

# Initial state QED radiation at next-to-leading logarithmic accuracy for future $e^+e^-$ colliders

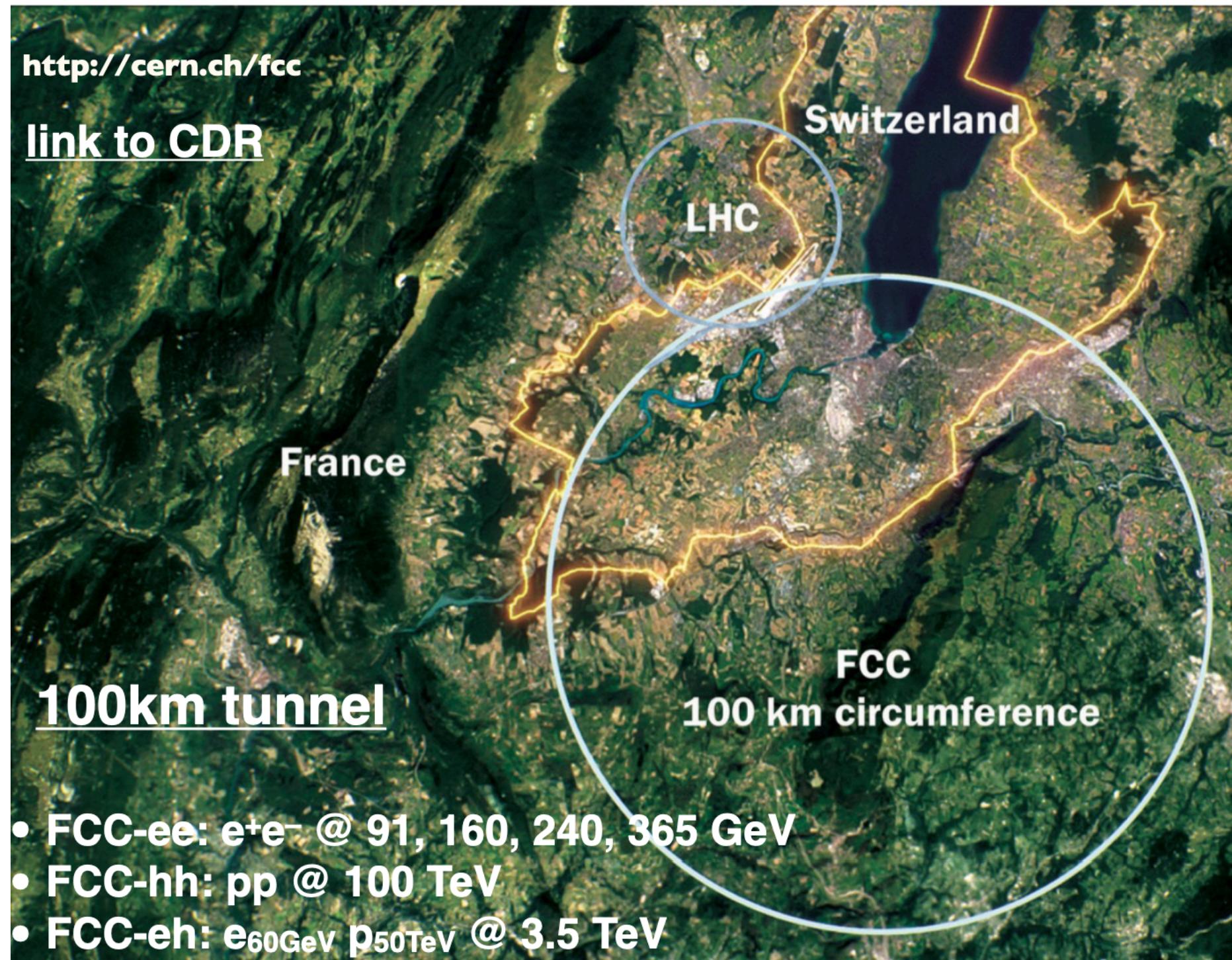
Giovanni Stagnitto  
(University of Zurich)

in collaboration with:  
V. Bertone, M. Cacciari, S. Frixione, M. Zaro, X. Zhao

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# Introduction

## Future Circular Collider



### Electro weak precision observables

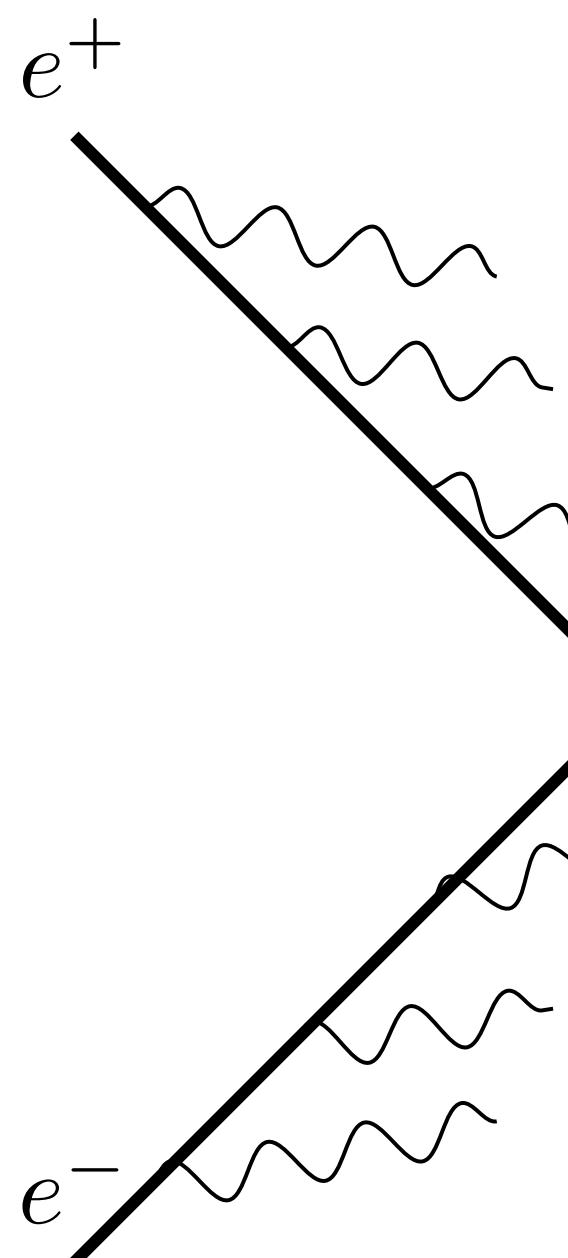
	Experiment uncertainty		Theory uncertainty	
	Current	CEPC	FCC-ee	Current
$M_W[\text{MeV}]$	15	0.5	0.4	4
$\Gamma_Z[\text{MeV}]$	2.3	0.025	0.025	0.4
$R_b[10^{-5}]$	66	4.3	6	10
$\sin^2 \theta_{\text{eff}}^l [10^{-5}]$	16	< 1	0.5	4.5

M. Mangano, "Why FCC?"  
Theory Colloquium, 15 June 2022, CERN  
<https://indico.cern.ch/event/1155782/>

Summary slides of week 1  
"Precision calculations for future  $e^+e^-$  colliders: targets and tools"  
7-17 June 2022, CERN  
<https://indico.cern.ch/event/1140580/>

# Initial state radiation (ISR)

Problem: presence in the cross section  $d\sigma_{e^+e^-}$  of **potentially large logarithms**, due to **collinear photon emissions** in the initial state

