}S - \frac{M_{S}^{2}}{2}S^{2} - \frac{\lambda_{2}}{4!}S^{4} \,, \quad \mathcal{L}_{F} = \bar{F}\left(iD\!\!\!\!/ - M_{F}\right)F$$

$$\mathcal{L}_{\mathrm{int}} = -y\,S\bar{F}P_{R}q - y^{*}S\bar{q}P_{L}F - \frac{\lambda_{3}}{2}S^{2}H^{\dagger}H \,,$$

coannihilation scenario $(M_{\eta} - M_{\chi})/M_{\chi} \lesssim 0.2$: $\langle \sigma v \rangle \approx \langle \sigma v \rangle_{\chi\chi} + \langle \sigma v \rangle_{\chi\eta} e^{-\Delta M/T} + \langle \sigma v \rangle_{\eta\eta} e^{-2\Delta M/T}$

in MSSM importance of coannihilations realized ling ago J. Edsjö and P. Gondolo [hep-ph/9704361]

simplified models with co-annihilators see Garny, Ibarra, Vogl [1503.01500]; De Simone, Jacques [1603.08002]

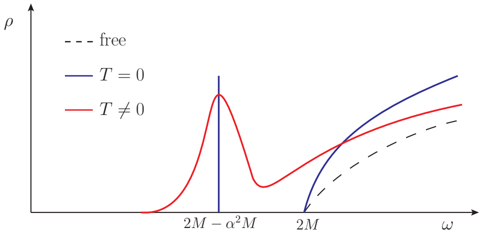
ρ and Schrödinger equation at $T \neq 0$

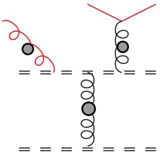
$$\omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + V(r,T)$$

- $V(r,T)$ and $\Gamma(r,T)$ from pNRQCD and the spectral function $\rho(E')$ is obtained from

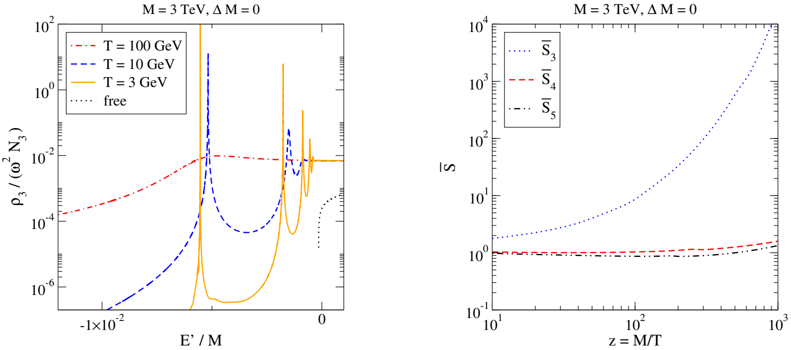
$$\left[H - i\Gamma(r,T) - E'\right] G(E';\mathbf{r},\mathbf{r'}) = N\delta^3(\mathbf{r}-\mathbf{r'}) \qquad \lim_{\mathbf{r},\mathbf{r'}\to 0} \mathrm{Im}\, G(E';\mathbf{r},\mathbf{r'}) = \rho(E')$$

$$\gamma \approx \left(\frac{MT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{2M}{T}} \int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{-\frac{E'}{T}} \rho(E') \Rightarrow \langle \sigma v \rangle \approx \frac{c}{M^2} \times \bar{S}(T)$$





SPECTRAL FUNCTION, BOUND STATES AND MELTING



- $\rho \to \bar{S}_i \to \langle \sigma v \rangle$, energy density from Boltzmann equation ($Y = n/s$) [ionization equilibrium kept]

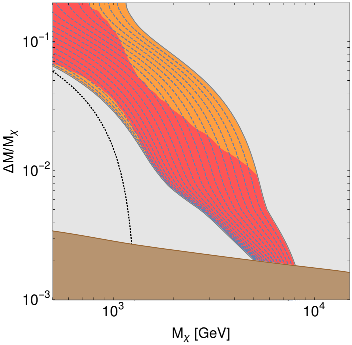
$$Y'(z) = -\langle \sigma_{\rm eff} v \rangle M m_{\rm Pl} \frac{c(T)}{\sqrt{24\pi\, e(T)}} \frac{Y^2(z) - Y^2_{\rm eq}(z)}{z^2}\Big|_{T=M/z}$$

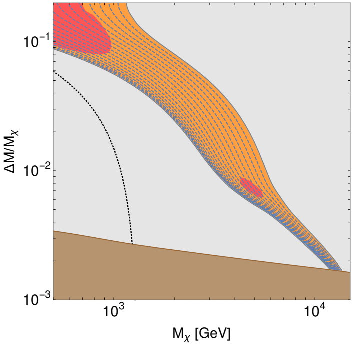
$$\langle \sigma_{\rm eff} v \rangle = \frac{2c_1 + 4c_2 N_c\, e^{-\Delta M/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c + 1)] N_c\, e^{-2\Delta M/T}}{(1 + N_c\, e^{-\Delta M_T/T})^2}$$

Parameter space for DM$_\chi$ + η S.B. and S. Vogl [1811.02581]

◇**valence-quark scenario**

$$\mathcal{L}_{\rm int} = -y\, \eta^\dagger \bar{\chi} P_R q - y^* \bar{q} P_L \chi \eta - \lambda_3 \eta^\dagger \eta H^\dagger H\,, \quad q \equiv u, d$$





- $2\Delta M > |E_1|$ S.B. and M. Laine [1801.05821], left $\lambda_3 = 0$, right $\lambda_3 = 1.5$, $y \in [0.1, 2]$
- dotted-black $y = 0.3$ (free); λ_3 boosts the annihilations because $c_3 \bar{S}_3 \approx (g_s^4 + \lambda_3^2)\bar{S}_3$
- Xenon1T sensitivity and DARWIN-like detector sensitivity

