Deviations from Isotropic Turbulence in Heavy-Ion Collision Experiments

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Motivation

In the initial stages after the heavy-ion collision, the system possesses a lot of fluctuations, e.g., energy density fluctuations, number density fluctuations, temperature fluctuations, etc., at different length scales. These can create disturbances in the flow. Signs of turbulence have been observed at the relativistic heavy-ion collision at high collision energies. We study the spectra of the initial fluctuations by studying the temperature and velocity distribution in the collision region using the relativistic hydrodynamic fluctuations formalism. While the velocity fluctuation can be studied for the initial stage, we study the temperature fluctuations only in the pre-equilibrium stage, where the temperature can be defined using the energy density and the concept of local thermal equilibrium. We study the scaling exponent of turbulence spectra in the two planes and find that there are significant departures from isotropic turbulence. We are interested to see whether the geometrical anisotropy is reflected in the anisotropic turbulence spectra generated in the initial plasma.

Extract Power Spectrum from Collision Data

* Turbulence is studied as fluctuations in the velocity field in the laminar flow.

$$\mathbf{u}(\mathbf{x}) = \mathbf{U}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}) \tag{1}$$

Here $\mathbf{u}(\mathbf{x}) =< u >= \lim_{\delta x \to 0} \int_V \frac{\mathbf{u} d^3x}{V}$, is the space average of velocity at a fixed time over volume V.

* The velocity correlation tensor for the turbulent velocity at two points denoted by **r** and **r+d** is given by,

$$R_{ij}(\mathbf{r}) =< \mathbf{u}'_i(\mathbf{x},t)\mathbf{u}'_j(\mathbf{x}+\mathbf{r},t) > \tag{2}$$

The R_{ij} is related to the energy spectrum tensor $E_{ij}(K)$ by,

$$E_{ij}(\mathbf{K}) = \frac{1}{(2\pi)^3}\int\int\int e^{-i\mathbf{K}.\mathbf{r}} R_{ij}(\mathbf{r}) d(\mathbf{r}) \tag{3}$$

* **Non-relativistic to Relativistic:** A fully relativistic turbulence has richer dynamics compared to the non-relativistic case. Since the particles are colliding with relativistic velocities along the z-axis, we need to take care of the Lorentz boost effect when we calculate the velocity correlation in the longitudinal plane.

$$R_{ij} = \Lambda(d/2)\Lambda(-d/2) < \mathbf{u}'_i(r-d/2), \mathbf{u}'_j(r+d/2) > \tag{4}$$

Here $\Lambda(\Delta x)$ is the boost which brings $u(x+\Delta x)$ to $u(x)$, $\Lambda(\Delta x)u(x+\Delta x) = u(x)$.

The correlator is boosted to the local reference frame of the midpoint between the two points whose correlation we are interested in. We obtain the $\Lambda(d/2)$ matrices for an infinitesimal boost. If $\phi_A(x)$ is a hydrodynamic fluctuation field, Phys. Rev. C **100**, 024910 (2019)

$$[\Lambda(\Delta x)\phi]_\mu = \phi_\mu - u_\mu(\Delta u.\phi) + \Delta u_\mu(u.\phi) \tag{5}$$

* **Different length scales:**

The structures of the rotating elements(eddies) in a turbulent system can be of different sizes. Thus we have different length scales in our problem. The largest eddy formed in the system can be of the largest length scale of the system. These large eddies extract kinetic energy from the mean flow and use it to develop angular momentum. In relativistic heavy-ion collisions, most of the energy gets converted into angular momentum and is eventually dissipated through the smaller eddies. This is known as the **energy cascade**. This energy cascade can be expressed in terms of the **Reynolds number**.

$$Re = \frac{F_i}{F_v} = \frac{\rho u l}{\mu_d} \tag{6}$$

Here $F_i = \rho l^3 \frac{u^2}{l}$ is the inertial force, and $F_v = \mu_d \frac{u}{l} l^2$ is the viscous force. In case of a large Reynolds number, the fluid viscosity is less dominant over the fluid inertia, and we get larger eddies. This is the regime of the Kolmogorov spectra.

- Here the cell size $= 0.3$ fm, and No. of cells in each direction$= 48$. System size $l = 14.4$ fm in each direction. The diameter of the Au nuclei is around 12 fm. $\Rightarrow$ $k_{min} \approx 0.5 fm^{-1}$
- The Kolmogorov length scale is defined as the length scale of the smallest eddy. This can be found out by making the Reynolds number very small in Eq. 6. It is given by,

$$\zeta = \left(\frac{\mu_k^3}{\epsilon_d}\right)^{1/4} \tag{7}$$

$$= lRe^{-3/4} \quad from\ conservation\ of\ energy \tag{8}$$

- Here the kinematic viscosity $\mu_k = \frac{\mu_d}{\rho} \approx 10^{-7}\frac{m^2}{s} \approx 1.69\ GeV^{-1}$, and $\epsilon_d \geq 2\ GeV/fm^3 \Rightarrow$ $k_{max} \approx 5$. SciPost Phys. 10, 118 (2021)
For QGP systems, $Re \geqq 8.52 \Rightarrow$ From Eq. 8, $k_{max} \geq 4$ Nuclear Physics B 921, 39-58 (2017)