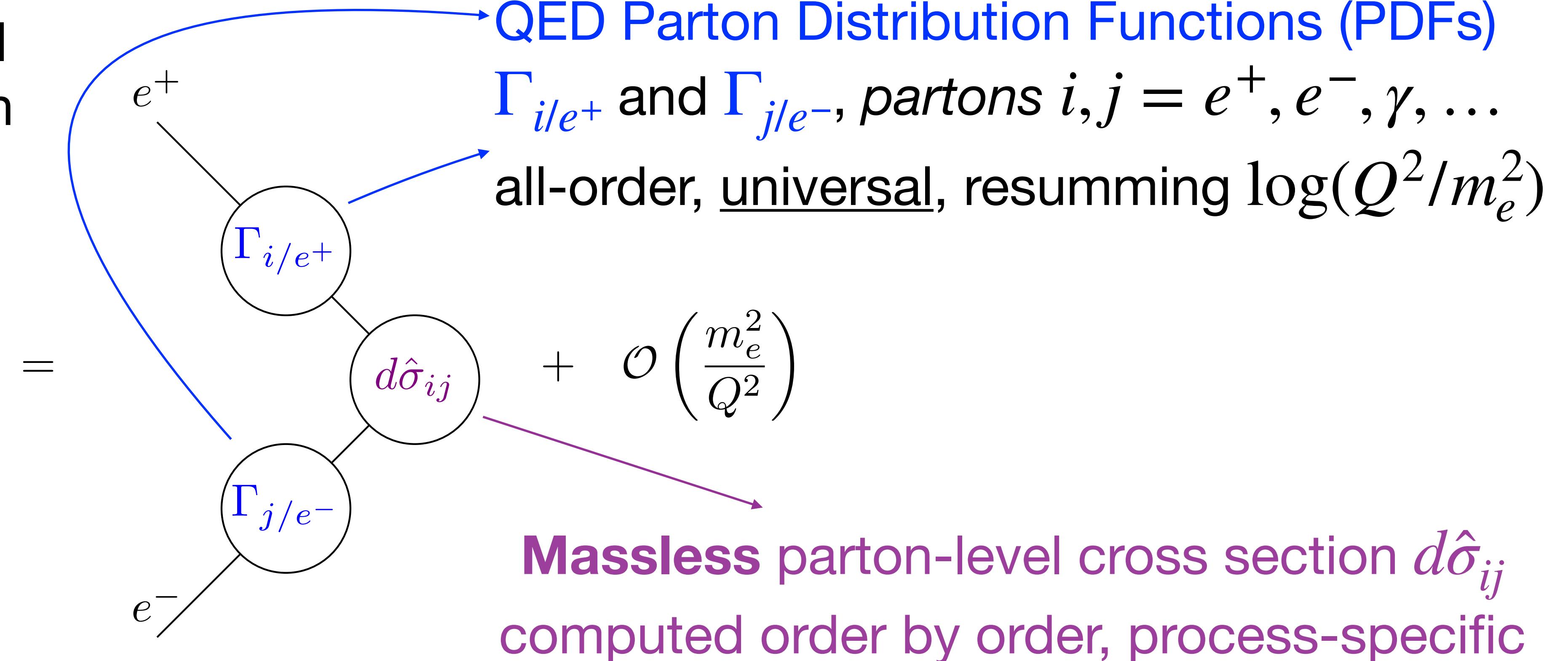
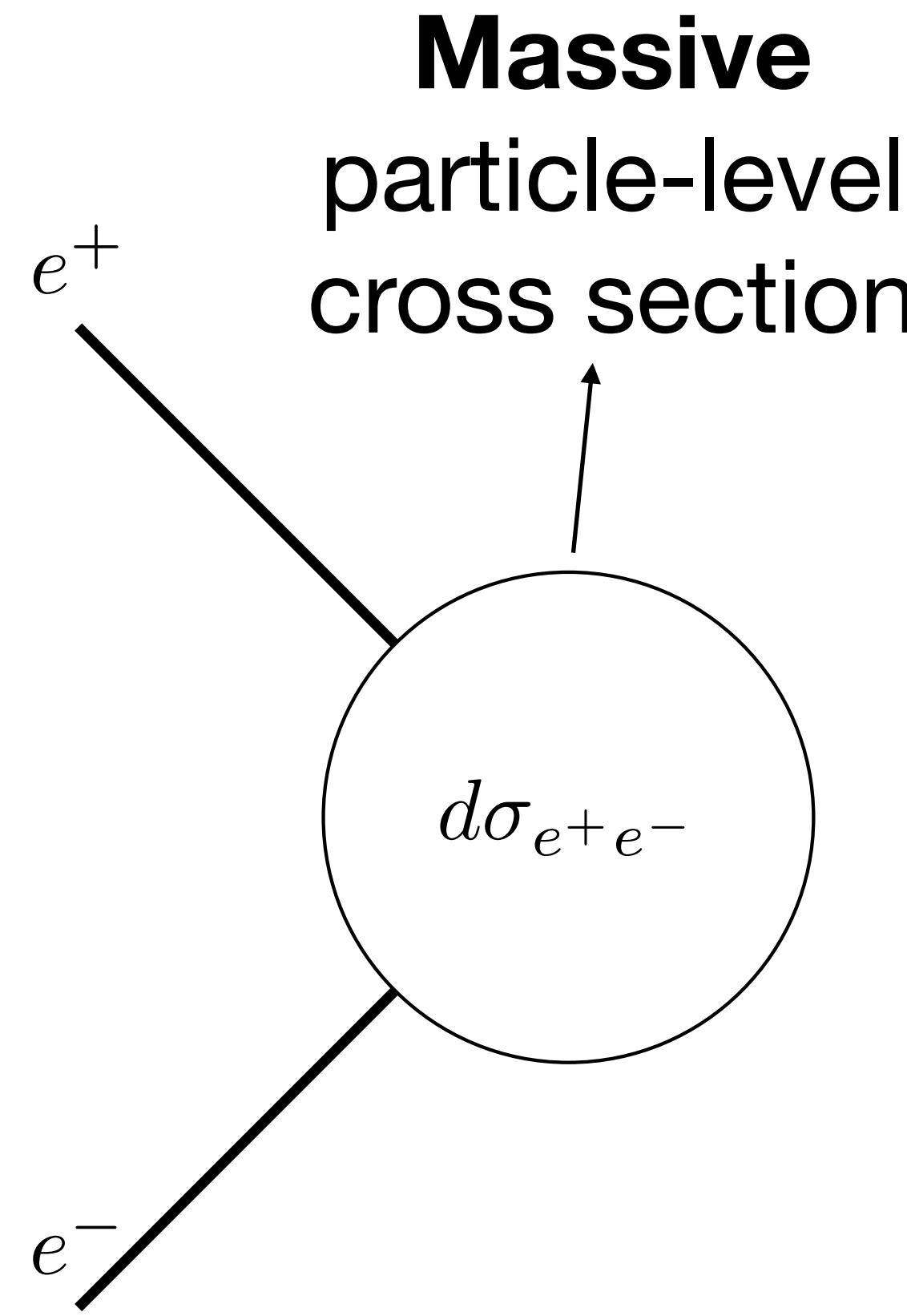


$$X \simeq \alpha^b \sum_{n=0}^{\infty} \alpha^n \left( c_0^{(n)} + c_1^{(n)} L + \dots + c_n^{(n)} L^n \right) \quad L = \log \left( \frac{Q^2}{m_e^2} \right)$$

$b$ : power of the  $\alpha$  in the Born process,  $m_e$ : electron mass  
 $Q^2$ : typical hard scale of the process e.g. c.o.m. energy squared  $s$

Basically all precision observables at  $e^+e^-$  colliders affected by ISR!