$M_\chi \simeq 1.2\,\text{TeV} \rightarrow M_\chi \simeq 2.0\,(3.1)\,\text{TeV}$ for $\Delta M/M_\chi = 10^{-2}$ and $\lambda_3 = 0.0\,(1.5)$

Parameter space for $\mathrm{DM}_S + F$ S.B. and S. Vogl [1907.05766]

◇ **valence-quark scenario**

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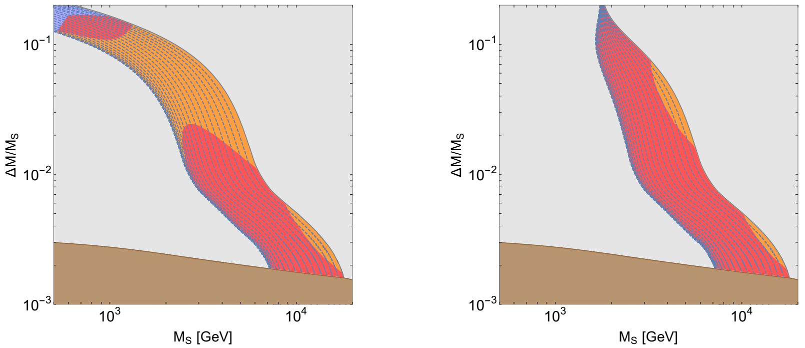


- small region sensitive to LHC searches for valence quarks; left $\lambda_3 = 0$, right $\lambda_3 = 1.5$, $y \in [0.1, 2]$
- throat-like behaviour for $\lambda_3 = 1.5$: recovering the importance of $SS \to H^\dagger H$ at large ΔM;
- **bound states** from antitriplet (FF): $3 \otimes 3 = \bar{3} \oplus 6$; running for $\lambda_3(2M_S) \to \lambda_3(\mu_p)$

Scalar and pseudoscalar mediators

Simplified model

$$\mathcal{L} = \bar{X}(i\partial\!\!\!/ - M)X + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 - \bar{X}(g + ig_5\gamma_5)X\phi + \mathcal{L}_{\text{portal}}\,,$$

M. B. Wise and Y. Zhang [1407.4121]; K. Kainulainen, K. Tuominen and V. Vaskonen [1507.04931]

- $M \gg M\alpha \sim Mv \gg \pi T \sim M\alpha^2 \sim Mv^2 \gg m$

$$L_{\text{pNRY}_{\gamma_5}} = \int d^3\boldsymbol{r}\, d^3\boldsymbol{R}\, \varphi^\dagger(\boldsymbol{r},\boldsymbol{R},t)\left\{ i\partial_0 + \frac{\boldsymbol{\nabla}_{\boldsymbol{r}}^2}{M} + \frac{\boldsymbol{\nabla}_{\boldsymbol{R}}^2}{4M} + \frac{\boldsymbol{\nabla}_{\boldsymbol{r}}^4}{4M^3} - V(\boldsymbol{p},\boldsymbol{r},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) \right.$$
$$\left. -2g\phi(\boldsymbol{R},t) - g\frac{r^i r^j}{4}\left[\nabla^i_R\nabla^j_R\,\phi(\boldsymbol{R},t)\right] - g\phi(\boldsymbol{R},t)\frac{\boldsymbol{\nabla}_{\boldsymbol{r}}^2}{M^2}\right\}\varphi(\boldsymbol{r},\boldsymbol{R},t) + L_{\text{scalar}}$$

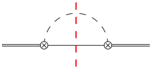
- Monopole, quadrupole and derivative interactions between the heavy pair and the mediator
- monopole contribution is zero $\langle\varphi_s|\varphi_p\rangle$, $\langle\varphi_s|\varphi_{s'}\rangle$, $\langle\varphi_b|\varphi_{b'}\rangle$

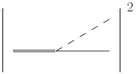
  exceptions for models with charged scalar and vector mediator, see R. Onacala and K. Petraki 1911.02605

Formation, Dissociation and annihilations

- $\varphi_s \to \varphi_b + \phi$

$$\sigma_{\rm bsf} v_{\rm rel}\big|_T = \sigma_{\rm bsf} v_{\rm rel}\big|_{T=0} \left[1 + n_{\rm B}(\Delta E_n^p)\right]$$





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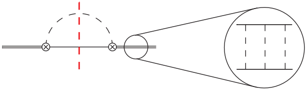

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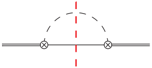
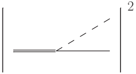
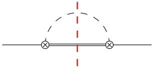
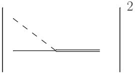




- $\varphi_b + \phi \to \varphi_b$

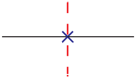
$$\Gamma^n_{\rm bsd} = \int_{|\boldsymbol{k}|\geq |E_n|} \frac{d^3\boldsymbol{k}}{(2\pi)^3}\; n_B(|\boldsymbol{k}|)\, \sigma^n_{\rm bsd}(|\boldsymbol{k}|)$$

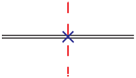




- Pair annihilations

$$\Gamma^{nS}_{\rm ann} = \langle nS|\, 2{\rm Im}(-\Sigma_{\rm ann})\, |nS\rangle$$
$$\sigma_{\rm ann} v_{\rm rel} = \langle \boldsymbol{p}|\, 2{\rm Im}(-\Sigma_{\rm ann})\, |\boldsymbol{p}\rangle$$





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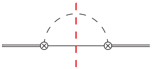
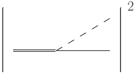
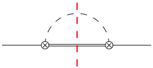
$$\sigma_{\rm bsf} v_{\rm rel}\big|_T = \sigma_{\rm bsf} v_{\rm rel}\big|_{T=0}\left[1+n_{\rm B}(\Delta E_n^p)\right]$$

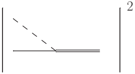




- $\varphi_b + \phi \to \varphi_b$

$$\Gamma^n_{\rm bsd} = \int_{|\boldsymbol{k}|\geq |E_n|} \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, n_B(|\boldsymbol{k}|)\, \sigma^n_{\rm bsd}(|\boldsymbol{k}|)$$