* In 1883, Kolmogorov hypothesized that the amount of energy in a turbulent flow carried by eddies of diameter D, gravitate towards $D^{5/3}$. But this is only valid within a specific range of length scales known as the inertial subrange. Thus in this range, Kolmogorov spectra will have a power-law nature where the kinetic energy is given by,

$$E(k) \approx k^{\nu} \approx k^{-5/3} \tag{9}$$

Longitudinal plane spectra

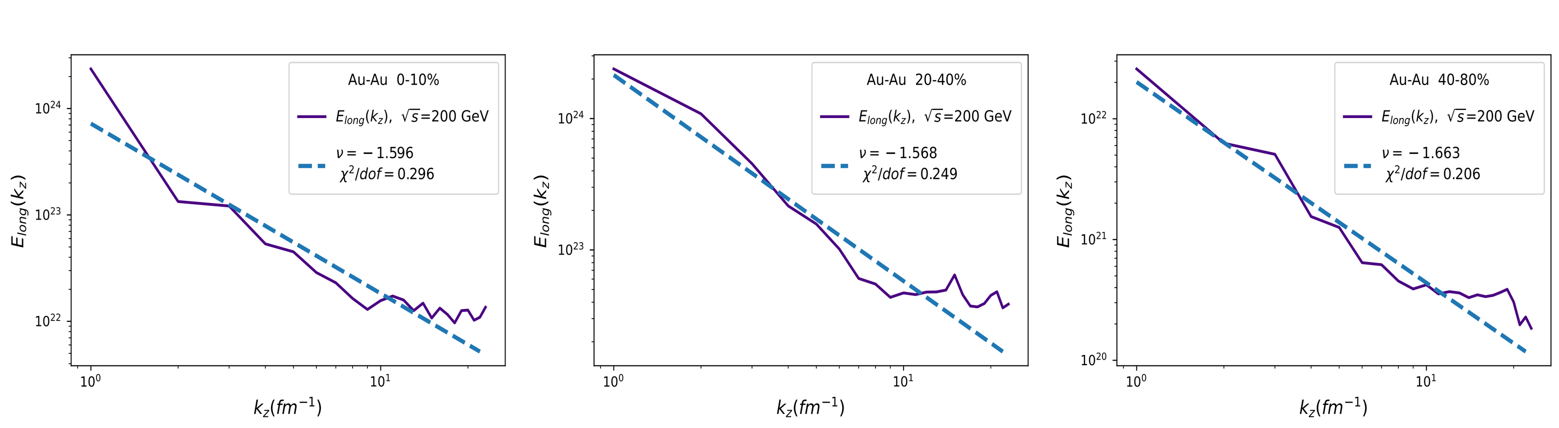

Fig. 1: Turbulence velocity spectra for the longi. plane at $\sqrt{s} = 200$ GeV. $\nu = -1.59, -1.57, -1.66$ for $0-10\%, 20-40\%, 40-80\%$ centrality collisions.

Transverse plane spectra

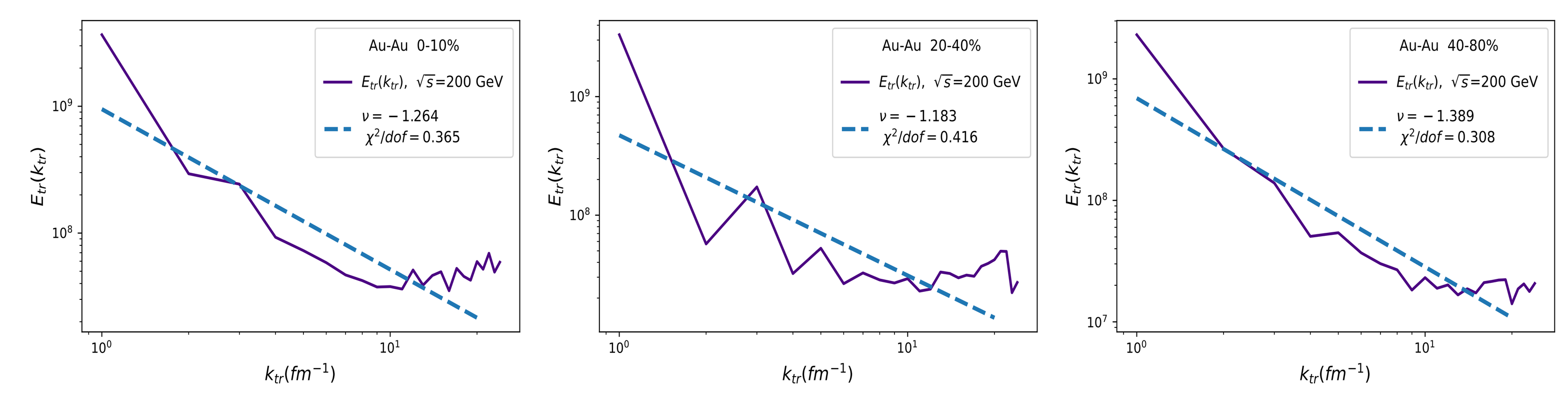

Fig. 2: Turbulence velocity spectra for the trans. plane at $\sqrt{s} = 200$ GeV. $\nu = -1.26, -1.18, -1.39$ for $0-10\%, 20-40\%, 40-80\%$ centrality collisions.