# Collinear factorisation



$$d\sigma_{e^+e^-} = \sum_{ij} \int dz_+ dz_- \Gamma_{i/e^+}(z_+, \mu^2, m_e^2) \Gamma_{j/e^-}(z_-, \mu^2, m_e^2) d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2) + \mathcal{O}(m_e^2/Q^2)$$

Charge-conjugation implies

$$\Gamma_{\alpha/e^-} = \Gamma_{\bar{\alpha}/e^+} \equiv \Gamma_\alpha$$

# QED PDFs $\Gamma_\alpha(z, \mu^2)$

Accuracy defined by counting the logs at the exponent:  
leading log (LL)  $[(\alpha L)^k]$ , next-to-leading log (NLL) [also  $\alpha(\alpha L)^{k-1}$ ], ...

Well-known LL result for  $\Gamma_{e^-}$ , which evolves a  $\delta(1 - z)$  at scale  $\mu_0^2 = m_e^2$ :

$$\Gamma_{e^-}^{\text{LL}}(z, \mu^2) = \frac{\exp \left[ (3/4 - \gamma_E)\eta \right]}{\Gamma(1 + \eta)} \eta(1 - z)^{-1+\eta} - \frac{1}{2}\eta(1 + z) + \mathcal{O}(\alpha^2), \quad \eta = \frac{\alpha}{\pi} \log \frac{\mu^2}{m_e^2} \equiv \frac{\alpha}{\pi} L$$

(All-order large-z bulk + fixed-order terms known up to high order.

Results with running  $\alpha$  and soft “-1” also available.)

In view of high-energy future colliders LL accuracy is insufficient and  
systematics not well defined in a LL-accurate picture

This work: **NLL PDFs, ready for sensible phenomenology**

# NLL-accurate QED PDFs

(Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688;  
Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.XXXX)

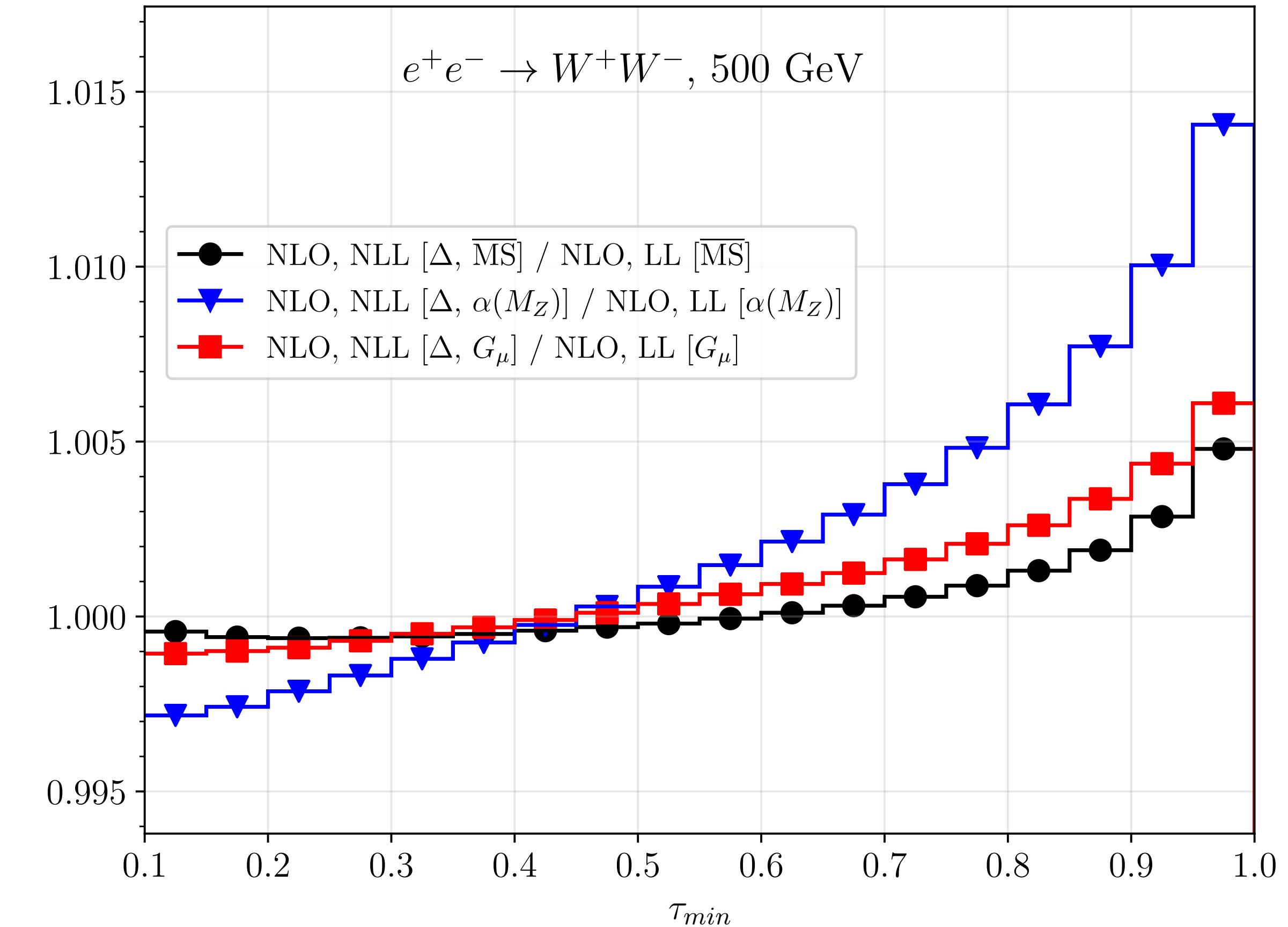
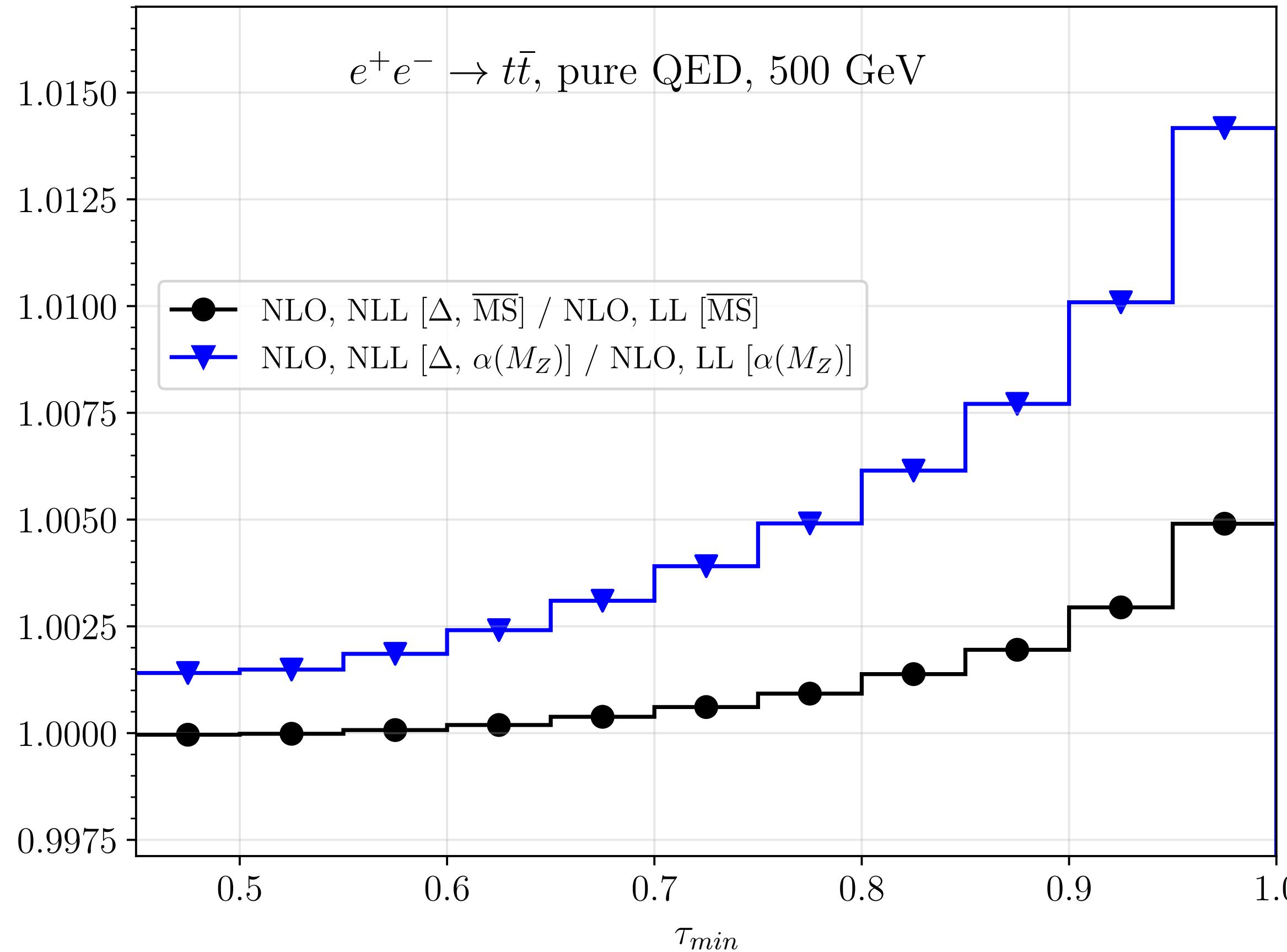
- NLO initial conditions at scale  $\mu_0^2 = m_e^2$  **evolved at NLL up to  $\mu^2$  with all fermion families** (lepton and quarks), in a variable flavour number scheme.
- PDFs in **three different renormalisation schemes**:  $\overline{\text{MS}}$  (where  $\alpha$  runs),  $\alpha(m_Z)$  and  $G_\mu$  (where  $\alpha$  is fixed); **two different factorisation schemes**:  $\overline{\text{MS}}$  and  $\Delta$  (DIS-like, with NLO initial condition maximally simplified).
- Solution built out of a numerical evolution, with a **switch to analytical expressions for  $z \rightarrow 1$** , where the electron PDF  $\Gamma_{e^-}$  features a power-like integrable singularity.
- **Photon-initiated partonic contributions** (through the photon PDF  $\Gamma_\gamma$ ) naturally included in the collinear framework at NLL.