- Pair annihilations

$$\Gamma^{nS}_{\rm ann} = \langle nS|\, 2{\rm Im}(-\Sigma_{\rm ann})\, |nS\rangle$$
$$\sigma_{\rm ann} v_{\rm rel} = \langle \boldsymbol{p}|\, 2{\rm Im}(-\Sigma_{\rm ann})\, |\boldsymbol{p}\rangle$$

- $\varphi_s \to \phi\phi$, $|R_{nS}(0)|^2 = 4/(n^3 a_0^3)$

$$\Gamma^{nS}_{\rm ann} = \frac{|R_{nS}(0)|^2}{\pi M^2}\left\{{\rm Im}[f(^1S_0)] + \frac{E_n}{M}{\rm Im}[g(^1S_0)]\right\} = \frac{M\,\alpha^4\alpha_5}{n^3}\left(1+\frac{\alpha^2}{3n^2}\right)$$

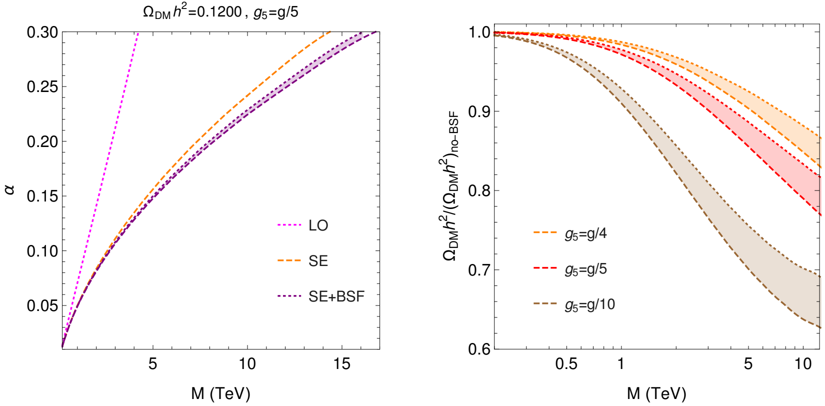
- $\varphi_s \to \phi\phi$, $|\mathcal{R}_0(0)|^2 = \frac{2\pi\zeta}{1-e^{-2\pi\zeta}}$

$$\sigma_{\rm ann} v_{\rm rel} = \frac{2\pi\alpha\alpha_5}{M^2}\left(1-\frac{v_{\rm rel}^2}{3}\right)S(\zeta) + \frac{(9\alpha^2-2\alpha\alpha_5+\alpha_5^2)v_{\rm rel}^2}{24M^2}S_p(\zeta)$$

Energy density

$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma_{\rm eff}\, v_{\rm rel}\rangle (n_X^2 - n_{X,{\rm eq}}^2)\,, \quad \langle \sigma_{\rm eff}\, v_{\rm rel}\rangle = \langle \sigma_{\rm ann}\, v_{\rm rel}\rangle + \sum_n \langle \sigma^n_{\rm bsf}\, v_{\rm rel}\rangle \frac{\Gamma^n_{\rm ann}}{\Gamma^n_{\rm ann} + \Gamma^n_{\rm bsd}}$$

one-single Boltzmann equation, see J. Ellis, F. Luo, and K. A. Olive [1503.07142]



Summary and Conclusions

- the freeze-out calculation is factorized into $\langle \sigma v \rangle \approx c_i \langle \mathcal{O}_i \rangle_T$
- $\langle \mathcal{O}_i \rangle_T$ from heavy pair soft/ultrasoft dynamics: potentials and rates from pNREFTs

$\rightarrow$ formation/melting of (many) bound states Sommerfeld effect for above-threshold scattering states

- inclusion of Landau damping and gluo-dissociation for some hierarchy of scales

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$\Rightarrow$ parameter space compatible with relic density **change substantially**: up to 18-20 TeV

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see also A. Mitridate, M. Redi, J. Smirnov and A. Strumia 1702.01141; J. Harz and K. Petraki 1805.01200. **[See talk by J. Harz]**

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- Scalar and pseudoscalar mediators: derivation of the NREFT and pNREFT
- Combination and interplay of scalar/pseudoscalar coupling

  bound-state formation $\Rightarrow$ $\mathcal{O}(40\%)$ **corrections to** $\boldsymbol{\Omega_{\rm DM} h^2}$

- worth a revisitation of the simplified model experimental constraints

Back-up slides

Self-interacting dark matter

It can relax inconsistencies about

- predictions of collisionless cold DM and the observed large-scale structures
- numbers of the galactic satellite haloes
- DM density profiles in the galaxies

- figure of merit for DM self-interaction is $\sigma_{\chi\chi}/M_\chi$
- in order to relax the tensions

$$\frac{\sigma_{\chi\chi}}{M_\chi} \approx 1\,\frac{\mathrm{cm}}{\mathrm{g}} \approx 2\times 10^{-24}\,\frac{\mathrm{cm}^2}{\mathrm{GeV}}$$

- this cross section is way larger than the one expected from electroweak physics

$$\frac{\sigma_{\chi\chi}}{M_\chi} \approx \times 10^{-38}\,\frac{\mathrm{cm}^2}{\mathrm{GeV}}\,, \mathrm{for}\, M_\chi \sim 100\mathrm{GeV}$$

- $\Rightarrow$ motivation for lighter scalar/gauge boson

A different rate equation?