Observations:

- For longitudinal spectra, the coefficient of the power spectrum is around $-1.6 \approxeq -5/3$ in all the cases. This is near the Kolmogorov limit. So, in this plane, the spectra resemble the Kolmogorov spectrum. Here the inertial force is greater than the dissipative force.
- The power-law exponent does not remain the same in all these cases for the transverse plane spectra. The exponents appear to be closer to $-4/3$ than the Kolmogorov value. Here, the dissipative forces are important and more dissipation occurs at the smaller length scales.
- The change in centrality makes the particles distributed anisotropically on the transverse plane. It causes an anisotropic pressure gradient on the transverse plane.
- We have also measured the exponent for the collision energies at $19.6\ GeV$, $39\ GeV$, $62.4\ GeV$, $100\ GeV$, $130\ GeV$, and $200\ GeV$ and the trend holds for all the RHIC energies.

Power spectrum of temperature fluctuations

- In the case of turbulent flow, the shear stress can be obtained from the equation of motion, where the tangential stress depends on the velocity change perpendicular to the flow direction and the total heat flow can be obtained from the conservation of energy where the heat flow occurs due to the presence of the temperature gradient in the direction perpendicular to the flow direction.
- One can obtain the power spectrum of the temperature fluctuations as long as we know the temperature at different points. To get the power spectrum, we have to obtain the energy of the system at different length scales. The temperature can be calculated using the equation,

$$\epsilon(x,y) = 12(4+3N_f)(\frac{T^4}{\pi^2}) \tag{10}$$

- We define $m(r) =< TT' >$ as the temperature correlation between the two given points, and the power spectrum for the temperature fluctuations is obtained from,

$$G(k) = \frac{2}{\pi}\int_0^\infty m(r) kr Sinkr dr \tag{11}$$

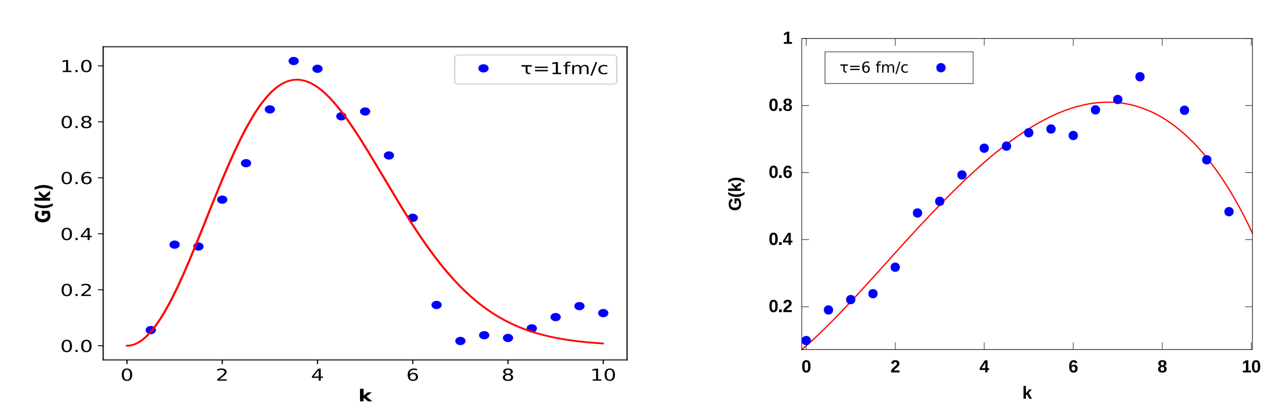

Fig. 3: The power spectrum of the temperature fluctuations for 200 GeV Au-Au central collision events at $\tau = 1\ fm/c$ and $6\ fm/c$

Observations:

- We find that though it is possible to fit the temperature spectrum by an approximate Gaussian curve, which is a characteristic of an isotropic system, fluctuations present at smaller length scales indicate the presence of anisotropies. This is further enhanced at later times when the spectrum is better fitted with a Poissonian q-Gaussian distribution. There is thus an anisotropy in the temperature fluctuation as well.
- The shift to smaller length scales indicates that energy is transferred to smaller eddies as time progresses. The length scales of temperature fluctuations are similar to the length scale calculated for the smallest eddies.

Reference: arXiv:2108.01847 [nucl-th]

My Other Recent Publications

1. *Machine Learning model driven prediction of the initial geometry in Heavy-Ion Collision experiments*, Abhisek Saha, Debasis Dan, Soma Sanyal, **Phys. Rev. C 00**, 004900 (2022)
2. *Temperature fluctuations, turbulence and Tsallis statistics in Relativistic Heavy Ion collisions*, Abhisek Saha, Soma Sanyal, **Mod. Phys. Lett. A** Vol. **36**, No. 22, 2150152 (2021)

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