# Studies on physical cross sections

- Computed in the MG5\_aMC framework, at NLO (EW) + NLL in  $e^+e^-$  collisions.
- Processes:
  - ▶  $e^+e^- \rightarrow q\bar{q}(\gamma)$  [pure QED, with real and virtual radiation limited to initial state]
  - ▶  $e^+e^- \rightarrow W^+W^-(X)$  [full EW]
  - ▶  $e^+e^- \rightarrow t\bar{t}(X)$  [full EW] and  $e^+e^- \rightarrow t\bar{t}(X)$  [pure QED]
- $\mu = \sqrt{s} = 500$  GeV (qualitatively similar results in the range 50-500 GeV)
- We focus on the cumulative cross section:

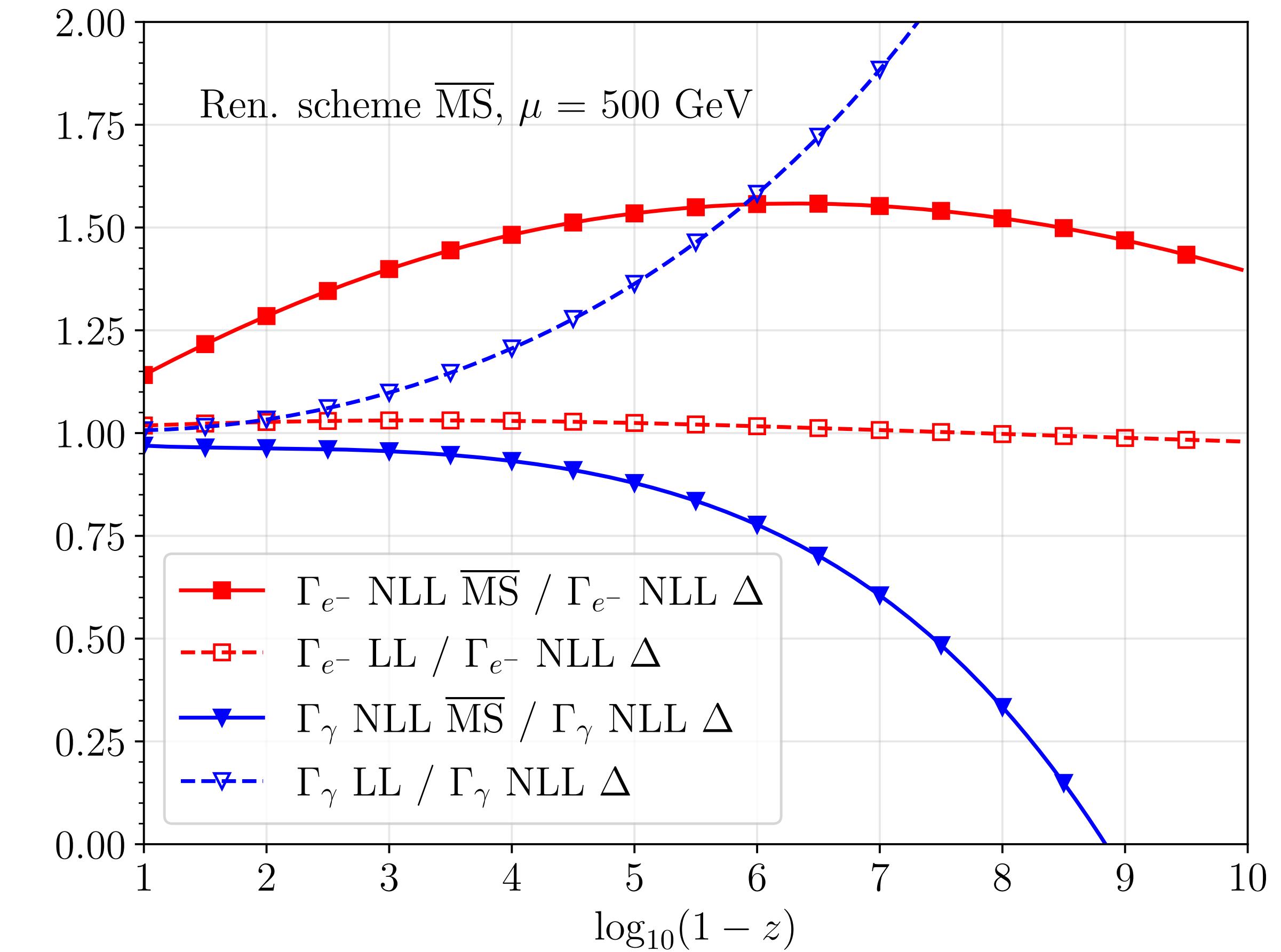
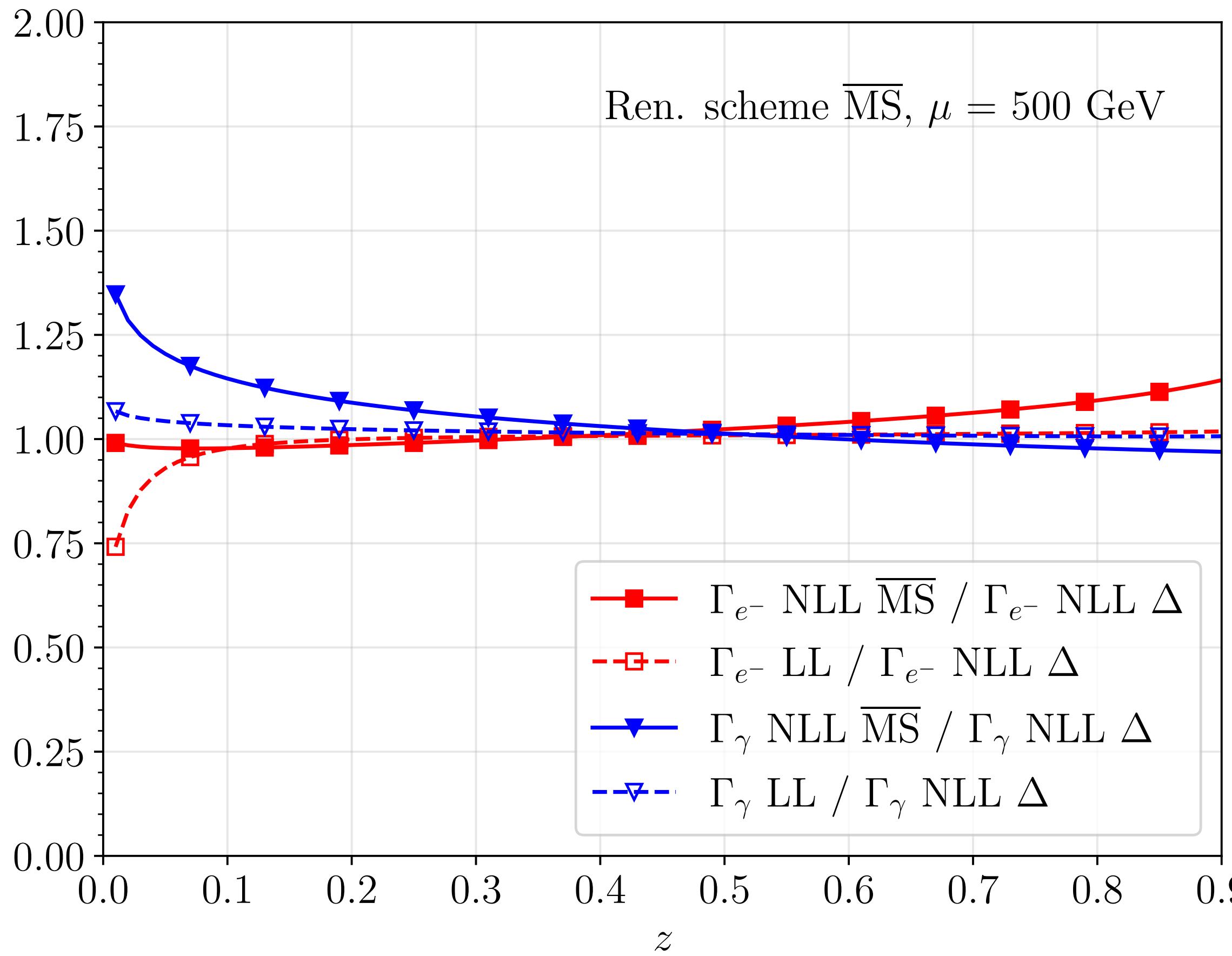
$$\sigma(\tau_{min}) = \int d\sigma \Theta(\tau_{min} \leq M_{p\bar{p}}^2/s), \quad p = q, t, W^+$$