- Recently an alternative form of the BE has been suggested T. Binder, L. Covi and K. Mukaida (2018)

$$\dot{n} + 3Hn = -\langle \sigma v \rangle (e^{2\beta\mu(n)} - 1) n_{\rm eq}^2$$

- μ couples to the total number of dark sector particles

- number density operator, total number of particles and eq. number density

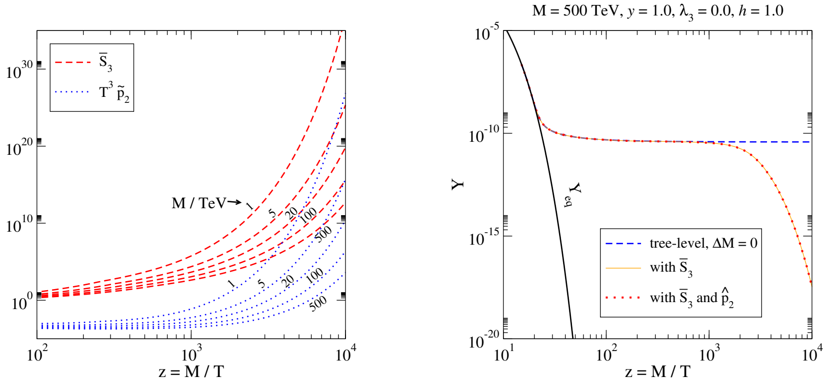
$$\hat{N} = \int_{\boldsymbol{x}} \hat{n}(\boldsymbol{x}) \, , \quad n_{\rm eq} = \lim_{\mu \to 0} \langle \hat{n} \rangle$$

- the density matrix has the form

$$\hat{\rho} = \frac{\exp[-\beta(\hat{H} - \mu \hat{N})]}{Z} \, , \quad Z = e^{p\beta V} \, , \quad n(\mu) = \frac{\partial p}{\partial \mu}$$

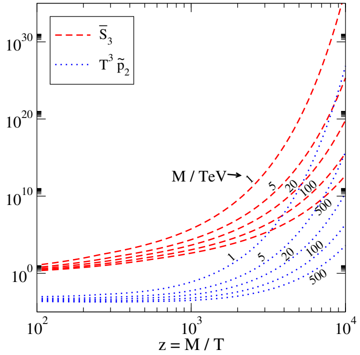
- expand pressure in the fugacity expansion $p = p_0 + p_1 e^{\beta\mu} + p_2 e^{2\beta\mu} + \cdots$, and obtain n

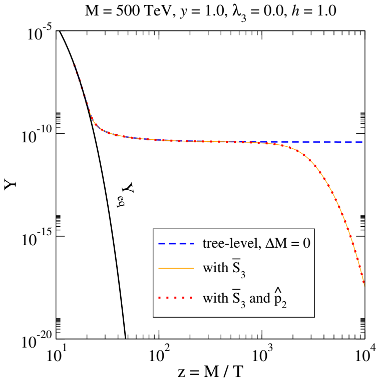
$$nT = p_1 e^{\beta\mu} + 2p_2 e^{2\beta\mu} + \cdots \, , \quad e^{\beta\mu} n_{\rm eq} \approx \frac{2n}{\sqrt{1 + 8\hat{p}_2 n}}$$



$$\hat{p}_2 \simeq \frac{N_c^2}{(N_c + e^{\beta \Delta M_T})^2}\tilde{p}_2 \,, \quad T^3 \tilde{p}_2 \equiv \frac{2}{N_c^2}\left(\frac{\pi T}{M}\right)^{3/2}(e^{\Delta E \beta} - 1)\,, \quad \bar{S}_3 \approx \left(\frac{4\pi}{MT}\right)^{3/2}\frac{e^{\Delta E \beta}}{\pi a^3}$$

Again the same problem





$$e^{\beta\mu} n_{\rm eq} \approx \frac{2n}{\sqrt{1+8\hat{p}_2 n}}\,, \quad 8\hat{p}_2 n = 8T^3\hat{p}_2 \frac{s}{T^3} Y\,, \quad \Omega_{\rm dm} h^2 = \frac{Y(z_f)M}{[3.645\times 10^{-12}\,{\rm TeV}]} \approx 0.12$$

From ρ to a Schrödinger equation

- non-relativistic dynamics:

$$\omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + V(r,T)$$

- the spectral function $\rho(E')$ is obtained from

$$\left[H - i\Gamma(r,T) - E'\right] G(E';\boldsymbol{r},\boldsymbol{r'}) = N\delta^3(\boldsymbol{r}-\boldsymbol{r'}) \qquad \lim_{\boldsymbol{r},\boldsymbol{r'}\to 0} \mathrm{Im}\, G(E';\boldsymbol{r},\boldsymbol{r'}) = \rho(E')$$

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- form inhomogeneous to homogeneous equation M.J. Strassler and M.E. Peskin (1991)

$$x \equiv \alpha M r \quad V \equiv \alpha^2 M \tilde{V}\,, \quad \Gamma \equiv \alpha^2 M \tilde{\Gamma}\,, \quad E' \equiv \alpha^2 M \tilde{E}'$$

- solve for the solution which is regular at the origin, $u_\ell(x) \sim x^{\ell+1}$ for $x \ll 1$

$$\left[-\frac{d^2}{dx^2} + \frac{\ell(\ell+1)}{x^2} + \tilde{V} - i\tilde{\Gamma} - \tilde{E}'\right] u_\ell(x) = 0$$

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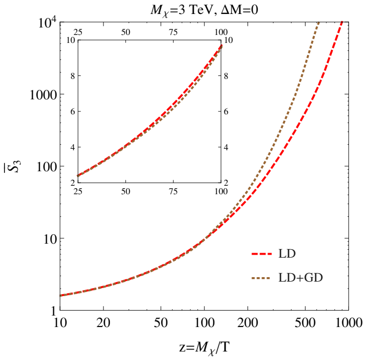
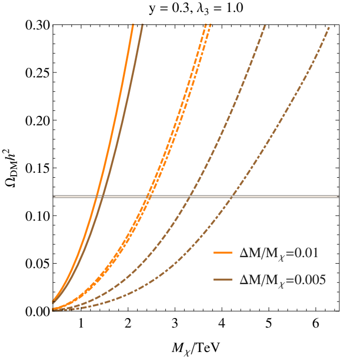
- the s-wave ($\ell = 0$) spectral function is obtained from

$$\rho(E') = \frac{\alpha\, N M^2}{4\pi} \int_0^\infty dx \mathrm{Im}\left[\frac{1}{u_0^2(x)}\right]$$