# Impact of NLL



**Non trivial pattern**, impossible to account in some universal manner.  
NLL-accurate PDFs are phenomenologically important for precision studies.

# Dependence on factorisation scheme



At the PDF level,  $\mathcal{O}(1)$  difference between  $\overline{\text{MS}}$  and  $\Delta$  scheme.  
Electron at NLL in the  $\Delta$  scheme closer to the LL value.

# Large- $z$ analytical expressions for $\Gamma_{e^-}$

$$\Gamma_{e^-}^{\text{NLL}}(z, \mu^2) = \frac{e^{-\gamma_E \xi_1} e^{\hat{\xi}_1}}{\Gamma(1 + \xi_1)} \xi_1 (1 - z)^{-1 + \xi_1} h(z, \mu^2)$$

$$\xi_1 = 2t + \mathcal{O}(\alpha^2)$$

$$\hat{\xi}_1 = \frac{3}{2}t + \mathcal{O}(\alpha^2)$$

$$t = \frac{1}{2\pi b_0} \log \frac{\alpha(\mu)}{\alpha(\mu_0)}$$

$$h^{\overline{\text{MS}}}(z, \mu^2) = 1 + \frac{\alpha(\mu_0)}{\pi} \left[ \left( \log \frac{\mu_0^2}{m^2} - 1 \right) \left( A(\xi_1) + \frac{3}{4} \right) - 2B(\xi_1) + \frac{7}{4} + \left( \log \frac{\mu_0^2}{m^2} - 1 - 2A(\xi_1) \right) \underline{\log(1-z)} - \underline{\log^2(1-z)} \right]$$

$$h^\Delta(z, \mu^2) = \frac{\alpha(\mu)}{\alpha(\mu_0)} + \frac{\alpha(\mu)}{\pi} \log \frac{\mu_0^2}{m^2} \left( A(\xi_1) + \log(1-z) + \frac{3}{4} \right)$$

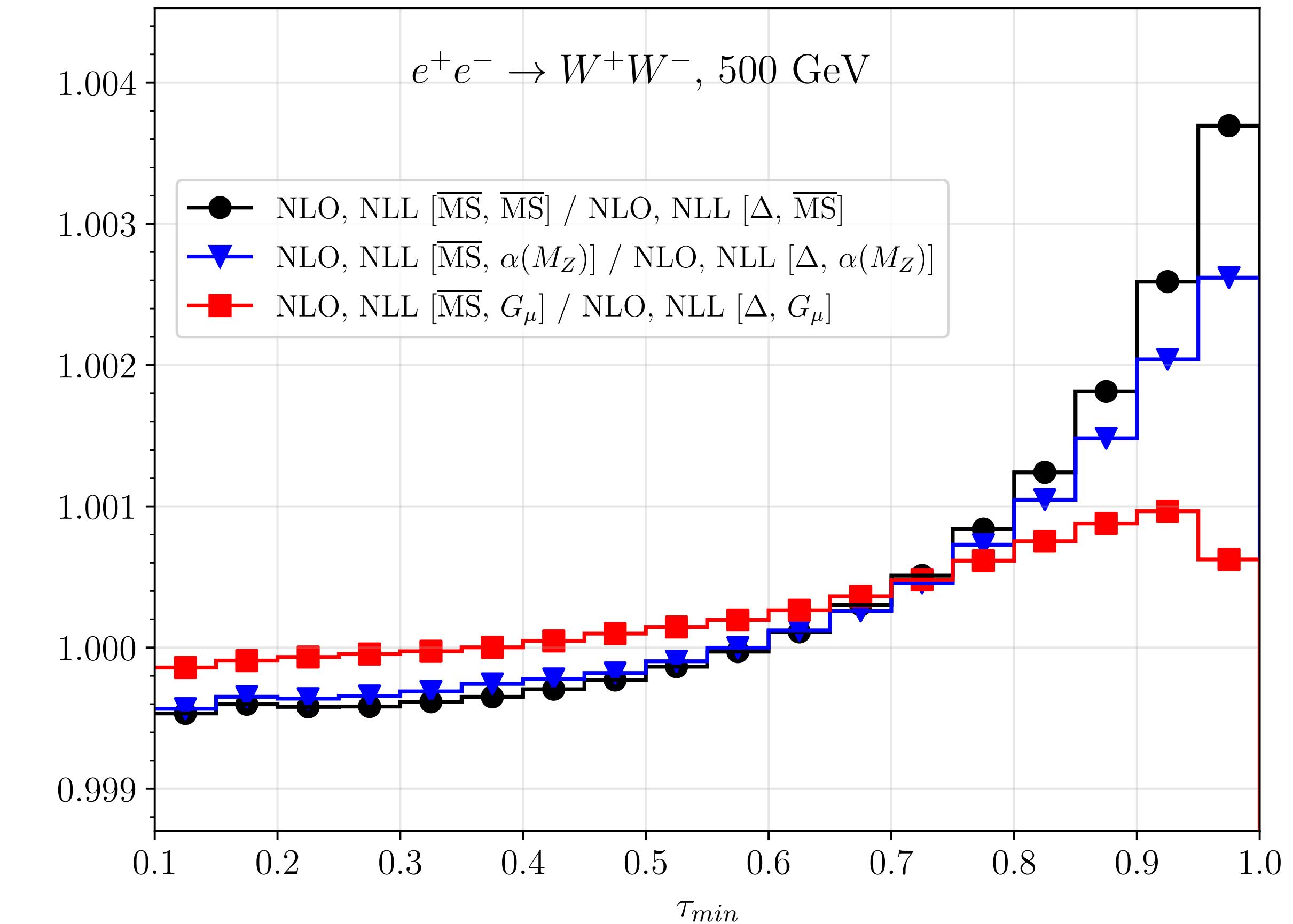
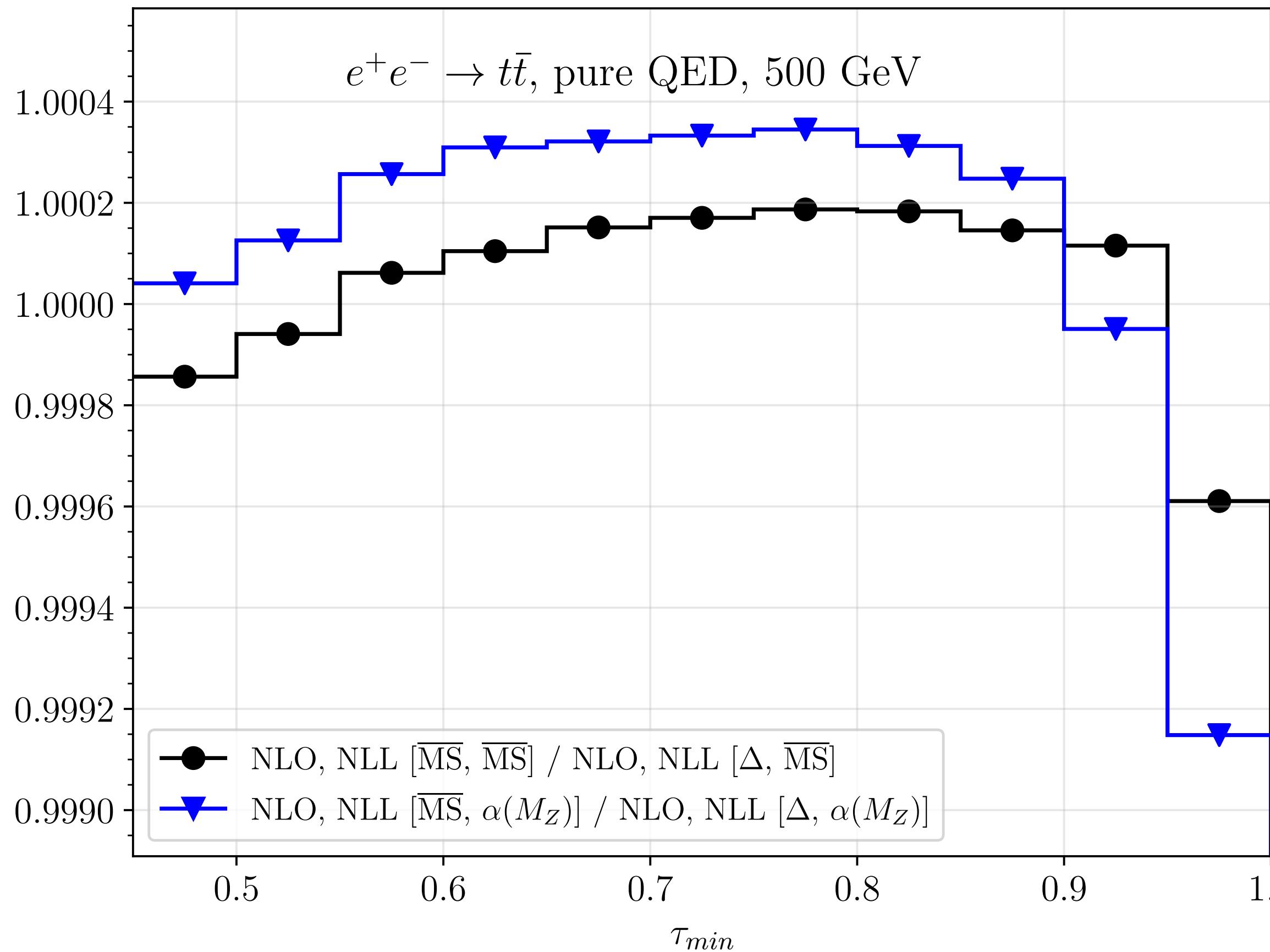
$$A(\xi_1) = \frac{1}{\xi_1} + \mathcal{O}(\xi_1)$$

$$B(\xi_1) = -\frac{\pi}{6} + 2\zeta_3 \xi_1 + \mathcal{O}(\xi_1^2)$$

**Logarithmic terms artefacts of the  $\overline{\text{MS}}$  fac. scheme, absent in the  $\Delta$  scheme.**

Here shown in the  $\overline{\text{MS}}$  ren. scheme and with a single-fermion family; evolution with multiple fermion families with their mass thresholds and different ren. schemes (e.g.  $\alpha(m_Z)$ ,  $G_\mu$ ) amount to a redefinition of  $\xi_1$  and  $\hat{\xi}_1$ .