Gluo-dissociation



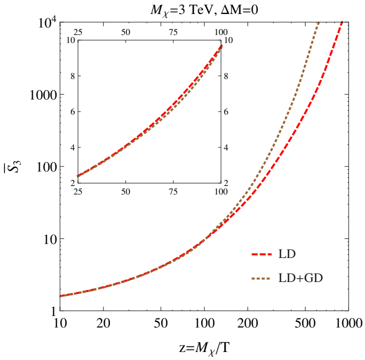


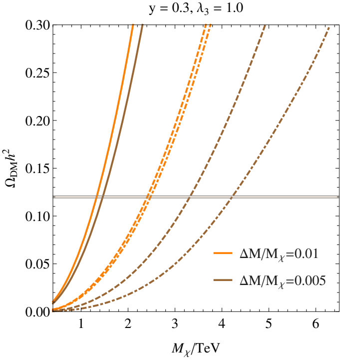
- borrow the result from heavy quarkonium $M \gg Mv \gg T \sim \Delta V$

$$\delta V_{\rm GD} = \frac{4}{3} C_F \frac{\alpha_s}{\pi} r^2\, T^2 \Delta V f(\Delta V/T)\,, \quad \Gamma_{\rm GD} = \frac{2}{3} C_F \alpha_s r^2 (\Delta V)^3 n_B(\Delta V),$$

N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky 0804.0993

Gluo-dissociation





- this is not a rigorous way of implementing it:
  other terms are missing beyond the static limit that are of the same order in the thermal width N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto and A. Vairo (2010)

$M_F v \sim T \gg M_D \gg \Delta V$

$$\delta V^q(r,T) = -\frac{C_F}{4}\,\alpha_s\, r\, m_{D,q}^2 - C_F\,\frac{3}{2\pi}\,\alpha_s\, r^2\, T\, m_{D,q}^2\, \zeta(3) + C_F\,\frac{\alpha_s\, m_{D,q}^2}{4\,\pi^2 r T^2}\int_0^\infty \frac{dx\, F^q(xrT)}{x\left(e^{x/2}+1\right)}\,,$$

$$F^q(u) = \left[-4 - 3u^2 + (u^2+4)\cos(u) + u\sin(u) + (6u+u^3)\,\mathrm{Si}(u)\right],$$

$$\delta V^g(r,T) = -\frac{C_F}{4}\alpha_s\, r\, m_{D,g}^2 - C_F\,\frac{\alpha_s\, r^2\, T\, m_{D,g}^2}{\pi}\,\zeta(3) + C_F\,\frac{\alpha_s\, m_{D,g}^2}{8\,\pi^2 r T^2}\int_0^\infty \frac{dx\, F^g(x\, rT)}{x\left(e^{x/2}-1\right)}\,,$$

$$F^g(u) = \left[-22 - 3u^2 + (u^2+10)\cos(u) + \left(u+\frac{12}{u}\right)\sin u + (u^3+12u)\mathrm{Si}(u)\right].$$

- $m_{D,q}^2 = (g_s^2 T^2 N_f T_F)/3$ and $m_{D,g}^2 = (g_s^2 T^2 N_c)/3$
- the result agrees with known limits
  $r\,T \ll 1$ and $r\,T \gg 1$ Brambilla, Ghiglieri, Petreczky, Vairo [0804.0993]

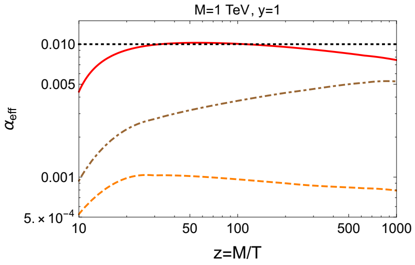
Higgs contribution

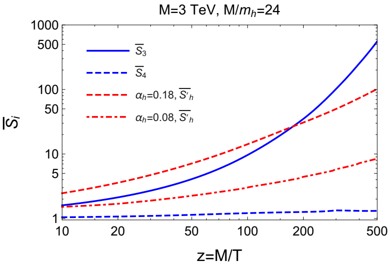
Simplified model case: $\mathcal{L}_{\rm INT} = -\lambda_3 \eta^\dagger \eta H^\dagger H$

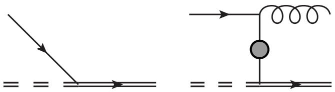
$$\mathcal{L}^{\rm NR}_{\rm int} = -\frac{\lambda_3 v_T}{2M}(\varphi^\dagger \varphi + \phi^\dagger \phi) h \,, \quad \alpha_{\rm eff} \equiv \frac{1}{4\pi}\left(\frac{\lambda_3 v_T}{2M}\right)^2$$

$$v_T^2 = \frac{1}{\lambda}\left[\frac{m_h^2}{2} - \frac{(g_1^2 + 3g_2^2 + 8\lambda + 4h_t^2)T^2}{16}\right]$$

- different situation if one takes $\mathcal{L}^{(2)}_{\rm int} = -g_h M_\eta \eta^\dagger \eta h + \dots$





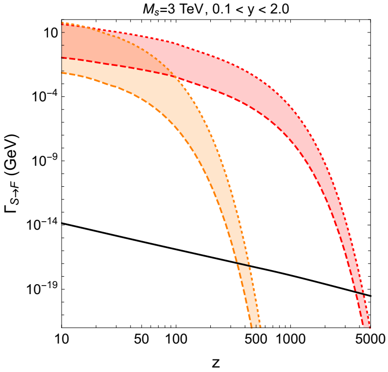


- $S + q \to F$ and $S + g + F$
- orange band: $\Delta M/M_S = 0.1$; red band: $\Delta M/M_S = 0.01$

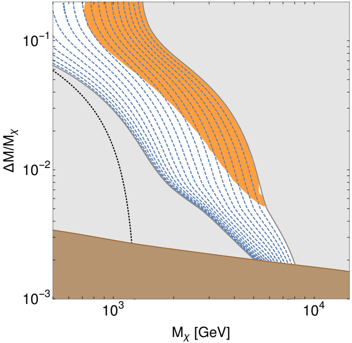
$$\Gamma_{2\to1} = \frac{|y|^2 N_c M_S}{4\pi}\left(\frac{\Delta M}{M_S}\right)^2 n_F(\Delta M)$$