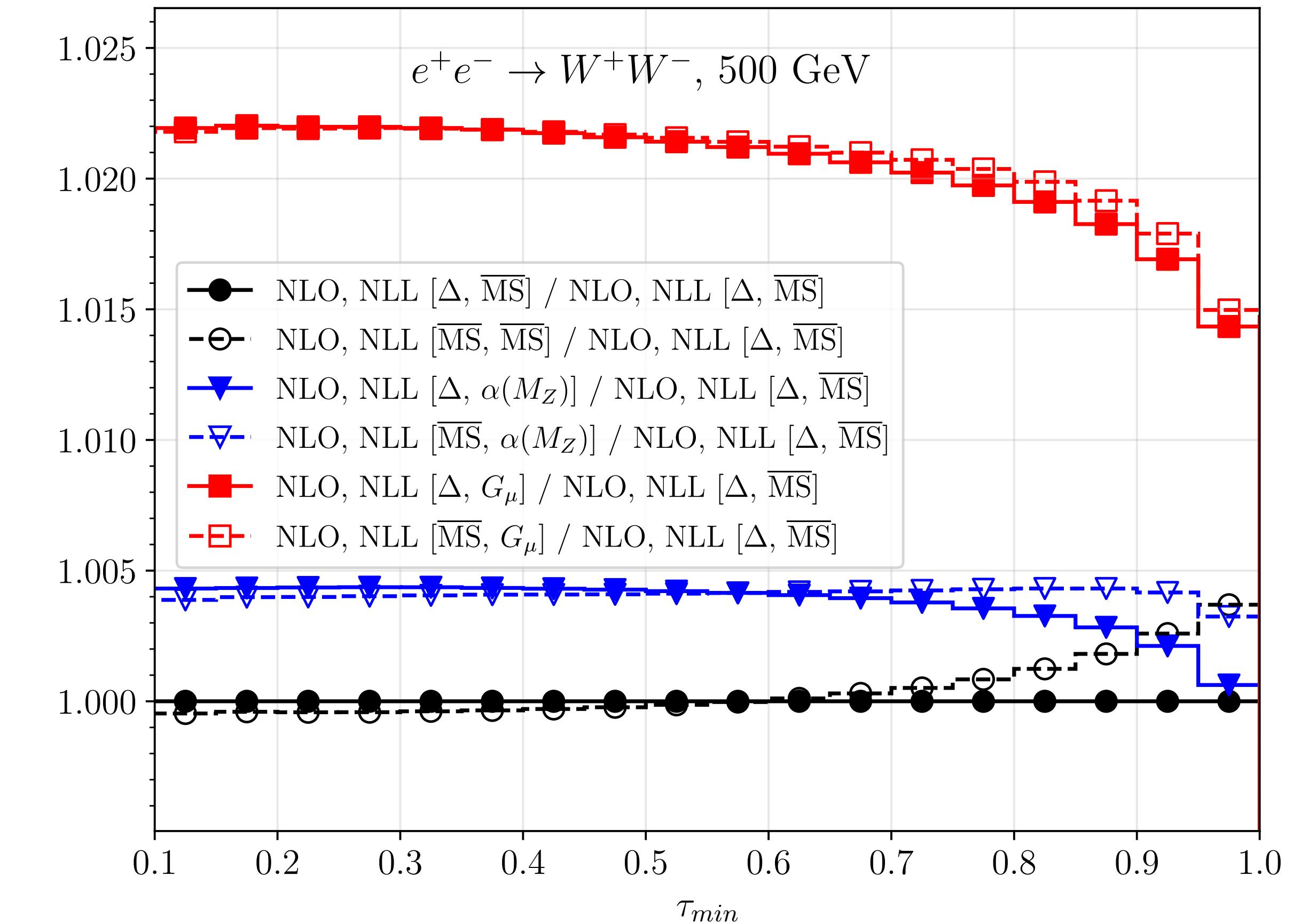
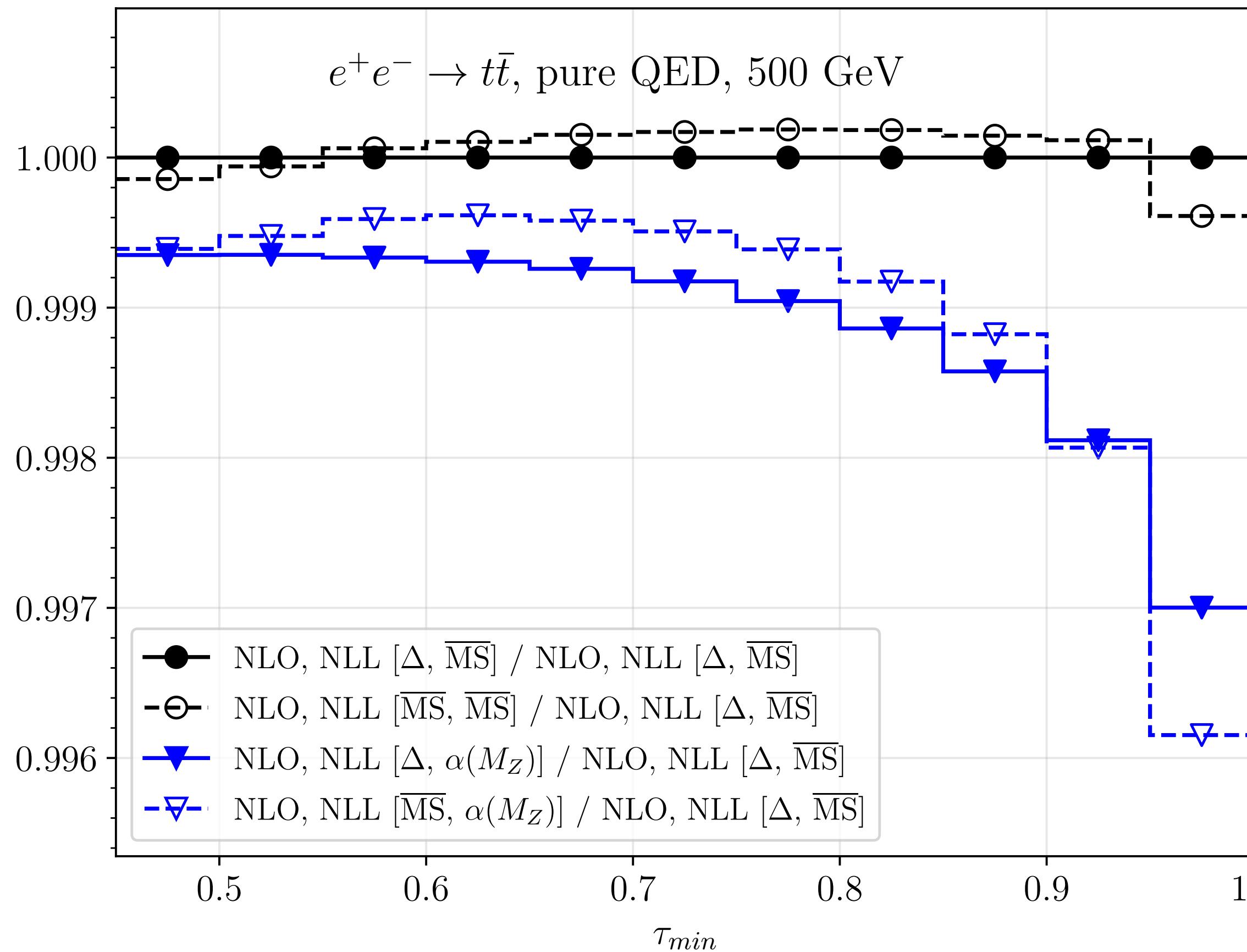
# Dependence on factorisation scheme



At the cross section level,  $\mathcal{O}(10^{-4} - 10^{-3})$  difference between fact. schemes.