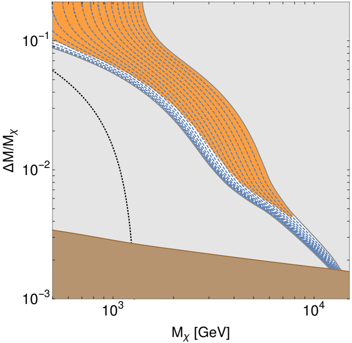
$$\Gamma_{2\to2} = \frac{|y|^2 N_c}{8M_S}\int_{\boldsymbol{p}} \frac{\pi m_q^2 n_F\left(\Delta M + \frac{p^2}{2M_S}\right)}{p(p^2+m_q^2)}$$

$$m_q = 2g_s^2 C_F \int_{\boldsymbol{q}} \frac{n_B(q)+n_F(q)}{q} = \frac{g_s^2 T^2 C_F}{4}$$

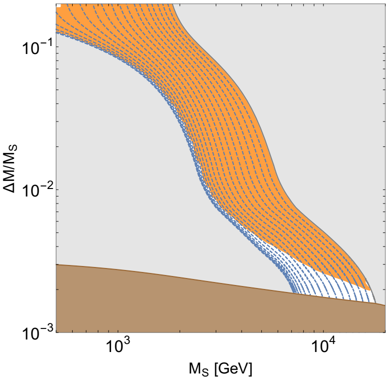


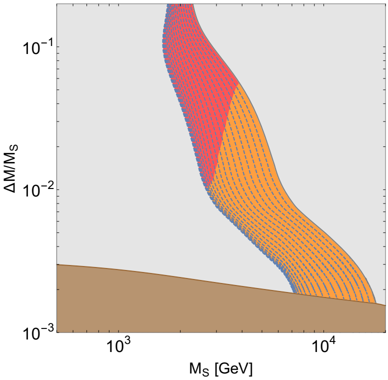
TOP-QUARK SCENARIO: MODEL(χ, η)





TOP-QUARK SCENARIO: MODEL(S, F)



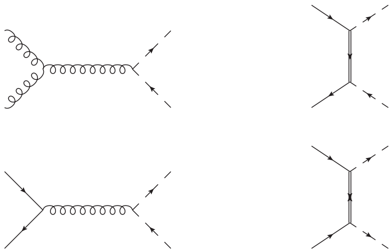


Experimental searches and relic density

- collider and direct detection experiments are sensitive to colored mediators
- relic density is almost flavour blind, whereas the quark flavour matters in the experimental searches

Collider searches at LHC: light quarks

- production channels: light quarks have significant parton luminosity



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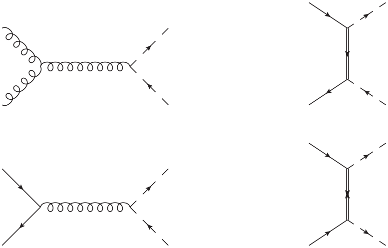
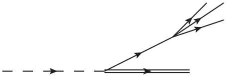


- Decay channel
  soft jets (ΔM small) and missing transverse energy
- FayRules $\rightarrow$ MadGraph5 $\rightarrow$ Pythia8
  based on ATLAS search [1711.03301]



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Collider searches at LHC: top quarks

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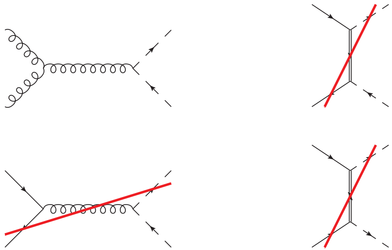

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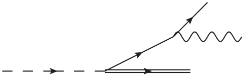
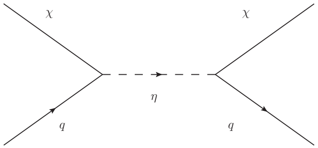
- Decay channels ($M_\eta < M_\chi + m_t$) b quarks and W (W*), then 3 or 4-body decays

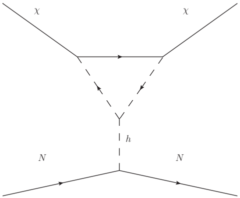


Direct detection: valence vs top quark

Valence quark

- dominant contribution to the effective DM-nucleon coupling arises from the **tree-level** exchange of η
- loop-induced coupling between the DM and the Higgs can become relevant for large λ_3, up to $\mathcal{O}(10\%)$





$$\frac{f_N}{m_N}\Big|_{\text{valence}} = -\sum_{q=u,d,s} f^N_{Tq}\left(\frac{M_\chi g_q}{2} - \frac{g_{h\chi\chi}}{2v_h m_h^2}\right) + f^N_{TG}\frac{2}{9}\frac{g_{h\chi\chi}}{2v_h\, m_h^2} - \sum_{q=u,d,s}(3q(2)+3\bar{q}(2))\frac{M_\chi g_q}{2}$$

Inert doublet model

- Supplement SM with χ SU(2) doublet, no coupling with fermions, unbroken vacuum
- We focus on the high-mass regime of the model: $M \gtrsim 530$ GeV

T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, 0903.4010

$$\begin{aligned}\mathcal{L}_\chi &= (D^\mu \chi)^\dagger (D_\mu \chi) - M^2 \chi^\dagger \chi \\ &- \left\{ \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 \, \phi^\dagger \phi \, \chi^\dagger \chi + \lambda_4 \, \phi^\dagger \chi \, \chi^\dagger \phi + \left[\frac{\lambda_5}{2} (\phi^\dagger \chi)^2 + h.c. \right] \right\}\end{aligned}$$

Electroweak thermal potentials

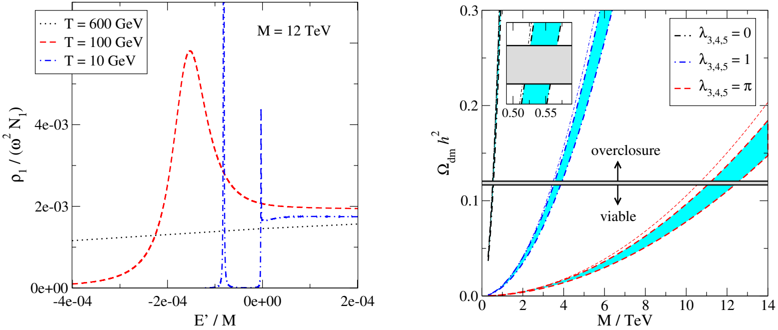
$$\mathcal{V}_{\rm W}(r) \equiv \frac{g_2^2}{4} \int_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \, i\langle W_0^+ W_0^- \rangle_{\rm T}(0,k) \quad \text{similar for } B^\mu \text{ and } W^3$$

$$i\langle W_0^+ W_0^- \rangle_T = \frac{1}{\boldsymbol{k}^2 + m_{\widetilde{W}}^2} - \frac{i\pi T}{k} \frac{m_{\rm E2}^2}{(\boldsymbol{k}^2 + m_{\widetilde{W}})^2}$$

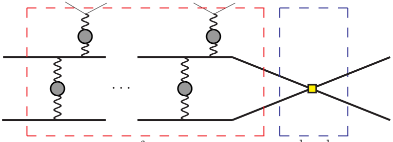
Bound states with electoweak gauge bosons

- the thermally modified Sommerfeld factors

$$\bar{S}_i = \frac{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} \rho(E') e^{-E'/T}}{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} \rho_{\text{free}}(E') e^{-E'/T}} = \left(\frac{4\pi}{MT}\right)^{\frac{3}{2}} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi}\, e^{[\mathrm{Re}\mathcal{V}_i(\infty) - E']/T}\, \frac{\rho_i(E')}{N_i}$$



- at small enough T bound states start to form and contribute to the annihilation cross section, up to 20% effect for large λ's S.B. and M. Laine (2017)

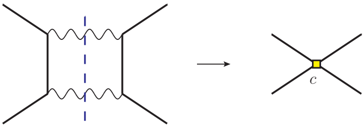


- $M \gg T, m_D, Mv, Mv^2$

- Annihilation of a heavy pair: DM-DM, with energies $\sim 2M$

$$\mathcal{O} = i\frac{c}{M^2}\psi^\dagger \chi \chi^\dagger \psi\,,\; c \approx \alpha^2 \quad \text{(inclusive s-wave annihilation)}$$

Caswell, Lepage (1985); Bodwin, Braaten, Lepage [hep-ph/9407339]



- $M \gg T \Rightarrow \Delta x \sim \frac{1}{k} \sim \frac{1}{M} \ll \frac{1}{T}$ local and insensitive to thermal scales

Beyond the free case: the spectral function

$$(\partial_t + 3H)n = -\Gamma_{\rm chem}(n - n_{\rm eq})\,, \quad \Gamma_{\rm chem} \approx \frac{8c}{M^2 n_{\rm eq}}\gamma \quad \text{where } \gamma = \langle \psi^\dagger \chi \chi^\dagger \psi \rangle_T$$

$$(\partial_t + 3H)n = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2) \quad \Rightarrow \langle \sigma v \rangle = \frac{\Gamma_{\rm chem}}{2 n_{\rm eq}}$$

Bodeker and Laine [1205.4987]; Kim and Laine [1602.08105]; Kim and Laine [1609.00474]

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- using $\Pi_<(\omega) = 2n_B(\omega)\int_{\boldsymbol{k}} \rho(\omega, \boldsymbol{k})$, with $\omega = E' + 2M + \frac{k^2}{4M}$

$$\gamma = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\Pi_<(\omega) = \int_{2M-\Lambda}^{\infty} \frac{d\omega}{\pi} e^{-\frac{\omega}{T}} \int_{\boldsymbol{k}} \rho(\omega, \boldsymbol{k}) + \mathcal{O}(e^{-4M/T})\,, \; \alpha^2 M \ll \Lambda \lesssim M$$

- $\boldsymbol{\rho}$ from the imaginary part of "Green's function" Laine [0704.1720], Burnier, Laine and Vepsälinen [0711.1743]

$$\left[H - i\Gamma(r, T) - E'\right] G(E'; \boldsymbol{r}, \boldsymbol{r'}) = N\delta^3(\boldsymbol{r} - \boldsymbol{r'}) \qquad \lim_{\boldsymbol{r},\boldsymbol{r'}\to 0} \mathrm{Im}\, G(E'; \boldsymbol{r}, \boldsymbol{r'}) = \rho(E')$$

- $H = -\frac{\nabla^2}{M} + V(r, T)$, $\Gamma(r, T)$ real scatterings with plasma particles

Hard annihilations and NREFT

- our master equation is $\boxed{\langle \sigma v \rangle = \frac{4}{n_{\rm eq}^2} \langle \mathrm{Im}\, \mathcal{L}_{\rm NREFT} \rangle}$
- Non-relativistic fields $\eta = \frac{1}{\sqrt{2M}}\left(\phi e^{-iMt} + \varphi^\dagger e^{iMt}\right)$ and $\chi = (\psi e^{-iMt}, -i\sigma_2 \psi^* e^{iMt})$

$$
\begin{aligned}
\mathcal{L}_{\rm NREFT} &= i \Big\{ c_1\, \psi_p^\dagger \psi_q^\dagger \psi_q \psi_p + c_2 \left(\psi_p^\dagger \phi_\alpha^\dagger \psi_p \phi_\alpha + \psi_p^\dagger \varphi_\alpha^\dagger \psi_p \varphi_\alpha \right) \\
&+ c_3\, \phi_\alpha^\dagger \varphi_\alpha^\dagger \varphi_\beta \phi_\beta + c_4\, \phi_\alpha^\dagger \varphi_\beta^\dagger \varphi_\gamma \phi_\delta\, T^a_{\alpha\beta} T^a_{\gamma\delta} + c_5 \left(\phi_\alpha^\dagger \phi_\beta^\dagger \phi_\beta \phi_\alpha + \varphi_\alpha^\dagger \varphi_\beta^\dagger \varphi_\beta \varphi_\alpha \right) \Big\}
\end{aligned}
$$