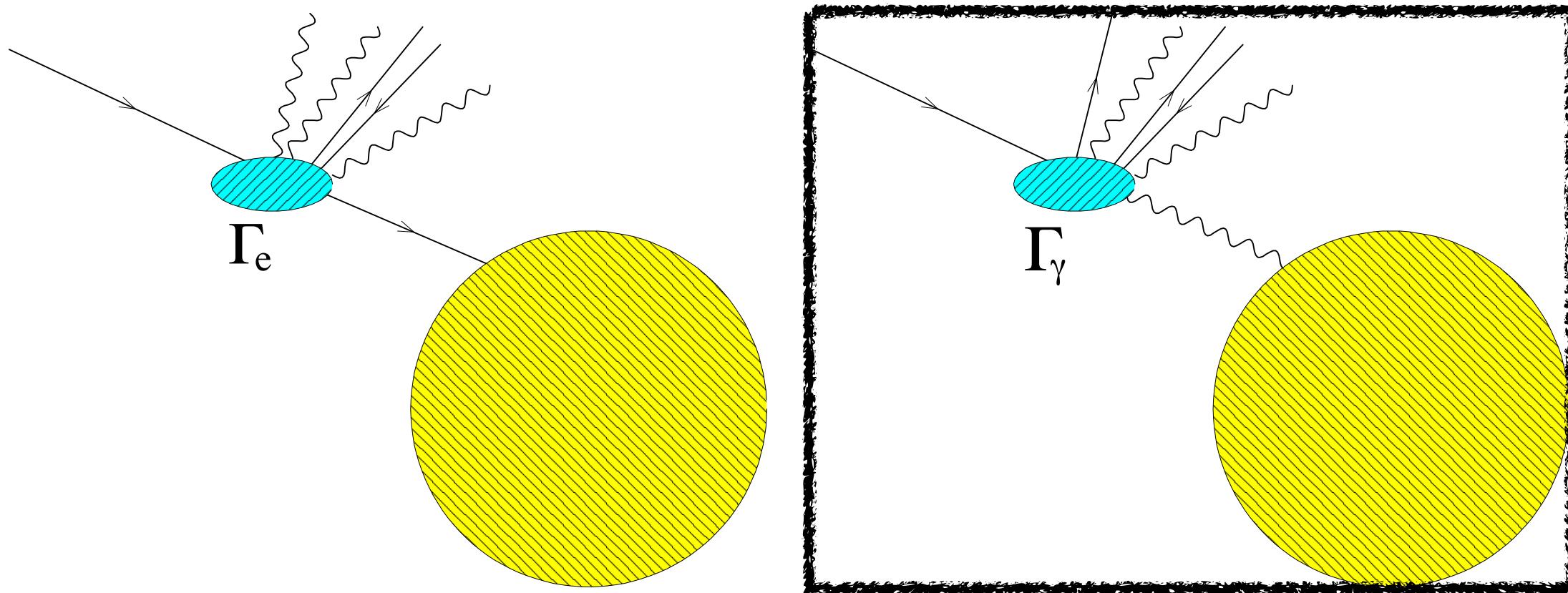
Large cancellations in the  $\overline{\text{MS}}$  fact. scheme.

# Dependence on renormalisation scheme



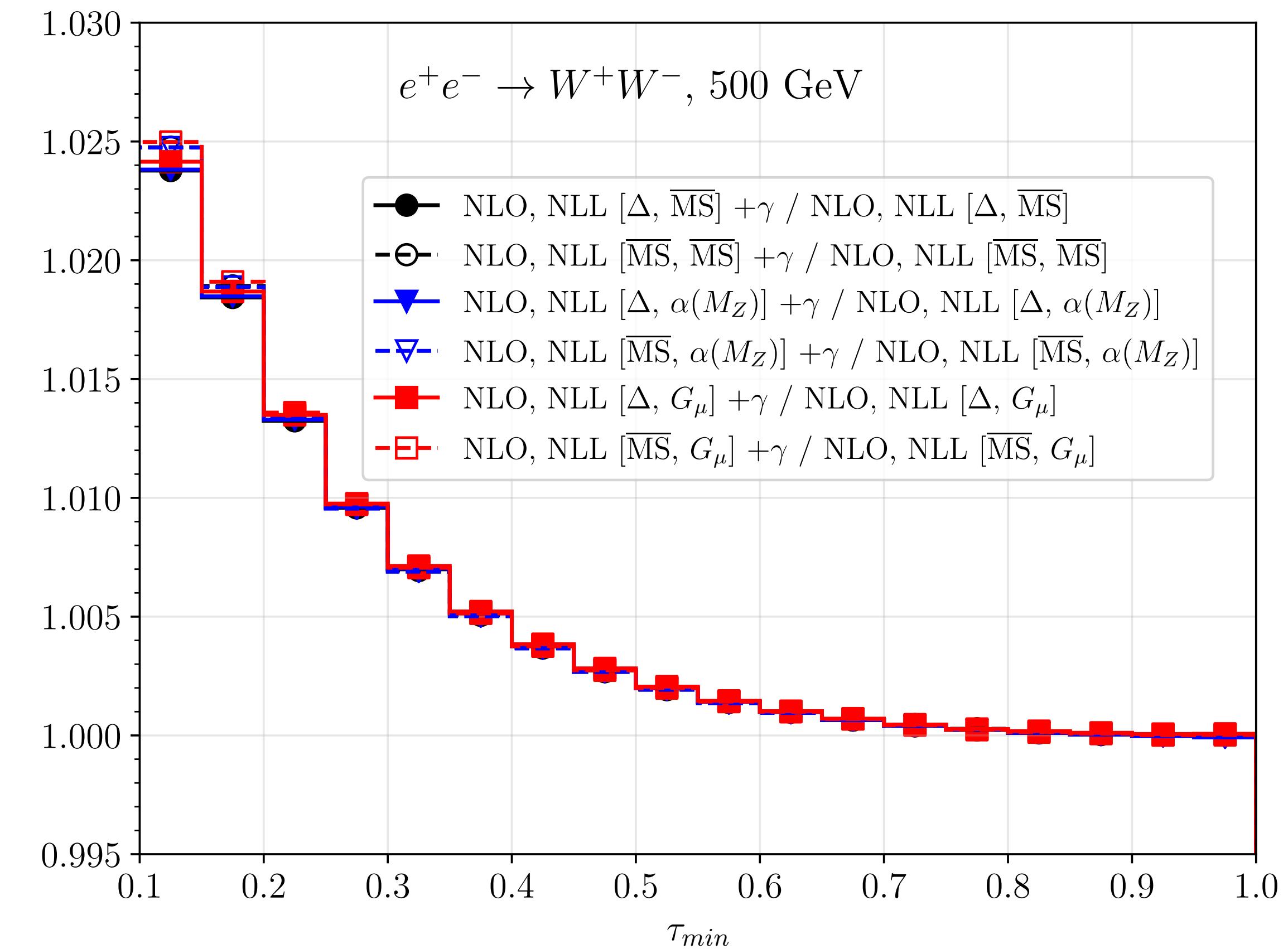
Ren. scheme dependence significantly **larger** than the fact. scheme one.  
Mostly a normalisation effect.

# Impact of photon-induced contributions



- At LO, i.e.  $\mathcal{O}(\alpha^2)$ , both  $W^+W^-$  and  $t\bar{t}$  feature a  $\gamma\gamma$  channel.
- Photon PDF  $\Gamma_\gamma$  only suppressed by a power of  $\alpha$  w.r.t.  $\Gamma_{e^-}$ , and peaked at small- $z$  values.

Both effects can lead to **physical effects**  
e.g.  $W^+W^-$  at small  $\tau_{min}$ .



# Conclusions

- **First NLO+NLL predictions** at high-energy  $e^+e^-$  colliders, improving on accuracy but also important for an assessment of sources of theoretical uncertainties.
- **Impact of NLL PDFs local** both in shape and size, hence impossible to account in a universal manner.
- Quite a few combinations of factorisation ( $\Delta$  vs  $\overline{\text{MS}}$ ) and renormalisation ( $\overline{\text{MS}}$ ,  $\alpha(m_Z)$ ,  $G_\mu$ ) schemes have been assessed in the context of NLL computations.
- **Photon-induced contributions not negligible** and properly included in a NLL-accurate evolution.
- **Public code for NLL PDFs**, eMELA: <https://github.com/gstagnit/eMELA>. PDFs with beamstrahlung effects are also provided.