Last term in $\mathcal{L}_{\rm NREFT}$ is relevant because for $\eta\eta \to qq$ (due to χ exchange)

$$
\begin{aligned}
c_1 &= 0 \,, \quad c_2 = \frac{|y|^2 (|h|^2 + g_s^2 C_F)}{128\pi M^2} \,, \\
c_3 &= \frac{1}{32\pi M^2} \left(\lambda_3^2 + \frac{g_s^4 C_F}{N_c} \right) \,, \quad c_4 = \frac{g_s^4 (N_c^2 - 4)}{64\pi M^2 N_c} \,, \quad c_5 = \frac{|y|^4}{128\pi M^2}
\end{aligned}
$$

S.B. and M. Laine [1801.05821]

Thermal potentials and $\bar{S}_i$

$$\langle \sigma_{\rm eff}\, v \rangle = \frac{2c_1 + 4c_2 N_c\, e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c+1)] N_c\, e^{-2\Delta M_T/T}}{\left(1 + N_c\, e^{-\Delta M_T/T}\right)^2}$$

- with $\Delta M_T \equiv \Delta M + \frac{(g_s^2 C_F + \lambda_3) T^2}{12M} - \frac{g_s^2 C_F m_D}{8\pi}$

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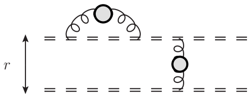
- potential from the static limit of the HTL resummed temporal gluon propagator



$$v(r) \equiv \frac{g_s^2}{2} \int_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \left[\frac{1}{\boldsymbol{k}^2 + m_D^2} - i\frac{\pi T}{k} \frac{m_D^2}{(\boldsymbol{k}^2 + m_D^2)^2}\right]$$

$$m_D = g_s \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}\, T$$

pNCQD for colored scalars

Integrating out $1/r \sim m_D$ from NRQCD$_{\rm HTL}$ Brambilla, Ghiglieri, Petreczky, Vairo [0804.0993]

$$\mathcal{L}_{\rm pNRQCD_{m_D}} = \mathcal{L}_{\rm gauge} + \int d^3r {\rm Tr} \left\{ S^\dagger \left[\partial_0 - V_s - \delta M_s\right] S + O^\dagger \left[D_0 - V_o - \delta M_o\right] O + \Sigma^\dagger \left[D_0 - V_\Sigma - \delta M_\Sigma\right] \Sigma \right\}$$

- with equal mass shifts $\delta M_s = \delta M_o = \delta M_\Sigma = -\alpha_s C_F \left(m_D + iT\right)$ and potentials

$$V_s(r) = \alpha_s C_F \left[-\frac{e^{-m_D r}}{r} + iT\, \Phi_r(m_D r) \right] , \quad V_o(r) = \frac{\alpha_s}{2N_c} \left[\frac{e^{-m_D r}}{r} - iT \Phi_r(m_D r) \right]$$

$$V_\Sigma(r) = \frac{\alpha_s C_F}{N_c + 1} \left[\frac{e^{-m_D r}}{r} - iT \Phi_r(m_D r) \right]$$

- where $\Phi(m_D r) = \frac{2}{m_D r} \int_0^\infty dz \, \frac{\sin(z m_D r)}{(1+z^2)^2}$ Burnier, Laine and Vepsälïnen [0711.1743]

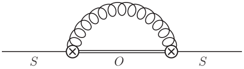
Fermion coannihilator and pNRQCD

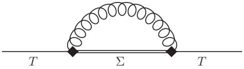
$$\mathcal{L}^{F\bar{F}}_{\text{pNRQCD}} = \int d^3r \, \text{Tr}\left\{ \text{S}^\dagger \left[i\partial_0 - \mathcal{V}_s - \delta M_s \right] \text{S} + \text{O}^\dagger \left[iD_0 - \mathcal{V}_o - \delta M_o \right] \text{O} \right\}$$
$$+ \int d^3r \left(\text{Tr}\left\{ \text{O}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\, \text{S} + \text{S}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\, \text{O} \right\} + \frac{1}{2}\text{Tr}\left\{ \text{O}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\, \text{O} + \text{O}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\, \text{O} \right\} \right) + \cdots$$

$$\mathcal{L}^{FF}_{\text{pNRQCD}} = \int d^3r \, \text{Tr}\left\{ \text{T}^\dagger \left[iD_0 - \mathcal{V}_{\text{T}} - \delta M_T \right] \text{T} + \Sigma^\dagger \left[iD_0 - \mathcal{V}_\Sigma - \delta M_\Sigma \right] \Sigma \right\}$$
$$+ \int d^3r \sum_{a=1}^{8} \sum_{\ell=1}^{3} \sum_{\sigma=1}^{6} \left[\left(\boldsymbol{\Sigma}^\sigma_{ij} T^a_{jk} \mathbf{T}^\ell_{ki} \right) \Sigma^{\sigma\,\dagger} \boldsymbol{r} \cdot g\boldsymbol{E}^a\, \text{T}^\ell - \left(\mathbf{T}^\ell_{ij} T^a_{jk} \boldsymbol{\Sigma}^\sigma_{ki} \right) \text{T}^{\ell\,\dagger} \boldsymbol{r} \cdot g\boldsymbol{E}^a\, \Sigma^\sigma \right] + \cdots$$

$\Rightarrow$ **bound states** from singlet **and** antitriplet

[for fermion-fermion pNRQCD see Brambilla, Rosch, Vairo [hep-ph/0506065]]





- HTL $2 \to 2$ complemented with the case $M_F v \sim T \gg m_D \gg E$

  imaginary part from Brambilla, Escobedo, Ghiglieri, Vairo [1303.6097], real part from S.B. and S. Vogl [1907.05766]

  for the abelian case see Escobedo and Soto [0804.0691]